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Semiclassical Approach to Structure Functions at Small x^*

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Abstract

Inclusive and diffractive structure functions for electron-proton scattering are calculated in a semiclassical approach at large momentum transfer Q^2 and small values of the scaling variable x . The basic process is the production of a quark-antiquark pair in the colour field of the proton. The structure functions are expressed in terms of Wilson lines along the classical trajectories of quark and antiquark passing through the colour field, and their covariant derivatives. Based on some rather general assumptions on properties of the colour field, inclusive and diffractive structure functions are evaluated in terms of four field dependent constants.

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1 Introduction

The events with a large gap in rapidity, observed in deep inelastic electron-proton scattering at HERA [1, 2], represent a puzzling phenomenon. The separation of a colour neutral cluster of partons from the proton, which then fragments independently of the proton remnant, is a non-perturbative process. As the data show, the cross section for these “diffractive” events is not suppressed at large values of Q^2 relative to the inclusive cross section. All this is difficult to understand in the framework of perturbative QCD and the parton model, which has been so successful in describing many phenomena of strong interactions at short distances.

Various phenomenological models have been proposed which can account for some aspects of the rapidity gap events. They combine ideas from the quark-parton model and vector meson dominance [3], Regge theory and perturbative QCD [4]-[10], or use effective lagrangians together with vector meson dominance [11]. In a recent paper [12] we have shown that the global properties of the rapidity gap events can be understood based on electron-gluon scattering as underlying partonic process, provided a soft “colour rotation” [13] transforms the produced quark-antiquark pair into a colour singlet. Based on the same idea a detailed description of the final state has been developed [14]. The importance of the gluon in diffractive processes has been anticipated in previous work on the pomeron in QCD [15].

So far, however, there exists no procedure for calculating the diffractive structure function in QCD in a systematic way. It is the goal of this paper to make some progress in this direction. In view of the phenomenological success of the results presented in [12] we shall start from the production of a quark-antiquark pair as basic process. We are particularly interested in the non-perturbative mechanism which yields colourless diffractive final states. As a first approximation we shall therefore consider the extreme case where also the partonic gluon in electron-gluon scattering is soft, so that it cannot be distinguished from the soft gluons which provide the colour neutralization. In this limit the diffractive process becomes the production of a quark-antiquark pair in the colour field of the proton, where the classical colour field represents a state with a high density of gluons. We shall work in the proton rest frame where pair production of quarks is known to be the dominant process [16, 17].

A formalism to treat soft colour interactions in high energy processes has been devel-

oped by Nachtmann [18]. So far, it has been applied to elastic hadron-hadron scattering at high energies [18, 19]. Similar ideas have been pursued in connection with heavy quark production in hadron collisions [20]. In [18] the propagation of quarks in a soft colour field is calculated in an eikonal approximation which allows to relate quark-quark scattering amplitudes to Wilson lines. In the following we shall use and further develop this approach to describe diffractive processes in deep inelastic scattering. This requires a derivative expansion starting with the eikonal approximation and a proper treatment of the colour field inside the proton.

The paper is organized as follows. In sect. 2 quark and antiquark wave functions in a soft colour field are evaluated in a high energy expansion. Sect. 3 deals with the pair production cross section, which is obtained in terms of Wilson lines and their covariant derivatives. In sect. 4 explicit expressions are derived for inclusive and diffractive structure functions, making some assumptions about the colour field inside the proton. Sect. 5 contains a more general discussion of the connection between inclusive and diffractive structure functions, and in sect. 6 we summarize our results.

2 High energy expansion for quark wave functions

The goal of this paper is to calculate the cross section for the production of a quark-antiquark pair by a virtual photon in an external colour field. According to the LSZ-formalism, this requires knowledge of the wave functions of the outgoing quark and antiquark with 4-momenta l' and l , respectively, which are given by

$$\bar{\psi}_u(x) = - \int d^4y e^{il'y} \bar{u}(l') i \overleftrightarrow{\partial}_y S_F(y, x) \quad , \quad \psi_v(x) = \int d^4y S_F(x, y) i \overleftrightarrow{\partial}_y v(l) e^{ily} \quad (1)$$

Here S_F stands for the Feynman propagator in the colour background field $G(x)$. Assuming, that the field G contains no Fourier modes with frequencies of order l_0 , l'_0 or larger, the Feynman propagator can be replaced by the advanced propagator (cf. [18]). Hence, these wave functions can equally well be defined as solutions of the Dirac equation

$$\not{D} \psi_v(x) = 0 \quad , \quad \bar{\psi}_u(x) \overleftarrow{\not{D}} = 0 \quad , \quad (2)$$

satisfying the boundary conditions

$$\psi_v(x) \rightarrow v(l) e^{ilx} \quad , \quad \bar{\psi}_u(x) \rightarrow e^{il'x} \bar{u}(l') \quad \text{for} \quad x_0 \rightarrow \infty \quad . \quad (3)$$

Here the covariant derivatives are given by

$$D_\mu = \partial_\mu + iG_\mu \quad , \quad \overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu - iG_\mu \quad , \quad G_\mu = \frac{1}{2}\lambda^a G_\mu^a \quad , \quad (4)$$

where G_μ^a is the colour field, and the λ^a (a=1...8) are the SU(3) generators.

We are interested in the wave function of an outgoing quark (antiquark) whose energy is very large compared to the scale on which the colour field varies. This suggests an expansion of the form,

$$\psi_v(x) = e^{il \cdot x} (V_0 + V_1 + \dots) v(l) \quad , \quad (5)$$

$$\bar{\psi}_u(x) = \bar{u}(l') (U_0 + U_1 + \dots) e^{il' \cdot x} \quad , \quad (6)$$

where V_{n+1} (U_{n+1}) is suppressed by one power of l_0 (l'_0) with respect to V_n (U_n). $v(l)$ and $u(l')$ are the usual spinors for massless fermions, satisfying

$$\not{l} v(l) = 0 \quad , \quad \bar{u}(l') \not{l}' = 0 \quad . \quad (7)$$

Inserting the expansion (5) into the Dirac equation, one finds

$$i\not{D} (V_0 + V_1 + \dots) v(l) = \not{l} (V_0 + V_1 + \dots) v(l) \quad . \quad (8)$$

This leads to the recursion relations for the matrices V_n ($n > 0$),

$$l \cdot D V_0 = 0 \quad , \quad (9)$$

$$\not{l} V_n = i\not{D} V_{n-1} \not{l} \quad . \quad (10)$$

In order to solve these equations it is convenient to introduce the projection operators

$$P_+ = \frac{1}{2l_0} \not{l} \gamma_0 \quad , \quad P_- = \frac{1}{2l_0} \gamma_0 \not{l} = 1 - P_+ \quad . \quad (11)$$

The matrices V_n can now be written as

$$V_n = \sum_{i,j=+,-} V_n^{ij} \quad , \quad \text{where} \quad V_n^{ij} = P_i V_n P_j \quad . \quad (12)$$

Obviously, only V^{++} and V^{-+} contribute in Eq. (10). Hence we can set $V^{+-} = V^{--} = 0$. For the two remaining terms one obtains from Eq. (10),

$$V_n^{-+} = \frac{i}{2l_0} \gamma_0 \not{D} V_{n-1} P_+ \quad , \quad l \cdot D V_n^{++} = -\frac{1}{2} \not{l} \not{D} V_n^{-+} \quad . \quad (13)$$

