

August 25, 1995
SLAC-PUB-95-6874

ANOTHER GLANCE AT THE RAINBOW[†]*

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ABSTRACT

Inflationary cosmology predicts the generation of both density perturbation and relic gravitons. Tests of these predictions, especially the relic gravitons, will be very crucial to the verification of inflation. Efforts have been made to distinguish different inflation models using observed anisotropies of the cosmic microwave background radiation. However, the fact that both scalar and tensor modes contribute to the Sachs-Wolfe effect renders such determination difficult. We point out that, mediated by the primordial magnetic field, the relic gravitons can resonantly convert into photons at the same frequencies. A measurement of the spectrum of these long wavelength EM waves can help to determine the power law index directly, which will distinguish different inflationary models. This opens up a new window for another glance at “gravity’s rainbow”.

submitted to
General Relativity and Gravitation

[†] This Essay was awarded with the Fourth Prize by the Gravity Research Foundation for the 1995 Essay Competition.

^{*} Work supported by Department of Energy contract DE-AC03-76SF00515.

The inflationary Universe scenario^[1], which revolutionized our thinking about the very early Universe, makes but three robust predictions: (1) a flat Universe; (2) a nearly scale-invariant spectrum of adiabatic density perturbations; (3) the presence of a spectrum of relic gravitons. Since non-inflation theories can also provide scenarios for the first two, a test of the third prediction should be most decisive^[2]. Efforts have been made towards distinguishing the density perturbation (scalar mode) contribution from the relic graviton (tensor mode) contribution in the cosmic microwave background radiation (CMBR) fluctuations^[3]. This can be used to measure the relic graviton spectrum, or the “gravity’s rainbow”^[4], and thereby test different inflationary models^[4,5]. It was concluded that the observation from the Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) at large angular scales^[6] alone cannot distinguish whether the scalar mode or the tensor mode dominates^[4,5]: CMBR anisotropy measurements at smaller angular scales are needed. Before projects such as LIGO and VIRGO^[7] that can provide direct measurements of the gravitational waves, one wonders if there exists any other imprints of the relic gravitons, so as to offer another glance at the spectrum of this beautiful rainbow, and to enhance our understanding of the inflation.

It has been recently pointed out^[8] that, due to the coupling between the thermal CMBR photons and the background primordial magnetic field in the post-decoupling (or recombination) epoch, the thermal photons can resonantly convert into gravitons with the same frequency, causing a fluctuation in the number and energy flux. Using the observed CMBR fluctuation as a bound, a constraint on the primordial field strength was derived^[8], which is reasonably consistent with that deduced from other astrophysical considerations. Since this effect also allows for

the relic gravitons to convert into photons, it provides the possibility of testing different models of inflation by directly measuring the EM waves converted from the relic gravitons.

Gertsenshtein^[9] first pointed out that a propagating EM wave can couple its field-strength tensor $F_{\mu\nu}$ to that of a transverse background EM field to give rise to a nontrivial energy-momentum stress tensor, which serves as a source for the linearized Einstein equation to excite a gravitational wave. In quantum language this corresponds to a mixing between the propagating photon and a graviton via a Yukawa-type coupling mediated by a virtual photon from the background field.

For a mixed photon-graviton state traversing a constant magnetic field with strength B at an angle Θ , the strength of the mixing is determined by the ratio of the coupling $\Delta_M \approx (B \sin \Theta / M_P)$ and the mass splitting between the photon and the graviton states $\Delta_i(\omega) = (n_i - 1)\omega$, where M_P is the Planck mass and n_i is the refractive index. For a weak mixing, i.e. $\Delta_M / \Delta_i(\omega) \ll 1$, the photon-graviton degeneracy is removed. In this case the transition probability is

$$P(\gamma \rightarrow g) = 4 \left(\frac{\Delta_M}{\Delta_i(\omega)} \right)^2 \sin^2(\Delta_i(\omega)z/2) \quad . \quad (1)$$

