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# Collective Effects in Isochronous Storage Rings

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## ABSTRACT

We studied the collective instabilities in isochronous storage rings using a linac-type analysis. Simple criteria for avoiding the longitudinal and transverse instabilities are developed by employing a two-particle model. Numerical examples show that these conditions do not impose serious performance restrictions for two of the currently proposed isochronous storage rings.

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## I. INTRODUCTION

It has been suggested that ultra-short electron beam bunches can be stored in a quasi-isochronous storage ring whose momentum slip factor $\eta$ is designed to be very small [1]. Since the peak current is high, the collective instabilities are one of the limiting factors in the operation of the quasi-isochronous storage rings.

In discussing the collective instabilities, we need to distinguish different regimes according to the relative magnitudes of the period of the synchrotron oscillation $\tau_{\rm syn}$ and the radiation damping time $\tau_{\rm rad}$. The quantities $\tau_{\rm syn}$ and $\tau_{\rm rad}$ are given by [2]

$$\tau_{\rm syn} = 2\pi\sqrt{\frac{2\pi R E_0}{\eta c e \omega_{\rm rf} V_{\rm rf}}}, \tag{1}$$

where $2\pi R$ is the storage-ring circumference, $E_0$ is the design particle energy, $c$ is the speed of light, $e$ is the electron charge, $\omega_{\rm rf}$ and $V_{\rm rf}$ are the angular frequency and integrated voltage per revolution of the rf cavities, and

$$\tau_{\rm rad} = \frac{4\pi}{c C_\gamma E_0^3 < G^2 >}, \tag{2}$$

where $C_\gamma = 8.85 \times 10^{-5}$ m-GeV$^{-3}$, $G = 1/\rho$, $\rho$ is the bend radius, and the angular brackets imply taking the average over the ring circumference.

In the regime where

$$\tau_{\rm syn} \ll \tau_{\rm rad}\ , \tag{3}$$

the instability mechanisms are the usual microwave instabilities, discussed extensively in the literature (for a review, see [3]). A storage ring for which the inequality (3) is satisfied is referred to as the conventional storage ring in this paper.

The synchrotron oscillation period is proportional to $1/\sqrt{\eta}$. When $\eta$ becomes sufficiently small, therefore, we have the opposite regime

$$\tau_{\rm syn} \gg \tau_{\rm rad}. \tag{4}$$

In this regime the internal longitudinal motion of particles in the bunch can be neglected. A storage ring for which the inequality (4) is satisfied is referred to as the isochronous storage ring in this paper. In an isochronous storage ring, the usual analysis of microwave instabilities breaks down. The instability mechanisms are now replaced by the linac collective effects and the analyses have also to be replaced.

In this note we consider the collective instabilities in an isochronous storage ring for the case when the wakefields are short-ranged so that we only have to worry about single-bunch, single-turn wakefields. We further simplify the analysis by employing a two-particle model.

The damping time is different for the longitudinal and transverse oscillation. The radiation damping time $\tau_{\rm rad}$ in the above should therefore be interpreted as the energy damping time $\tau_{\rm rad,E} = \tau_{\rm rad}/J_{\rm E}$ in discussing the longitudinal synchrotron oscillation, and as the betatron (horizontal or vertical) damping time $\tau_{{\rm rad},\beta} = \tau_{\rm rad}/J_\beta$ in discussing the transverse betatron oscillations. Here the quantities $J$s are the damping partition numbers given by $J_E \approx 2$ and $J_\beta \approx 1$.

The longitudinal and transverse collective effects for isochronous storage rings are discussed in Section II. Two numerical examples, one for a $\Phi$ factory and another for a free electron laser device (FEL), are included in Section III.

## II. ANALYSIS IN ISOCHRONOUS STORAGE RING

In this section we consider the collective effects in an isochronous storage ring, where the inequality (4) is valid.

