

Factorial and Cumulant Moments in $e^+e^- \rightarrow$ Hadrons at the Z^0 Resonance*

The SLD Collaboration^o

Stanford Linear Accelerator Center, Stanford University,
Stanford, CA 94309

Submitted to Physics Letters B

*Work supported by: Department of Energy contracts DE-FG02-91ER40676 (BU), DE-FG03-92ER40701 (CIT), DE-FG03-91ER40618 (UCSB), DE-FG03-92ER40689 (UCSC), DE-FG03-93ER40788 (CSU), DE-FG02-91ER40672 (Colorado), DE-FG02-91ER40677 (Illinois), DE-AC03-76SF00098 (LBL), DE-FG02-92ER40715 (Massachusetts), DE-AC02-76ER03069 (MIT), DE-FG06-85ER40224 (Oregon), DE-AC03-76SF00515 (SLAC), DE-FG05-91ER40627 (Tennessee), DE-AC02-76ER00881 (Wisconsin), DE-FG02-92ER40704 (Yale); National Science Foundation grants PHY-91-13428 (UCSC), PHY-89-21320 (Columbia), PHY-92-04239 (Cincinnati), PHY-88-17930 (Rutgers), PHY-88-19316 (Vanderbilt), PHY-92-03212 (Washington); the UK Science and Engineering Research Council (Brunel and RAL); the Istituto Nazionale di Fisica Nucleare of Italy (Bologna, Ferrara, Frascati, Pisa, Padova, Perugia); and the Japan-US Cooperative Research Project on High Energy Physics (Nagoya, Tohoku).