If the path z is much longer than the *oscillation length*, $l_{osc} = 2\pi/\Delta_i(\omega)$, then the probability $P \approx 2(\Delta_M/\Delta_i(\omega))^2 \ll 1$. On the other hand, the maximum mixing occurs when $\Delta_i = 0$ (i.e., $k = \omega/c$). Here the degeneracy between the photon and the graviton states is reinstated, and the two are in resonance. Then,

$$P(\gamma \rightarrow g) = \sin^2(\Delta_M z) \quad . \quad (2)$$

In this case a complete transition is possible. In the typical situation, however,

the coupling is so weak that for any physically realistic distance the argument can never reach $\pi/2$.

In our case, we are interested in an expanding Universe. Then for a mildly inhomogeneous, or slowly time-varying field, we have in general^[10]

$$P(\gamma \rightarrow g) = \left| \int_0^z dz' \Delta_M(z') \exp \left\{ -i \int_0^{z'} \Delta_i(z'') dz'' \right\} \right|^2, \quad (3)$$

Note that if $\Delta_i \neq 0$, and $\int \Delta_i dz \lesssim \int \Delta_M dz \ll 1$, then Eq.(3) reduces to $|\int \Delta_M dz|^2 \sim \Delta_M^2 z^2$. This is to say that for a given external field and distance z , there is a broad resonance frequency window which satisfies the condition $\Delta_i(\omega_{\text{res}} \pm \Delta\omega) \ll \pi/z$, and within this window the conversion probability is essentially independent of the photon frequency.

In the post recombination era, $n_i - 1$ is dominated by the atomic polarizability. Thus $\Delta_i(\omega) \sim 2\pi r_e n_a \omega / (\omega_0^2 - \omega^2)$, where r_e is the classical electron radius, n_a is the atomic number density, and ω_0 is the atomic resonant frequency. At the recombination time t_* , the typical photon energy is $T_* \sim 0.3$ eV, and the gas density is $n_{a*} \sim 10^3 \text{ cm}^{-3}$. With $\omega_0 \sim 13.6$ eV, the change in the refractive index gives $\Delta_i(x = \omega/T_* = 1) \sim 6 \times 10^{-17} \text{ cm}^{-1}$. When taking into account the Hubble expansion, the corresponding oscillation length $l_{osc}^*(x = 1) \sim 10^{18} \text{ cm} \ll H_*^{-1} \sim 10^{24} \text{ cm}$. Therefore the resonance window covers only the infrared and the ultraviolet regimes of the CMBR spectrum. For example, for $L_* \sim H_*^{-1}$ the resonance window only covers the $x \lesssim 10^{-5}$ and $x \gtrsim 10^8$ limits of the CMBR spectrum. We emphasize, however, that the induced fluctuation inside these two regimes is essentially *frequency-independent*.

There are several arguments for the existence of an intergalactic magnetic field. For example, to obtain the observed high energy cosmic rays ($E > 10^{20}$ eV), one would need an intergalactic magnetic field with strength of the order $\sim 10^{-7} - 10^{-9}$ G at scales $L_1 \sim 100$ Mpc to confine the accelerated particles^[11]. There have been many proposals regarding the primordial origin of this magnetic field^[12,13,14]. Let us assume that there indeed exist large magnetic domains with size L_* and strength B_* at the recombination time t_* . Although the magnetic domain size can in principle be larger than the Hubble radius H_*^{-1} for models where the magnetic field “seeds” are generated during inflation^[12], for the sake of discussion we shall only focus on the case $L_* \lesssim H_*^{-1}$, the largest possible causally connected domain at the decoupling time. A photon with its frequency inside the resonance window that traverses these magnetic domains has a finite probability of turning into a graviton. This “leakage” of photons into gravitons leads to a fluctuation in the CMBR flux^[8]:

$$P_{rms}(\gamma \rightarrow g) = \langle \delta \rho_\gamma(x) / \rho_\gamma(x) \rangle \sim \frac{3}{2\sqrt{14}} \left(\frac{t_*}{L_*} \right)^{1/2} \frac{B_*^2 L_*^2}{M_P^2} \quad , \quad L_* \lesssim H_*^{-1} \quad , \quad (4)$$

where $\rho_\gamma(x) = (T^4/\pi^2)x^3/(e^x - 1)$.

