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IMAGE TUNING TECHNIQUES FOR ENHANCING THE PERFORMANCE OF PURE PERMANENT MAGNET UNDULATORS WITH SMALL GAP/PERIOD RATIOS*

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Abstract

The on-axis field of a small-gap undulator constructed out of pure permanent magnet (PM) blocks arranged in an alternating-dipole (i.e., 2 dipoles/period) array can be substantially varied by positioning monolithic permeable plates above and below the undulator jaws. This simple technique, which can be used to control the 1st harmonic energy in conventional synchrotron radiation (SR) or Free Electron Laser (FEL) applications requiring sub-octave tuning, can also be shown to suppress magnetic inhomogeneities that can contribute to the undulator's on-axis field errors. If a standard (4 vector rotations/period) Halbach undulator, composed of PM blocks with square cross sections, is rearranged into an alternating-dipole array with the same period, the peak field that can be generated with superimposed image plates can, for a certain range of magnet dimensions, exceed that of the pure-PM Halbach array. This design technique, which can be viewed as intermediate between the "pure-PM" and standard "hybrid/PM" configurations, provides a potentially cost-effective method of enhancing the performance of small-gap, pure-PM insertion devices. In this paper we report on the analysis and recent characterization of pure-PM undulator structures with superimposed image plates, and discuss possible applications to FEL research.

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1. Introduction

Linac-based FELs are often good candidates for small-gap undulator designs for which the ratio of gap to undulator period is a small number [1]. For permanent magnet (PM) undulators an optimal design is the standard Halbach structure, wherein the field directions in the top and bottom arrays are rotated successively by 90°, with four rotations/period. Such designs often utilize magnets with square cross sections, a strategy which allows the maximum sorting flexibility for field quality control. It is known that the Halbach array maximizes the on-axis field; i.e., if the same set of magnets is used to construct an alternating-dipole (A-D) undulator with the same period and gap, it will have a weaker on-axis field [2].

In recent work we have investigated the effect of placing Fe image planes at varying distances above and below PM undulator structures of the A-D [3,4] and Halbach types. Elementary analysis predicts that the image planes will strongly enhance the on-axis field of the alternating-dipole device and only weakly affect the on-axis field of the Halbach structure, a result consistent with the much higher field strength of the A-D undulator in regions above and below its magnet arrays. Both experimental and numerical investigations of these configurations are described.

2. The experimental magnetic structures

For the magnetic field tests a set of 40 inexpensive permanent magnets (7/8" x 3/8" x 15/8") with 0.3 T remanent fields (B_r) were purchased from Radio Shack (RASH). First, an A-D undulator was assembled in a wooden fixture and characterized, then a Halbach structure. In both structures the gap was set to 6 mm. Side views of the magnet layouts are shown in Figs. 1 and 2. Using 1/8" Teflon spacers, two steel plates were placed in proximity to the upper and lower jaws (dimension t). Three cases were tested: 1) no steel plates; 2) t=1/8" ; and 3) t=0. The on-axis field measurements, taken with a scanning Hall probe, are shown in Figs. 3 and 4. In both steel-free cases, the effect of the magnets' poor quality on the field profile is evident. For the A-D structure, however, both the field amplitude and quality are seen to be strongly enhanced by the proximity of the Fe planes. For the given magnet piece and gap dimensions, the A-D field amplitude varied by 25-30% and the Halbach configuration by less than 4% vs. the full variation of t (0 to ∞).