Inverting the differential operator $l \cdot D$ with appropriate boundary conditions the solution of these equations can be written in the compact form

$$V_n = \left(\frac{i}{2l \cdot D} \not{D} P_+ \not{D} \right)^n P_+ V_0 . \quad (14)$$

Finally, we have to solve the ordinary differential equation (9) for V_0 . The result is the well known non-abelian phase factor

$$V_0(x, l; G) = P \exp \left(\frac{i}{2} \int_{x_-}^{\infty} dx'_- G_+(x'_-, x_+, \vec{x}_\perp) \right) . \quad (15)$$

Here P denotes path ordering, and x^μ is decomposed with respect to l^μ ,

$$x_- = \frac{1}{l_0}(l_0 x^0 + \vec{x} \cdot \vec{l}) , \quad x_+ = \frac{1}{l_0}(l_0 x^0 - \vec{x} \cdot \vec{l}) , \quad \vec{x}_\perp \cdot \vec{l} = 0 . \quad (16)$$

The convention for plus- and minus-components is chosen in a way that stresses their similarity to the light-cone coordinates used later on. Note, however, that they are not identical, since we will define the x^3 -axis anti-parallel to the photon momentum below (cf. Fig. 1).

Eqs. (5) and (14)-(16) give the wave function of an antiquark propagating in a colour field G_μ^a , which at large times approaches the wave function of a free antiquark with momentum l . The wave function is given as a power series in D/l_0 where D stands for a covariant derivative.

The wave function of an outgoing quark can be obtained in a completely analogous fashion. The result reads explicitly

$$U_n = U_0 P_- \left(\overleftarrow{\not{D}} P_- \overleftarrow{\not{D}} \frac{i}{2\overleftarrow{D} \cdot l'} \right)^n , \quad (17)$$

$$U_0(x, l'; G) = \exp \left(\frac{i}{2} \int_{\infty}^{x_-} dx'_- G_+(x'_-, x_+, \vec{x}_\perp) \right) . \quad (18)$$

In these expressions projection operator and components of x^μ are defined with respect to the quark momentum l' .

In the following we shall have to manipulate expressions involving the non-abelian phase factors V_0 and U_0 . We therefore list some useful formulae which can be derived by standard methods. Let $U(z, y)$ be the phase factor, path-ordered along a straight line from y to z ,

$$U(z, y) = P \exp \left(-i \int_y^z dx^\mu G_\mu \right) . \quad (19)$$

Then the covariant derivative with respect to the upper end point is given by

$$D_\mu(z)U(z, y) = -U(z, y)(z - y)^\nu \int_0^1 t dt U_t^{-1} G_{\mu\nu}(y + t(z - y)) U_t, \quad (20)$$

where

$$U_t = U(y + t(z - y), y). \quad (21)$$

Note, that the covariant derivative in the direction of the straight line vanishes.

The path-ordered integral around a triangle can be expressed in terms of the field strength as follows,

$$\begin{aligned} U_\Delta(z, y, x) &= U(x, z)U(z, y)U(y, x) \\ &= P \exp \int_0^1 du b^\mu K_\mu(u), \end{aligned} \quad (22)$$

with

$$K_\mu(u) = -a^\nu \int_0^1 t dt U_{ut}^{-1} G_{\mu\nu}(x + t(a + ub)) U_{ut}, \quad U_{ut} = U(x + t(a + ub), x), \quad (23)$$

and $a = y - x$, $b = z - y$. For one infinitesimal distance, $b = \epsilon \hat{b}$, this reduces to

$$U_\Delta = 1 - \epsilon \hat{b}^\mu a^\nu \int_0^1 t dt U_t^{-1} G_{\mu\nu}(x + ta) U_t. \quad (24)$$

As expected, in this case $U_\Delta - 1$ is proportional to the enclosed infinitesimal area.

3 Pair production in a colour field

3.1 Matrix elements

We are interested in the diffractive and inclusive structure functions of the proton at large momentum transfer of the electron and at small x ,

$$-q^2 = Q^2 \gg \Lambda^2, \quad x = \frac{Q^2}{2m_p q_0} \ll 1, \quad (25)$$

where Λ is the QCD scale parameter and q_0 is the photon energy in the proton rest frame. Here, the photon energy is much larger than the QCD scale Λ and, at low x , the kinematically required momentum transfer to the proton is very small. Hence, it appears reasonable to describe the photon-proton interaction as pair production in a classical

colour field. In case of the diffractive structure function, we shall consider final states with invariant mass

$$M^2 = (l + l')^2 = O(Q^2), \quad (26)$$

where l' and l are the 4-momenta of quark and antiquark, respectively (cf. Fig. 1). Our goal is to calculate the leading term of the cross section in the limit $x \ll 1$ and $\Lambda^2/Q^2 \ll 1$.

The S-matrix element for pair production is given by the expression

$$S_{fi} = ie\epsilon_\mu \int d^4x e^{-iqx} \bar{\psi}_u \gamma^\mu \psi_v \equiv ie\epsilon_\mu S^\mu. \quad (27)$$

Here ψ_u and ψ_v are the wave functions of the produced quark and antiquark, respectively. They have been calculated in the previous section and they contain the dependence on the colour field. To leading order one has (cf. eqs. (5),(6)),

$$S_{(0)ab}^\mu = \left(\int d^4x e^{i\Delta x} \bar{u}(l') U_0(x) \gamma_\mu V_0(x) v(l) \right)_{ab}, \quad (28)$$

where

$$\Delta = l + l' - q, \quad (29)$$

and the indices ab denote the colour of the created state.

For the evaluation of longitudinal and transverse structure functions the combinations $S_\mu^* S^\mu$ and $S_0^* S^0$ will be needed. Summing over the spins of quark and antiquark, one easily obtains,

$$S_{(0)ab}^\mu S_{(0)\mu}^* = -8(l'l') |\tilde{f}(\Delta)_{ab}|^2 = -4M^2 |\tilde{f}(\Delta)_{ab}|^2, \quad (30)$$

with

$$\tilde{f}(\Delta)_{ab} = \int d^4x e^{i\Delta x} f(x)_{ab} = \int d^4x e^{i\Delta x} (U_0(x) V_0(x))_{ab}. \quad (31)$$

Note, that here and below the dependence of the matrices U , V and f on l' and l , as described in the previous section, is not shown explicitly. As it is obvious from Eq. (30), the leading term, which is formally of order $l_0 l'_0 \sim (q_0)^2$ in the high energy limit, turns out to be only of order $(q_0)^0$ due to the contraction of the 4-vectors l and l' . Therefore the formally next-to-leading and next-to-next-to-leading contributions are competitive and have to be calculated.