K. Abe,⁽²⁹⁾ I. Abt,⁽¹⁴⁾ C.J. Ahn,⁽²⁶⁾ T. Akagi,⁽²⁷⁾ N.J. Allen,⁽⁴⁾ W.W. Ash,^{(27)†}
 D. Aston,⁽²⁷⁾ K.G. Baird,⁽²⁴⁾ C. Baltay,⁽³³⁾ H.R. Band,⁽³²⁾ M.B. Barakat,⁽³³⁾
 G. Baranko,⁽¹⁰⁾ O. Bardon,⁽¹⁶⁾ T. Barklow,⁽²⁷⁾ A.O. Bazarko,⁽¹¹⁾ R. Ben-David,⁽³³⁾
 A.C. Benvenuti,⁽²⁾ G.M. Bilei,⁽²²⁾ D. Bisello,⁽²¹⁾ G. Blaylock,⁽⁷⁾ J.R. Bogart,⁽²⁷⁾
 T. Bolton,⁽¹¹⁾ G.R. Bower,⁽²⁷⁾ J.E. Brau,⁽²⁰⁾ M. Breidenbach,⁽²⁷⁾ W.M. Bugg,⁽²⁸⁾
 D. Burke,⁽²⁷⁾ T.H. Burnett,⁽³¹⁾ P.N. Burrows,⁽¹⁶⁾ W. Busza,⁽¹⁶⁾ A. Calcaterra,⁽¹³⁾
 D.O. Caldwell,⁽⁶⁾ D. Calloway,⁽²⁷⁾ B. Camanzi,⁽¹²⁾ M. Carpinelli,⁽²³⁾ R. Cassell,⁽²⁷⁾
 R. Castaldi,^{(23)(a)} A. Castro,⁽²¹⁾ M. Cavalli-Sforza,⁽⁷⁾ A. Chou,⁽²⁷⁾ E. Church,⁽³¹⁾
 H.O. Cohn,⁽²⁸⁾ J.A. Coller,⁽³⁾ V. Cook,⁽³¹⁾ R. Cotton,⁽⁴⁾ R.F. Cowan,⁽¹⁶⁾ D.G. Coyne,⁽⁷⁾
 G. Crawford,⁽²⁷⁾ A. D'Oliveira,⁽⁸⁾ C.J.S. Damerell,⁽²⁵⁾ M. Daoudi,⁽²⁷⁾ R. De Sangro,⁽¹³⁾
 P. De Simone,⁽¹³⁾ R. Dell'Orso,⁽²³⁾ M. Dima,⁽⁹⁾ P.Y.C. Du,⁽²⁸⁾ R. Dubois,⁽²⁷⁾
 B.I. Eisenstein,⁽¹⁴⁾ R. Elia,⁽²⁷⁾ E. Etzion,⁽⁴⁾ D. Falciai,⁽²²⁾ C. Fan,⁽¹⁰⁾ M.J. Fero,⁽¹⁶⁾
 R. Frey,⁽²⁰⁾ K. Furuno,⁽²⁰⁾ T. Gillman,⁽²⁵⁾ G. Gladding,⁽¹⁴⁾ S. Gonzalez,⁽¹⁶⁾
 G.D. Hallewell,⁽²⁷⁾ E.L. Hart,⁽²⁸⁾ A. Hasan,⁽⁴⁾ Y. Hasegawa,⁽²⁹⁾ K. Hasuko,⁽²⁹⁾ S. Hedges,⁽³⁾
 S.S. Hertzbach,⁽¹⁷⁾ M.D. Hildreth,⁽²⁷⁾ J. Huber,⁽²⁰⁾ M.E. Huffer,⁽²⁷⁾ E.W. Hughes,⁽²⁷⁾
 H. Hwang,⁽²⁰⁾ Y. Iwasaki,⁽²⁹⁾ D.J. Jackson,⁽²⁵⁾ P. Jacques,⁽²⁴⁾ J. Jaros,⁽²⁷⁾ A.S. Johnson,⁽³⁾
 J.R. Johnson,⁽³²⁾ R.A. Johnson,⁽⁸⁾ T. Junk,⁽²⁷⁾ R. Kajikawa,⁽¹⁹⁾ M. Kalelkar,⁽²⁴⁾
 H. J. Kang,⁽²⁶⁾ I. Karliner,⁽¹⁴⁾ H. Kawahara,⁽²⁷⁾ H.W. Kendall,⁽¹⁶⁾ Y. Kim,⁽²⁶⁾
 M.E. King,⁽²⁷⁾ R. King,⁽²⁷⁾ R.R. Kofler,⁽¹⁷⁾ N.M. Krishna,⁽¹⁰⁾ R.S. Kroeger,⁽¹⁸⁾
 J.F. Labs,⁽²⁷⁾ M. Langston,⁽²⁰⁾ A. Lath,⁽¹⁶⁾ J.A. Lauber,⁽¹⁰⁾ D.W.G.S. Leith,⁽²⁷⁾
 M.X. Liu,⁽³³⁾ X. Liu,⁽⁷⁾ M. Loreti,⁽²¹⁾ A. Lu,⁽⁶⁾ H.L. Lynch,⁽²⁷⁾ J. Ma,⁽³¹⁾ G. Mancinelli,⁽²²⁾
 S. Manly,⁽³³⁾ G. Mantovani,⁽²²⁾ T.W. Markiewicz,⁽²⁷⁾ T. Maruyama,⁽²⁷⁾ R. Massetti,⁽²²⁾
 H. Masuda,⁽²⁷⁾ E. Mazzucato,⁽¹²⁾ A.K. McKemey,⁽⁴⁾ B.T. Meadows,⁽⁸⁾ R. Messner,⁽²⁷⁾
 P.M. Mockett,⁽³¹⁾ K.C. Moffeit,⁽²⁷⁾ B. Mours,⁽²⁷⁾ D. Muller,⁽²⁷⁾ T. Nagamine,⁽²⁷⁾
 S. Narita,⁽²⁹⁾ U. Nauenberg,⁽¹⁰⁾ H. Neal,⁽²⁷⁾ M. Nussbaum,⁽⁸⁾ Y. Ohnishi,⁽¹⁹⁾
 L.S. Osborne,⁽¹⁶⁾ R.S. Panvini,⁽³⁰⁾ H. Park,⁽²⁰⁾ T.J. Pavel,⁽²⁷⁾ I. Peruzzi,^{(13)(b)}
 M. Piccolo,⁽¹³⁾ L. Piemontese,⁽¹²⁾ E. Pieroni,⁽²³⁾ K.T. Pitts,⁽²⁰⁾ R.J. Plano,⁽²⁴⁾
 R. Prepost,⁽³²⁾ C.Y. Prescott,⁽²⁷⁾ G.D. Punkar,⁽²⁷⁾ J. Quigley,⁽¹⁶⁾ B.N. Ratcliff,⁽²⁷⁾
 T.W. Reeves,⁽³⁰⁾ J. Reidy,⁽¹⁸⁾ P.E. Rensing,⁽²⁷⁾ L.S. Rochester,⁽²⁷⁾ P.C. Rowson,⁽¹¹⁾
 J.J. Russell,⁽²⁷⁾ O.H. Saxton,⁽²⁷⁾ S.F. Schaffner,⁽²⁷⁾ T. Schalk,⁽⁷⁾ R.H. Schindler,⁽²⁷⁾
 B.A. Schumm,⁽¹⁵⁾ A. Seiden,⁽⁷⁾ S. Sen,⁽³³⁾ V.V. Serbo,⁽³²⁾ M.H. Shaevitz,⁽¹¹⁾ J.T. Shank,⁽³⁾
 G. Shapiro,⁽¹⁵⁾ S.L. Shapiro,⁽²⁷⁾ D.J. Sherden,⁽²⁷⁾ K.D. Shmakov,⁽²⁸⁾ C. Simopoulos,⁽²⁷⁾
 N.B. Sinev,⁽²⁰⁾ S.R. Smith,⁽²⁷⁾ J.A. Snyder,⁽³³⁾ P. Stamer,⁽²⁴⁾ H. Steiner,⁽¹⁵⁾ R. Steiner,⁽¹⁾
 M.G. Strauss,⁽¹⁷⁾ D. Su,⁽²⁷⁾ F. Suekane,⁽²⁹⁾ A. Sugiyama,⁽¹⁹⁾ S. Suzuki,⁽¹⁹⁾ M. Swartz,⁽²⁷⁾
 A. Szumilo,⁽³¹⁾ T. Takahashi,⁽²⁷⁾ F.E. Taylor,⁽¹⁶⁾ E. Torrence,⁽¹⁶⁾ A.I. Trandafir,⁽¹⁷⁾
 J.D. Turk,⁽³³⁾ T. Usher,⁽²⁷⁾ J. Va'vra,⁽²⁷⁾ C. Vannini,⁽²³⁾ E. Vella,⁽²⁷⁾ J.P. Venuti,⁽³⁰⁾
 R. Verdier,⁽¹⁶⁾ P.G. Verdini,⁽²³⁾ S.R. Wagner,⁽²⁷⁾ A.P. Waite,⁽²⁷⁾ S.J. Watts,⁽⁴⁾
 A.W. Weidemann,⁽²⁸⁾ E.R. Weiss,⁽³¹⁾ J.S. Whitaker,⁽³⁾ S.L. White,⁽²⁸⁾ F.J. Wickens,⁽²⁵⁾
 D.A. Williams,⁽⁷⁾ D.C. Williams,⁽¹⁶⁾ S.H. Williams,⁽²⁷⁾ S. Willocq,⁽³³⁾ R.J. Wilson,⁽⁹⁾
 W.J. Wisniewski,⁽²⁷⁾ M. Woods,⁽²⁷⁾ G.B. Word,⁽²⁴⁾ J. Wyss,⁽²¹⁾ R.K. Yamamoto,⁽¹⁶⁾
 J.M. Yamartino,⁽¹⁶⁾ X. Yang,⁽²⁰⁾ S.J. Yellin,⁽⁶⁾ C.C. Young,⁽²⁷⁾ H. Yuta,⁽²⁹⁾ G. Zapalac,⁽³²⁾
 R.W. Zdarko,⁽²⁷⁾ C. Zeitlin,⁽²⁰⁾ Z. Zhang,⁽¹⁶⁾ and J. Zhou,⁽²⁰⁾

