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Neutral Higgs CP Violation at $\mu^+\mu^-$ colliders

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Abstract: CP violating asymmetries in both the production and the decays of s-channel neutral Higgs bosons at $\mu^+\mu^-$ colliders can be large. If the CP violation occurs in the muon-Higgs coupling, one observes a production asymmetry with transversely polarized $\mu^+\mu^-$ beams. Likewise if the CP violation occurs in the top-Higgs or τ-Higgs coupling, one observes an azimuthal decay asymmetry in $\mu^+\mu- \to t\bar{t}$ (or $\tau^+\tau^-$). CP studies at such colliders allow a uniquely clean way of deducing the underlying CP phases; in turn these parameters would significantly improve our understanding of baryogenesis.

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The Large Hadron Collider (LHC) currently being planed at CERN may provide a reach for new physics of up to a few TeV. It is to be hoped that this hadronic capability will be complemented with leptonic machines.

Due to synchrotron radiation effects it is probably impractical to construct an e^+e^- storage ring (such as LEP) at these higher energies. Two possible leptonic colliders at the TeV energy scale that have been suggested are a colliding beam e^+e^- machine (Next Linear Collider or NLC), which would operate on principles similar to the current SLC and a Muon Collider (MUC). Since synchrotron radiation is so much smaller at a MUC compared to an electron machine, a MUC has the advantage that muon beams could be re-circulated in a storage ring and thus muons could be reused several hundred times before their eventual decay. Consequently MUCs have been receiving increasing attention in the past few years [1]–[5]. By the same token, NLC has been the subject of intense research effort [6] in the past decade and its feasibility is no longer widely questioned whereas for MUCs many challenges in accelerator design still need to be overcome [1]–[4].

For most physics applications muon collisions and electron collisions are virtually identical. Hence factors such as cost, and the engineering problems involved in building such colliders are likely to be the determining factor in their eventual construction. If a MUC is eventually constructed, a possible physics bonus may be the ability to study neutral Higgs bosons in the s-channel [1, 5].

In this paper, we shall assume that at least one neutral Higgs scalar, which we will denote $\mathcal{H}$, will be discovered in the mass range $100 \leq m_{\mathcal{H}} \leq 1000 GeV$ and that the muon collider will be tuned to study such a resonance in the s-channel so that $\sqrt{s} = m_{\mathcal{H}}$. In this way the feeble coupling of the μ to the Higgs may be partially compensated by the resonance enhancement and so the signal of an s-channel Higgs boson may be studied in detail. Bearing this in mind we consider how the CP phase of its couplings to fermions may be investigated. We find that the method gives a uniquely clean way of deducing CP violating phases in the coupling of the neutral Higgs to leptons and to the top quark.

As is well known, at the present time, the only form of CP violation experimentally observed is in the K^0 system. It is thought that these results can be explained through the standard model's phase in the CKM matrix [7], a hypothesis which will be tested at B factories now being built. However, strong theoretical arguments exist that suggest that additional forms of CP violation are needed to explain the baryon asymmetry of the universe [8]. Indeed one class of models which has been shown to give reasonable baryon asymmetries are models involving CP violation in the Higgs sector.

Motivated by these considerations we shall investigate two methods of observing a CP violating signal in such a Higgs resonance at a MUC. First, if the μ beams are transversely polarized at different inclinations, there is a CP violating production asymmetry proportional to $\sin\phi_\mu$, ϕ_μ being the azimuthal angle between the polarization directions. This is sensitive to CP violation in the Higgs-$\mu\mu$ coupling. The second observable we consider is an analogous asymmetry in $\mu^+\mu^- \to t\bar{t}$ (or $\tau^+\tau^-$). In the latter case, for instance, the signal is proportional to $\sin\phi_t$ where ϕ_t is the azimuthal angle between the polarization of the top quarks; ϕ_t may easily be determined if the tops decay leptonically. Polarization of the colliding muons is not a necessity for this study, although longitudinal polarization of the beam can be used to reduce the background. In this case one is sensitive to CP violation in the Higgs-$t\bar{t}$ (or $\tau^+\tau^-$) coupling which relates directly to some proposed mechanisms of baryogenesis in the early universe [8].

In all three cases mentioned in the preceding paragraph the asymmetries may be large. However, to a certain extent, they will be obscured by standard model backgrounds, as we will discuss below. We find that observation of these asymmetries will certainly be possible if the Higgs is relatively light or narrow leading to a relatively large production rate. Furthermore, in many theoretical scenarios, even if the Higgs is not that light, it can have a large production rate and it can lead to appreciable asymmetries.

Although the underlying model we have in mind may contain several Higgs doublets we would like to cast our analysis in a relatively model independent way. Let us assume that a single Higgs is under experimental study which we will denote $\mathcal{H}$. We will parameterize its coupling to a fermion f by

$$C_{\mathcal{H}ff} = C^0_{ff}\chi_f e^{i\gamma_5\lambda_f} \tag{1}$$

where C^0_{ff} is the coupling in the standard model (with one Higgs doublet). In such theories with an extended Higgs sector the field $\mathcal{H}$ may either be a scalar H or a pseudoscalar A. If $\mathcal{H} = A$ then it does not couple to two gauge bosons while if $\mathcal{H} = H$ we will parameterize the coupling to two vector bosons ($VV = ZZ$ or WW) as:

$$C_{\mathcal{H}VV} = C^0_{VV}\cos\alpha. \tag{2}$$

where C_{VV}^0 is the coupling of the SM Higgs to the gauge bosons and α is the angle between the observed Higgs $\mathcal{H}$ and the orientation of the vacuum in the Higgs space.

To serve as an illustration we will consider three sample models for the $\mathcal{H}$ in question:

- 1: $\mathcal{H} = H$ with $\alpha = \lambda_f = \pi/4$ and $\chi_f = 1$ for all fermions.
- 2: $\mathcal{H} = A$ with $\lambda_f = \pi/4$ and $\chi_f = 1$ for all fermions.
- 3: $\mathcal{H} = A$ with $\lambda_f = \pi/4$ and $\chi_l = \chi_d = 5$ and $\chi_u = 1/5$.