Hence, we start from the high energy expansion of S^μ ,

$$S^\mu = S_{(0)}^\mu + S_{(1)}^\mu + S_{(2)}^\mu + \dots, \quad (32)$$

where

$$S_{(1)}^\mu = \int d^4x e^{i\Delta x} \bar{u}(l') (U_1 \gamma^\mu V_0 + U_0 \gamma^\mu V_1) v(l) \quad (33)$$

$$S_{(2)}^\mu = \int d^4x e^{i\Delta x} \bar{u}(l') (U_1 \gamma^\mu V_1 + U_2 \gamma^\mu V_0 + U_0 \gamma^\mu V_2) v(l). \quad (34)$$

The first three terms of the corresponding expansion of the squared S-matrix element,

$$S^\mu S_\mu^* = A_0 + A_1 + A_2 + \dots \quad (35)$$

have to be taken into account. A_0 is given by Eq. (30). The two following terms are

$$\begin{aligned} A_1 &= 2\text{Re} S_{(0)}^\mu S_{(1)\mu}^*, \\ A_2 &= S_{(1)}^\mu S_{(1)\mu}^* + 2\text{Re} S_{(0)}^\mu S_{(2)\mu}^*. \end{aligned} \quad (36)$$

A_1 can be easily evaluated as follows. Ordinary Dirac algebra manipulations give

$$S_{(0)}^{\mu*} S_{(1)\mu} = -2\tilde{f}(\Delta)_{ab}^* \int d^4x e^{i\Delta x} \text{tr} [\not{l}' (U_0 V_1 + U_1 V_0)_{ab}]. \quad (37)$$

Using Eq. (14) for V_1 together with the easily derived relation

$$\not{D} P_+ \not{D} P_+ V_0 = \not{D}^2 P_+ V_0, \quad (38)$$

and the corresponding expressions for U_1 and U_0 , one obtains

$$\begin{aligned} \text{tr} [\not{l}' (U_0 V_1 + U_1 V_0)] &= \frac{i}{2} U_0 \text{tr} \left[\not{l}' \left(\frac{1}{lD} \not{D}^2 P_+ + P_- \overleftarrow{\not{D}}^2 \frac{1}{l\overline{D}} \right) \right] V_0 \\ &= 2i U_0 \left(-l'D - l\overline{D} + M^2 \mathcal{O}\left(\frac{D}{l}\right) \right) V_0 \\ &= -2i(l+l')^\mu \partial_\mu (U_0 V_0) + M^2 \mathcal{O}\left(\frac{D}{l}\right). \end{aligned} \quad (39)$$

Here the colour indices have been dropped for brevity. Note, that the inverse differential operator has disappeared. After performing a partial integration the leading order term $\sim (q_0)^0$ reads

$$A_1 = 4(Q^2 + M^2) |\tilde{f}(\Delta)_{ab}|^2, \quad (40)$$

which is very similar to the expression (30) for A_0 .

The calculation of A_2 can be carried out using the same technique and the formulae of the last section. It turns out to be convenient to introduce the total momentum $L \equiv l+l'$ and the momentum fraction α ,

$$l \equiv \alpha L + a, \quad l' \equiv (1-\alpha)L - a, \quad a_0 \equiv 0. \quad (41)$$

α and the four-vector a are defined by Eq. (41). From the relation $\vec{a}^2 = \alpha(1 - \alpha)M^2$ it is clear that to leading order in our high energy expansion

$$l \approx \alpha L \quad , \quad l' \approx (1 - \alpha)L . \quad (42)$$

Since A_2 is formally of order $(q_0)^0$ its leading part is already sufficient for our purposes. Therefore, the approximation eqs. (42) may be used. A straightforward calculation yields the result

$$A_2 = 4 \left[\frac{1 - \alpha}{\alpha} \tilde{g}_{R\mu}(\Delta)_{ab}^* \tilde{g}_R^\mu(\Delta)_{ab} + \frac{\alpha}{1 - \alpha} \tilde{g}_{L\mu}(\Delta)_{ab}^* \tilde{g}_L^\mu(\Delta)_{ab} \right] - 8 \text{Re} \tilde{f}(\Delta)_{ab}^* \tilde{h}(\Delta)_{ab} , \quad (43)$$

where $\tilde{g}_{R\mu}$, $\tilde{g}_{L\mu}$ and \tilde{h} are the Fourier transforms of (cf. eq. (31))

$$g_{R\mu}(x)_{ab} = (U_0(x) D_\mu V_0(x))_{ab} \quad , \quad g_{L\mu}(x)_{ab} = (U_0(x) \overleftarrow{D}_\mu V_0(x))_{ab} \quad (44)$$

and

$$h(x)_{ab} = (U_0(x) \overleftarrow{D}_\mu D^\mu V_0(x))_{ab} . \quad (45)$$

Now the leading contribution to $S^\mu S_\mu^*$ is explicitly given by Eq. (35).

The calculation of $S^0 S_0^*$ is much simpler. Here, contrary to $S^\mu S_\mu^*$, the leading term is of order $(q_0)^2$ and there is no suppression after a contraction of Lorentz indices. One finds,

$$S^0 S_0^* = 8q_0^2 |\tilde{f}(\Delta)_{ab}|^2 \alpha(1 - \alpha) . \quad (46)$$

3.2 Cross section

Consider the production of a quark-antiquark pair by a virtual photon with 3-momentum \vec{q} in the time interval $-T/2 < x_0 < T/2$. The standard formula for the production cross section reads

$$d\sigma(\gamma^* \rightarrow q\bar{q}) = \frac{1}{2|\vec{q}|T} \int dX |S_{fi}|^2 , \quad (47)$$

where the phase space element is

$$dX = \frac{d^3\vec{l}}{(2\pi)^3 2l_0} \frac{d^3\vec{l}'}{(2\pi)^3 2l'_0} = \frac{d\Delta_+ d\Delta_- d^2\Delta_\perp d\alpha}{8(2\pi)^5} . \quad (48)$$

Here a change of variables has been performed from the 3-momenta of quark and antiquark to the momentum transfer in light-cone coordinates, $\Delta_\pm = \Delta^0 \pm \Delta^3$, and the

momentum fraction α (cf. (41)). Furthermore, one angular integration has been carried out. Without contraction with the photon polarization vector, one has (cf. eq. (27))

$$d\sigma^{\mu\nu} = \frac{\pi e^2}{|\vec{q}|} \frac{1}{2\pi T} \int dX S^\mu S^{*\nu} , \quad (49)$$

with

$$S^\mu = \int d^4x e^{i\Delta x} F^\mu(x; \Delta, \alpha) . \quad (50)$$

Here the function F stands for the expressions f, g and h introduced in Eqs. (31) and (44), (45) above. The dependence on the quark-antiquark configuration, and therefore on Δ and α , is explicitly kept.

It is useful to also write the photon 4-momentum $q^\mu = (q_0, \vec{0}, -\sqrt{q_0^2 + Q^2})$ (cf. Fig. 1) in light-cone coordinates,

$$q = (q_+, q_-, \vec{q}_\perp) = \left(-\frac{Q^2}{2q_0}, 2q_0, \vec{0} \right) . \quad (51)$$

In the proton rest frame the large component is q_- . The momentum transferred by the smoothly varying external field is small, $\Delta = O(\Lambda)$. Hence, $\Delta_- \ll q_-$ is irrelevant for the quark-antiquark configuration. This is also clear from the expression for the opening angle θ , which, in the configuration with $\vec{l}_\perp + \vec{l}'_\perp = 0$, reads

$$\theta^2 \simeq \frac{M^2}{q_0^2 \alpha (1 - \alpha)} , \quad (52)$$

with the invariant mass M^2 given by

$$M^2 + Q^2 = q_+ \Delta_- + q_- \Delta_+ + \Delta^2 \simeq 2q_0 \Delta_+ . \quad (53)$$

We choose the coordinate system such that the proton is at the origin, and $x_\parallel \equiv x^3$ is the distance between the proton and the point where the pair is created. Since the quark-antiquark configuration is essentially independent of Δ_- , the same can be expected to hold for $F(x; \Delta, \alpha)$. Hence, we have

$$\begin{aligned} F(x; \Delta, \alpha) &\equiv F(x_+, x_\parallel, x_\perp; \Delta_+, \Delta_-, \Delta_\perp, \alpha) \\ &\simeq F(x_+, x_\parallel, x_\perp; \Delta_+, \Delta_\perp, \alpha) . \end{aligned} \quad (54)$$

