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FLUCTUATIONAL INTERFEROMETRY FOR MEASUREMENT OF SHORT PULSES OF INCOHERENT RADIATION *

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Abstract

A novel method for measuring ultrashort pulses of incoherent radiation is proposed. The method is based on detecting the fluctuations of the visibility of interference fringes in a two-beam interferometer. It is shown that the dispersion of the fluctuations is proportional to the convolution integral of the instantaneous intensity in the pulse. Using statistical analysis of the interference patterns allows, in many practically important cases, to restore the shape of the pulse.

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The necessity to measure ultrashort pulses of incoherent radiation is encountered in many fields of pure and applied research. Such pulses can be produced by high-energy short bunches of charged particles [1], or bremsstrahlung from a laser generated hot plasma [2]. Nowadays, short pulses of incoherent radiation can only be resolved by a streak camera with the time resolution not exceeding fractions of a picosecond. Advancing to shorter pulses in future accelerators [3] makes it necessary to search for higher time resolution and less expensive diagnostic techniques.

In this Letter, we propose a novel method of interferometric technique for measuring ultrashort pulses of incoherent radiation. The method is based on the observation of the interferometric fringes produced by each single bunch in a two beam interferometer. It is conventionally assumed that incoherent light does not form interferometric pattern beyond its coherence length. This is, however, only true for a quasicontinous sources of light and the detector that averages over many coherence times. If the pulse length is commensurable with the coherence length of the radiation, the interference pattern produced by a single bunch can be detected. The visibility of this pattern would average to zero over many pulses, however, it is possible to measure *fluctuations of the visibility* and, using statistical analysis to determine standard deviation for these fluctuations. We show that the dispersion of the fluctuations of the visibility is directly related to the convolution integral of the instantaneous intensity in the pulse. Measurement of this function allows to recover the pulse structure in many practically important cases. Note that this kind of measurements does not require a fast detector and can be performed with a time resolution much lower than the pulse duration.

We assume that the electromagnetic field of the radiation can be described in terms of the classical field, which is true if the number of photons in the coherence volume is much greater than unity. Incoherent pulses of radiation are characterized by that the length of the pulse τ_p is much longer than the reciprocal effective spectral width $\Delta\omega, \Delta\omega\tau_p \gg 1$. Let us denote the coherence time $\tau_{coh} \sim \Delta\omega^{-1}$, associated with the frequency spread $\Delta\omega$, and introduce the ratio $N = \tau_p/\tau_{coh}$ equal to the number of coherent slices within the pulse. The parameter N determines the level of fluctuations in the measurements that we discuss below. In many cases, the natural spectral width $\Delta\omega$ is so large that N greatly exceeds unity, and the relative fluctuations are negligible. In order to increase their role, we need to control the spectral width $\Delta\omega$ using a filter. In what follows, $\Delta\omega$ represents the spectral width of the pulse after passing through the filter with the central frequency ω_0. Fig. 1 shows a possible realization of an incoherent electric field $E(t)$ for a pulse with the instantaneous intensity $I(t) \equiv \left\langle |E(t)|^2 \right\rangle = I_0 \exp\left(-t^2 \big/ 2\tau_p^2\right)$ and $N = 10$. As is seen from this figure, the pulse is made of about ten independent coherent parts each of which has a random amplitude and phase. A detector measuring the integrated intensity of the pulse $\int_{-\infty}^{\infty} |E(t)|^2 \, dt$ would show fluctuations from one pulse to another with a relative variation of the order of $N^{-1/2}$. Finding the amplitude of these fluctuations, in principle, allows to evaluate the parameter N and, for a given $\Delta\omega$, determine the length of the pulse $\tau_p \sim N/\Delta\omega$. Conversely, if the length of the pulse is known, this kind of measurement can be used to find the spectral width $\Delta\omega$. Note that in addition to pulse-to-pulse fluctuations, one can use several nonoverlapping filters at different central frequencies to produce independent fluctuations in the same pulse. For a wide-spectrum pulses, using many filters would allow an approximate evaluation of the pulse length even in a one-pulse measurement.