The last case is motivated by models of the type that, for instance, are used for the Higgs sector for supersymmetric (SUSY) scenarios [5]. In such a case one might want to partially explain the large top mass by making the corresponding vacuum expectation value (vev) large giving rise to $\chi_l = \chi_d = v_2/v_1 = 1/\chi_u$ for large v_2/v_1. Although minimal SUSY models do not have CP violation in the Higgs sector, more complicated Higgs sectors or non-minimal SUSY models do readily admit CP violation [10].

It is difficult to observe the effects of the Higgs resonance except if the accelerator is precisely on shell [1, 5]. If s is the center of mass energy and $s = m_{\mathcal{H}}^2$ then defining $B_\mu = Br(\mathcal{H} \to \mu^+\mu^-)$ and $\sigma_{\mathcal{H}} = \sigma(\mu^+\mu^- \to \mathcal{H})$, the cross section for Higgs production is:

$$\sigma_{\mathcal{H}} = \frac{4\pi}{m_{\mathcal{H}}^2} B_\mu. \tag{3}$$

It is useful to compare this with $\sigma_0 = \sigma(\mu^+\mu^- \to \gamma^* \to e^+e^-)$ so we define

$$R(\mathcal{H}) = \frac{\sigma_{\mathcal{H}}}{\sigma_0} = \frac{3}{\alpha_e^2} B_\mu \tag{4}$$

where α_e is the electromagnetic coupling. It is clear in the above that the key to having a large rate for Higgs production is for its width to be as small as possible (thereby increasing B_μ). The main possible decay modes of the Higgs which we consider are $\mathcal{H} \to ZZ$, $\mathcal{H} \to W^+W^-$, $\mathcal{H} \to t\bar{t}$, $\mathcal{H} \to b\bar{b}$, and $\mathcal{H} \to \tau^+\tau^-$. The last two become important for the case when $m_{\mathcal{H}} < 2m_W$. The decay rates to these modes, given the above couplings, can be readily calculated by using the results that exist in the literature [9]:

$$\begin{aligned}
\Gamma(\mathcal{H}\to t\bar{t}) &= \frac{3g_W^2 m_t^2 m_{\mathcal{H}}}{32\pi m_W^2}\beta_t\left[\beta_t^2+(1-\beta_t^2)\sin\lambda_t\right]\chi_t^2 K_t \\
\Gamma(\mathcal{H}\to b\bar{b}) &= \frac{3g_W^2 m_b^2 m_{\mathcal{H}}}{32\pi m_W^2}\chi_b^2 K_b \\
\Gamma(\mathcal{H}\to ZZ) &= \frac{g^2}{128\pi}\frac{m_{\mathcal{H}}^3}{m_Z^2}\beta_Z(\beta_Z^2+12\frac{m_Z^4}{m_{\mathcal{H}}^4})\cos^2\alpha \\
\Gamma(\mathcal{H}\to WW) &= \frac{g^2}{64\pi}\frac{m_{\mathcal{H}}^3}{m_W^2}\beta_W(\beta_W^2+12\frac{m_W^4}{m_{\mathcal{H}}^4})\cos^2\alpha
\end{aligned} \tag{5}$$

where $\beta_i = \sqrt{1-4m_i^2/m_{\mathcal{H}}}$. The ZZ and WW modes are only relevant if $\mathcal{H}=H$. In these equations K_b, K_t are QCD corrections given, for example, in Ref. [9].

In comparison, the width of the Higgs decay to $\mu^+\mu^-$ is

$$\Gamma(\mathcal{H}\to\mu^+\mu^-) = \frac{g_W^2 m_\mu^2 m_{\mathcal{H}}}{32\pi m_W^2}\chi_\mu^2 \tag{6}$$

In Figure 1 we present the value of $R(\mathcal{H})$ at $s=m_{\mathcal{H}}^2$ assuming that $\alpha=45°$ for the $\mathcal{H}=H$ and $\mathcal{H}=A$ cases as a function of $m_{\mathcal{H}}$.

In deriving equation (4) above we assumed that the energy of the collider could be controlled to a precision much finer than the width of the Higgs. While this should be generally true if $m_{\mathcal{H}}\gg 2m_W$ or $2m_t$, below this threshold it may well not be the case. To take this into account let us assume that the actual value of s is uniformly distributed in the range:

$$m_{\mathcal{H}}^2(1-\delta) < s < m_{\mathcal{H}}^2(1+\delta) \tag{7}$$

The beam spread thus defined will lead to an observed rate of Higgs production described by:

$$\tilde{R}(\mathcal{H}) = \left[\frac{\Gamma_{\mathcal{H}}}{m_{\mathcal{H}}\delta}\arctan\frac{m_{\mathcal{H}}\delta}{\Gamma_{\mathcal{H}}}\right]R(\mathcal{H}) \tag{8}$$

In Figure 1 we also show $\tilde{R}(\mathcal{H})$ using the value of $\delta=10^{-3}$.

Let us now consider the production of $\mu^+\mu^-\to\mathcal{H}$ with transversely polarized μ beams. We take the z-axis in the center of mass frame to be the direction of the μ^- beam to be polarized in the x direction. Assume further that the polarization of the μ^+ beam is inclined at an angle of ϕ_μ. The cross section in this case becomes:

$$\sigma(\phi_\mu) = (1 - \cos 2\lambda_\mu \cos\phi_\mu + \sin 2\lambda_\mu \sin\phi_\mu)\sigma_0 \tag{9}$$

where σ_0 is the unpolarized cross section. Then the CP odd asymmetry in the production will be given by:

$$A_\mu \equiv \frac{\sigma(90^\circ) - \sigma(-90^\circ)}{\sigma(90^\circ) + \sigma(-90^\circ)} = \sin 2\lambda_\mu \tag{10}$$

Thus the underlying CP violating phase λ_μ becomes cleanly measurable. Since this phase (λ_μ)is unconstrained, large asymmetries approaching 100% are possible. As we shall discuss below, this asymmetry will be diluted somewhat by the standard model backgrounds.

Let us now turn our attention to the analogous type of asymmetry in the decays $\mathcal{H} \to t\bar{t}$ or $\tau^+\tau^-$ [11]. In order to observe a CP violating correlation in the spins of the top quarks or τ-leptons, we clearly need to look at the correlations among their decay products.