Using this property of the function F , we can carry out the integrations over Δ_- , x_+ and x'_+ in Eq. (49). Dropping Lorentz indices for brevity, one obtains

$$d\sigma \simeq \frac{\pi e^2}{|\vec{q}|} \frac{1}{4(2\pi)^5 T} \int d\Delta_+ d^2\Delta_\perp d\alpha I(\Delta, \alpha) , \quad (55)$$

where

$$\begin{aligned}
I(\Delta, \alpha) &= \int d^4x \int d^4x' \delta(x_+ - x'_+) e^{i(\frac{x_- \Delta_+}{2} - x_\perp \Delta_\perp)} e^{-i(\frac{x'_- \Delta_+}{2} - x'_\perp \Delta_\perp)} F(x; \Delta, \alpha) F^*(x'; \Delta, \alpha) \\
&= \int d^2x_\perp dx_+ \int_{-T/2+x_+}^{T/2+x_+} dx_\parallel \int d^2x'_\perp dx'_+ \int_{-T/2+x'_+}^{T/2+x'_+} dx'_\parallel \delta(x_+ - x'_+) \\
&\quad e^{-i(x_\parallel \Delta_+ + x_\perp \Delta_\perp)} e^{i(x'_\parallel \Delta_+ + x'_\perp \Delta_\perp)} F(x; \Delta, \alpha) F^*(x'; \Delta, \alpha) \\
&= \int_{-\infty}^{+\infty} dx_+ \left| \int_{-T/2+x_+}^{T/2+x_+} dx_\parallel \int d^2x_\perp e^{-i(x_\parallel \Delta_+ + x_\perp \Delta_\perp)} F(x_+, x_\parallel, x_\perp; \Delta_+, \Delta_\perp, \alpha) \right|^2 \\
&= \int_{-T/2}^{+T/2} dx_+ \left| \int_{-\infty}^{+\infty} dx_\parallel \int d^2x_\perp e^{-i(x_\parallel \Delta_+ + x_\perp \Delta_\perp)} F(x_+, x_\parallel, x_\perp; \Delta_+, \Delta_\perp, \alpha) \right|^2. \tag{56}
\end{aligned}$$

The last equality holds if only a finite range $L \ll T$ contributes to the integration over x_\parallel . Technically, this can be realized by introducing some smooth suppression of F for $x_\parallel \rightarrow \infty$, which is removed after the limit $T \rightarrow \infty$ has been taken.

The cross section now reads

$$d\sigma \simeq \frac{1}{T} \int_{-T/2}^{T/2} dx_+ \left\{ \frac{\pi e^2}{|\vec{q}|} \frac{1}{4(2\pi)^5} \int d\Delta_+ d^2\Delta_\perp d\alpha \left| \int d^3x e^{-i(x_\parallel \Delta_+ + x_\perp \Delta_\perp)} F(x_+, \vec{x}; \Delta, \alpha) \right|^2 \right\}. \tag{57}$$

This expression is an average over all times x_+ at which the proton is tested by the quark-antiquark pair. For a smoothly varying colour field the x_+ -dependence should not matter and the corresponding integral is trivial. This yields the final result

$$d\sigma \simeq \frac{\pi e^2}{|\vec{q}|} \frac{1}{4(2\pi)^5} \int d\Delta_+ d^2\Delta_\perp d\alpha \left| \int d^3x e^{-i(x_\parallel \Delta_+ + x_\perp \Delta_\perp)} F(\vec{x}; \Delta, \alpha) \right|^2, \tag{58}$$

which is similar to the cross section for pair production in a static colour field. The only difference lies in the Fourier transform of the function F , where Δ_+ occurs instead of Δ_\parallel .

The expressions for $S^\mu S_\mu^*$ and $S^0 S_0^*$ obtained in Sect. 3.1 contain the 4-dimensional Fourier transforms \tilde{f} , \tilde{g} and \tilde{h} . They have to be replaced by the 3-dimensional Fourier transforms appearing in Eq. (58), when used in expressions for cross sections in the next section.

4 Inclusive and diffractive structure functions

4.1 Relations between different formfactors

Before discussing the actual functional form of f , g_R , g_L and h , introduced in Sect. 3.1, we shall derive some relations between terms involving g_R and g_L . For brevity, colour indices will be dropped throughout this subsection.

As we shall see in the following subsection, the Fourier integrals \tilde{f} , $\tilde{g}_{L,R}$ and \tilde{h} receive negligible contributions from the region where the spatial point x lies inside the proton. Therefore, one may use the approximation

$$g_{R\mu}(x) \simeq U_0(x)\partial_\mu V_0(x) \quad , \quad g_{L\mu}(x) \simeq U_0(x)\overleftarrow{\partial}_\mu V_0(x). \quad (59)$$

Since l is a light-like vector with negligible transverse components, and $l^\mu g_\mu = 0$, Eq. (43) can be written as

$$A_2 = -4 \frac{1-\alpha}{\alpha} \sum_{i=1}^2 |\tilde{g}_{Ri}(\Delta)|^2 - 4 \frac{\alpha}{1-\alpha} \sum_{i=1}^2 |\tilde{g}_{Li}(\Delta)|^2 - 8 \text{Re} \tilde{f}(\Delta)^* \tilde{h}(\Delta). \quad (60)$$

In the following, we mean by \tilde{g}_R , \tilde{g}_L , \tilde{f} and \tilde{h} the 3-dimensional Fourier transforms in the sense of Eq. (58) which occur in the cross sections. Correspondingly, A_i , $i = 0, 1, 2$, are the expressions defined in Sect. 3.1 in terms of the 3-dimensional Fourier transforms.

It is convenient to further specify the coordinate system. In addition to choosing the x^3 -axis anti-parallel to the photon momentum, we assume the plane spanned by \vec{l} and \vec{l}' to be orthogonal to the x^1 -axis, neglecting the small transverse momentum transfer. Defining θ_1 , θ_2 and θ to be the angles between \vec{l}' and \vec{q} , between \vec{q} and \vec{l} and between \vec{l}' and \vec{l} , respectively, and writing $-i(x_\parallel \Delta_+ + x_\perp \Delta_\perp) = ix\Delta$ for brevity, one has

$$\begin{aligned} i\Delta_+ \int e^{ix\Delta} U_0 V_0 &= \int e^{ix\Delta} \partial_3 (U_0 V_0) \\ &= \int e^{ix\Delta} (\theta_1 U_0 \overleftarrow{\partial}_2 V_0 - \theta_2 U_0 \partial_2 V_0) \\ &= \int e^{ix\Delta} (-i\Delta_2 \theta_1 U_0 V_0 - \theta_1 U_0 \partial_2 V_0 - \theta_2 U_0 \partial_2 V_0) \\ &= - \int e^{ix\Delta} (i\Delta_2 \theta_1 U_0 V_0 + \theta U_0 \partial_2 V_0). \end{aligned} \quad (61)$$