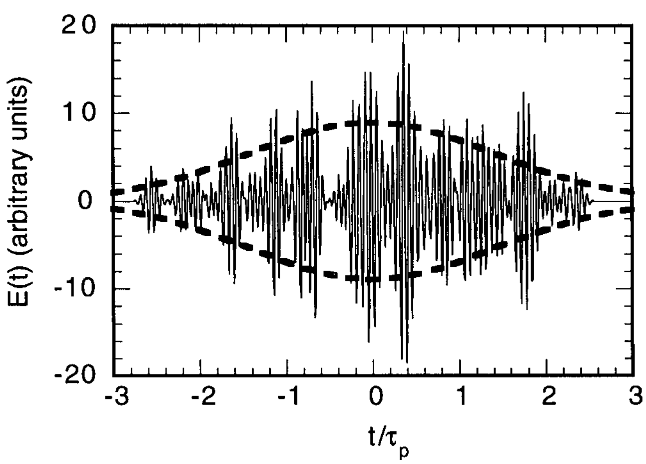

Fig. 1. Electric field of a pulse of incoherent radiation as a function of time. The ratio $\Delta\omega/\omega_0$ = 0.1, and the parameter N = 10. The dashed lines show $\sqrt{I(t)}$.

A more detailed information about the pulse can be obtained from the normalized coherence function $\gamma(t) \equiv \Gamma(t)/\Gamma(0)$, where

$$\Gamma(\tau) = \int_{-\infty}^{\infty} E(t)\, E^*(t-\tau)\, dt. \tag{1}$$

The absolute value of $\gamma(t)$ can be found from the measurement of the visibility of interference fringes generated by a splitting of the pulse in a two-beam interferometer. For a given τ, $\tau < \tau_p$, the functions $E(t)$ and $E^*(t-\tau)$ in Eq. (1) partially overlap, and the number of coherent slices n in the overlapping region is approximately equal to $n \approx (\tau_p - \tau)/\tau_{coh} \leq N$. The visibility of the interferometric pattern will be determined by a superposition of n random sets of fringes each of which is generated by an interfering pair of coherent slices. Because of the randomness of these patterns, the resulting visibility will fluctuate with a rms value of the order of $n^{-1/2}$. Obviously, the statistical properties of these fluctuations, as a function of the time delay τ, will be related to the intensity distribution in the pulse; as shown below, the rms value of the fluctuations of the visibility for a given τ is proportional to the convolution of the intensity $I(t)$ in the pulse. For the values of N in the range from about 10 to 100, the expected fluctuations will be approximately 30 to 10 percents, and can easily be detected even for low-intensity beams.

The above consideration assumed that the quantum fluctuations of the number of photons in the detector are negligible. If this is not the case, these fluctuations will superimpose on the random fluctuations described above. Note, however, that the information about the pulse shape will still be present in the interferometric pattern, although, a larger statistics will be needed to suppress the additional noise introduced by the quantum fluctuations.

To relate the fluctuations of Γ with the pulse parameters, we represent the electric field of the pulse in the detector $E(t)$ as a product of two functions,

$$E(t) = A(t)\, e(t)\,, \tag{2}$$

where $e(t)$ is a stationary complex-valued stochastic process [4], and $A(t)$ denotes a (deterministic) complex amplitude of the pulse, $A(t) = \sqrt{I(t)}e^{-i\varphi(t)}$, where $I(t)$ is the instantaneous radiation intensity and $\varphi(t)$ is the phase of the pulse envelope. The characteristic time of the variation of both functions $A(t)$ and $\varphi(t)$ is of the order of the pulse duration τ_p. Related to the fluctuational part of the field $e(t)$ is a correlation function $K(\tau)$,

$$K(\tau) = \langle e(t)\, e^*(t-\tau)\rangle\,, \tag{3}$$

where the angular brackets denote ensemble average. The Fourier transform of $K(t)$ gives the spectrum of the radiation in the pulse; it oscillates with the frequency ω_0 and falls off on the scale of the coherence time τ_{coh}. The coherence function (1) is then equal,

$$\Gamma(\tau) = \int\limits_{-\infty}^{\infty} A(t)\, A^*(t-\tau)\, e(t)\, e^*(t-\tau)\, dt. \tag{4}$$

Using Eq. (3), we can easily find the average value of $\Gamma(\tau)$,

$$\langle\Gamma(\tau)\rangle = \mathrm{K}(\tau) \int\limits_{-\infty}^{\infty} A(t)\, A^*(t-\tau)\, dt \approx \mathrm{K}(\tau) \int\limits_{-\infty}^{\infty} I(t)\, dt. \tag{5}$$

Being proportional to $K(t)$,$\langle\Gamma(t)\rangle$ approaches zero when $\tau > \tau_{coh}$, and because τ_{coh} is much smaller than the characteristic time of variation of the functions $A(t)$, we neglected τ in its argument. Eq. (5) represents a result of the conventional interference theory, which tells that the average interference pattern is observable within the coherence time of the incident light. Measuring $\langle\Gamma(\tau)\rangle$ provides the information about the correlation function $K(\tau)$ only, and hence the spectrum of the pulse; the function $I(t)$ cannot be extracted from $\langle\Gamma(\tau)\rangle$.