- (1) Adelphi University, Garden City, New York 11530
(2) INFN Sezione di Bologna, I-40126 Bologna, Italy
(3) Boston University, Boston, Massachusetts 02215
(4) Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom
(5) California Institute of Technology, Pasadena, California 91125
(6) University of California at Santa Barbara, Santa Barbara, California 93106
(7) University of California at Santa Cruz, Santa Cruz, California 95064
(8) University of Cincinnati, Cincinnati, Ohio 45221
(9) Colorado State University, Fort Collins, Colorado 80523
(10) University of Colorado, Boulder, Colorado 80309
(11) Columbia University, New York, New York 10027
(12) INFN Sezione di Ferrara and Università di Ferrara, I-44100 Ferrara, Italy
(13) INFN Lab. Nazionali di Frascati, I-00044 Frascati, Italy
(14) University of Illinois, Urbana, Illinois 61801
(15) Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(16) Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(17) University of Massachusetts, Amherst, Massachusetts 01003
(18) University of Mississippi, University, Mississippi 38677
(19) Nagoya University, Chikusa-ku, Nagoya 464 Japan
(20) University of Oregon, Eugene, Oregon 97403
(21) INFN Sezione di Padova and Università di Padova, I-35100 Padova, Italy
(22) INFN Sezione di Perugia and Università di Perugia, I-06100 Perugia, Italy
(23) INFN Sezione di Pisa and Università di Pisa, I-56100 Pisa, Italy
(24) Rutgers University, Piscataway, New Jersey 08855
(25) Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX United Kingdom
(26) Sogang University, Seoul, Korea
(27) Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(28) University of Tennessee, Knoxville, Tennessee 37996
(29) Tohoku University, Sendai 980 Japan
(30) Vanderbilt University, Nashville, Tennessee 37235
(31) University of Washington, Seattle, Washington 98195
(32) University of Wisconsin, Madison, Wisconsin 53706
(33) Yale University, New Haven, Connecticut 06511

† Deceased

(a) Also at the Università di Genova

(b) Also at the Università di Perugia

ABSTRACT

We present the first experimental study of the ratio of cumulant to factorial moments of the charged-particle multiplicity distribution in high-energy particle interactions, using hadronic Z^0 decays collected by the SLD experiment at SLAC. We find that this ratio, as a function of the moment-rank q , decreases sharply to a negative minimum at $q = 5$, which is followed by quasi-oscillations. These features are insensitive to experimental systematic effects and are in qualitative agreement with expectations from next-to-next-to-leading-order perturbative QCD.

1 Introduction

One of the most fundamental observables in high-energy particle interactions is the multiplicity of particles produced in the final state. A large body of experimentally measured multiplicity distributions has been accumulated in a variety of hard processes[1]. The Poisson distribution (PD) does not describe the shapes of multiplicity distributions measured in e^+e^- , pp , and $p\bar{p}$ collisions, implying non-random particle-production mechanisms, but elucidation of the relationship between the measured shapes and the underlying dynamics has proven to be problematic.

At present the theory of strong interactions, Quantum Chromodynamics (QCD) [2], cannot be used to calculate distributions of final-state hadrons since the mechanism of hadron formation has not been understood quantitatively. However, perturbative QCD can be applied to calculate some properties of the cascade of gluons radiated by the partons produced in a hard scattering process. If there is a simple relationship between the distributions of partons and detected final state particles, as follows for example from the ansatz of local parton-hadron duality (LPHD) [3], then such calculations may be expected to reproduce some features of experimental data. An early calculation [4] in the leading double-logarithmic approximation (DLA) was successful in describing the energy dependence of the average multiplicity, as well as the energy independence of the “KNO distribution” [5] of $n/\langle n \rangle$, the multiplicity scaled by its average value. However, the width predicted by this calculation is much larger than that of experimentally observed multiplicity distributions [4]. It has been suggested that the inclusion of higher-order terms in perturbative QCD calculations should reduce the predicted width of the multiplicity distribution [6, 7], although no such calculation has yet been achieved. However, the ratio of cumulant to factorial moments has recently been proposed [8] as a sensitive measure of the shape of multiplicity distributions and has been found to be calculable in higher-order perturbative QCD.

Factorial moments have been used to characterize cascade phenomena in various scientific fields [9]. The factorial moment of rank q is defined [9]

$$F_q \equiv \frac{\langle n(n-1)\dots(n-q+1) \rangle}{\langle n \rangle^q}, \quad (1)$$

where n is the particle multiplicity of an event and $\langle n \rangle$ is the average multiplicity in the event sample. The cumulant moments K_q are related to the F_q by [10]

$$F_q = \sum_{m=0}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m, \quad (2)$$

and $F_0 = F_1 = K_1 \equiv 1$. While the DLA QCD calculation predicts [8] that the ratio $H_q \equiv K_q/F_q$ decreases as q^{-2} , the inclusion of higher orders yields more striking behavior. A calculation in the next-to-leading logarithm approximation (NLA) predicts [11] a minimum in H_q at $q \approx 5$ and a positive constant value for $q \gg 5$, while the next-to-next-to-leading logarithm approximation (NNLA) predicts [12] that this minimum is negative and is followed by quasi-oscillations about zero. These predictions are illustrated in Fig. 1.

In a previous study [13] H_q were calculated using published multiplicity distributions from e^+e^- and $p\bar{p}$ collisions, and features qualitatively similar to those predicted by the NNLA calculation were observed. This was a significant result, supporting not only QCD at the parton level, but also the notion of LPHD. However, no account was taken of experimental systematic effects or of the correlations, both statistical and systematic, between values of H_q at different ranks q . In addition, some H_q values derived from data from similar experiments were apparently inconsistent. Furthermore, it was subsequently shown [14] that the observed features could be induced by the effective truncation of the multiplicity distribution inherent in a measurement using a finite data sample.

In this letter we present the first experimental determination of the ratio of cumulant to factorial moments of the charged-particle multiplicity distribution in high-energy particle interactions, using hadronic decays of Z^0 bosons produced in e^+e^- annihilations. We study systematic effects in detail, in particular the influence of truncation of the distribution, and investigate the correlations between moments of different rank. We compare our measurements with the predictions of perturbative QCD, and also with two widely used distributions predicted by phenomenological models of particle production.