In order to obtain the second equality, one has to observe that in our coordinate system, moving e.g. the quark-line, associated with U_0 , by some small amount ϵ in x^3 -direction

is equivalent to moving it by the amount $\theta_1\epsilon$ in x^2 -direction. From Eq. (61) we obtain the relation

$$\left| \int e^{ix\Delta} U_0 \partial_2 V_0 \right| = \frac{\Delta_+}{\theta} |\tilde{f}| + \frac{\Delta_2 \theta_1}{\theta} |\tilde{f}|. \quad (62)$$

The first term on the r.h.s. of Eq. (62) gives a contribution of the same order of magnitude as the formally leading terms A_0 and A_1 . However, the contribution proportional to the transverse momentum transfer Δ_2 can be neglected. Obviously, as long as α and $1 - \alpha$ are $O(1)$, it is suppressed with respect to the leading contributions by a factor $|\Delta_2|/Q \sim \Lambda/Q$. This suppression is not so obvious if $\alpha \ll 1$, since then the prefactor $(1 - \alpha)/\alpha$ in Eq. (60) becomes large. Yet this enhancement is completely compensated by the factor θ_1/θ , since in this limit $\theta_1/\theta = \theta_1/(\theta_1 + \theta_2) \approx \theta_1/\theta_2 \approx \alpha/(1 - \alpha)$. These relations can be read off from the vector parallelogram corresponding to $\vec{l} + \vec{l}' = \vec{q}$. Hence, we obtain for the leading contribution

$$|\tilde{g}_{R2}(\Delta)| \simeq \left| \int e^{ix\Delta} U_0 \partial_2 V_0 \right| \simeq \frac{\Delta_+}{\theta} |\tilde{f}|. \quad (63)$$

The term in Eq. (60) involving \tilde{g}_{R1} can not be directly related to \tilde{f} . However, for very small and very large values of x_{\parallel} , one expects

$$\left| \int e^{ix\Delta} U_0 \partial_1 V_0 \right| \simeq \left| \int e^{ix\Delta} U_0 \partial_2 V_0 \right|. \quad (64)$$

For sufficiently small x_{\parallel} one has $U_0 \simeq 1$, which restores the rotational invariance in the transverse plane. For sufficiently large x_{\parallel} only the quark or the antiquark trajectory penetrates the proton field. Hence, either $U_0 = 1$ or $V_0 = 1$, thus again restoring rotational invariance.

The above considerations suggest to replace \tilde{g}_{R1} by a function \tilde{f}' , defined in analogy to Eq. (63),

$$|\tilde{g}_{R1}(\Delta)| = \left| \int e^{ix\Delta} U_0 \partial_1 V_0 \right| = \frac{\Delta_+}{\theta} |\tilde{f}'|. \quad (65)$$

The functions \tilde{f} and \tilde{f}' are then expected to have a similar asymptotic behaviour.

The arguments used above to rewrite the terms involving \tilde{g}_R also apply to the terms involving \tilde{g}_L .

Combining the results of Sect. 3.1 for A_0 and A_1 with Eqs. (52), (60), (63) and (65), we obtain for the leading contribution to $S^\mu S_\mu^*$,

$$A_0 + A_1 + A_2 = Q^2 \left[4|\tilde{f}|^2 - \frac{1 - 2\alpha(1 - \alpha)}{\beta(1 - \beta)} (|\tilde{f}|^2 + |\tilde{f}'|^2) \right] - 8\text{Re}\tilde{f}\tilde{f}'^* \tilde{h}, \quad (66)$$

where

$$\beta = \frac{Q^2}{Q^2 + M^2} \quad (67)$$

is the parameter conventionally introduced in diffractive deep-inelastic scattering. The corresponding expression for $S^0 S_0^*$ is given in Eq. (46). Inserting these expressions in Eq. (58) yields the $\gamma^* p$ cross sections

$$d\sigma_{\mu\mu}^{\mu} = \frac{\pi e^2}{|\vec{q}|} \frac{1}{4(2\pi)^5} \int d^2\Delta_{\perp} d\Delta_+ d\alpha \times \left\{ Q^2 \left[4|\tilde{f}|^2 - \frac{1-2\alpha(1-\alpha)}{\beta(1-\beta)} (|\tilde{f}|^2 + |\tilde{f}'|^2) \right] - 8\text{Re}\tilde{f}^* \tilde{h} \right\}, \quad (68)$$

$$d\sigma_{00} = \frac{\pi e^2}{|\vec{q}|} \frac{2}{(2\pi)^5} \int d^2\Delta_{\perp} d\Delta_+ d\alpha q_0^2 |\tilde{f}|^2 \alpha(1-\alpha). \quad (69)$$

From these two cross sections one can obtain transverse and longitudinal structure functions in the usual way.

4.2 Averages over the proton colour field

All the information on the photon-proton interaction is contained in the functions $\tilde{f}(\Delta)_{ab}$, $\tilde{f}'(\Delta)_{ab}$ and $\tilde{h}(\Delta)_{ab}$, which occur in Eqs. (68) and (69). As discussed in Sect. 3.2, the 3-dimensional Fourier transforms with respect to x_{\parallel} and x_{\perp} , as defined in Eq. (58), are needed for the cross sections and the related structure functions. In this section several general features of these functions will be discussed, which will allow us to evaluate the inclusive and diffractive structure functions in terms of several unknown constants. Our main assumptions are that the field strength $G_{\mu\nu}(x)$ vanishes outside a region of size $\sim 1/\Lambda$, and that it varies smoothly on a scale Λ .

Consider first the dependence on the transverse coordinates x_{\perp} . For $f(x)_{ab}$, $a \neq b$, to be non-zero, at least one of the two fermion lines of Fig. 1 has to pass through the region with non-zero field, which has the transverse size $\sim 1/\Lambda$. Hence, integration over the transverse coordinates can be expected to yield

$$\int d^2x_{\perp} e^{-ix_{\perp}\Delta_{\perp}} f(x_{\parallel}, x_{\perp}; \Delta_+, \Delta_{\perp}, \alpha)_{ab} \simeq \frac{\pi}{\Lambda^2} \exp\left(-\frac{\Delta_{\perp}^2}{4\Lambda^2}\right) f_{\parallel}(x_{\parallel}; \Delta_+, \alpha)_{ab}. \quad (70)$$

This relation becomes exact for $f(x; \Delta, \alpha) = f_{\parallel}(x_{\parallel}; \Delta_+, \alpha) \exp(-x_{\perp}^2 \Lambda^2)$, but its qualitative features can be expected to hold in general. The case $a = b$ can be treated completely analogous, after replacing f_{aa} by $f_{aa} - 1$. This is possible since the constant does not contribute to the Fourier transform for $\Delta_+ \neq 0$.

According to eqs. (22) and (23), given at the end of Sect. 2, the function f_{ab} integrates the colour field strength in the double shaded area of Fig. 1. Outside the shaded area the Wilson lines of quark and antiquark may be connected by a space-like Wilson line in the quark-antiquark plane, yielding a Wilson triangle. The light-like vector a^μ essentially points along the light-cone axis $x_+ = x^0 + x^3 = \text{const.}$, and the space-like vector b^μ may be chosen orthogonal to the longitudinal axis. The component of the field strength tensor, which is integrated over the double shaded area, reads

$$G_{+,2\ ab} = E_{2\ ab} - B_{1\ ab}, \quad (71)$$

where we have chosen the 1-axis to be perpendicular to the quark-antiquark plane, $G_{0i} = E_i$ and $G_{ij} = -\epsilon_{ijk}B_k$.