Consider first the determination of the spin of a fermion in general. Suppose f decays to XY where X, Y may be a single particle or a multiparticle state. Let ϵ_X^f be the "analyzing power", i.e., the degree to which the momentum of X is correlated with the spin of f, defined by

$$\epsilon_X^f \equiv 3 < cos\theta_X > . \tag{11}$$

where θ_X is the angle between the spin of f and $\vec{P}_X$, in the f rest frame.

We discuss the decay of the top quark first. If the top decays semi-leptonically, $t \to bW \to bl^+\nu_l$ (for $l = e,\ \mu$) then $\epsilon_l^t = 1$ [12]. For hadronic decays of the top one could obtain $\epsilon = 1$ if one could tag which jet was the $\bar{d}$ type (thus playing the role of the positron). Experimentally this is hard to do. We will therefore only use the inclusive W momentum to determine the polarization for the hadronic decays. The analyzing power in this case is thus:

$$\epsilon_W^t = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} \approx 0.39 \tag{12}$$

Next consider the case of the τ lepton. If the τ decays semi-leptonically, then $\epsilon_l^\tau = -\frac{1}{3}$ [13] (note that if one could observe the $\bar{\nu}_l$ then $\epsilon = 1$). In [13] it is shown that the inclusive hadronic decays of the τ can give $\epsilon_h^\tau = -.42$ (h stands for inclusive hadron states). If one analyses the detailed structure of the hadronic decays one can obtain better results [14, 15] but we will just use this number in our discussions to follow.

Let us define a coordinate system in the Higgs center of mass frame where the z' axis is in the direction of the f (i.e., t or τ) momentum. Let us now consider the f decays via $f \to X_i Y_i$ and the $\bar{f}$ decays $\bar{f} \to \bar{X}_j \bar{Y}_j$. We define the angle ϕ_{ij} to be the azimuthal angle between the p_{Xi} and the $p_{\bar{X}j}$ projected into the $x' - y'$ plane:

$$\sin(\phi_{ij}) = \frac{\vec{p}_{Xi} \times \vec{p}_{\bar{X}j} \cdot \vec{p}_f}{|\vec{p}_{Xi}|\ |\vec{p}_{\bar{X}j}|\ |\vec{p}_f|} \tag{13}$$

Taking into account ϵ_i^f, ϵ_j^f the differential distribution in ϕ_{ij} is (note that $\epsilon^{\bar{f}} = -\epsilon^f$):

$$\frac{d\Gamma}{\Gamma\ d\phi_{ij}} = 1 + \frac{\pi^2}{16}\epsilon_i^f \epsilon_j^f \rho_f \cos 2\lambda_f \cos\phi_{ij} + \frac{\pi^2}{16}\epsilon_i^f \epsilon_j^f \eta_f \sin 2\lambda_f \sin\phi_{ij} \tag{14}$$

In eqn. (14), for $f = t$ (so that the threshold mass effects must be retained), ρ and η are given by:

$$\begin{aligned} \rho_t &= \frac{1 - \beta_t^2 - (1 + \beta_t^2)\cos 2\lambda_t}{\cos 2\lambda_t (1 + \beta_t^2 - (1 - \beta_t^2)\cos 2\lambda_t)} \\ \eta_t &= \frac{\beta_t}{1 - (1 - \beta_t^2)\cos^2\lambda_t} \end{aligned} \tag{15}$$

On the other hand, if $f = \tau$ then the above simplifies to $\rho_\tau = \eta_\tau = 1$. The resultant CP violating asymmetry which we will consider is:

$$A_{ij}^f = \frac{\Gamma(\sin\phi_{ij} > 0) - \Gamma(\sin\phi_{ij} < 0)}{\Gamma(\sin\phi_{ij} > 0) + \Gamma(\sin\phi_{ij} < 0)} \tag{16}$$

Thus using the above distribution we find:

$$A_{ij}^f = \frac{\pi}{8}\epsilon_i^f \epsilon_j^f \eta_f \sin 2\lambda_f \tag{17}$$

Of course we would like to optimally combine the resultant asymmetries from different pairs of modes $\{ij\}$. In order to do this we would like to choose weights w_{ij} normalized by:

$$\sum w_{ij} B_i B_j = \sum B_i B_j \tag{18}$$

where B_i is the branching ratio of $f \to i$ and the summation is over all the modes under observation. Thus the total asymmetry

$$\mathcal{A}^f = \sum w_{ij} A^f_{ij} B_i B_j \tag{19}$$

is to be maximized. This is done by taking $w_{ij} \propto A^f_{ij}$ [16]. Applying this to the asymmetries defined in (16) we find that the total asymmetry is given by

$$\mathcal{A}^f = \frac{\pi}{8} (\mathcal{E}^f)^2 \eta_f \sin 2\lambda_f \tag{20}$$

where

$$\mathcal{E}^f = \sqrt{\sum B_i (\epsilon^f_i)^2} \tag{21}$$

Selecting the top quark and the τ lepton modes mentioned above we obtain numerically (using $m_t = 176 GeV$, $B^\tau_l = .36$, $B^\tau_h = .64$, $B^t_e + B^t_\mu = \frac{2}{9}$ and $B^t_h + B^t_\tau = \frac{7}{9}$):

$$\mathcal{E}^t = .58 \qquad \mathcal{E}^\tau = .39 \tag{22}$$

In the case of top decay, if the quark or the anti-quark decay semi-leptonically, the 4-momenta of the undetected neutrino(s) may be reconstructed by using the mass constraints of the top and the W. For the decay of the τ lepton, it is less obvious that this can be done. For instance, if one or both of the τ leptons decays leptonically, there are at least 3 ν's in the final state and the kinematics does not provide sufficient restrictions to reconstruct the event. Even if both τ's decay hadronically, although it would appear that there is enough information to reconstruct the event, in fact solving the kinematics gives rise to a "quadratic ambiguity" resulting in an ambiguity in the sign of $\sin \phi_{ij}$. Therefore with this information alone the asymmetry $\mathcal{A}^f$ cannot be evaluated.

These problems with the τ may be resolved as follows given a reasonable vertex detection capability.