Now, we want to show that the fluctuations of $\Gamma(\tau)$ around its average value indeed carry the information about $I(t)$. To this end, we calculate the absolute value of the dispersion $d_\Gamma(\tau)$ of the fluctuations of the absolute value of $\Gamma(t)$. For $d_\Gamma(\tau)$ we have

$$\begin{aligned} d_\Gamma(\tau) &\equiv \left\langle \left|\Gamma(\tau) - \langle\Gamma(\tau)\rangle\right|^2 \right\rangle \\ &= \left\langle \left|\Gamma(\tau)\right|^2 \right\rangle - \left|\langle\Gamma(\tau)\rangle\right|^2 \\ &= \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} dt dt' A(t)\, A^*(t-\tau)\, A^*(t')\, A(t'-\tau) \times \\ &\quad \langle e(t)\, e^*(t-\tau)\, e^*(t')\, e(t'-\tau)\rangle - |K(\tau)|^2 \times \\ &\quad \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} dt dt' A(t)\, A^*(t-\tau)\, A^*(t')\, A(t'-\tau). \end{aligned} \tag{6}$$

For the normal stochastic process, the fourth order correlators are reduced to the sum of the products of the second or correlators, yielding

$$\begin{aligned}&\left\langle e\left(t\right)e^{*}\left(t-\tau\right)e^{*}\left(t'\right)e\left(t'-\tau\right)\right\rangle\\ =&\left\langle e\left(t\right)e^{*}\left(t-\tau\right)\right\rangle\left\langle e^{*}\left(t'\right)e\left(t'-\tau\right)\right\rangle\\ +&\left\langle e\left(t\right)e^{*}\left(t'\right)\right\rangle\left\langle e^{*}\left(t-\tau\right)e\left(t'-\tau\right)\right\rangle\\ =&\left|K\left(\tau\right)\right|^{2}+\left|K\left(t-t'\right)\right|^{2}.\end{aligned}\tag{7}$$

The first term in Eq. (7) appears in the theory of Hunbury Brown – Twiss interferometry [5], however, it does not contain information about the pulse shape. It will be canceled by the last term in Eq. (6). Putting the second term in Eq. (7) into Eq. (6) and using the fact that $K\left(\tau\right)$ is a narrow function in comparison with $A(t)$ gives the final result,

$$d_{\Gamma}\left(\tau\right)=\int\limits_{-\infty}^{\infty}\left|K\left(\xi\right)\right|^{2}d\xi\times\int\limits_{-\infty}^{\infty}dtI\left(t\right)I\left(t-\tau\right).\tag{8}$$

Note that $d_{\Gamma}\left(\tau\right)$ is proportional to the convolution of the intensity $I\left(t\right)$. Precise determination of a pulsewidth requires further knowledge about the shape of $I(t)$, however, if $I(t)$ were known to be symmetric, its shape could be deduced from its convolution function [6].