3. Field Simulations and Analysis

The field of a rectangular rare-earth permanent magnet can be accurately represented by two equivalent magnetic charge sheets located on the faces perpendicular to the easy axis [5]. Integrating over a sheet parallel to the x-y plane, bounded by the intervals [x1,x2] and [y1,y2], and located at {((x1+x2)/2),((y1+y2)/2),z0}, we find the following expressions for the magnetic field components:

$$B_x = a \cdot \ln\left[\frac{(y-y2)-\sqrt{(x-x2)^2+(y-y2)^2+(z-z0)^2}}{(y-y1)-\sqrt{(x-x2)^2+(y-y1)^2+(z-z0)^2}} \cdot \frac{(y-y1)-\sqrt{(x-x1)^2+(y-y1)^2+(z-z0)^2}}{(y-y2)-\sqrt{(x-x1)^2+(y-y2)^2+(z-z0)^2}}\right], \tag{1}$$

$$B_y = a \cdot \ln\left[\frac{(x-x2)-\sqrt{(x-x2)^2+(y-y2)^2+(z-z0)^2}}{(x-x1)-\sqrt{(x-x1)^2+(y-y2)^2+(z-z0)^2}} \cdot \frac{(x-x1)-\sqrt{(x-x1)^2+(y-y1)^2+(z-z0)^2}}{(x-x2)-\sqrt{(x-x1)^2+(y-y1)^2+(z-z0)^2}}\right], \tag{2}$$

and

$$\begin{aligned} B_z = a \cdot \{ & \tan^{-1}\left(\frac{(x-x1)\cdot(y-y1)}{(z-z0)\cdot\sqrt{(x-x1)^2+(y-y1)^2+(z-z0)^2}}\right) - \\ & \tan^{-1}\left(\frac{(x-x1)\cdot(y-y2)}{(z-z0)\cdot\sqrt{(x-x1)^2+(y-y2)^2+(z-z0)^2}}\right) - \tan^{-1}\left(\frac{(x-x2)\cdot(y-y1)}{(z-z0)\cdot\sqrt{(x-x2)^2+(y-y2)^2+(z-z0)^2}}\right) + \\ & \tan^{-1}\left(\frac{(x-x2)\cdot(y-y2)}{(z-z0)\cdot\sqrt{(x-x2)^2+(y-y2)^2+(z-z0)^2}}\right)\}, \end{aligned} \tag{3}$$

where a is a constant proportional to the PM's remanent field, B_r. The field of an arbitrarily oriented sheet can be obtained from eqs. (1-3) by a straightforward linear transformation of the coordinates of the observation point. Using the appropriate pair of these expressions for each magnet piece, the field of any real Fe-free PM undulator is easily simulated by linear superposition.

The field expressions for the magnetic structures with added image planes (of sufficient size and sufficiently large and linear relative permeability) are similarly easy to derive [6]. For the A-D array the effect of the image planes is equivalent to superimposing the fields of an infinite number of undulators identical to the first (real) one, but with linearly-increasing gapsizes. In practice, even for very small gaps, only the first several superimposed undulators need to be considered, due to the rapid exponential rolloff of the additional undulator fields with gaps greater than λ_u. The same basic method can be applied to the Halbach structure with image planes, but the vertically-oriented lattice should be treated independently of the horizontally-oriented pieces, due to the alternating field reversal of the former vs. the latter. In all cases, then, expressions (1) through (3) can be linearly superimposed to simulate the fields of all the configurations addressed in this study.

In Fig. 5 we plot the calculated curves equivalent to: 1) the measured A-D curves from the bottom of Fig. 3, and 2) the Halbach structure's curves from the bottom of Fig. 4. Apart from the differences in profile detail stemming from the poor field quality of the experimental magnets, it is evident that the variation of the theoretical curves vs. tuning gap generally corroborates the experimental measurements.

In the limit of infinitely long magnets, an analytical comparison of the 1st field harmonic of a general Halbach structure vs. that of an A-D undulator using tuning plates at t=0 can be made using the well-known 2-D formula derived by Halbach [2]; viz., $B_0 = 2B_r e^{-\pi g/\lambda_u}\left(1-e^{-2\pi h/\lambda_u}\right)\sin c\left(M^{-1}\right)$. Here B_0 is the on-axis field amplitude, h is the height of the PM pieces, M is the number of field rotations/period, and each magnet touches its nearest neighbors. With image plates in proximity, we find the following modified expressions for the 1st field harmonics of the Halbach (M=4) and A-D (M=2) structures:

$$B_0 = \frac{4\sqrt{2}}{\pi} B_r e^{\pi(h+2t)/\lambda_u} [\sinh\{\pi h/\lambda_u\}/\sinh\{\pi(g+2h+2t)/\lambda_u\}], \tag{4}$$

$$B_0 = \frac{8}{\pi} B_r \cosh\{\pi(h+2t)/\lambda_u\}\cdot[\sinh\{\pi h/\lambda_u\}/\sinh\{\pi(g+2h+2t)/\lambda_u\}]. \tag{5}$$