Suppose X_i is a particle emerging from the τ^- and $\bar{X}_j$ is a particle emerging from the τ^+. In general the tracks of these two particles will not intersect. Let us define the vector v_{ij} to be the displacement from the track of X_j to the track of X_i at either their closest approach or the closest approach inferred from extrapolating the portion of the tracks which may be observed. Typically the magnitude of v_{ij} will be of the order of $2c\tau_\tau = 180\mu m$ (in spite of the fact that the displacement of the decaying τ's is of order $2c\tau_\tau\gamma = O(1cm)$). We then define the angle $\tilde{\phi}_{ij}$ to be the difference in the azimuthal angle between $\vec{p}_{Xi}$ and $\vec{p}_{\bar{X}j}$ with respect to an axis in the v_{ij} direction.

Recall that the asymmetry A_{ij} is just the expectation value of $\sigma(\sin(\phi_{ij}))$ where:

$$\sigma(x) = \begin{cases} +1 & if \quad x > 0 \\ -1 & if \quad x < 0 \end{cases} \tag{23}$$

A vertex detector may allow the determination of $\sigma(\sin\phi)$ in a $\tau^+\tau^-$ event using one or more of the following four methods:

- 1: Since the displacement of the τ decay from the interaction point is on the order of $c\tau_\tau\gamma = O(1cm)$ it is not unreasonable to assume that, depending on detector design, in many events either the τ^+ or the τ^- will form a track in the vertex detector. If this is the case then the z' axis is determined and ϕ_{ij} is obtained directly from the momenta of X_i and $\bar{X}_j$.

- 2: If X_i or $\bar{X}_j$ consist of multiple charged tracks, running these tracks back to their common decay vertex again determines the z' axis if the machine is configured so that the interaction region is in a single point. More generally if the interaction region is not a well defined point, one may use the decay vertex and the combined momentum of the multiple tracks to define the trajectory of the center of gravity of the system and use method (3) below treating this trajectory as an effective single track.

- 3: If both X_i and $\bar{X}_j$ are single charged tracks (e.g., $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$ or $\tau^- \to \pi^-\nu_\tau$) then

  $$\sigma(\sin\phi_{ij}) = \sigma(\sin\tilde{\phi}_{ij}) \tag{24}$$

  so that the exact decay vertex need not be identified. To see why (24) should be true, let $\vec{x}_1$ be any point on the track of X_i and $\vec{x}_2$ be any point on the track of $\bar{X}_j$. If $\vec{u} = \vec{x}_1 - \vec{x}_2$ then $\vec{u}\cdot(\vec{p}_i \times \vec{p}_j) = \vec{v}_{ij}\cdot(\vec{p}_i \times \vec{p}_j)$.

If the τ^- decays at time t_1 and the τ^+ at time t_2 then the vertex points $\vec{V}_1$, $\vec{V}_2$ are $\vec{V}_1 = m_\tau^{-1} t_1 \vec{p}_{\tau^-}$ and $\vec{V}_2 = m_\tau^{-1} t_2 \vec{p}_{\tau^+}$ therefore $\vec{V}_1 - \vec{V}_2 = m_\tau^{-1}(t_1 + t_2)\vec{p}_{\tau^-}$ for the τ^- and τ^+ respectively. Since the point $\vec{V}_1$ lies on the X_i track and the point $\vec{V}_2$ lies on the X_j track,

$$
\begin{aligned}
\sigma(\sin\tilde{\phi}_{ij}) &= \sigma(\vec{v}_{ij} \cdot \vec{p}_i \times \vec{p}_j) \\
&= \sigma((\vec{V}_1 - \vec{V}_2) \cdot \vec{p}_i \times \vec{p}_j) \\
&= \sigma(\vec{p}_\tau^- \cdot \vec{p}_i \times \vec{p}_j) \\
&= \sigma(\sin\phi_{ij})
\end{aligned}
\tag{25}
$$

- 4: If both τ's decay hadronically we can use the method of [17]. In this instance the hadronic states will contain charged tracks $X_i \to X_{i,1} \ldots X_{i,n}$ and $\bar{X}_j \to X_{j,1} \ldots X_{j,m}$. For any pair of charged tracks $\{X_{i,k}, \bar{X}_{j,l}\}$ we define $\tilde{\phi}_{i,k\ j,l}$ as above. In this case we first kinematically solve for the neutrino momenta up to the "quadratic ambiguity" mentioned above. Note that this is in fact a parity ambiguity so that only one of the reconstructions will give the correct values for $\sin\tilde{\phi}_{i,k\ j,l}$. The false reconstruction will give exactly the wrong sign for each of these angles providing multiple checks on the parity of the reconstruction.

The only decay combination of modes which is not obviously susceptible to methods 2–4 (although method 1 will work in all cases given a vertex detector sufficiently close to the beam axis) is $X_i = \pi^-\pi^0$ and $\bar{X}_j = l^+$ (or vice versa). Let us assume that the 2π mode is dominated by the ρ resonance. Since, using equation (12) generalized to $\tau \to \rho\nu$ gives $\epsilon_\rho^\tau = .45$ so that $\epsilon_\rho^\tau \epsilon_l^\tau \approx .15 \approx (\mathcal{E}^\tau)^2$. Thus totally ignoring these modes is roughly equivalent to reducing the cross section by $2B_\rho^\tau B_l^\tau \approx 10\%$.

The above asymmetries must be observed against the CP even standard model backgrounds $\mu^+\mu^- \to b\bar{b}$, $t\bar{t}$, ZZ and W^+W^-. Therefore, using the expressions for the rates for these reactions given in Ref. [18], we numerically study their impact on the CP violating signals that we are seeking.

When looking for decay asymmetries of the Higgs the ability to control the longitudinal polarization of the beams may help to increase the signal over the background [19]. This is because the Higgs production flips the chirality of the muon while all standard model interactions preserve it. Thus, for instance, if both beams are left polarized with polarization P then the

Higgs production is enhanced by $1+P^2$ while the background is reduced by $1-P^2$ [19]. In the numerical results below we will illustrate the effects of polarization for a few cases.