For small areas the integral f_{ab} is proportional to the area. Since we are considering invariant masses $M^2 = O(Q^2)$, the opening angle θ is always small. Hence, f_{\parallel} rises linearly in x_{\parallel} and θ in the range $0 < x_{\parallel} \ll 1/\theta\Lambda$ ($a \neq b$),

$$f_{\parallel}(x_{\parallel}; \Delta_+, \alpha)_{ab} \simeq C_{ab} x_{\parallel} \theta\Lambda. \quad (72)$$

For $x_{\parallel} \sim 1/\theta\Lambda$, the area reaches the size of the proton area $\sim 1/\Lambda^2$. Since the average field strength is $\sim \Lambda^2$, one has $f_{\parallel}(1/\theta\Lambda) = O(1)$, and therefore $C = O(1)$. Also, for $x_{\parallel} \gg 1/\theta\Lambda$, f_{\parallel} must be bounded by a constant $O(1)$. Eq. (72) then implies for the Fourier transform in the range $\Delta_+ \gg \theta\Lambda$,

$$\tilde{f}_{\parallel}(\Delta_+, \alpha)_{ab} \simeq \frac{C_{ab}}{\Delta_+} \frac{\theta\Lambda}{\Delta_+}. \quad (73)$$

This behaviour is intuitively clear since high frequency modes are only present due to the onset of the linear rise at $x_{\parallel} = 0$. In the range $\Delta_+ \ll \theta\Lambda$, the Fourier transform \tilde{f}_{\parallel} is bounded by $\sim 1/\Delta_+$ on dimensional grounds.

As suggested by Eq. (73) it proves convenient to introduce the variable

$$y^2 \equiv \left(\frac{\theta\Lambda}{\Delta_+} \right)^2 = \frac{z^2}{\alpha(1-\alpha)}, \quad (74)$$

with

$$z^2 = 4\beta(1-\beta) \frac{\Lambda^2}{Q^2}. \quad (75)$$

Note, that up to corrections of relative order θ , the x_{\parallel} -dependence of $f_{\parallel}(x_{\parallel}; \Delta_+, \alpha)$ is a dependence on the product θx_{\parallel} . This is true, since after the x_{\perp} -integration the transverse

distance at which the two quarks penetrate the proton, i.e. θx_{\parallel} , is the only relevant parameter.

As a result, the Fourier transform \tilde{f}_{\parallel} can only depend in a non-trivial way on the dimensionless variable y . Therefore we introduce a dimensionless function \bar{f} , requiring

$$\tilde{f}_{\parallel}(\Delta_+, \alpha)_{ab} \simeq \frac{1}{\Delta_+} \bar{f}(y)_{ab}, \quad (76)$$

where $\bar{f}(y)$ has the properties (cf. (72),(73)),

$$\bar{f}(0)_{ab} = 0, \quad \left. \frac{\partial}{\partial y} \bar{f}(y)_{ab} \right|_{y=0} = C_{ab} = O(1), \quad |\bar{f}(y)_{ab}| < O(1) \quad \text{as } y \rightarrow \infty. \quad (77)$$

In analogy to the function $\bar{f}(y)_{ab}$, one can define functions $\bar{f}'(y)_{ab}$ and $\bar{h}(y)_{ab}$ starting from the functions $\tilde{f}'(\Delta)_{ab}$ and $\tilde{h}(\Delta)_{ab}$ which occurred in the previous subsection.

We are now ready to evaluate the wanted structure functions. In the proton rest frame they are related to the cross sections by the well known relations

$$F_{\Sigma} = -\frac{Q^2}{2\pi e^2} \sigma^{\mu}_{\mu}, \quad (78)$$

$$F_L = \frac{Q^4}{\pi e^2 q_0^2} \sigma_{00}, \quad (79)$$

$$F_2 = F_{\Sigma} + \frac{3}{2} F_L. \quad (80)$$

Since the diffractive structure functions are usually defined in terms of x , Q^2 and $\xi \equiv x/\beta = \Delta_+/m_p$, we change variables from Δ_+ to ξ and also from α to y . Performing the Δ_{\perp} -integration in Eqs. (68) and (69) and dropping colour-indices for the rest of this subsection, one obtains

$$dF_{\Sigma} = \frac{\beta d\xi}{4\pi^2 \xi} \int_{2z}^{\infty} \frac{dy}{y^2 \sqrt{y^2 - 4z^2}} \times \left[\left(1 - \frac{2z^2}{y^2}\right) (|\bar{f}|^2 + |\bar{f}'|^2) - 4\beta(1-\beta)|\bar{f}|^2 + \frac{8\beta(1-\beta)}{Q^2} \text{Re} \bar{f}^* \bar{h} \right], \quad (81)$$

$$dF_L = \frac{4\beta^2(1-\beta)d\xi}{\pi^2 \xi} \int_{2z}^{\infty} \frac{z^2 dy}{y^4 \sqrt{y^2 - 4z^2}} |\bar{f}|^2. \quad (82)$$

Finally, the integral over ξ has to be carried out.

The behaviour of the structure functions at large Q^2 can be found by calculating the y -integrals in the above two formulas in the limit $z \rightarrow 0$ (cf. (75)). As an illustration we discuss the integral

$$J = \int_{2z}^{\infty} \frac{dy}{y^2 \sqrt{y^2 - 4z^2}} |\bar{f}|^2. \quad (83)$$

Splitting J into the two contributions

$$\begin{aligned} J &= J_1 + J_2, \\ J_1 &= \int_{2z}^{\infty} \frac{dy}{\sqrt{y^2 - 4z^2}} \frac{1}{(1 + y^2)} |C|^2, \\ J_2 &= \int_{2z}^{\infty} \frac{dy}{y^2 \sqrt{y^2 - 4z^2}} \left(|\bar{f}|^2 - \frac{y^2}{1 + y^2} |C|^2 \right), \end{aligned} \quad (84)$$

the limit $z \rightarrow 0$ can be performed for J_2 , while the first part, which can be calculated explicitly, diverges logarithmically in this limit. Therefore, dropping all contributions vanishing for $z \rightarrow 0$,

$$J = |C|^2 \ln \frac{1}{z} + \mathcal{F}[\bar{f}], \quad (85)$$

where the functional \mathcal{F} is defined by

$$\mathcal{F}[\bar{f}] = \int_0^{\infty} \frac{dy}{y^3} \left(|\bar{f}|^2 - \frac{y^2}{1 + y^2} |C|^2 \right). \quad (86)$$

The other contributions in Eqs. (81) and (82) can be evaluated in the same way. Since the function $h(x)$ contains two covariant derivatives, and therefore two gauge potentials, one expects that \bar{h} is bounded by $\text{const.} \times \Lambda^2$. In Eq. (68) the term involving \bar{h} is suppressed by $1/Q^2$ and does therefore not contribute in the limit $z \rightarrow 0$. Using the relation $|C'_{ab}| = |C_{ab}|$ for the derivative of \bar{f}' at $y = 0$, which follows from the validity of Eq. (64) in the region of small x_{\parallel} , we finally obtain

$$dF_{\Sigma} = \frac{\beta d\xi}{4\pi^2 \xi} \left[\left\{ (2 - 4\beta(1 - \beta)) \ln \frac{1}{z} - 1 \right\} |C|^2 + (1 - 4\beta(1 - \beta)) \mathcal{F}[\bar{f}] + \mathcal{F}[\bar{f}'] \right] \quad (87)$$

$$dF_L = \frac{\beta^2(1 - \beta)d\xi}{\pi^2 \xi} |C|^2. \quad (88)$$

To conclude this subsection, let us recall our main assumptions which led to this result. We have assumed that the proton colour field is confined to a region of size $\sim 1/\Lambda$, that it varies smoothly on a scale Λ , and that \bar{f} and \bar{f}' increase linearly with the probed area of the proton for small longitudinal separation.