### A. Head-Tail Energy Split

The main longitudinal collective effect in a linac, and in an isochronous storage ring, is to cause a head-tail energy split. We consider a two-particle model. Let the bunch head and the bunch tail have longitudinal coordinates $z = \frac{1}{2}\ell_z$ and $z = -\frac{1}{2}\ell_z$ relative to the bunch center. The equation for the energy error of the bunch head is

$$\dot{\delta}_{\rm head} = -\frac{2}{\tau_{\rm rad,E}}\delta_{\rm head} + \frac{eV_{\rm rf}\omega_{\rm rf}\ell_z}{4\pi R E_0}. \tag{5}$$

The energy distribution of the stored beam reaches an equilibrium by a balance between radiation damping and the energy gain from the rf cavities. The bunch head then has an equilibrium energy error

$$\delta_{\mathrm{head}} = \frac{eV_{\mathrm{rf}}\omega_{\mathrm{rf}}\ell_z\tau_{\mathrm{rad,E}}}{8\pi R E_0} \ . \tag{6}$$

Similarly, the equation for the bunch-tail energy error is [3]

$$\dot{\delta}_{\mathrm{tail}} = -\frac{2}{\tau_{\mathrm{rad,E}}}\delta_{\mathrm{tail}} - \frac{eV_{\mathrm{rf}}\omega_{\mathrm{rf}}\ell_z}{4\pi R E_0} - \frac{N r_0 c W_0'}{4\pi R\gamma} \tag{7}$$

where $N$ is the number of electrons in the bunch, $r_0$ is the electron classical radius, $W_0'$ is the longitudinal wakefunction (created by the bunch head and seen by the bunch tail) integrated over the storage-ring circumference, and $\gamma$ is the Lorentz energy factor of the stored electrons. In equilibrium, we have

$$\delta_{\mathrm{tail}} = -\frac{eV_{\mathrm{rf}}\omega_{\mathrm{rf}}\ell_z\tau_{\mathrm{rad,E}}}{8\pi R E_0} - \frac{N r_0 c W_0'\tau_{\mathrm{rad,E}}}{8\pi R\gamma} \tag{8}$$

In Eqs. (5) and (7) we have ignored the energy loss due to the self-wakes of the macro particles.

The head-tail energy split is therefore, using Eqs. (6) and (8),

$$\Delta\delta = \delta_{\mathrm{head}} - \delta_{\mathrm{tail}} = \Delta\delta_{\mathrm{rf}} + \Delta\delta_Z, \tag{9}$$

where

$$\Delta\delta_{\mathrm{rf}} = \frac{eV_{\mathrm{rf}}\omega_{\mathrm{rf}}\ell_z\tau_{\mathrm{rad,E}}}{4\pi R E_0}, \tag{10}$$

and

$$\Delta\delta_Z = \frac{N r_0 c W_0'\tau_{\mathrm{rad,E}}}{8\pi R\gamma} \ . \tag{11}$$

The first term, $\Delta\delta_{\mathrm{rf}}$, is independent of the wakefields, and is there to maintain an equilibrium energy distribution of the bunch. The second term, $\Delta\delta_Z$, is wake dependent. In the design of the isochronous storage ring, $\Delta\delta$ needs to be within tolerance.

Equation(10) can be written as

$$\eta c\tau_{\mathrm{rad,E}}\Delta\delta_{\mathrm{rf}} = 2\pi^2(\tau_{\mathrm{rad,E}}/\tau_{\mathrm{syn}})^2\ell_z. \tag{12}$$

The left-hand side of the equation is the distance a particle with an energy error $\Delta\delta_{\rm rf}$ travels in one damping time, which is much smaller than the bunch length $\ell_z$ in the present case where Eq. (4) is valid. Thus the relative motion of the particles in the bunch can indeed be neglected. The isochronous storage ring needs to be designed in such a way that $\Delta\delta_{\rm rf}$ is within the momentum aperture of the ring.