Let R_j^0 be the standard model contribution to a specific final state j. Let $\mathcal{L}_0$ be the integrated luminosity per year of a given accelerator. For a final state f let us denote by $y_f^{(3\sigma)}$ the number of years needed to accumulate a 3-σ signal for the CP asymmetry. This will be given by

$$y_f^{(3\sigma)} = 9\frac{R_j^0 + R_{\mathcal{H}} Br(\mathcal{H}\to j)}{A_j^2 R_{\mathcal{H}}^2 Br(\mathcal{H}\to j)^2 \sigma(\mu^+\mu^- \to e^+e^-)\mathcal{L}_0} \tag{26}$$

We will use the notation $y_i^{3\sigma}$ for the CP asymmetry in the production obtained by monitoring the decay of the Higgs to the final state i and $\hat{y}_j^{3\sigma}$ for the azimuthal asymmetries described above by observing the decay of the state j (i.e., $j = t\bar{t}$ or $\tau\tau$).

In the following we will take the luminosity to be $10^{34}\ cm^{-2}s^{-1}$ so assuming that a year is $10^7 s$ (taking 1/3 efficiency), $\mathcal{L}_0 = 10^{41}\ cm^{-2}yr^{-1}$. Numerical results for the three cases of Higgs couplings given below eqn. (2) are summarized in Figures 1–4. For definiteness in all the cases we will take $\lambda_f = \pi/4$.

In Figure 1 we consider the overall Higgs production cross sections in relation to $\mu^+\mu^- \to \gamma^* \to e^+e^-$. In each of the three cases we show $R_{\mathcal{H}}$ and $\tilde{R}_{\mathcal{H}}$ with $\delta = 10^{-3}$. Note the pronounced effect of the vev ratio in case (3) due to the fact that the decay to $t\bar{t}$ is suppressed and the $b\bar{b}$ and $\mu\mu$ modes are enhanced.

In Figure 2 we consider $y^{3\sigma}$ for production asymmetries as monitored by various final states in case (1). Below the WW threshold $y^{3\sigma}$ is in the $10^{-3} - 10^{-4}$ range so observation should be relatively easy. Indeed, in this instance, even a machine luminosity of 10^{31}–10^{32} cm^{-2}s^{-1} may be adequate. Above this threshold $y_{WW}^{3\sigma}$ rapidly increases and passes through 1 at $m_{\mathcal{H}} \approx$ 250 GeV.

In Figure 3 we consider the same quantities for case (2). Here we take $\delta = 10^{-3}$ in all the curves. In this case the WW and ZZ thresholds are absent so a small value of $y^{3\sigma}$ persists up to the $t\bar{t}$ threshold. So again for this mass range the production asymmetry is a very sensitive probe and observation may be feasible even with a machine of $\mathcal{L} \sim 10^{32}$ cm^{-2}s^{-1}. After the $t\bar{t}$ threshold $y_{t\bar{t}}^{3\sigma}$ rapidly grows passing through 1 around $m_{\mathcal{H}} \approx 400$ and observation starts to become rather difficult. We also show the decay asymmetries $\hat{y}^{3\sigma}_{\tau\tau}$ and $\hat{y}^{3\sigma}_{t\bar{t}}$. $\hat{y}^{3\sigma}_{\tau\tau}$ is about 10 below the $t\bar{t}$ threshold. $\hat{y}^{3\sigma}_{t\bar{t}}$ grows rapidly from 1 to 100 above threshold. The effect of polarization is also illustrated in this figure for these two instances of $\hat{y}^{3\sigma}_{\tau\tau}$ and $\hat{y}^{3\sigma}_{t\bar{t}}$. The lower dotted and dashed dotted curves are with $P = 0.9$ (the upper ones with $P = 0$). We thus see that polarization can improve the effectiveness of the decay asymmetries by about one order of magnitude.

In Figure 4 we plot the same quantities for case (3). In this case $y^{3\sigma}$ ranges between 10^{-5} and 10^{-3} over the entire mass range so the production asymmetries are excellent probes for $m_{\mathcal{H}} \lesssim 800$ GeV. Indeed for this case the production asymmetries appear detectable even with $\mathcal{L} \sim 10^{31}$ cm^{-2}s^{-1}. The decay asymmetry $\hat{y}^{3\sigma}_{t\bar{t}}$ is around 10^{-2} and $\hat{y}^{3\sigma}_{\tau\tau}$ is in the range .1–1 below the $t\bar{t}$ threshold and 1–100 above it. We have also shown the effect of using polarization $P = .9$ on $\hat{y}^{(3\sigma)}_{\tau\tau}$ and $\hat{y}^{(3\sigma)}_{tt}$. Again polarization seems to improve the situation by about an order of magnitude. Thus the decay asymmetry in the $\tau\tau$ channel is quite effective for $m_{\mathcal{H}} \lesssim 300$ GeV; for a heavier $\mathcal{H}$ the $t\bar{t}$ channel seems very promising and $\mathcal{L} \sim 10^{32}$ cm^{-2}s^{-1} may well be adequate.

Summarizing, this study shows that production asymmetries represent a promising way to study CP violation in the couplings of $\mathcal{H}$ in all the three cases for at least some range of Higgs masses. Decay asymmetries appear less effective and probably are most useful in models where there is a large ratio between the vacuum expectation values. Longitudinal polarization of the muon beams helps to significantly reduce the backgrounds to the decay asymmetries and can render them quite viable in a variety of Higgs models.

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Figure Captions

Figure 1:

The value for $R_{\mathcal{H}}$ as a function of $m_{\mathcal{H}}$ for the following three cases: (1) $\mathcal{H}=H$, $\alpha=\pi/4$ and $\chi_f=1$ (solid) (2) $\mathcal{H}=A$ and $\chi_f=1$ (dashes) (3) $\mathcal{H}=A$ and $\chi_l=\chi_d=5$ and $\chi_u=1/5$ (dots). In each case the upper branch represents the result for $\sqrt{s}=m_{\mathcal{H}}$ while the lower branch is the result with an energy spread given by $\delta=10^{-3}$.

