# Tau Polarization Asymmetry in $B \rightarrow X_{s} \tau^{+} \tau^{-}$* 

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#### Abstract

Rare $B$ decays provide an opportunity to probe for new physics beyond the Standard Model. In this paper, we propose to measure the tau polarization in the inclusive decay $B \rightarrow X_{s} \tau^{+} \tau^{-}$and discuss how it can be used, in conjunction with other observables, to completely determine the parameters of the flavor-changing low-energy effective Hamiltonian. Both the Standard Model and several new physics scenarios are examined. This process has a large enough branching fraction, $\sim \mathrm{few} \times 10^{-7}$, such that sufficient statistics will be provided by the B-Factories currently under construction.


[^0]The recent first observation[1] of the inclusive and exclusive radiative decays $B \rightarrow$ $X_{s} \gamma$ and $B \rightarrow K^{*} \gamma$ have placed the study of rare $B$ decays on a new footing. These flavor changing neutral current (FCNC) transitions provide fertile testing ground for the Standard Model (SM) and offer a complementary strategy in the search for new physics by probing the indirect effects of new particles and interactions in higher order processes. In particular, the probing of loop induced couplings can provide a means of testing the detailed structure of the SM at the level of radiative corrections where the Glashow-Iliopoulos-Maiani cancellations are important. These first measurements have restricted the magnitude of the electromagnetic penguin transition, resulting in bounds on the value[2] of the ratio of Cabibbo-Kobayashi-Maskawa (CKM) weak mixing matrix elements $\left|V_{t s} / V_{c b}\right|$, as well as providing powerful constraints on new physics[3] which in some classes of models complement or surpass the present bounds obtainable from direct collider searches.

The study of rare $B$ decays can be continued with the analysis of the higher order process $B \rightarrow X_{s} \ell^{+} \ell^{-}$. The experimental situation for these decays is very promising[4], with $e^{+} e^{-}$and hadron colliders closing in on the observation of exclusive modes with $\ell=e$ and $\because \mu$ final states, respectively. These transitions proceed via electromagnetic and $Z$ penguin as well as $W$ box diagrams, and hence can probe different coupling structures than the pure electromagnetic process $B \rightarrow X_{s} \gamma$. Investigation of this decay mode offers exciting possibilities as various kinematic distributions associated with the final state lepton pair, such as the lepton pair invariant mass spectrum and the lepton pair forward-backward asymmetry, can also be measured in addition to the total rate. These distributions are essential in separating the short distance FCNC processes from the contributing long range physics[5]. In particular, it has been shown $[6,7]$ that the lepton pair forward-backward asymmetry is sizable for large values of the top-quark mass and is highly sensitive to contributions from new physics. Ali et al.[7] have proposed a program to use these distributions, as well as the
total rate for $B \rightarrow X_{s} \gamma$, to determine the sign and magnitude of each class of short distance FCNC contribution in a model independent fashion. Unfortunately, this does not provide enough observable quantities to uniquely determine each contribution. Here, we propose a new observable, the tau polarization asymmetry for the decay $B \rightarrow X_{s} \tau^{+} \tau^{-}$. We will show that this asymmetry also has a large value for top-quarks in the mass range observed[8] at the Tevatron, and will be measurable with the high statistics available at the B-Factories presently under construction. The tau polarization asymmetry (and the $M_{\tau \tau}$ spectrum) together with the above kinematic distributions, will then provide a complete arsenal for a stringent test of the SM.

The transition rate for $B \rightarrow X_{s} \ell^{+} \ell^{-}$, including QCD corrections, is computed[9] via an operator product expansion based on the the effective Hamiltonian,

$$
\begin{equation*}
\mathcal{H}_{e f f}=\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \sum_{i=1}^{10} C_{i}(\mu) \mathcal{O}_{i}(\mu) \tag{1}
\end{equation*}
$$

which is evolved from the electroweak scale down to $\mu \sim m_{b}$ by the Renormalization Group Equations. Here $V_{i j}$ represents the relevant CKM factors, and the $\mathcal{O}_{i}$ are a complete set of renormalized dimension five and six operators involving light fields which govern $b \rightarrow s$ transitions. This basis (involving left-handed fields only) consists of six four-quark operators $\mathcal{O}_{1-6}$, the electro- and chromo-magnetic operators respectively denoted as $\mathcal{O}_{7,8}$, $\mathcal{O}_{9} \sim e \bar{s}_{L \alpha} \gamma_{\mu} b_{L \alpha} \bar{\ell} \gamma^{\mu} \ell$, and $\mathcal{O}_{10} \sim e \bar{s}_{L \alpha} \gamma_{\mu} b_{L \alpha} \bar{\ell} \gamma^{\mu} \gamma_{5} \ell$. For $B \rightarrow X_{s} \ell^{+} \ell^{-}$, this effective Hamiltonian leads to the matrix element (neglecting the strange quark mass)