Assuming the longitudinal wake is due to a longitudinal impedance $Z_0^{\|}/n$, then

$$W_0' \approx \frac{cR}{b^2}\frac{Z_0^{\|}}{n} \ , \tag{13}$$

where $b$ is the storage-ring beam pipe radius, and $n$ is the impedance frequency in units of revolution frequency. The contribution to the head-tail energy split due to the wakefield then reads

$$\Delta\delta_Z \approx \frac{Nr_0c\tau_{\rm rad,E}}{2b^2\gamma}\frac{1}{Z_0}\frac{Z_0^{\|}}{n} \ , \tag{14}$$

where $Z_0 = 377\ \Omega$.

### B. Beam Break-Up Instability

The main transverse collective effect in a linac is to cause a beam break-up instability. In the beam break-up instability, the bunch head executes a simple betatron oscillation without being affected by the wakefields. The bunch tail sees the wakefield left behind by the bunch head, and is driven resonantly by it. Using a two-particle model, the betatron oscillation of the bunch tail grows by a factor of $\Upsilon$ per turn, where [3]

$$\Upsilon = -\frac{Nr_0W_1\beta_Z}{4\gamma} \ , \tag{15}$$

with $W_1$ being the transverse wake function integrated over the storage-ring circumference, and $\beta_Z$ is the beta function at the location of the impedance.

For stability, the growth of the bunch tail must be suppressed by radiation damping. This leads to the stability criterion

$$\frac{T_0}{\tau_{\rm rad,\beta}} > \Upsilon \ , \tag{16}$$

where $T_0$ is the revolution period.

We assume the wakefields are produced by a transverse impedance $Z_1^\perp$. For a short bunch with $\ell_z \ll b$, the wakefield seen by the bunch tail (which trails the bunch head by a distance $\ell_z$) is proportional to $\ell_z$. The transverse wake function $W_1$ seen by the bunch tail is approximately

$$W_1 \approx -\frac{c\ell_z}{b^2} Z_1^\perp \ . \tag{17}$$

For our purpose it is more convenient to relate $Z_1^\perp$ to the longitudinal impedance $Z_0^\parallel/n$ by the approximate relation

$$Z_1^\perp = \frac{2R}{b^2}\frac{Z_0^\parallel}{n} \ . \tag{18}$$

Combining Eqs. (16), (17), and (18) then gives the stability criterion

$$\frac{Z_0^\parallel}{n} < Z_0 \frac{\gamma b^4}{cNr_0\beta_Z\ell_z\tau_{\mathrm{rad},\beta}} \ . \tag{19}$$

It should be mentioned that the short length of the bunch helps to reduce the beam break-up effect. This reduction, as seen in Eq. (17), is due to the fact that the transverse wakefield is smaller for short bunches.

## III. NUMERICAL EXAMPLES

### A. UCLA $\Phi$ Factory

A design of a high-luminosity $\Phi$ factory based on a small value of $\eta$ is described in [4]. The ring consists of four cells, each containing two 47-degree bending sections of $\rho = 0.425$ m and one $-4$-degree inverted bending section of $\rho = -1.7$ m. By controlling the dispersion in the inverted bending section, the momentum slip factor $\eta$ is variable between $-0.005$ and 0.008. Other relevent parameters are $2\pi R = 32.7$ m, rf frequency = 499 MHz, $V_{\mathrm{rf}} = 0.1$ MV, and $E_0 = 0.51$ GeV. With six bunches in the ring, each with $N = 1.33 \times 10^{11}$ electrons, and $\beta^* = \sigma_z = 0.4$ cm, where $\beta^*$ is the beta function at the interaction point, the luminosity becomes $1.6 \times 10^{33}$ /cm$^2$/s.

We find $\tau_{\rm rad,E} \approx 3.7$ ms. If the ring were conventional, the energy spread would be calculated according to

$$\sigma_\delta = \sqrt{C_q \frac{< G^3 > \gamma^2}{J_{\rm E} < G^2 >}}, \tag{20}$$

where $C_q = 3.84 \times 10^{-13}$ m. We then obtain $\sigma_\delta = 6.7 \times 10^{-4}$. The momentum slip factor necessary for $\sigma_z = 0.4$ cm is, using the expression

$$\sigma_z = \frac{\eta c \tau_{\rm syn}}{2\pi}\sigma_\delta = \sqrt{\frac{2\pi R \eta c E_0}{e\omega_{\rm rf} V_{\rm rf}}}\sigma_\delta \tag{21}$$

is found to be $\eta = 2.2 \times 10^{-3}$. Inserting this into Eq. (1), we obtain $\tau_{\rm syn} = 56\mu$s. Since $\tau_{\rm syn} \ll \tau_{\rm rad,E}$, the ring in this parameter regime is conventional although the bunch length $\sigma_z$ is likely to be much shorter than the pipe radius $b$.