4.3 Results for inclusive and diffractive structure functions

The inclusive structure functions can now be obtained from Eqs. (87), (88) by substituting $d\xi/\xi = -d\beta/\beta$, performing the β -integration in the kinematical limits $x < \beta < 1$, and summing over the colours in the final state. Dropping contributions suppressed by x and introducing the two constants

$$C_1 = \sum_{a,b} |C_{ab}|^2 \quad , \quad C_2 = \sum_{a,b} \left\{ \frac{13}{6} |C_{ab}|^2 + \frac{1}{2} \mathcal{F}[\bar{f}_{ab}] + \frac{3}{2} \mathcal{F}[\bar{f}'_{ab}] \right\} \quad , \quad (89)$$

which are expected to be $O(1)$, one obtains

$$F_2(x, Q^2) = \frac{1}{6\pi^2} \left(C_1 \ln \frac{Q^2}{4\Lambda^2} + C_2 \right) \quad , \quad (90)$$

$$F_L(x, Q^2) = \frac{1}{6\pi^2} C_1 \quad . \quad (91)$$

The leading contribution as $x \rightarrow 0$ is non-singular. Since our semiclassical approach in its present form does not contain any QCD-radiation effects, this is not surprising. As expected, the semiclassical approximation is meaningful in the regime of very high parton densities, and it therefore leads to a result for which the limit of $x \rightarrow 0$ exists. Note, that Eqs. (90) and (91) are very similar to results obtained for pair production in an external field in quantum electrodynamics [21].

In the range of x presently probed at HERA, an increase of the structure function with decreasing x is theoretically expected [22, 23] and experimentally observed [24, 25]. In order to obtain a realistic description of structure functions in this range of x , radiation effects have to be taken into account, which may be possible following the approach of Lipatov [26] and Balitsky [27].

The $\ln Q^2$ term in F_2 is due to the integration over all possible configurations of quark-antiquark pairs, from symmetric pairs with $\alpha \approx 1/2$ to extremely asymmetric pairs with $\alpha \approx \Lambda^2/Q^2$ (or $1 - \alpha \approx \Lambda^2/Q^2$) very small. In the latter case one particle carries most of the momentum of the photon. The absence of such a logarithm in F_L can be traced back to the factor $\alpha(1 - \alpha)$ in Eq. (46), which suppresses the contribution of asymmetric pairs. A similar observation has been made in [28] in connection with the wave function of the longitudinal photon.

It is also very instructive to compare this with the origin of the $\ln Q^2$ term in the usual perturbative treatment in the infinite momentum frame. In the contribution to F_2

from photon-gluon fusion the appearance of the logarithm is due to configurations where the produced quark or antiquark is almost collinear with the gluon. In the proton rest frame this corresponds to a configuration where one of the produced particles is relatively soft.

Diffractive events are expected to occur whenever the quark-antiquark pair is created in a colour-singlet state. In this case the proton remnant is also in a singlet and no colour flow occurs between the remnant, most probably still a proton, and the created fast moving pair. Hence, in the hadronic energy flow a large gap in rapidity is expected.

In order to obtain the diffractive structure functions, one has to project on colour-singlet final states in Eqs. (87) and (88). This amounts to the replacement

$$\bar{f}_{ab} \rightarrow \frac{1}{\sqrt{3}} \text{tr} \bar{f} = \frac{1}{\sqrt{3}} \sum_a \bar{f}_{aa}, \quad (92)$$

and the corresponding substitutions for \bar{f}' and \bar{h} .

It is now important to observe that, in contrast to $f_{\parallel}(x_{\parallel})_{ab}$ the function $\text{tr} f_{\parallel}(x_{\parallel})$ does not exhibit the linear rise at small x_{\parallel} , as considered in Eq. (72). This is easily understood remembering that the contribution to $f(x)_{ab}$ linear in θ is also linear in the field strength $G^{\mu\nu}_{ab}$, which is traceless (cf. Sect. 2). Therefore, $\text{tr} f_{\parallel}(x_{\parallel})$ is rising like $(x_{\parallel})^2$ at small x_{\parallel} . For the Fourier transform this means, that $\text{tr} \bar{f}(y) \sim y^2$ for $y \ll 1$, i.e. $\text{tr} C = 0$.

Since the diffractive structure functions are defined by cross-sections differential in ξ , no integration needs to be performed in Eqs. (87) and (88). F_2^D and F_L^D are then obtained by dropping terms proportional to $|C|^2$ and by substituting for \bar{f}_{ab} , \bar{f}'_{ab} the correctly normalized traces. This yields

$$F_2^D(x, Q^2, \xi) = \frac{\beta}{4\pi^2 \xi} [\{1 - 4\beta(1 - \beta)\} C_3 + C_4], \quad (93)$$

$$F_L^D(x, Q^2, \xi) = 0, \quad (94)$$

where

$$C_3 = \mathcal{F}\left[\frac{1}{\sqrt{3}} \text{tr} \bar{f}\right], \quad C_4 = \mathcal{F}\left[\frac{1}{\sqrt{3}} \text{tr} \bar{f}'\right]. \quad (95)$$

have been introduced. Note, that the vanishing of $\text{tr} C$ simplifies the form of the functional \mathcal{F} , resulting in

$$C_3 = \frac{1}{3} \int_0^{\infty} \frac{dy}{y^3} |\text{tr} \bar{f}|^2, \quad C_4 = \frac{1}{3} \int_0^{\infty} \frac{dy}{y^3} |\text{tr} \bar{f}'|^2. \quad (96)$$

Due to the slower rise of $\text{tr}\bar{f}(y)$ at small y , the contribution of symmetric quark-antiquark pairs is suppressed and only asymmetric pairs are relevant for the leading twist contribution to the diffractive structure function. This explains the absence of a term $\sim \ln(Q^2/\Lambda^2)$ in F_2^D and the vanishing of F_L^D .

In summary, we arrive at a clear picture of the final states in ordinary deep inelastic and diffractive deep inelastic events. Symmetric and asymmetric pairs contribute to F_2 . This leads to a contribution growing logarithmically with Q^2 . In the diffractive case, the contribution from symmetric pairs is suppressed, implying the absence of a term $\sim \ln(Q^2/\Lambda^2)$. For both, the inclusive and the diffractive longitudinal structure function, the contribution of asymmetric pairs is suppressed. As a result, F_L contains no term $\sim \ln(Q^2/\Lambda^2)$, and F_L^D is suppressed by $1/Q^2$, i.e. of higher twist.

5 On the relation between F_2 and F_2^D

The above results are similar to those obtained from the simple partonic picture of [12], as far as the relation between the inclusive structure function F_2 and the diffractive structure function F_2^D is concerned. In both models F_2^D is related to F_2 by some constant connected with the colour-singlet requirement for the produced quark-antiquark pair, and in both cases the slope in x of F_2 is larger by one unit than the slope of F_2^D in ξ .

This relation between the slopes of the two structure functions has first been proposed in [11] based on a picture of diffraction as scattering on wee parton lumps inside the proton. In the present section we will argue, that there is a rather general class of models for diffractive deep inelastic scattering, which should reproduce the above relation.

