Swamp plots for Dynamic Aperture studies of PEP-II Lattices *

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Abstract

With a newly developed algorithm using resonance basis Lie generators and their evaluation with action-angle Poisson bracket maps (nPB tracking) we have been able to perform fast tracking for dynamic aperture studies of PEP-II lattices as well as incorporate lattice nonlinearities in beam-beam studies[1]. We have been able to better understand the relationship between dynamic apertures and the tune shift and resonance coefficients in the generators of the one-turn maps[2]. To obtain swamp plots (dynamic aperture vs. working point) of the PEP-II lattices, we first compute a one-turn resonance basis map for a nominal working point and then perform nPB tracking by switching the working point while holding fixed all other terms in the map. Results have been spot-checked by comparing with element-by-element tracking.

I. INTRODUCTION

An adequate dynamic aperture is a basic requirement for an acceptable accelerator lattice. Conventionally, to check the dynamic aperture of a lattice one would take the following numerical steps: (1) choose a working point (horizontal and vertical betatron tunes) and make the best effort to optimize the bare lattice; (2) assign systematic and random multipole errors and random misalignment of the magnets; (3) make suitable corrections to the orbit, tunes, and chromaticities; (4) track particles (with synchrotron oscillations) to determine the dynamic aperture.

For the dynamic aperture studies of the PEP-II B-factory[3] High-Energy Ring HER[4] and Low-Energy Ring (LER)[5] lattices, we usually select a working point at $\nu_x = 0.57$ and $\nu_y = 0.64$ and track enough particles (electrons for the HER or positrons for the LER), element-by-element for 1024 turns to determine the dynamic aperture (if a particle survives for 1024 turns after injection, it would survive for longer due to synchrotron radiation damping). Typical dynamic aperture plots can be found in these proceedings[4], [5]. Each of such element-by-element dynamic aperture calculations takes a few hours of CPU time in a RISC Workstation using a tracking program called Despot.

Although a dynamic aperture plot at one working point can help in making an appraisal of lattice nonlinear performance, it offers limited information. For example, during the course of updating and improving the PEP-II lattices, we found, in some cases, that the dynamic apertures were not adequate at the nominal working point (0.57, 0.64) while we would find that they were very good at other working points such as (.78, .82) or (.715, .735). In some occasions, particularly for the HER lattices, we even found that some dynamic apertures were extremely small at (.57, .64) due to synchro-betatron resonances, but were adequate at a working point slightly different from (.57, .64). Thus, it is preferable and even necessary to understand lattice behavior for a broad range of working tunes. Dynamic aperture displayed on a reasonable mesh in the tune plane have been called swamp plots.

However, it is virtually impossible to obtain swamp plots for the PEP-II lattices with element-element trackings since it would require months of computer time. Aimed at not only fast tracking but also understanding in detail the lattice nonlinearities, we developed a fast tracking method using resonance basis Hamiltonians, which we call nPB tracking[1]. The nPB tracking provides a factor of 100 in tracking speed and also offers insight into the underlying physics. Map tracking for proton accelerators[6][7][8] is a different problem since there is no significant synchrotron radiation damping.

II. THE nPB TRACKING

To perform nPB tracking, we first write out a resonance basis map file by taking the following steps: (1) obtain a one-turn transverse Taylor map and a one-turn time-of-flight map at a preset suitable order with respect to the closed orbit, treating the off-momentum δ as a parameter; (2) select terms in the time-of-flight map (neglect insignificant terms) and write them (the coefficients) in a file; (3) calculate the linear normalization transformation matrices and write the two 4 X 5 similarity transformation matrices in the file; (4) perform similarity transformation of the nonlinear one-turn Taylor map and then make a Deprit-type Lie transformation such that in the linearly normalized space, the map can be represented by $Re^{-:H(x,p_x,y,p_y,\delta):}$, where R is a rotation that depends on the transverse and vertical tunes only, and H is a discrete δ-type Hamiltonian for the nonlinear perturbation; write the working tunes in the file; (5) transform the discrete Hamiltonian $H(x,p_x,y,p_y,\delta)$ from the Cartesian coordinate space to the action-angle variable space such that $H = H(\theta_x, J_x, \theta_y, J_y, \delta) = H_T(J_x, J_y, \delta) + H_R(\theta_x, J_x, \theta_y, J_y, \delta)$ and write out all terms (coefficients) in the file, where H_T, in which every term depends on the actions (J_x, J_y) and the parameter (δ) only, i.e. not depend on the angles, contains tune shift information[9] while H_R, in which every term depends on the angles and actions, contains the resonance strength information.