The condition for the amplitude (B_0) of the A-D undulator to exceed that of the Halbach device is readily expressed using the ratio of the above two expressions to yield $\exp(-2\pi(h+2t)/\lambda_u)) \geq \sqrt{2}-1$, or $(h+2t)/\lambda_u < 0.1403$. This indicates that an image-tuned A-D structure using 8 square magnets/period (viz., $h/\lambda_u = 0.125$) can, in principle, always be made to outperform the corresponding image-tuned Halbach configuration. Performing a similar comparison of the image-tuned A-D undulator (with t=0) vs. an Fe-free Halbach structure yields the corresponding conditions $h/\lambda_u < 0.1403(g/\lambda_u \to \infty)$, and $h/\lambda_u < 0.1954(g/\lambda_u \to 0)$. Consequently, for small g/λ_u ratios and h/λ_u ratios in the 0.19-0.25 range, the image-tuned A-D structure will be marginally outperformed by the Fe-free Halbach configuration, and more substantially so for $h/\lambda_u > 0.25$.

We next use our 3-D field equations to simulate the $h/\lambda_u = 0.125$ case using the structures shown in Fig. 6. In Fig. 7 the calculated field amplitudes down the axes of both devices are plotted. The curves, whose amplitudes can be corroborated with equs. (4-5) and a hand calculator, demonstrate that with the use of tuning plates the A-D structure's peak field can be made to match or exceed that of the standard Fe-free Halbach undulator.

4. Conclusions

Based on the experimental measurements and computer simulations, the following conclusions can be drawn:

1) For K>2, the image-tuning method can be used to provide a 1st harmonic tuning variation of up to an octave or more in an A-D structure. The motion of tuning plates can in principle be made simpler than the variation of the undulator gap.

2) if sufficiently long magnets with square cross sections and $h \le \lambda_u / 8$ are employed, the A-D structure (with image plates in close proximity to the magnets) can develop an on-axis field with a substantially higher 1st harmonic than the alternative Halbach configuration with or without plates.

3) The more restricted sorting flexibility of the A-D structure on the quality of its field is partially compensated by the suppression of randomly-located field errors by the image plates.

4) The relative advantages of the A-D undulator with image-plate tuning decrease rapidly vs. the Halbach device with increasing height of the PM pieces.

5) For the small gap/period ratios acceptable for linac-driven FEL undulators, image-plate tuning of the Halbach configuration for ($h / \lambda_u \le 0.25$) can provide a field variation that, while substantially smaller than that of the A-D device, can still be of potential interest for limited-range spectral tuning.

In future work, the effect of image planes on different classes of field errors and on the higher harmonic content of the undulator field will be studied. Straightforward extensions of the image formalism to account for finite and non-linear permeability effects will also be explored.

5. Acknowledgments

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6. References

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Figure Captions

Figure 1. Radio Shack (RASH) PM Halbach structure with superimposed image plates at distance t from the magnets.

Figure 2. RASH alternating-dipole (A-D) undulator with superimposed image plates.