2 Charged Multiplicity Analysis

Hadronic decays of Z^0 bosons produced by the SLAC Linear Collider (SLC) were collected with the SLC Large Detector (SLD) [15]. The trigger and initial selection of hadronic events are described in [16]. The analysis used charged tracks measured in the central drift chamber (CDC) [17] and vertex detector (VXD) [18]. A set of cuts was applied to the data to select well-measured tracks and events well contained within the detector acceptance. Charged tracks were required to have: a distance of closest approach transverse to the beam axis within 5 cm, and within 10 cm along the axis from the measured interaction point; a polar angle, θ , with respect to the beam axis within $|\cos\theta| < 0.8$; and a momentum transverse to the beam axis greater than 0.15 GeV/c. Events were required to have: a minimum of five such tracks; a thrust-axis [19] direction within $|\cos\theta_T| < 0.71$; and a total visible energy of at least 20 GeV, which was calculated from the selected tracks assigned the charged pion mass. A total of 86,679 events from the 1993 to 1995 SLC/SLD runs survived these cuts and were included in this analysis. The efficiency for selecting hadronic events satisfying the $|\cos\theta_T|$ cut was estimated to be above 96%. The background in the selected event sample was estimated to be $(0.3 \pm 0.1)\%$, dominated by $Z^0 \rightarrow \tau^+\tau^-$ decays, and was subtracted statistically from the observed multiplicity distribution.

The experimentally observed charged-particle multiplicity distribution was corrected for effects introduced by the detector, such as geometrical acceptance, track-reconstruction efficiency, and additional tracks from photon conversions and particle interactions in the detector materials, as well as for initial-state photon radiation and the effect of the cuts listed above. The charged multiplicity of an event was defined to include all promptly produced charged particles, as well as those produced in the decay of particles with lifetime $< 3 \cdot 10^{-10}$ s. A two-stage correction was calculated using Monte Carlo simulated hadronic Z^0 decays produced by the JETSET 6.3 [20] event generator, subjected to a detailed simulation of the SLD and reconstructed in the same way as the data. Each MC event passing the event-selection cuts yielded a number of generated tracks n_g and a number of observed tracks n_o , which were used to form the matrix

$$M(n_g, n_o) = \frac{N(n_g, n_o)}{N_{obs}^{MC}(n_o)}, \quad (3)$$

where $N(n_g, n_o)$ is the number of MC events with n_g generated tracks and n_o observed tracks, and $N_{obs}^{MC}(n_o)$ is the number of MC events with n_o observed tracks. For each n_o , a sum of three Gaussians was fitted to $M(n_g, n_o)$ and this parametrization was used in the correction. The effects of the event-selection cuts and of initial-state radiation

were corrected using factors

$$C_F(n_g) = \frac{P^{true}(n_g)}{P^{sel}(n_g)}, \quad (4)$$

where $P^{true}(n_g)$ is the normalized simulated multiplicity distribution generated without initial-state radiation and $P^{sel}(n_g)$ is the normalized distribution for those events in the fully-simulated sample that passed the selection cuts.

Both corrections were applied to the experimentally observed multiplicity distribution $P^{exp}(n_o)$ to yield the corrected distribution:

$$P^{cor}(n) = C_F(n) \cdot \sum_{n_o} M(n, n_o) \cdot P^{exp}(n_o), \quad (5)$$

which is shown with statistical errors only in Fig. 2a. The factorial moments F_q , cumulant moments K_q , and their ratios H_q were calculated from this distribution according to Eqs. 1 and 2. The resulting H_q up to rank $q = 17$ are shown in Fig. 3 and listed in Table 1. As q increases, the value of H_q falls rapidly (inset of Fig. 3), reaches a negative minimum at $q = 5$, and then oscillates about zero with a positive maximum at $q = 9$ and a second negative minimum at $q = 13$. The statistical and systematic errors are strongly correlated between ranks as we now discuss.

3 Statistical and Systematic Errors

Statistical errors and correlations were studied by analyzing simulated multiplicity distributions. The H_q were calculated from 10 Monte Carlo samples of the same size as the data sample and 20 multiplicity distributions generated according to the measured distribution. For each H_q the standard deviation in these 30 samples was taken as the statistical error, and is listed in Table 1. In each case the H_q exhibited the same behavior as those calculated from the data, although the value of H_5 and the apparent phase of the quasi-oscillation for $q \geq 8$ were found to be sensitive to statistical fluctuations. that the observed features might result from a statistical fluctuation by generating 10,000 multiplicity distributions according to Poisson and negative-binomial distributions (see below) with the same mean value as our corrected multiplicity distribution. In no case did any sample exhibit either a minimum near $q = 5$ or quasi-oscillations at higher q .

Experimental systematic effects were also investigated. An important issue is the simulation of the track-reconstruction efficiency of the detector. The H_q were found to be sensitive to the global efficiency, which was tuned in the simulation so that our average corrected multiplicity equalled the value measured in hadronic Z^0 decays [21]. The H_q resulting from a variation in the global efficiency of $\pm 1.7\%$, corresponding

to the error on the measured average multiplicity, are shown in Fig. 4. There is an asymmetric effect on the value of H_5 and on the apparent phase of the quasi-oscillation. For each q the difference between the H_q with increased and decreased efficiency was assigned as a symmetric systematic uncertainty.

It is important to consider the dependence of the track reconstruction efficiency on multiplicity. Our simulated efficiency is 91.5% for tracks crossing at least 40 of the 80 layers of the CDC, and is independent of n_g within $\pm 0.5\%$. Varying the efficiency for $n_g > 20$ by $\pm 0.5\%$ caused a change of $\pm 4\%$ in H_5 , and negligible changes for $q > 5$. This change was assigned as a systematic uncertainty.

Variation of the form of the parametrization of the correction matrix M was found to affect mainly the amplitude of the quasi-oscillation for $q \geq 8$. Application of the unparametrized version of the matrix $M(n_g, n_o)$ produced the largest such effect, which is shown in Fig. 4. This change was conservatively assigned as a symmetric systematic uncertainty to account for possible mismodelling of the off-diagonal elements of the matrix. The effect on the H_q of variation of the parameters of the three-Gaussian fits to M within their errors increases with increasing q , becoming the dominant uncertainty for $q \geq 16$.

The effects on the H_q of wide variations in the criteria for track and event selection were found to be small compared with those due to the above sources. The effect of including values of the multiplicity distribution at $n=2$ and $n=4$, taken from the JETSET model, in the calculation of the moments is also small. Varying the estimated level of non-hadronic background, which appears predominantly in the low-multiplicity bins, by $\pm 100\%$ produces a negligible change in the H_q .