Once the resonance basis map is written in a file, we are ready to perform the nPB tracking by first read in the resonance basis map from the file. After initializing the particle phase-space Cartesian coordinates, we then perform turn-by-turn nPB tracking following the steps: (a) transform the phase-space coordinates into the linearly nor-

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malized space (always kept in Cartesian coordinate space); (b) advance the phase of the particles through rotation R that only depends on the working tunes; (c) perform the Lie transformation $e^{-:H(\theta_x,J_x,\theta_y,J_y,\delta):}X$ for nonlinear perturbation of the coordinates by directly making Poisson bracket expansions up to a preset n^{th} Poisson bracket (thereby the name of nPB tracking), where X represents each of the phase-space Cartesian coordinates x, p_x, y, p_y which are considered as functions of action-angle variables in Poisson bracket calculations; (d) transform the phase-space coordinates in the linearly normalized space back to the original space; (e) update the off-momentum δ with the time-of-flight map and then go on to the next turn.

III. RELIABILITY OF THE nPB TRACKING

In performing nPB tracking, one has to be careful in two regards. First, it is not 100% accurate since the map is truncated at a moderate order. Second, it is not 100% symplectic since one does not carry the Poisson bracket expansion to the infinite order. Such inaccuracy and non-symplecticity have been controversial in the use of one-turn maps for long-term tracking. However, it is well understood that the required accuracy and symplecticity depend on circumstances[7]. For the PEP-II lattice dynamic aperture studies (only 1024 turns), through numerous testing we have concluded that a map of 10^{th} order in Hamiltonian (9^{th} order in Taylor expansion) is accurate enough and 3 Poisson bracket expansion of the Lie transformation is accurate and symplectic enough. It takes about 1 minute with such 10^{th}-order, 3PB tracking in a RISC Workstation to obtain a dynamic aperture plot at a given working point.

IV. PEP-II SWAMP PLOTS

To obtain swamp plots for the PEP-II lattices, we have followed exactly the nPB tracking procedures described in Section II except that we would increment the working tunes in Step (b) to obtain dynamic apertures throughout the tune plane. It should be noted that except for working tunes, all other linear and nonlinear terms in the resonance basis map are held fixed. This is equivalent to that of the element-by-element tracking by inserting an exactly matched linear trombone to switch the working tunes without further changing the lattice.

Two typical swamp plots of the HER 60^o lattices are shown in Figures 1 & 2. Figure 1 shows the dynamic apertures for a $\beta_y^* = 2cm$ HER lattice with interlaced sextupoles while Figure 2 shows the dynamic apertures for a $\beta_y^* = 1.5cm$ HER lattice with β-beat semi-local correction. Before we performed the swamp plots, we had wondered which lattice was better. Now, it is clear as shown in the figures that $\beta_y^* = 1.5cm$ lattice has better dynamic apertures throughout the tune plane. The predominant dynamic apertures for the interlaced lattice ($\beta_y^* = 2cm$) are between 10σ and 14σ while they are between 15σ and 19σ for the β-beat semi-local correction lattice ($\beta_y^* = 1.5cm$). Note that the dynamic aperture is below 10σ on where no symbol is shown.

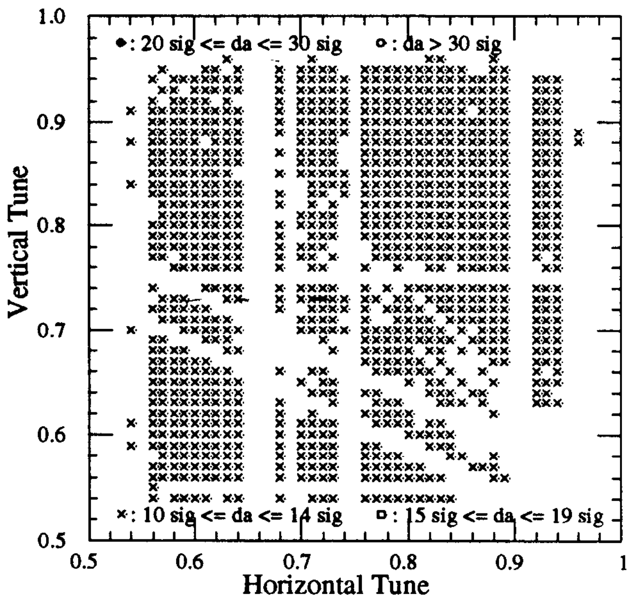

Figure. 1. Swamp plot for the HER lattice with interlaced sextupoles, $\beta_y^* = 2$ cm

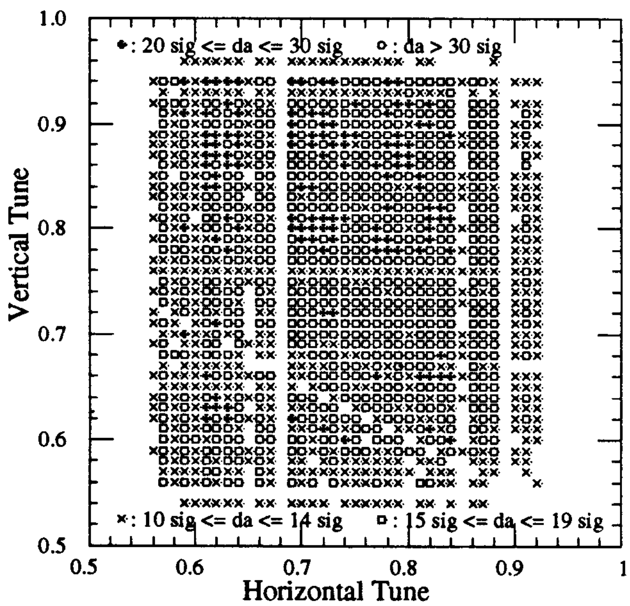

Figure. 2. Swamp plot for the HER lattice with beta-bump correction, $\beta_y^* = 1.5$ cm