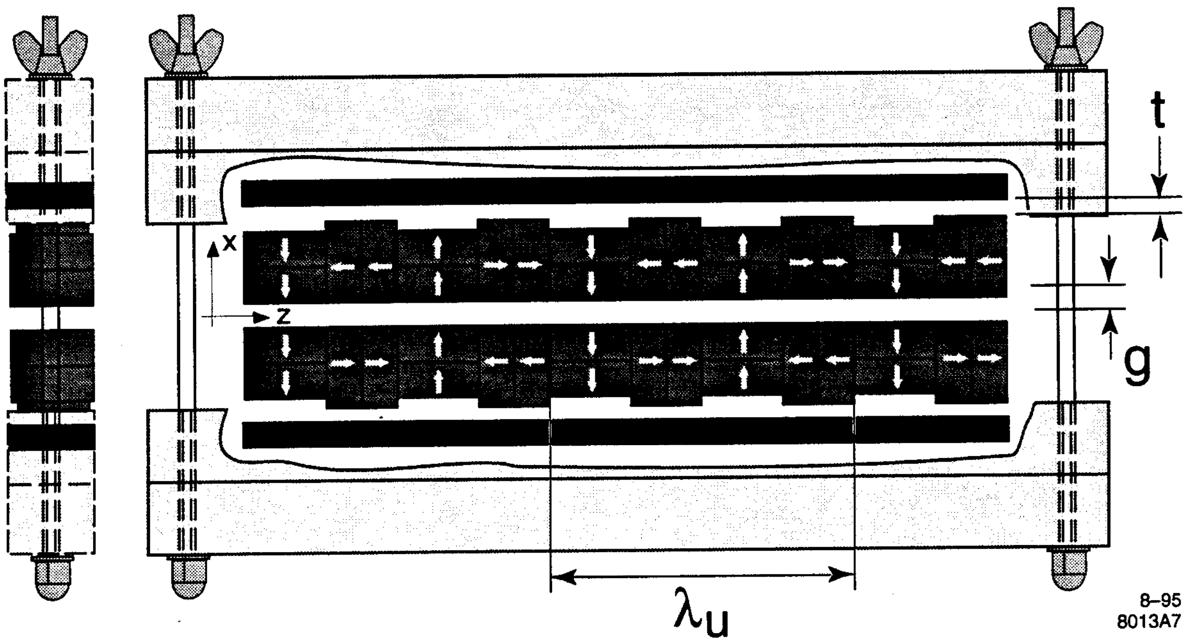
Figure 3. Measured field amplitude and quality variation of the on-axis field in a RASH A-D undulator vs. proximity (t) of image planes. Bottom graph shows detail of peak A in top graph. λu=44.4 mm; g=6 mm. Magnet pieces 22.2 mm x 9.5 mm x 46.7 mm.

Figure 4. Measured field amplitude and quality variation of the on-axis field in a RASH Halbach undulator vs. proximity of image planes. Bottom graph shows detail of peak B in top graph. λu=82.4 mm; g=6 mm. Magnet pieces 22.2 mm x 9.5 mm x 46.7 mm.

Figure 5. Calculated field profiles corresponding to peak regions A (top) and B (bottom) in Figs. 2 and 3.

Figure 6. A-D PM undulator with superimposed image plates (right) contrasted with Fe-free Halbach PM undulator (right) with the same gap and period. Both undulators are constructed out of identical magnet pieces.

