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ON THE $|\Delta I| = \frac{1}{2}$ RULE IN THE STANDARD MODEL*

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Abstract

We consider the $K \to \pi\pi$ amplitudes in the Standard Model. We show that the infamous $|\Delta I| = \frac{1}{2}$ rule can be explained by using Padé Approximants to sum the diverging QCD perturbation series.

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1 Introduction

It has been known for over 30 years that the $K \rightarrow \pi\pi$ decay amplitudes obey an approximate $|\Delta I| = \frac{1}{2}$ selection rule. The $|\Delta I| = \frac{1}{2}$ amplitude is much larger than the $|\Delta I| = \frac{3}{2}$ amplitude. Although the $|\Delta I| = \frac{3}{2}$ amplitude is explained in the Standard Model, theoretical calculations fail to reproduce the observed enhancement in $|\Delta I| = \frac{1}{2}$ amplitudes of K-decays by more than an order of magnitude.

In fact Pich *et al.* [1] conclude:

> "Our conclusion that $|\Delta I| = \frac{1}{2}$ transitions in K-decays pose a serious problem to a 'natural' understanding within the framework of the Standard Model. We find a serious discrepancy which in order to be solved requires in our opinion a rather subtle mechanism in the strong interaction dynamics, or perhaps, new physics."

Although Stech [2] has claimed to have resolved the problem, his calculation involves diquark-anti-diquark operators in a phenomenological model and has not been accepted by the physics community [3]. Our calculations, however, are strictly within the Standard Model.

Quantitatively the situation is described in terms of the coupling constants as follows [1]:

$$\left[g^{(1/2)}\right]_{\text{exp}} = \left[g_8^{(1/2)} + g_{27}^{(1/2)}\right]_{\text{exp}} = 5.1 \tag{1}$$

$$\left[g_{27}^{(3/2)}\right]_{\text{exp}} = 0.16\ . \tag{2}$$

The theoretical estimates obtained in Ref. [1] are:

$$\left[g_8^{(1/2)}\right] = 0.40 \pm 0.10 \tag{3}$$

$$\left[g_{27}^{(1/2)}\right] = 3.2 \pm 0.8 \times 10^{-2} \tag{4}$$

$$\left[g_{27}^{(3/2)}\right] = 0.17\ . \tag{5}$$

It can be seen from Eqs. (2) and (5) that the $|\Delta I| = \frac{3}{2}$ amplitude is well understood. However the $|\Delta I| = \frac{1}{2}$ amplitude in Eqs. (3) and (4) disagrees with the experimental value in Eq. (1) by more than an order of magnitude.

The QCD corrections have been calculated [4, 5] to $\mathcal{O}(\alpha_s^2)$.

$$f(\alpha_s) = 1 + \frac{117501}{4840}\left(\frac{\alpha_s}{\pi}\right) + 470.72\left(\frac{\alpha_s}{\pi}\right)^2 \tag{6}$$

where $f(\alpha_s)$ represents the gluonic corrections to the two-point function

$$466(q^2) = i\int d^4x\, e^{iqx}\left\langle 0|T(Q_6(x)\, Q_6^{\dagger}(0))|0\right\rangle \tag{7}$$

due to the so-called "Penguin Diagrams." If we use the value of α_s at a few GeV

$$0.19 \leq \alpha_s \leq 0.31 \tag{8}$$

or

$$0.060 \leq \frac{\alpha_s}{\pi} \leq 0.10 \tag{9}$$

one sees that the series for $f(\alpha_s)$ is diverging

$$f(\alpha_s) = 1 + 1.46 + 1.69 + \ldots \tag{10}$$

at the lower limit of Eq. (9). The first reaction is to throw up our hands and say that the series explodes and thus is meaningless. However we will show that the series in Eq. (6) is, in fact, meaningful and can be summed by using Padé Approximants (PA). We have used PA recently to estimate the next unknown term and the sum of the series (Full Padé) in many examples in QED, QCD, Atomic Physics and Statistical Physics (as well as Applied Mathematics). See Refs. [6] through [14].

From Eqs. (1), (3) and (4) we see that we need an enhancement of 12.8 $\pm$ 3.2. From Eq. (6) the series to be analyzed is given by

$$S = \sum S_n X^n \tag{11}$$

where $S_0 = 1,\ S_1 = 24.3,\ S_2 = 470.7$ and $X = \alpha_s/\pi$, where X is given in Eq. (9). We now estimate the next term in this series using our method of Padé Approximants. The $[N/M]$ PA is the ratio of 2 polynomials, R_N and Q_M, where R_N is of degree N and Q_M is of degree M. From the [1/1] we obtain

$$S_3 = 9118 \tag{12}$$

while from the [0/2] we get

$$S_3 = 8527\ . \tag{13}$$

We take the average of Eq. (12) and (13),

$$S_3 = 8800 \pm 600\ . \tag{14}$$

Now using S_0, S_1, S_2 and S_3 we construct the [1/2] PA and the [2/1] PA. The reuslts are given in Table I. It can be seen that we can obtain large enhancement for reasonable values of α_s.

Thus we see that our PA method allows us to sum a perturbation series which appears to be "blowing up" and enables us to understand the large enhancement of the $|\Delta I| = \frac{1}{2}$ amplitude within the Standard Model.

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Table I – Padé estimates for the "sum" of the series in Eq. (6).

$X = \alpha_s/\pi$	[1/2]	[2/1]
0.06	−12.9	−11.4
0.07	−5.17	−4.76
0.08	−3.40	−3.13
0.09	−2.63	−2.40
0.10	−2.21	−1.98

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