$$
\begin{equation*}
M=\frac{\sqrt{2} G_{F} \alpha}{\pi} V_{t b} V_{t s}^{*}\left[C_{9}^{e f f} \bar{s}_{L} \gamma_{\mu} b_{L} \bar{\ell} \gamma^{\mu} \ell+C_{10} \bar{s}_{L} \gamma_{\mu} b_{L} \bar{\ell} \gamma^{\mu} \gamma_{5} \ell-C_{7} m_{b} \bar{s}_{L} i \sigma_{\mu \nu}^{q^{2}} b_{R} \bar{\ell} \gamma^{\mu} \ell\right] \tag{2}
\end{equation*}
$$

where $q^{2}$ is the momentum transferred to the lepton pair. The Wilson coefficients $C_{i}$ of the $b \rightarrow s$ operators are evaluated perturbatively at the electroweak scale where the matching
conditions are imposed and are then evolved down to the renormalization scale $\mu$. $C_{7-10}\left(M_{W}\right)$ are given by the Inami-Lim functions $[10], C_{2}\left(M_{W}\right)=-1$, and $C_{1,3-6}\left(M_{W}\right)=0$. The expressions for the QCD-renormalized coefficients $C_{i}(\mu)$ are given explicitly in Refs. [7, 9]. The effective coefficient of $\mathcal{O}_{9}$ is defined by $C_{9}^{\text {eff }}(\mu) \equiv C_{9}(\mu)+Y\left(\mu, q^{2}\right)$ where the function $Y$ contains the contributions from the one-loop matrix element of the four-quark operators and can be found in Refs. [7, 9]. We note that $Y\left(\mu, q^{2}\right)$ contains both real and imaginary contributions (the imaginary piece arises when the c-quarks in the loop are on-shell). The differential branching fraction for $B \rightarrow X_{s} \tau^{+} \tau^{-}$is then

$$
\begin{align*}
\frac{d B\left(B \rightarrow X_{s} \tau^{+} \tau^{-}\right)}{d \hat{s}}=B(B \rightarrow & X \ell \bar{\nu}) \frac{\alpha^{2}}{4 \pi^{2}} \frac{\left|V_{t b} V_{t s}^{*}\right|^{2}}{\left|V_{c b}\right|^{2}} \frac{(1-\hat{s})^{2}}{f(z) \kappa(z)}\left[1-\frac{4 x}{\hat{s}}\right]^{1 / 2}\left\{\left[\left|C_{9}^{e f f}\right|^{2}-\left|C_{10}\right|^{2}\right] 6 x\right. \\
& +\left[\left|C_{9}^{e f f}\right|^{2}+\left|C_{10}\right|^{2}\right]\left[(\hat{s}-4 x)+\left(1+\frac{2 x}{\hat{s}}\right)(1+\hat{s})\right]  \tag{3}\\
& \left.+12 C_{7} \mathcal{R e} C_{9}^{e f f}\left(1+\frac{2 x}{\hat{s}}\right)+\frac{4\left|C_{7}\right|^{2}}{\hat{s}}\left(1+\frac{2 x}{\hat{s}}\right)(2+\hat{s})\right\}
\end{align*}
$$

with all Wilson coefficients evaluated at $\mu \sim m_{b}, \hat{s} \equiv q^{2} / m_{b}^{2}, x \equiv m_{\tau}^{2} / m_{b}^{2}, z \equiv m_{c} / m_{b}$, and $f(z)$ and $\kappa(z)$ represent the phase space and QCD corrections[11], respectively, to the semileptonic rate. This agrees with the literature in the zero lepton mass limit. The differential branching fraction is scaled to that of the semi-leptonic decay $B \rightarrow X \ell \nu$ to remove the uncertainties associated with the overall factor of $m_{b}^{5}$ and to reduce the ambiguities involved with the imprecisely determined CKM factors.
$B \rightarrow X_{s} \ell^{+} \ell^{-}$also receives large long distance contributions from the tree-level processes $B \rightarrow K^{(*)} \psi^{\left({ }^{( }\right)}$followed by $\psi^{\left({ }^{\prime}\right)} \rightarrow \ell^{+} \ell^{-}$. These pole contributions are incorporated into the lepton pair invariant mass spectrum following the prescription in Ref. [5], where both on- and off-shell vector mesons are considered by employing a Breit-Wigner form for