The anisotropy of such a fluctuation is associated with the only physical scale of the process, namely the bubble size L_* at t_* . Thermal photons arriving at our detector from different angles have crossed different sets of randomly oriented bubbles. So the flux varies at the scale of the bubble size across the sky. For an observer at present, this bubble size has been Hubble-expanded to $L_1 \sim (t_1/t_*)^{2/3} L_*$.

It is clear that the maximum allowed photon-graviton conversion-induced fluctuation can never exceed the observed CMBR fluctuation. Since for the Sachs-

Wolfe effect^[15] we have $\langle \delta \rho_\gamma / \rho_\gamma \rangle_{SW} = (x/(1-e^{-x}))\langle \delta T/T \rangle$, the constraint should be set by the measurements at low frequencies, where the fluctuation, $\langle \delta \rho_\gamma / \rho_\gamma \rangle_{SW} \approx \langle \delta T/T \rangle$, is a minimum.

$$\frac{B_*}{B_c} \lesssim 0.14 \frac{M_p}{m} \frac{\lambda_c}{t_*^{1/4} L_*^{3/4}} \sqrt{\langle \delta T/T \rangle} \quad , \quad L_* \lesssim H_*^{-1} \quad , \quad (5)$$

where $B_c = m^2/e \approx 4.4 \times 10^{13} \text{G}$ is the Schwinger critical field strength. Note that the anisotropy scale $L_1 \sim (t_1/t_*)^{2/3} H_*^{-1} \sim 280 \text{Mpc}$ (the Hubble-expanded horizon size at t_*) corresponds to a coherence angle $\theta_c \sim 1.5^\circ$. scale^[16], which gives $\langle \delta T/T \rangle \sim 1 \times 10^{-5}$, we find $B_* \lesssim 0.03 \text{G}$.

This bound is reasonably consistent^[8] with that deduced from the Big Bang Nucleosynthesis (BBN)^[17]. Moreover, based on Hogan's theory^[18], this field strength corresponds to an intergalactic field of $\sim 10^{-9} \text{G}$ at present, consistent with arguments based on the observed high energy cosmic rays^[11]. Therefore even though this is an upper bound, there is a good reason to assume that $B_* \sim 0.03 \text{G}$ at the scale $L_* \sim H_*^{-1}$. In fact, as we will see later, the exact magnitude of this field is not essential in our effort to distinguish different inflationary models.

Now we look at the conversion of relic gravitons into photons. It can be shown that prior to the decoupling, e.g., during the e - p plasma epoch, the magnetic field and the plasma density are both so high that the resonance window is very narrow around the resonance frequency at any given time: $\omega_{res}(t) = \sqrt{90\pi/7\alpha} [B_c/B(t)] \omega_p(t)$, where ω_p is the plasma frequency. In turn the time for a photon to remain in resonance, or the so-called level crossing, $\Delta t \sim [\sqrt{90\pi/7\alpha} B_c/B(t) (\pi t/\omega_p(t))]^{1/2}$, is very short. As a result the resonant conversion is negligible. Thus the relic graviton spectrum is well preserved until the decoupling time.