Figure 7. Calculated on-axis field profiles for the Halbach and A-D undulators in Fig. 6, with λu=80 mm and g=3 cm.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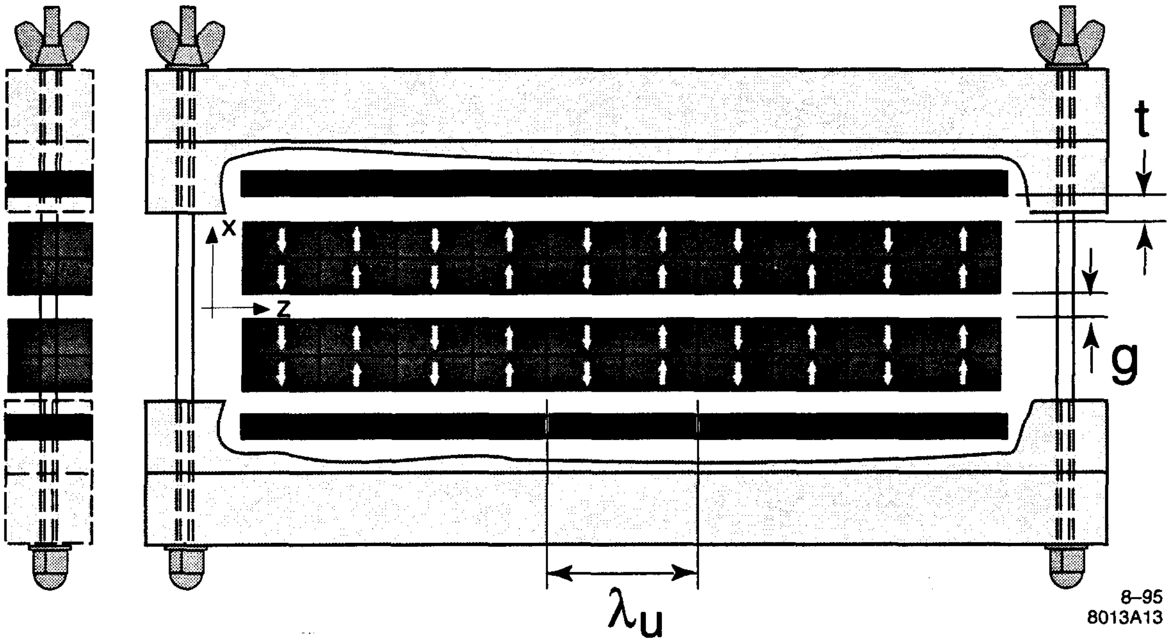
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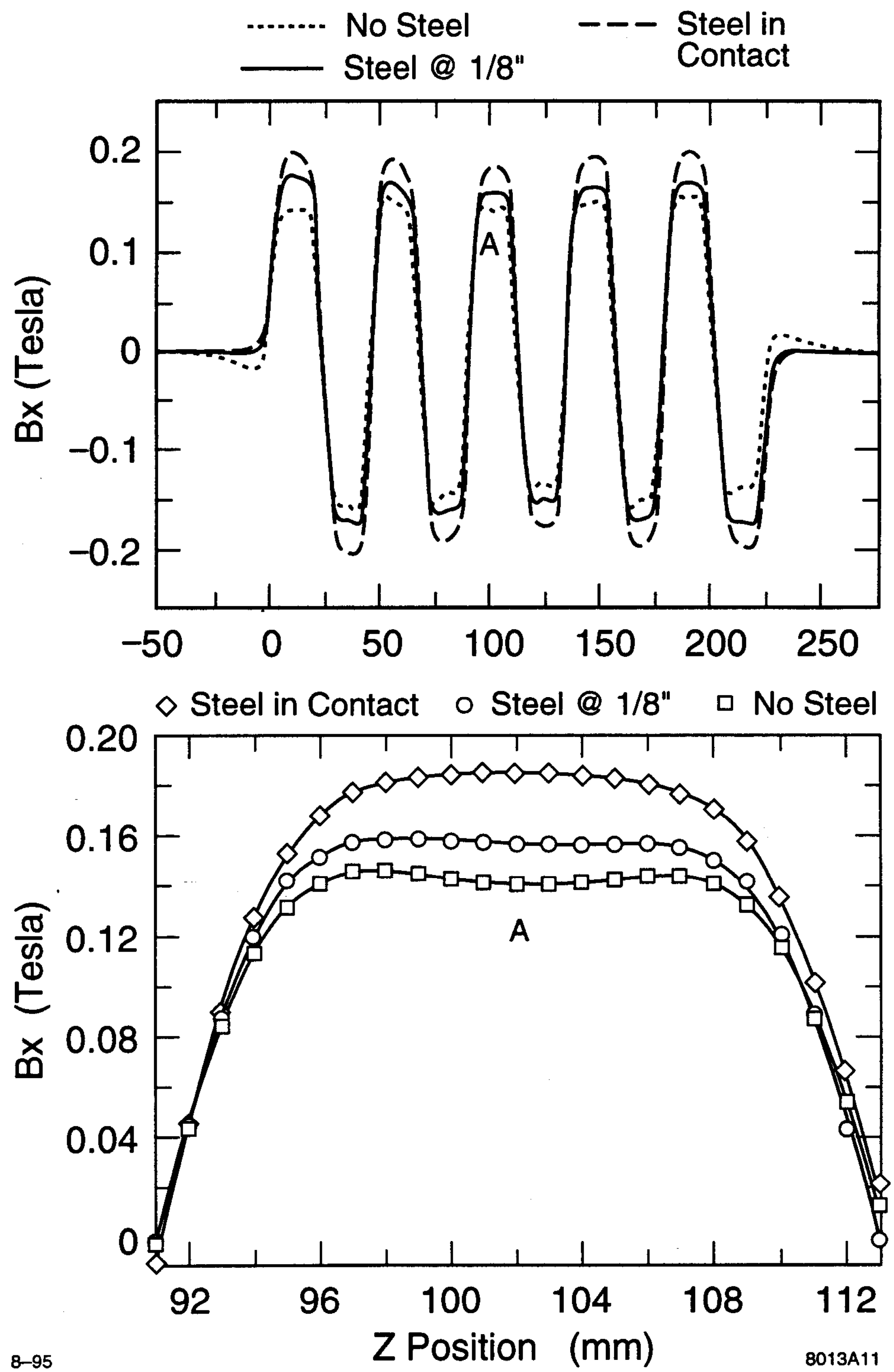
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Steel @ 1/8"
Steel in Contact
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0
−0.1
−0.2
Bx (Tesla)
A
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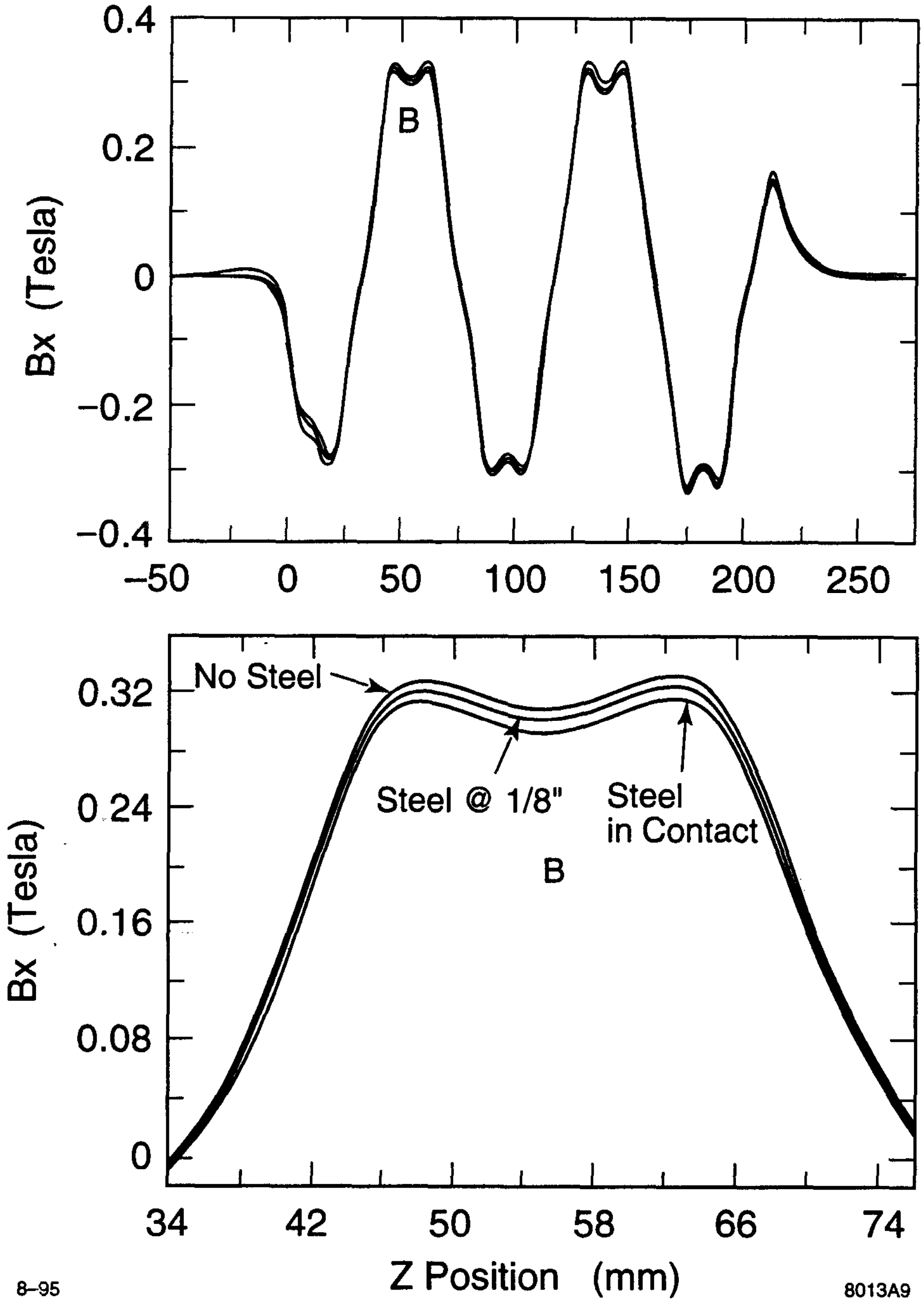
0
50
100
150
200
250
◇ Steel in Contact
○ Steel @ 1/8"
□ No Steel
0.20
0.16
0.12
0.08
0.04
0
Bx (Tesla)
A
92
96
100
104
108
112
Z Position (mm)
8–95
8013A11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0.2
0
−0.2
−0.4
Bx (Tesla)
B
−50
0
50
100
150
200
250
No Steel
Steel @ 1/8"
Steel
in Contact
0.32
0.24
0.16
0.08
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42
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8–95
8013A9