The uncertainties from the above systematic sources were added in quadrature to derive a systematic error on each H_q , which is listed in Table 1. All of our studies showed a clear first minimum in H_q at $q = 5$ followed by quasi-oscillations for $q \geq 8$. The value of H_5 has a total uncertainty of $\pm 13\%$ that is strongly correlated with similar errors on H_6 and H_7 and with an uncertainty in the phase of the quasi-oscillation of ∓ 0.2 units of rank. There is an uncertainty on the amplitude of the quasi-oscillation of $\pm 15\%$ that is essentially independent of the other errors. From these studies we conclude that the steep decrease in H_q for $q < 5$, the negative minimum at $q = 5$, and the quasi-oscillation about zero for $q \geq 8$ are well-established features of the data.

4 Comparison of the H_q with QCD Predictions

We have compared these results with the qualitative predictions of perturbative QCD discussed in Section 1. Figure 1 shows that the DLA QCD calculation predicts no negative values of H_q and is inconsistent with the data. The NLA and NNLA calculations

q	H_q (10^{-4})	Statistical error	Systematic error
2	411.00	2.96	11.13
3	54.41	1.40	5.61
4	5.15	0.74	0.93
5	-4.08	0.40	0.51
6	-3.40	0.28	0.39
7	-1.40	0.20	0.32
8	0.08	0.14	0.10
9	0.91	0.12	0.16
10	0.84	0.10	0.19
11	0.10	0.08	0.09
12	-0.66	0.10	0.15
13	-0.83	0.09	0.21
14	-0.18	0.10	0.13
15	0.89	0.16	0.26
16	1.50	0.19	0.45
17	0.61	0.26	0.37

Table 1: Ratio of cumulant to factorial moments, H_q . The errors are strongly correlated between ranks as discussed in the text.

predict [13] a steep decrease in H_q to a minimum at

$$q_{min} = \left[\frac{96\pi}{121\alpha_s(Q^2)} \right]^{1/2} + \frac{1}{2}.$$

For $\alpha_s(M_Z^2)$ measured in Z^0 decays [16] $q_{min} \approx 5$. These features are seen in the data. For $q > 5$, the NLA calculation predicts that H_q increases toward a constant value, which is not consistent with the data, whereas the NNLA calculation predicts quasi-oscillations in H_q in agreement with the data.

The moment ratios are thus seen to be a sensitive discriminator between QCD calculations at different orders of perturbation theory. We conclude that the H_q calculated for gluons in the next-to-next-to-leading logarithm approximation of perturbative QCD describe the shape of the observed multiplicity distribution, whereas the available calculations at lower order do not.

5 Comparison with phenomenological models

Measured multiplicity distributions have been compared extensively with the predictions of phenomenological models. We consider two such predicted distributions. The negative binomial distribution (NBD)

$$P_n(\langle n \rangle, k) = C_{k+n-1}^m \left(\frac{\langle n \rangle}{\langle n \rangle + k} \right)^n \left(\frac{k}{\langle n \rangle + k} \right)^k, \quad (6)$$

where $\langle n \rangle$ and k are free parameters, is predicted [22] by models in which the hard interaction produces several objects, sometimes identified with the partons in a QCD cascade, each of which decays into a number of particles. The log-normal distribution (LND)

$$P_n(\mu, \sigma, c) = \int_n^{n+1} \frac{N}{n' + c} \exp\left(-\frac{(\ln(n' + c) - \mu)^2}{2\sigma^2}\right) dn', \quad (7)$$

where μ , σ , and c are free parameters, is predicted [23] by models in which the particles result from a scale-invariant stochastic branching process, which might be related to the parton branchings in a QCD cascade.

Considering statistical errors only, we performed least-squares fits of the NBD and LND to our corrected multiplicity distribution. These fitted distributions and their normalized residuals are shown in Figs. 2a and 2b, respectively. Both provide reasonable descriptions of the data, with χ^2/ndf of 68.0/24 and 30.5/23, respectively. Although the NBD has a high χ^2 and shows structure in the residuals in the core of the distribution, it is difficult to exclude without a thorough understanding of the uncorrelated component of the systematic errors. These results are in agreement with those from a previous analysis [24].

The PD and the phenomenological distributions differ markedly in their moment structure: for the PD, $H_q = 0$ for all q ; for the fitted NBD, H_q is positive and falls as q^{-25} ; for the fitted LND, H_q falls with increasing q to a negative minimum at $q = 6$ and then oscillates about zero. It was recently argued [14] that the truncation of the large- n tail of the multiplicity distribution due to finite data-sample size could lead to quasi-oscillations in H_q similar to those observed in the data. We calculated H_q values from the fitted distributions over the multiplicity range observed in the data, $6 \leq n \leq 54$, and the results are displayed in Fig. 5. The truncated PD are found to produce features similar to those in the data, but with much smaller amplitudes. The amplitudes are not sensitive to the exact value of the truncation point and we conclude that the moment ratios predicted by the PD and NBD are inconsistent with the data. The LND predictions are insensitive to the truncation point and show the same qualitative features as the data. However, the first minimum is smaller in amplitude and is at $q = 6$.

The quasi-oscillation for $q \geq 8$ has similar amplitude and period, and is displaced by about one unit from the data. The moment ratios H_q are thus seen to provide a sensitive test of phenomenological models.

6 Conclusion

In conclusion, we have conducted the first experimental study of the ratio H_q of cumulant to factorial moments of the charged-particle multiplicity distribution in high-energy particle interactions, using hadronic Z^0 decays. We find that H_q decreases sharply with increasing rank q to a negative minimum at $q = 5$, followed by quasi-oscillations; we show these features to be insensitive to statistical and experimental systematic effects.

The predictions of perturbative QCD in the next-to-next-to-leading-logarithm approximation are in agreement with the features observed in the data, supporting both the validity of QCD at the parton level and the notion that the observable final state reflects the underlying parton structure. Calculations in the leading double-logarithm and next-to-leading-logarithm approximations are not sufficient to describe the data. The Poisson and negative binomial distributions do not predict these features. The log-normal distribution predicts features similar to those of the data, scale-invariant production mechanism in detail. We conclude that the moment ratios H_q of the charged-particle multiplicity distribution provide a sensitive test both of perturbative QCD and of phenomenological models.

Acknowledgements

We thank the personnel of the SLAC accelerator department and the technical staffs of our collaborating institutions for their efforts, which resulted in the successful operation of the SLC and the SLD. We thank I. Dremin for useful discussions.