We have performed computer simulation of the fluctuations of the interference pattern produced by light pulses in a two-beam interferometer. The code generates the function $\mathrm{Re}\Gamma\left(\tau\right)$ resulting from the interference of a pulse with intensity $I\left(t\right)$ = $I_{0}\exp\left(-t^{2}\middle/2\tau_{p}^{2}\right)$ of partially coherent light having a Gaussian spectrum of width $\Delta\omega$ at the frequency ω_{0}. The normalized correlation function for the Gaussian spectrum is $K\left(\tau\right)/K\left(0\right)=\exp(-\Delta\omega^{2}\tau^{2}/2)$. We have chosen the frequency ω_{0} = 3.1015 s^{-1} corresponding to the wavelength 630 nm, and the pulse length τ_{p} = 30 femtosecond. The spectral width was assumed $\Delta\omega/\omega_{0}$ = 0.1. Fig. 2 shows the function $\left[\mathrm{Re}\Gamma\left(\tau\right)\right]^{2}$ for a single pulse similar to that shown in Fig. 1. The peak at the origin (τ < 10 fs) occurs in accordance with Eq. (5) for the average value $\langle\Gamma\ (t)\rangle$, $|\ \langle\Gamma\ (\tau)\rangle\ |\propto\exp(-\Delta\omega^{2}\tau^{2}/2)$. The rest of the plot, corresponding to τ > 10 fs, represents a fluctuation of $\Gamma\left(\tau\right)$ with an average value equal to zero. Fig. 3 shows $\left[\mathrm{Re}\Gamma\left(\tau\right)\right]^{2}$ averaged over 100 pulses along with the theoretical dependence given by Eq. (8). We changed the vertical scale on this plot to emphasize the fluctuational part of $\Gamma(\tau)$; the peak at the origin representing $\langle\Gamma(t)\rangle$ is cut off in our plot. As we see, on the average, $[\mathrm{Re}\Gamma(\tau)]^2$ agrees with the convolution function of the intensity of the pulse. This example demonstrates that in the visible light one can easily measure pulse lengths in the range of tens of femtosecond. For shorter wavelengths, one can achieve even better time resolution, with the principal limitation of the method being several periods of oscillations.

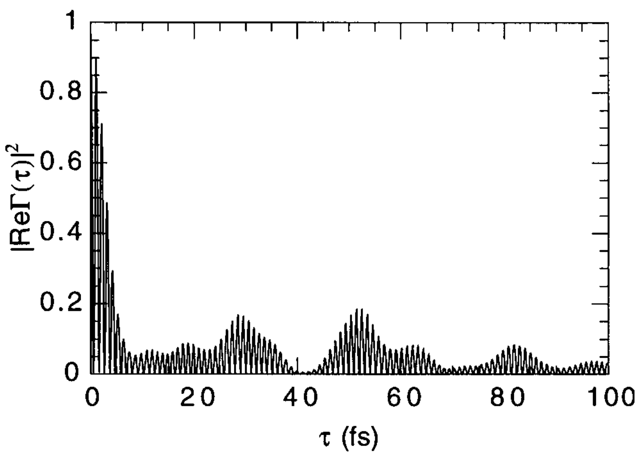

Fig. 2. $[\mathrm{Re}\Gamma(\tau)]^2$ for a single pulse.

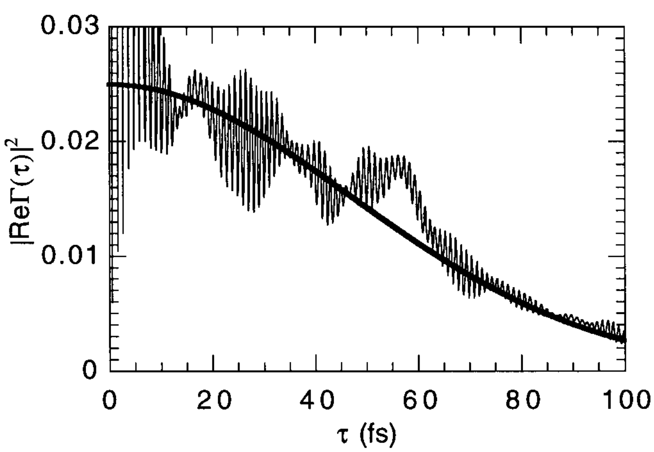

Fig. 3. $[\mathrm{Re}\Gamma(\tau)]^2$ averaged over 100 pulses (thin line) and the theoretical value proportional to the convolution of the intensity, given by Eq. (8) (heavy line).

Practical realization of the interferometry in the proposed method will depend substantially on the duration of the pulses. For short pulses (less then a picosecond) one can use

Michelson interferometer and observe interference fringes with a CCD detector. For longer pulses, (a few picosecond and more) a scheme using a blazed diffraction grating [7] can be utilized.

In summary, a technique is proposed, capable of measuring short pulses of incoherent radiation. It is based on the observation and statistical analysis of fluctuations of the visibility of the interferometric pattern produced by single bunches. The method can be realized with a photodetector that, by itself, cannot resolve the pulse length. It is best suited for repetitive pulses (e.g., radiation of bunches in a circular accelerator) when statistics can be accumulated over many pulses.

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