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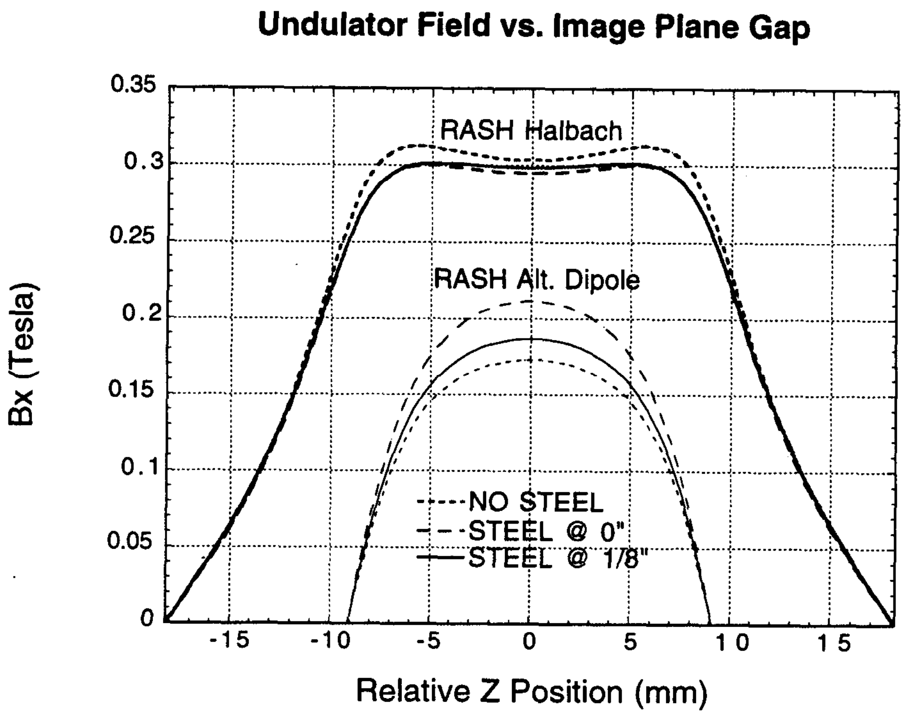