Since in small- x events the kinematically required momentum transfer to the proton-target is relatively small, it is natural to think of these events in terms of a virtual photon cross-section $\sigma_T(\gamma^*p \rightarrow p'X)$. Here p' is the proton remnant, which, in this picture, can be separated from the produced massive state X before hadronization. Assume that, with X being in a colour-singlet state, no hadronic activity develops between X and p' , and a rapidity gap event occurs. Now, neglecting the longitudinal contribution, the structure functions read

$$F_2(x, Q^2) = \frac{Q^2}{\pi e^2} \int_x^1 \frac{d\sigma_T(\gamma^*p \rightarrow p'X)}{d\xi} d\xi \tag{97}$$

$$F_2^D(x, Q^2, \xi) = \frac{Q^2}{\pi e^2} \cdot \frac{d\sigma_T(\gamma^* p \rightarrow p' X)}{d\xi} \cdot \mathcal{P},$$

where the probability for the proton remnant to be in a colour-singlet state is given by the factor \mathcal{P} . This results in the relation

$$F_2(x, Q^2) = \int_x^1 d\xi F_2^D(x, Q^2, \xi) \cdot \mathcal{P}^{-1} = x \int_x^1 d\beta \beta^{-2} F_2^D(x, Q^2, \xi) \cdot \mathcal{P}^{-1}. \quad (98)$$

We now assume that the ξ -dependence of F_2^D factorizes,

$$F_2^D(x, Q^2, \xi) = \xi^{-n} \hat{F}_2^D(\beta, Q^2), \quad (99)$$

with some number $n > 1$. This is consistent with data in a wide region of ξ (cf. [1, 2]). Dropping terms suppressed in the limit $x \rightarrow 0$, the relation between diffractive and inclusive structure function takes the form

$$F_2(x, Q^2) = x^{1-n} \int_0^1 d\beta \beta^{n-2} \hat{F}_2^D(\beta, Q^2) \cdot \mathcal{P}^{-1}. \quad (100)$$

At small x the photon energy q_0 in the proton rest frame is much larger than any other scale in the problem. Therefore it is natural to consider the limit $q_0 \rightarrow \infty$ and to try to understand the presence of a non-zero probability factor \mathcal{P} in this limit. Assuming that this small- x limit for \mathcal{P} has already been reached in the present measurements, \mathcal{P} is a function of β and Q^2 only. In this case Eq. (100) implies that the x -slope of F_2 and the ξ -slope F_2^D differ by one unit. It is interesting to observe that the result of this rather hand waving argument appears to agree well with the data on F_2^D [1, 2] and F_2 [24, 25] at small x .

6 Conclusions

A semiclassical approach has been developed for both inclusive and diffractive structure functions at small x . In this kinematic regime the momentum transfer to the proton, needed to produce a diffractive state with invariant mass $M = O(Q)$, is very small. Hence, it should be possible to describe deep inelastic scattering as quark-antiquark pair production in a classical colour field representing the proton. We have carried out such a calculation based on a high energy expansion for quark and antiquark wave functions in the presence of a colour background field, which is assumed to be sufficiently smooth

and localized within some typical hadronic size. In the high energy expansion corrections two orders beyond the leading eikonal approximation had to be considered to obtain the complete leading order result for the cross section. No expansion in the strong coupling constant α_s has been used. Instead of gluons, triangular averages of the colour field are the basic entities of the calculation.

The final result for the inclusive structure functions has been obtained neglecting all terms suppressed by x , Λ/Q or Λ/M . In this limit F_2 and F_L can be expressed in terms of two constants, the actual value of which depends on the details of the proton field. The inclusive structure functions are obtained in the limit $x \rightarrow 0$, for which no unitarity problem exists. F_2 grows logarithmically with Q^2 , whereas the longitudinal structure function F_L is independent of Q^2 . This can be traced back to the suppression of asymmetric configurations, with one relatively soft and one hard quark. It is also interesting to observe, that the enhancement of F_2 at large Q^2 is due to a term linear in the field strength, i.e. this effect would survive an expansion in α_s .

The diffractive structure functions have been calculated from the colour-singlet contribution to the above quark-antiquark pair production cross section. Since the interaction with the proton is generally very soft, it is natural to expect the appearance of a large rapidity gap event whenever a colour neutral pair has been created. The colour-singlet projection in the final state leads to the vanishing of the longitudinal structure function, $F_L^D = 0$, an effect connected with the already mentioned suppression of asymmetric configurations. Similar to the inclusive structure function, F_2^D can also be expressed in terms of two field dependent constants, given by integrals over certain non-abelian phase factors testing the proton field. F_2^D has no Q^2 -dependence and is given by some simple function of β multiplied with ξ^{-1} . The probably most important result of this paper is the leading twist behaviour of the diffractive structure function F_2^D . The exact ratio of diffractive and inclusive cross-sections depends on constants, sensitive to the details of the field, which have not been obtained explicitly. At high Q^2 diffraction is found to be suppressed by a factor of $\log Q^2$ with respect to the total cross section.

In the last section we have argued that the difference between the x -slope of F_2 and the ξ -slope of F_2^D by one unit is a generic feature of a large class of models. This relation between F_2 and F_2^D directly tests the hypothesis of a common mechanism for diffractive and ordinary deep inelastic scattering, with the colour state of the proton remnant being responsible for the distinction between the two event classes. The present calculation

shows the possibility of soft interactions adjusting the colour of the produced quark pair as well as the proton remnant to be in a singlet, resulting in leading twist diffraction. In this sense the semiclassical approach, although quite different from the partonic models [12, 14], supports the idea of soft interactions being responsible for diffraction in deep inelastic scattering at small x .

Finally, it has to be mentioned that several important problems require further study. First, the very applicability of the semiclassical approach to deep inelastic scattering has not been established rigorously. Second, even if this could be done, it would be necessary to integrate over the different field configurations forming the proton. Together with a group theoretical analysis, this could provide more information about the field dependent functions introduced above. Another important issue, not considered here, concerns hard gluon radiation in the process of pair creation. We expect that this effect will significantly modify the inclusive structure function at small x as well as the β -dependence of the diffractive structure function.

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Figure captions

Fig.1 Configuration space picture of quark-antiquark pair production by a virtual photon in an external colour field.

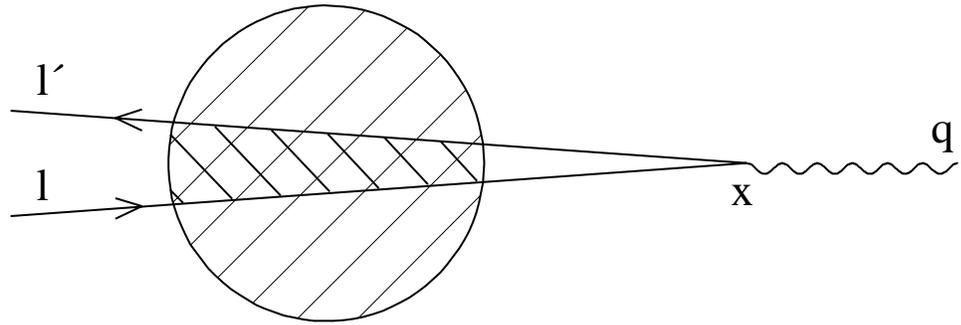


Fig. 1