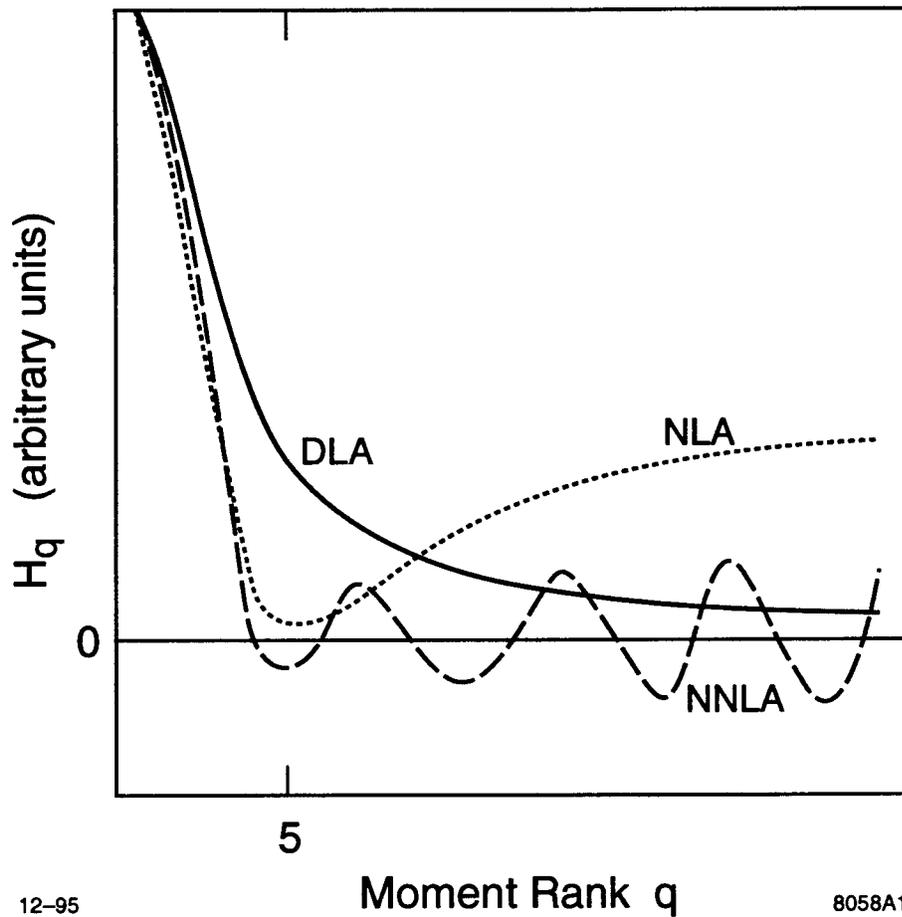
References

- [1] For reviews, see, for instance, G. Giacomelli, *Int. J. Mod. Phys.* **A5** (1990) 223; *Hadronic Multiparticle Production*, ed. P. Carruthers, World Scientific, Singapore (1990).
- [2] H. Fritzsch, M. Gell-Mann, and H. Leutwyler, *Phys. Lett.* **B47** (1973) 365; D.J. Gross and F. Wilczek, *Phys. Rev. Lett.* **30** (1973) 1343; H.D. Politzer, *Phys. Rev. Lett.* **30** (1973) 1346.

- [3] T.I. Azimov, Y.L. Dokshitzer, V.A. Khoze, and S.I. Troyan; *Z. Phys.* **C27** (1985) 65.
- [4] Yu.L. Dokshitzer, V.A. Khoze, S.I. Troyan, in: *Perturbative QCD*, ed. A.H. Mueller, World Scientific, Singapore (1989).
- [5] Z. Koba, M.B. Nielsen, P. Oleson, *Nucl. Phys.* **B240** (1972) 317.
- [6] E.D. Malaza and B.R. Webber, *Nucl. Phys.* **B267** (1986) 702.
- [7] Yu.L. Dokshitzer, *Phys. Lett.* **B305** (1993) 295.
- [8] I.M. Dremin, *Mod. Phys. Lett.* **A8** (1993) 2747.
- [9] A. Bialas and R. Peschanski, *Nucl. Phys.* **B273** (1986) 703.
- [10] I.M. Dremin and R.C. Hwa, *Phys. Rev.* **D49** (1994) 5805.
- [11] I.M. Dremin, *Phys. Lett.* **B313** (1993) 209.
- [12] I.M. Dremin and V.A. Nechitailo, *JETP Lett.* **58** (1993) 881.
- [13] I.M. Dremin et al., *Phys. Lett.* **B336** (1994) 119.
- [14] R. Ugoccioni, A. Giovannini, S. Lupia. *Phys. Lett.* **B342** (1995) 387.
- [15] SLD Design Report, SLAC Report 273 (1984).
- [16] SLD Collab., K. Abe, et al., *Phys. Rev.* **D51** (1995) 962.
- [17] M.D. Hildreth et al., *IEEE Trans. Nucl. Sci.* **42** (1994) 451.
- [18] C.J.S. Damerell et al., *Nucl. Inst. Meth.* **A288** (1990) 288.
- [19] E. Farhi, *Phys. Rev. Lett.* **39** (1977) 1587.
- [20] T. Sjöstrand and M. Bengtsson, *Comp. Phys. Comm.* **43** (1987) 367.
- [21] SLD Collab., K. Abe, et al., *Phys. Rev. Lett.* **72** (1994) 3145.
- [22] See e.g. A. Giovannini and L. Van Hove, *Z. Phys.* **C30** (1986) 391.
- [23] See e.g. S. Carius and G. Ingelman, *Phys. Lett.* **B252** (1990) 647.
- [24] ALEPH Collab., D. Decamp et al., *Phys. Lett.* **B273** (1991) 181.