Figure 2:

The value of $y^{(3\sigma)}$ for case 1 (see caption to Figure 1) assuming $\alpha=\pi/4=\lambda_\mu$. $y^{(3\sigma)}_{b\bar{b}}$ is shown with the solid line, $y^{(3\sigma)}_{WW}$ with the dot-dashed line, $y^{(3\sigma)}_{ZZ}$ with the dotted line, and $y^{(3\sigma)}_{t\bar{t}}$ with the dashed line. (Note $y^{3\sigma}$ is the number of years needed to accumulate a 3σ production asymmetry. Note also that the horizontal line, $y^{3\sigma}=1$, is drawn to serve as a point of reference.)

Figure 3:

The value of $y^{(3\sigma)}$ for case 2 assuming $\lambda_\mu=\lambda_\tau=\lambda_t=\pi/4$. $y^{(3\sigma)}_{b\bar{b}}$ is shown with the solid line, and $y^{(3\sigma)}_{t\bar{t}}$ with the dashed line. The results for decay asymmetries are also shown: $\hat{y}^{(3\sigma)}_{t\bar{t}}$ with the dot-dashed line and $\hat{y}^{(3\sigma)}_{\tau\tau}$ with the dotted line. In both of the latter cases the lower curve is for $P=.9$ and the upper curve is for $P=0$. See captions to Figures 1 and 2.

Figure 4:

The value of $y^{(3\sigma)}$ for case 3 assuming $\lambda_\mu=\lambda_\tau=\lambda_t=\pi/4$. $y^{(3\sigma)}_{b\bar{b}}$ is shown with the solid line, and $y^{(3\sigma)}_{t\bar{t}}$ with the dashed line. The results for decay asymmetries are also shown: $\hat{y}^{(3\sigma)}_{t\bar{t}}$ with the dot-dashed line and $\hat{y}^{(3\sigma)}_{\tau\tau}$ with the dotted line. Again in both of the latter cases the lower curve is for $P=.9$ and the upper curve is for $P=0$. See captions to Figures 1 and 2.