The relic gravitons generated during the inflation epoch can be described by a scale-free power spectrum: $P(k) = Ak^{n-1}$. The exponent $n = 1$ corresponds to an exactly scale invariant, Harrison-Zel'dovich spectrum. New inflation^[19] and chaotic inflation^[20] result in a nearly scale-invariant spectrum with logarithmic deviations, whereas the extended inflation^[21] predicts $n \lesssim 0.84$. The CMBR anisotropy measured by COBE^[6] can be fit to this scale-free spectrum and gives $n = 1.1 \pm 0.6(1\sigma \text{ limit})$, which cannot help to further distinguish among the inflationary models. The lower edge of the scale-free or nearly scale-free relic graviton spectrum at present is around $\lambda_{min} \sim 10^7 \text{ cm}$, which corresponds to our normalized frequency $x_{max} = c/(\lambda_{min}T_1) \sim 5 \times 10^{-8}$, where $T_1 \simeq 2.7^\circ \text{ K}$ is the CMBR temperature at present. Thus the entire range of the spectrum lies inside the window of resonance photon-graviton conversion.

Let us introduce the magnetic energy density in units of the critical energy density, ρ_c^* , at t_* : $\Omega_{mag}^* = B_*^2/8\pi\rho_c^*$. For the curvature signature $k = 0$ and the isotropic pressure $p = 0$ we have, from the Friedmann equation, $H_*^2 = (8\pi/3)G\rho_c^*$. Inserting these into Eq.(4), we get

$$P_{rms}(g \rightarrow \gamma) \sim \frac{9}{4\sqrt{7}}\Omega_{mag}^*(H_*L_*)^{3/2} \quad , \quad L_* \lesssim H_*^{-1} \quad . \quad (6)$$

Here the relation $H_*^{-1} \simeq 2t_*$ has been used. For $L_* \sim H_*^{-1}$, we assume that $B \sim 0.03 \text{ G}$. Thus $\Omega_{mag}^* \sim 10^{-5}$, and from Eq.(6) we find $P_{rms}(g \rightarrow \gamma) \sim 10^{-5}$ as well.

Constraints on the graviton density at the maximum wavelength ($\lambda_1 \sim H_1^{-1}$) gives the maximum possible energy density $\Omega_{GW} \sim 10^{-14}$ at present^[2]. To account for the tilt of the graviton spectrum, we parameterize the variation by the function

$(x_{max}/x)^{n-1}$. We thus expect the density of the photons (EM waves) resonantly converted from the relic gravitons to be

$$\delta\Omega_{EM}(x) \simeq P_{rms}(g \rightarrow \gamma)\Omega_{GW}\left(\frac{x_{max}}{x}\right)^{n-1}. \quad (7)$$

Note that although $P_{rms}(g \rightarrow \gamma)\Omega_{GW} \sim 10^{-19}$, $\delta\Omega_{EM}$ is still about 6 orders of magnitude above the CMBR density at x_{max} . Any detection of EM waves in this frequency regime will be a direct signal from the relic gravitons. The power law index n can be extracted by measuring the variations of $\delta\Omega_{EM}(x)$ in a range of frequencies without the need for the exact value on $P_{rms}(g \rightarrow \gamma)$.

In addition to the nearly scale-free spectrum, one also expects abundant gravitons generated through the bubble collisions in the extended inflation model^[2], with the peak of the spectrum at $\lambda \sim 10^8\text{cm}$ and a normalized energy density $\Omega_{BC} \sim 10^{-5}$. This spectrum should presumably leave a much more pronounced imprint on the resonant photon-graviton conversion signal.

We have shown that measurements of the EM waves resonantly converted from the relic gravitons may help to determine the spectrum power law index, which distinguishes among various inflation models. In addition the relic gravitons generated by bubble collisions should also leave an imprint through this effect. Unlike the case of CMBR fluctuation induced by the Sachs-Wolfe effect, where the tensor mode is mixed with the scalar mode rendering the extraction of the individual mode difficult, in our case the effect is sensitive only to the tensor mode. This will open a new window for another, more direct, glance at gravity's rainbow. This is a very exciting prospect. For a scenario as promising as the

inflationary Universe, it surely would help if we are able to examine it from a different perspective.

ACKNOWLEDGEMENT

I thank David Schramm for the stimulating discussion that triggered this work, and Anatoly Spitkovsky for reading the manuscript.

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