Figure captions

1. Functional form of perturbative QCD predictions of the ratio H_q of cumulant to factorial moments in the leading double-logarithm (solid line), next-to-leading-logarithm (dotted line) and next-to-next-to-leading-logarithm (dashed line) approximations. The vertical scale and relative normalizations are arbitrary.
2. a) The corrected charged-particle multiplicity distribution. The open circles at $n = 2, 4$ are the predictions of the JETSET Monte Carlo. The solid and dashed lines represent fitted negative-binomial and log-normal distributions, respectively. The normalized residuals are shown in b). The fits yielded parameter values of $k = 24.9$ and $\langle n \rangle = 20.7$ for the NBD and $\mu = 3.52$, $\sigma = 0.175$ and $c = 13.4$ for the LND. The errors are statistical only.
3. Ratio of cumulant to factorial moments, H_q , as a function of the moment rank q . The error bars are statistical and are strongly correlated between ranks.
4. Examples of systematic effects on H_q . The data points show the H_q with statistical errors derived using the standard correction. The dotted (dashed) line connects H_q values derived with an increase (decrease) of 1.7% in the simulated track reconstruction efficiency. The solid line connects H_q values derived using the unparametrized correction matrix.
5. Comparison of the H_q measured in the data (dots with statistical errors) with the predictions of truncated Poisson (dotted line joining the values at different q), negative binomial (dashed line) and log-normal (dot-dashed line) distributions.



12-95

Moment Rank q

8058A1

Fig. 1

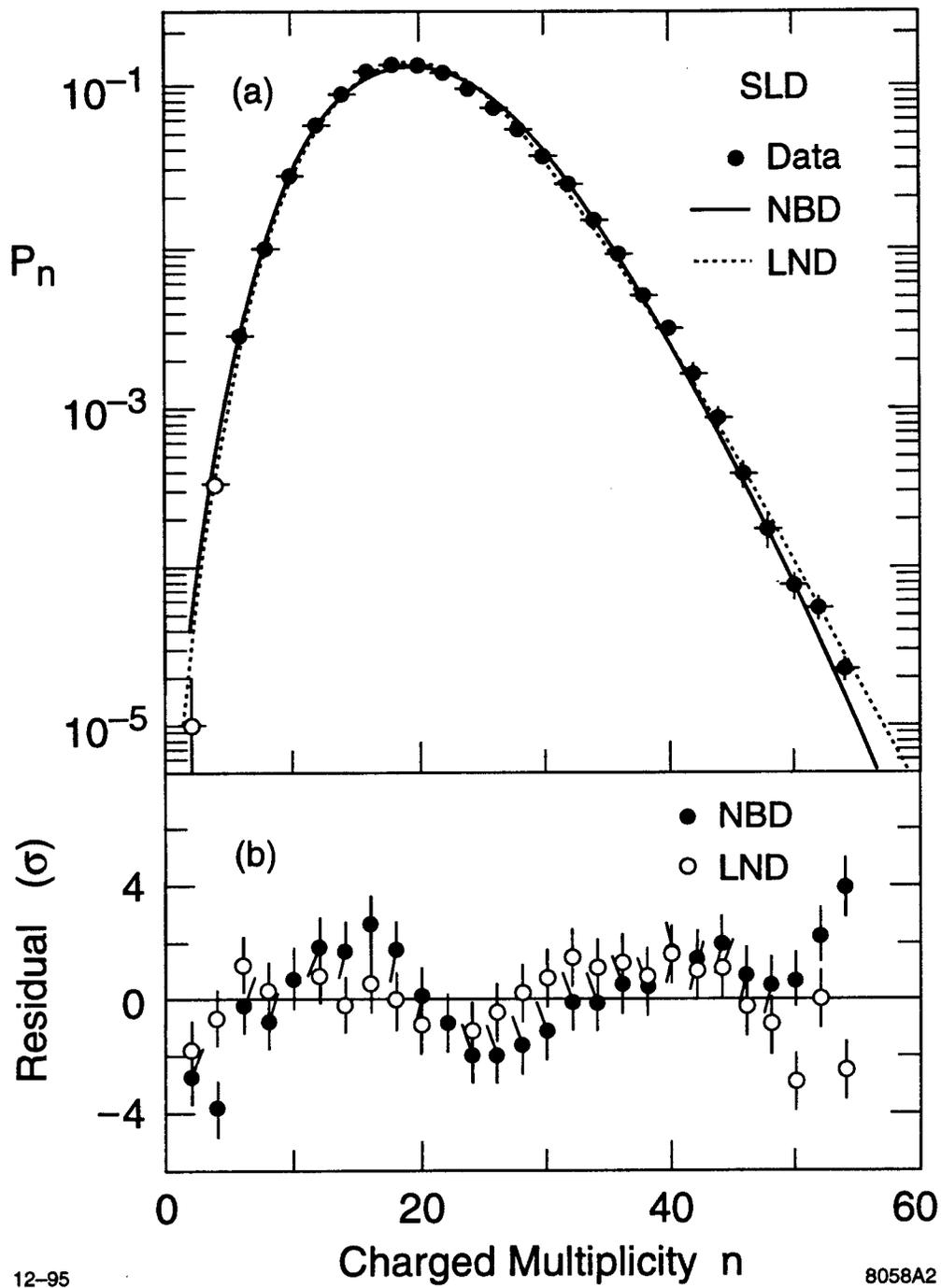
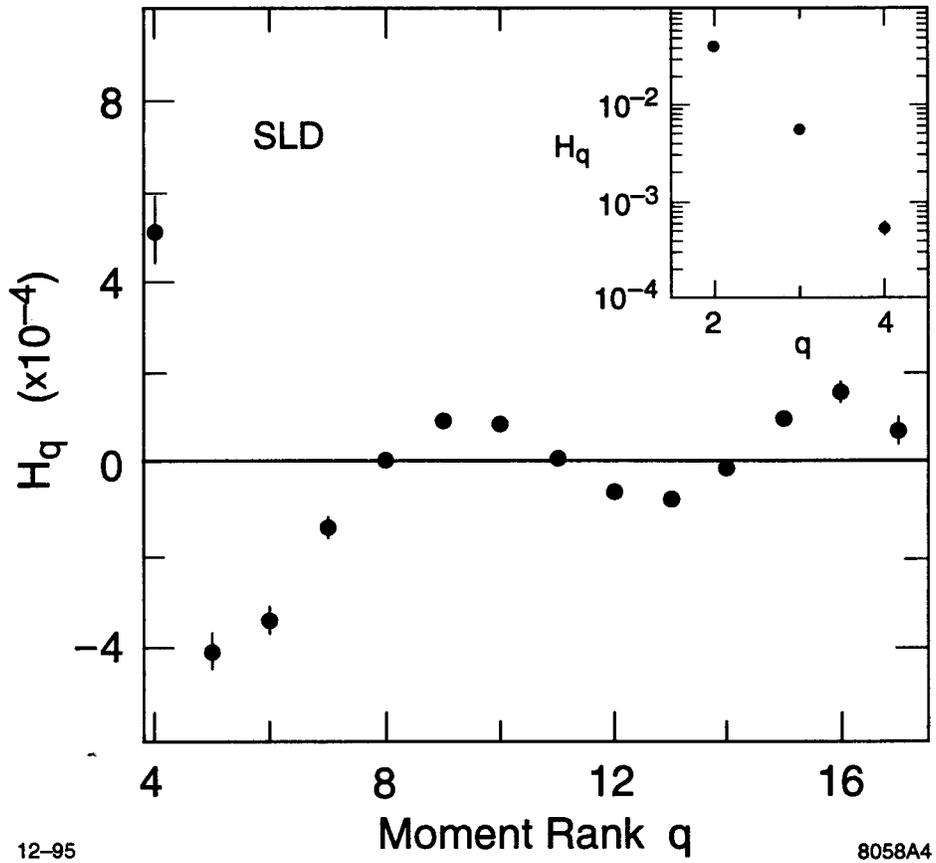