For a conventional ring with very short bunches, it is conceivable that one may obtain the microwave instability criterion by using the result for an isochronous ring, Eq. (19), but with $\tau_{\rm rad,\beta}$ replaced by $\tau_{\rm syn}$. The threshold of the transverse microwave instability is then given by

$$\frac{Z_0^{\|}}{n} < Z_0 \frac{\gamma b^4}{cNr_0\beta_Z\sigma_z\tau_{\rm syn}} \ . \tag{22}$$

With similar substitution to Eq. (14), we obtain the contribution to the energy spread due to the impedance effect in a conventional storage ring:

$$\Delta\delta_Z \approx \frac{Nr_0c\tau_{\rm syn}}{2b^2\gamma}\frac{1}{Z_0}\frac{Z_0^{\|}}{n} \ . \tag{23}$$

We assume $Z_0^{\|}/n \approx 0.2\Omega$, which is the value obtained in the Advanced Light Source in Berkeley. Taking $b = 2$ cm, we obtain from Eq. (23) that $\Delta\delta_Z = 4.2 \times 10^{-3}$. Thus the impedance contribution to the energy spread is about seven times larger than the natural energy spread, implying that the bunch length will be seven times longer than the zero current value of 0.4 cm. On the other hand, the right-hand side of Eq. (22) using $\sigma_z = 2.8$ cm is about 7 $\Omega$; the transverse microwave effect is not important, although strictly speaking, Eq. (22) no longer applies because $\sigma_z$ is now comparable to $b$.

### B. A Proposed Quasi-Isochronous Ring at ETL

An experimental ring, possibly for FEL application, to be built at Electrotechnical Laboratory (ETL) in Japan was proposed recently [5]. The general idea of the ring is similar to that discussed above, each cell containing two 49-degree bending sections of $\rho = 1.5$ m and one $-8$-degree inverted bending section of $\rho = -10$ m. By controlling the dispersion in the inverted bending section, a momentum slip factor as small as $\eta = 2. \times 10^{-7}$ is contemplated. Other parameters are $2\pi R = 82.4$ m, rf frequency $= 502$ MHz, $V_{\rm rf} = 1$ MV, and $E_0$ up to 1.5 GeV. If we use the formulae for conventional ring, we obtain $\sigma_z = 51\mu$m. However, we also obtain $\tau_{\rm syn} = 5.1$ ms, which is longer than $\tau_{\rm rad,E} \approx 1.25$ ms and $\tau_{{\rm rad},\beta} \approx 2.5$ ms. Thus, the ring is in the isochronous regime, in which case the short bunch length is not the result of the radiation damping, it must be injected from the beginning.

The energy acceptance of the ring is about 1%. Taking $\ell_z \approx 2\sigma_z$, we find from Eq. (10) that $\Delta\delta_{\rm rf} = 1.6 \times 10^{-3}$, which is negligible. Assuming again that $b = 2$ cm and $Z_0^{\parallel}/n \approx 0.2\Omega$, we compute from Eq. (14) the number of electrons per bunch $N$ corresponding to the case $\Delta\delta_Z$ equals the enegy acceptance 1%, and find $N = 4.2 \times 10^{10}$. This would be more than would be required for most applications. With this value of $N$, the inequality (24) becomes $0.2\Omega < 19.6\Omega/\beta_Z[{\rm m}]$. Since the average value of the horizontal or vertical beta function of the ring is less than 20 m, this inequality is also easily satisfied, provided $Z_0^{\parallel}/n$ is controlled to 0.2 $\Omega$.

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