| $\ell$ | $4 x \leq \hat{s} \leq 1$ | $0.6 \leq \hat{s} \leq 1$ |
| :--- | :--- | :---: |
| $e$ | $1.2 \times 10^{-5}$ | $8.5 \times 10^{-7}$ |
| $\mu$ | $1.0 \times 10^{-5}$ | $8.5 \times 10^{-7}$ |
| $\tau$ | $5.4 \times 10^{-7}$ | $4.3 \times 10^{-7}$ |

Table 1: Integrated branching fractions for $B \rightarrow X_{s} \ell^{+} \ell^{-}$for the total and high dilepton mass regions.
the resonance propagator. This produces an additional contribution to $C_{9}^{\text {eff }}$ of the form

$$
\begin{equation*}
\frac{-3 \pi}{\alpha^{2} m_{b}^{2}} \sum_{V_{i}=\psi, \psi^{\prime}} \frac{M_{V_{i}} \Gamma\left(V_{i} \rightarrow \ell^{+} \ell^{-}\right)}{\left(\hat{s}-M_{V_{i}}^{2} / m_{b}^{2}\right)+i \Gamma_{V_{i}} M_{V_{i}} / m_{b}^{2}} \tag{4}
\end{equation*}
$$

The resulting differential branching fraction for $B \rightarrow X_{s} \ell^{+} \ell^{-}$, with and without the long distance resonance contributions, is presented in Fig. 1a for both $\ell=e$ and $\tau$, taking $m_{t}=180 \mathrm{GeV}, m_{b}=4.87 \mathrm{GeV}$, and $z=0.316$. We see that the pole contributions clearly overwhelm the branching fraction near the $\psi$ and $\psi^{\prime}$ peaks, and that there is significant interference between the dispersive part of the resonance and the short distance contributions. However, suitable $\ell \bar{\ell}$ invariant mass cuts can eliminate the resonance contributions, and observations away from these peaks cleanly separate out the short distance physics. This divides the spectrum into two distinct regions[7], (i) low-dilepton mass, $4 x \leq \hat{s} \leq M_{\psi}^{2} / m_{b}^{2}-\delta$, and (ii) high-dilepton mass, $M_{\psi^{\prime}}^{2} / m_{b}^{2}+\delta \leq \hat{s} \leq \hat{s}_{\max }$, where $\delta$ is to be matched to an experimental cut. The integrated branching fractions (without the pole contributions) for $\ell=e, \mu, \tau$ are presented in Table 1 for both the total and high dilepton mass regions of $\hat{s}$. We note that the branching fraction for $B \rightarrow X_{s} \tau^{+} \tau^{-}$is comparable to that for $\ell=e, \mu$ in the clean $\hat{s}$ region above the $\psi^{\prime}$ resonance. The exclusive decay $B \rightarrow K \tau^{+} \tau^{-}$has been considered in Ref. [12], where the exclusive branching fraction was found to be $\sim 50 \%$ of the inclusive; this places $B\left(B \rightarrow K \tau^{+} \tau^{-}\right)$in the range $(1-2) \times 10^{-7}$.

The tau polarization asymmetry is defined as

$$
\begin{equation*}
P_{\tau}(\hat{s}) \equiv \frac{d B_{\lambda=-1}-d B_{\lambda=+1}}{d B_{\lambda=-1}+d B_{\lambda=+1}} \tag{5}
\end{equation*}
$$

where $d B$ represents the differential $B \rightarrow X_{s} \tau^{+} \tau^{-}$branching fraction. The spin projection operator is represented as $\left(1+\gamma_{5} \delta\right) / 2$, with the normalized dot product being defined as $\hat{\mathbf{s}} \cdot \hat{p}=\lambda= \pm 1$ with the $-(+)$ sign corresponding to the case where the spin polarization is anti-parallel (parallel) to the direction of the $\tau^{-}$momentum. This corresponds to the usual definition of a polarization asymmetry, given in terms of couplings, i.e., $(L-R) /(L+R)$, in the massless case. For the process $B \rightarrow X_{s} \tau^{+} \tau^{-}$this asymmetry is then calculated to be