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FIGURE 1

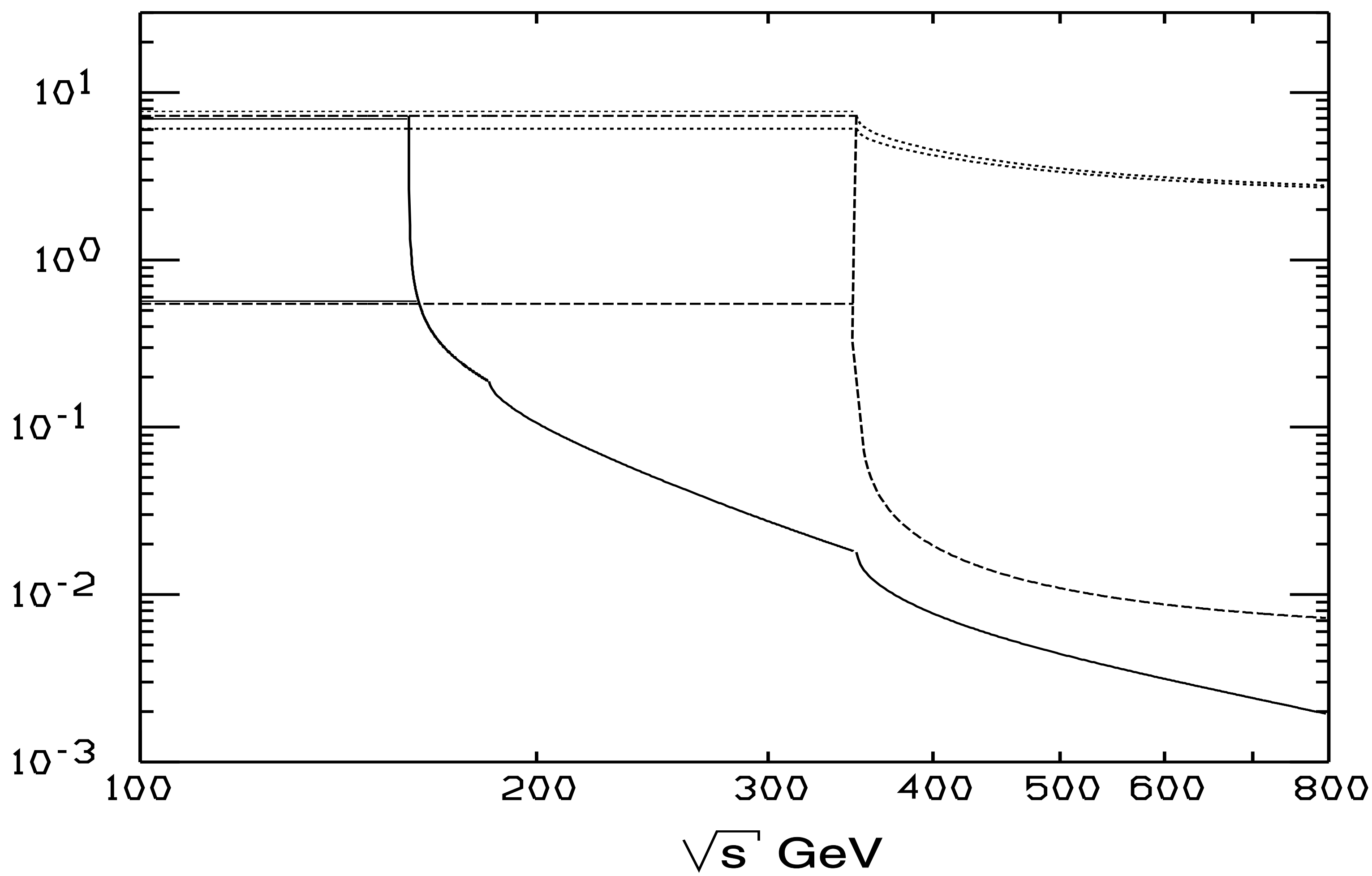
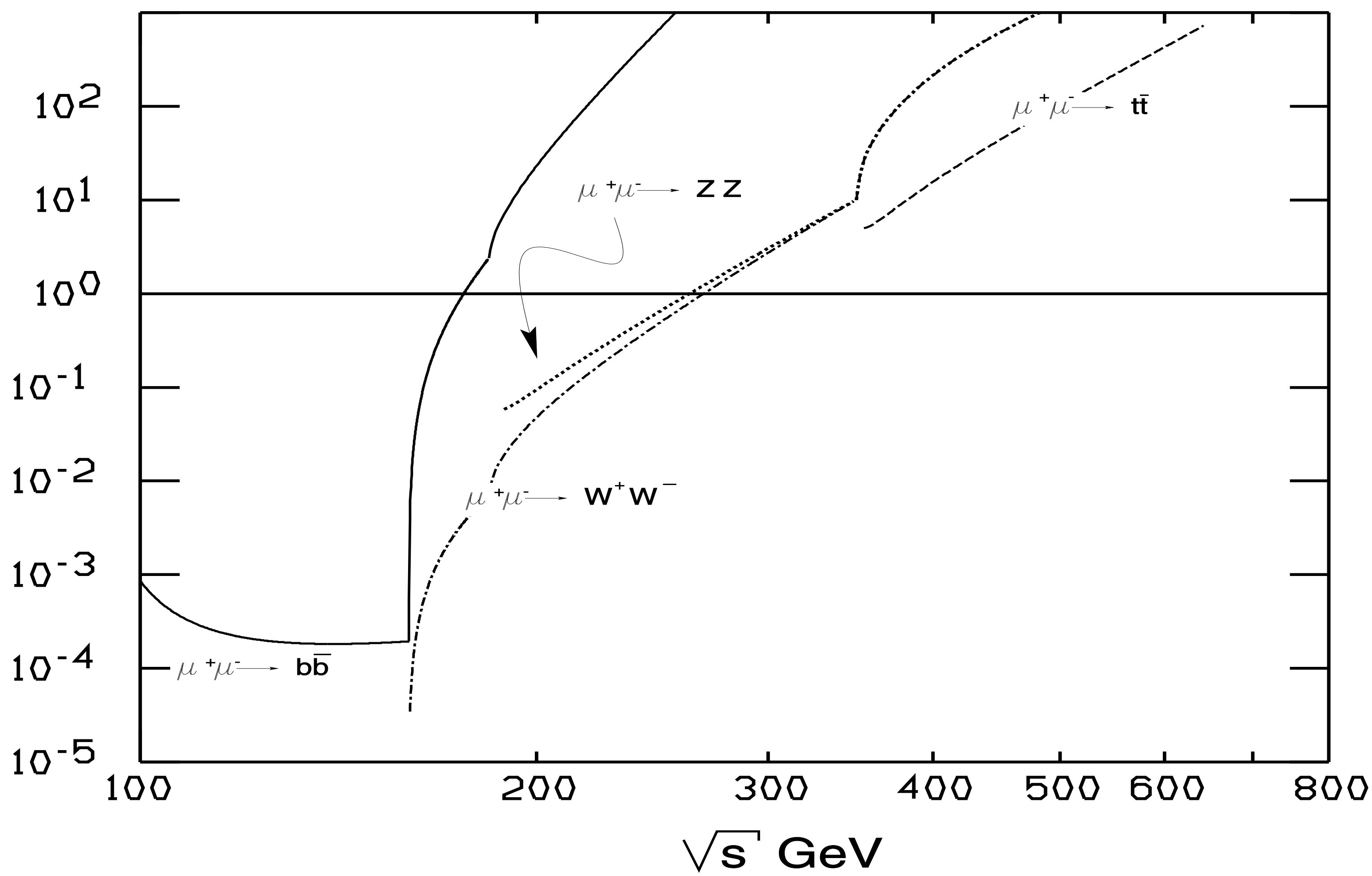
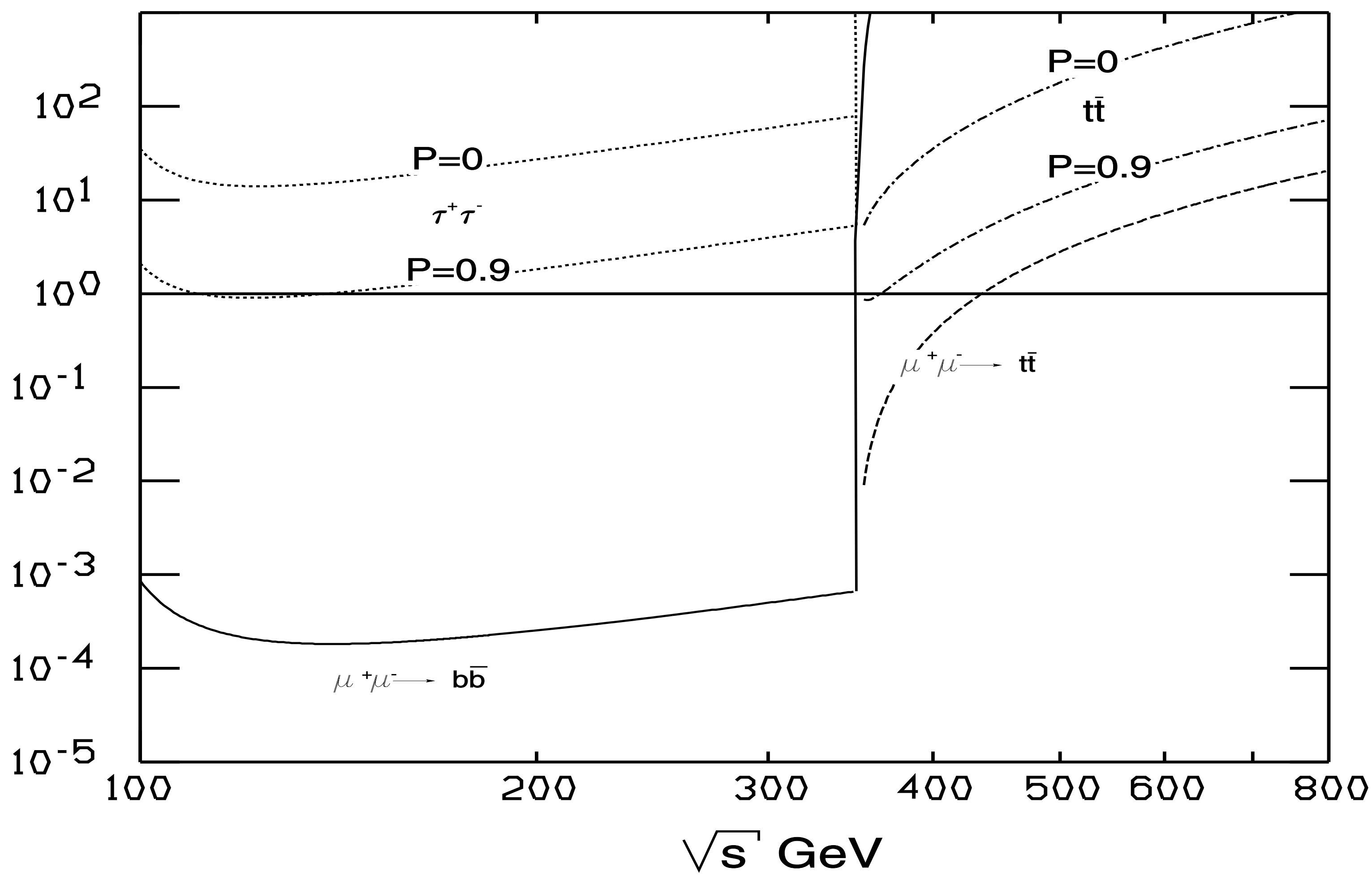