Fig. 2



12-95

8058A4

Fig. 3

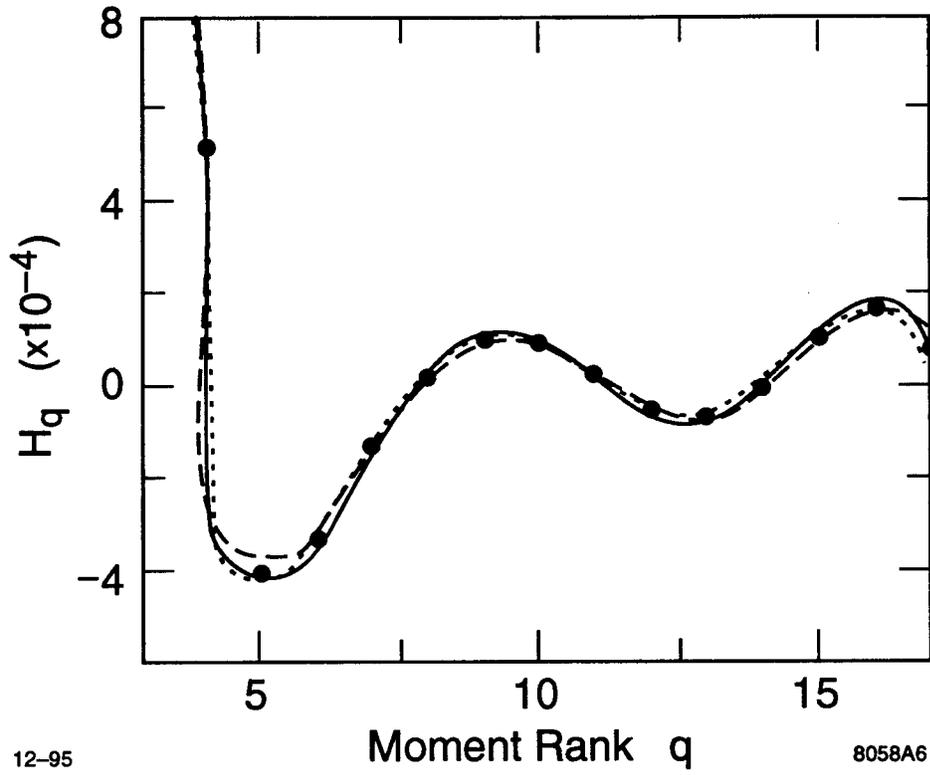
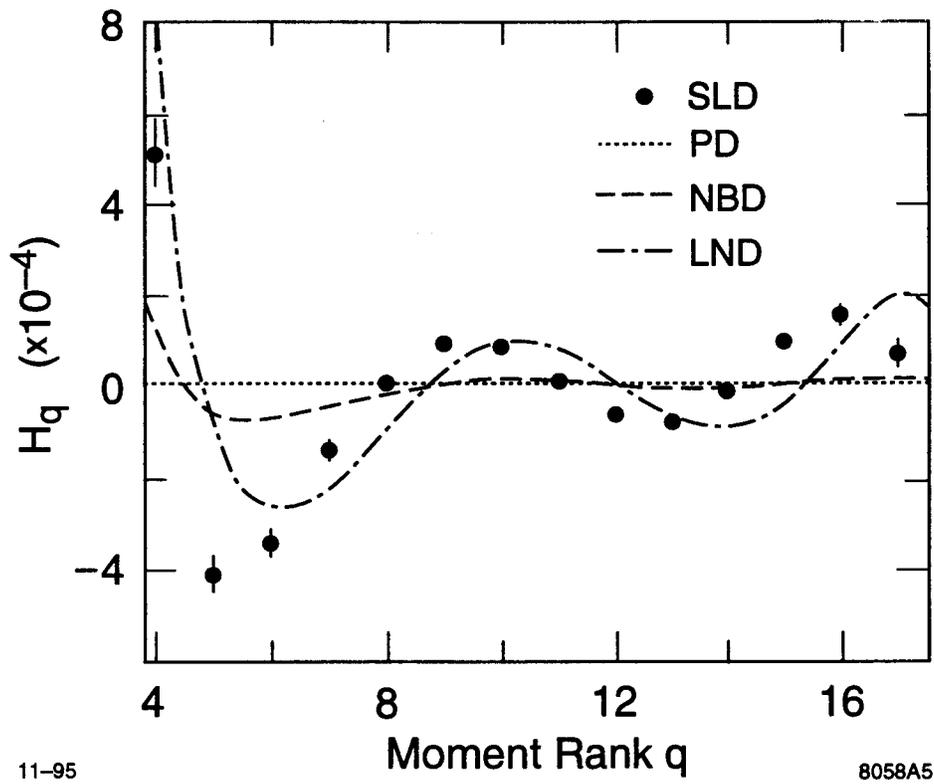


Fig. 4



11-95

8058A5

Fig. 5