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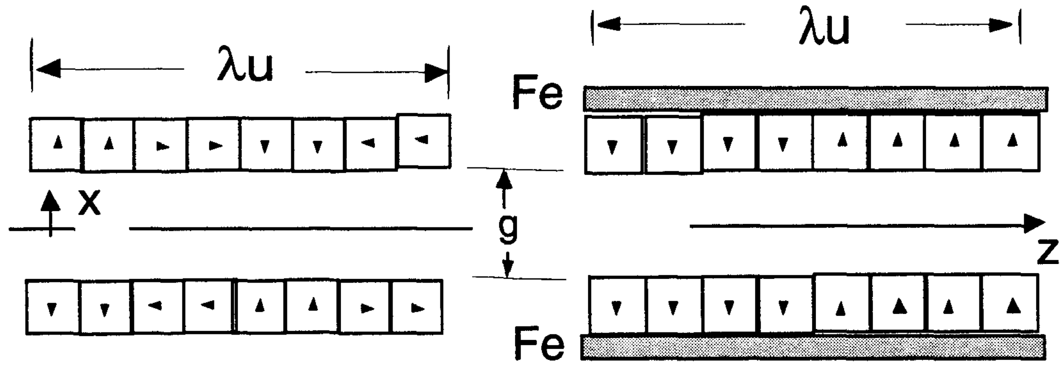