FIGURE 2



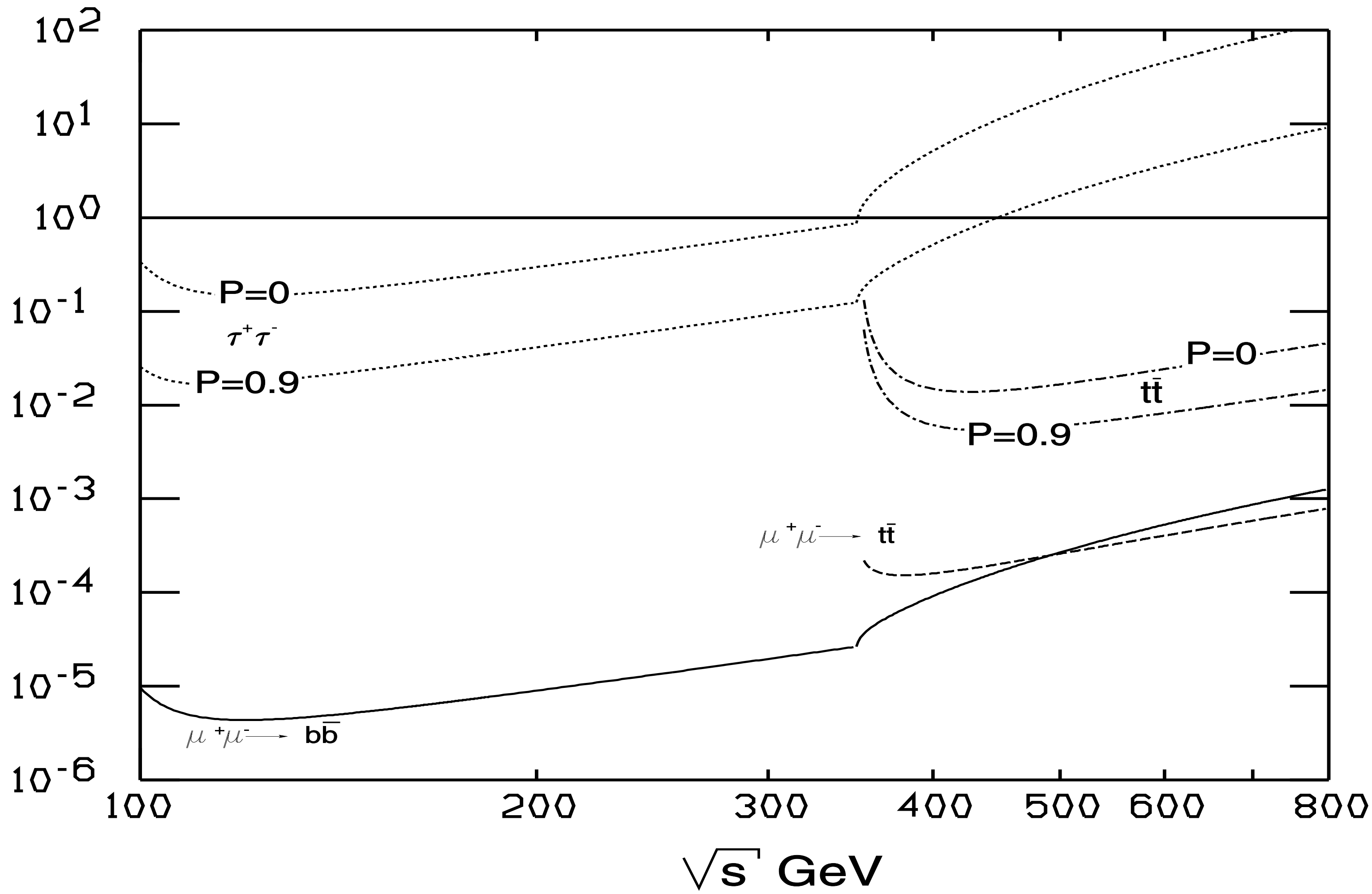
$\mu^+\mu^- \longrightarrow ZZ$
$\mu^+\mu^- \longrightarrow t\bar{t}$
$\mu^+\mu^- \longrightarrow W^+W^-$
$\mu^+\mu^- \longrightarrow b\bar{b}$
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
100
200
300
400
500
600
800
$\sqrt{s}$ GeV

FIGURE 3



P=0
$\tau^+\tau^-$
P=0.9
P=0
$t\bar{t}$
P=0.9
$\mu^+\mu^- \longrightarrow t\bar{t}$
$\mu^+\mu^- \longrightarrow b\bar{b}$
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
100
200
300
400
500
600
800
$\sqrt{s}$ GeV

FIGURE 4



10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P=0
$\tau^+\tau^-$
P=0.9
P=0
$t\bar{t}$
P=0.9
$\mu^+\mu^- \longrightarrow t\bar{t}$
$\mu^+\mu^- \longrightarrow b\bar{b}$
100
200
300
400
500
600
800
$\sqrt{s}$ GeV