Another type of swamp plots we have used frequently is shown in Figure 3 for LER lattices. Dynamic apertures along the diagonal or slightly off-diagonal lines on the tune plane were obtained. As shown in the figure, we can easily conclude that the 90^o noninterlaced-sextupole lattice (dynamic apertures plotted on the left column) is a better choice than the 90^o interlaced-sextupole lattice (shown on the right column). This type of swamp plots is particularly useful during the course of lattice updating and improving since it takes less computer time than that of the full-tune-plane swamp plot while still offers adequate information for comparison of lattice nonlinear performance.

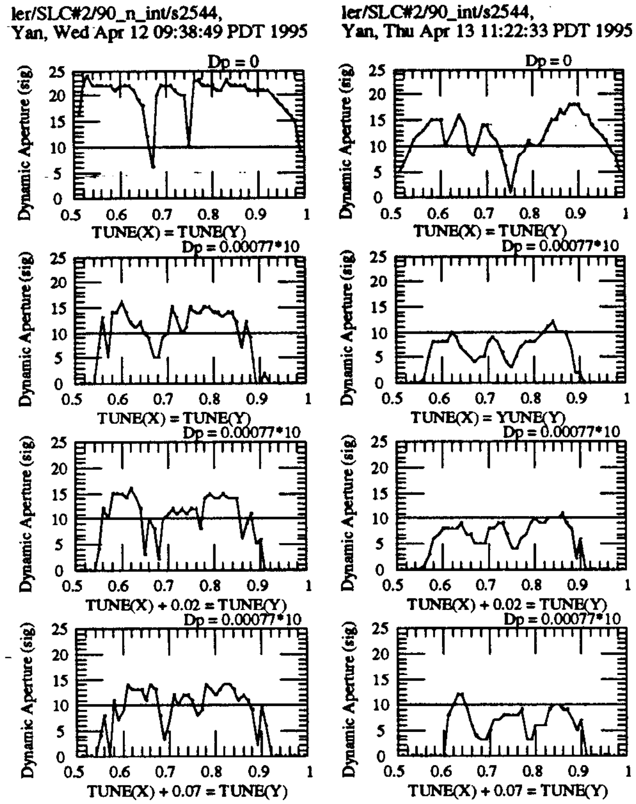

Figure. 3. Comparison of dynamic apertures between two lattices. Each column represents a lattice that is identified by the computer working directory shown on the top of the column.

Through use of swamp plots, one can also probe lattice nonlinearities. In particular, betatron and synchro-betatron resonances can be easily observed in a swamp plot with a fine mesh along the diagonal line on the tune plane as shown in Figure 4 for a 72° semi-local correction LER lattice with interlaced sextupoles. In Figure 4, one clearly sees the strong integer, half-integer and one-third-integer resonances. The one-fourth-integer resonance is not as strong as the one-third-integer resonance. The plot also shows clearly the side-band resonances due to synchrotron oscillations (the synchro-betatron resonances). The integer and half-integer resonances have strong side-band resonances as the dynamic apertures are 0 or very small for a big range of tunes near the integer and half-integer tunes. The strong one-third-integer resonance and related side-band resonances are a very nice clue for lattice improvement as a 72° lattice can possibly have a weak geometric nonlinearity up to 5th order.

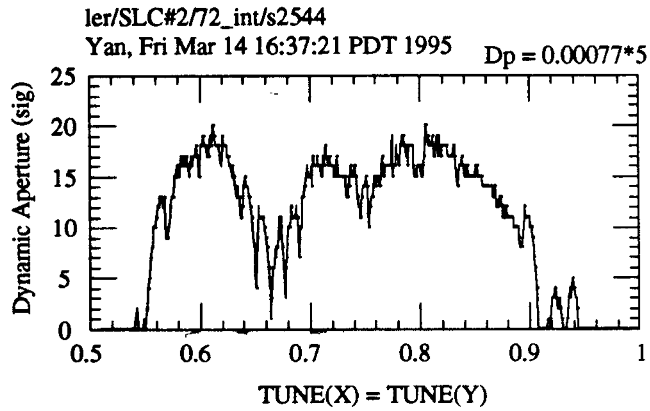

Figure. 4. Dynamic apertures along the diagonal line of the tune plane with a fine mesh for a 72° interlaced-sextupole LER lattice

V. SUMMARY

We have shown some typical swamp plots for the PEP-II lattice studies. These swamp plots have helped us in comparing lattices and often led us to the improvement of the lattices.

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