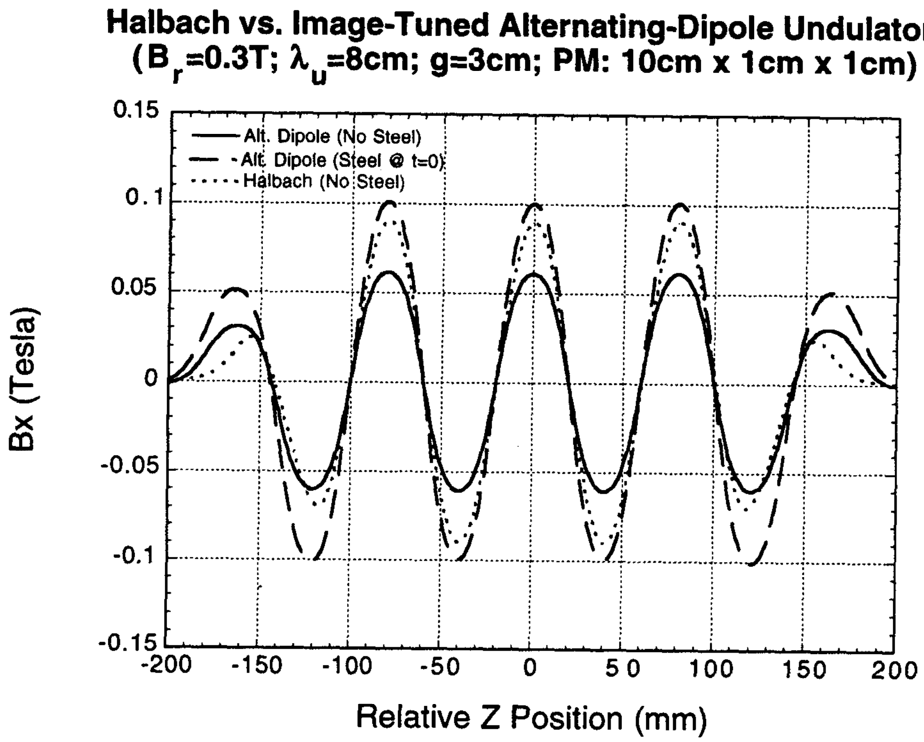
Halbach vs. Image-Tuned Alternating-Dipole Undulator
(B_r=0.3T; λ_u=8cm; g=3cm; PM: 10cm x 1cm x 1cm)
Alt. Dipole (No Steel)
Alt. Dipole (Steel @ t=0)
Halbach (No Steel)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-200
-150
-100
-50
0
50
100
150
200
Bx (Tesla)
Relative Z Position (mm)