$$
\begin{equation*}
P_{\tau}(\hat{s})=\frac{-2[1-4 x / \hat{s}]^{1 / 2} C_{10}\left[\mathcal{R e}^{e} C_{9}^{e f f}(1+2 \hat{s})+6 C_{7}\right]}{D} \tag{6}
\end{equation*}
$$

where $D$ is given by the expression in the curly brackets in Eq. (4). The tau polarization asymmetry is displayed as a function of $\hat{s}=q^{2} / m_{b}^{2}$ in Fig. 1b, with and without the long distance resonance contributions, and taking $m_{t}=180 \mathrm{GeV}$. We see that the asymmetry vanishes at threshold and grows with increasing $\hat{\boldsymbol{s}}$. The value of the total integrated asymmetry (i.e., averaged over the high dilepton mass region, $\hat{s} \geq 0.6$ ) is -0.484 . The experimentally relevant number of events required to measure an asymmetry $a$ at the $n \sigma$ level is $N=n^{2} / B a^{2}$, and is given here by $N=n^{2} /\left(4.3 \times 10^{-7}\right)(-0.484)^{2}=\left(n^{2}\right) 9.9 \times 10^{6}$ for the inclusive decay. The exclusive case of $B \rightarrow K \tau^{+} \tau^{-}$would then yield $N \sim\left(n^{2}\right) 1.9 \times 10^{7}$. This result demonstrates that $P_{\tau}$ will be accessible at the $5 \sigma$ level (even when $\tau$ identification efficiencies are taken into account) at the B-Factories under construction, which will produce $\sim 10^{9} B$ mesons per year.

We now explore the sensitivity of $P_{\tau}$ to new physics. We first investigate the influence of a change in sign of the short distance contributions to $C_{7-10}$ (holding the magni-
tudes constant). The results are shown in Fig. 2a, where the dashed, dash-dotted, dotted, solid, and long-dashed curves represent the polarization asymmetry with $C_{10}\left(M_{W}\right) \rightarrow$ $-C_{10}\left(M_{W}\right), C_{9,10}\left(M_{W}\right) \rightarrow-C_{9,10}\left(M_{W}\right), C_{9}\left(M_{W}\right) \rightarrow-C_{9}\left(M_{W}\right)$, the SM , and $C_{7,8}\left(M_{W}\right) \rightarrow$ $-C_{7,8}\left(M_{W}\right)$, respectively. We see that there is large sensitivity to any combination of sign changes in $C_{9,10}\left(M_{W}\right)$, but little variation to a sign change in the electro- and chromomagnetic operator coefficients. This is due to the fact that the operators $\mathcal{O}_{9,10}$ dominate the decay in the high $\hat{s}$ region. We next examine $P_{\tau}$ in two-Higgs-Doublet models of type II, where a charged Higgs boson participates in the decay via virtual exchange in the $\gamma, Z$ penguin and box diagrams. The modifications to the Wilson coefficients in this model are given in Deshpande et al.[13]. The resulting tau polarization asymmetry (with $\hat{s}=0.7$ and $m_{t}=180 \mathrm{GeV}$ ) for various values of the charged Higgs mass is presented in Fig. 2b as a function of $\tan \beta \equiv v_{2} / v_{1}$, the ratio of vacuum expectation values of the two doublets. We see that the effect of the $H^{ \pm}$is negligible for values of the parameters which are consistent with the present constraints from $B \rightarrow X_{s} \gamma[1,14]$, i.e., $\tan \beta \gtrsim 1$ and $m_{H^{ \pm}} \gtrsim 240 \mathrm{GeV}$. Finally, we study the effects of anomalous trilinear gauge boson cou$\because$ plings in $B \rightarrow X_{s} \ell^{+} \ell^{-}$. The dependence of the $C_{i}\left(M_{W}\right)$ on these anomalous couplings can be found in Ref. [15]. Figure 2c displays the deviation of $P_{\tau}$ (for $\hat{s}=0.7$ and $m_{t}=180 \mathrm{GeV}$ ) with non-vanishing values of the anomalous magnetic dipole and electric quadrupole $W W \gamma$ coupling parameters, $\Delta \kappa_{\gamma}$ and $\lambda_{\gamma}$, respectively, and of the parameter $g_{5}^{Z}$ which governs the term $i g_{5}^{Z} \epsilon^{\mu \nu \lambda \rho}\left(W_{\mu}^{\dagger} \partial_{\lambda} W_{\nu}-W_{\nu} \partial_{\lambda} W_{\mu}^{\dagger}\right) Z_{\rho}$ in the $W W Z$ Lagrangian. For the anomalous coupling parameters considered here, $B \rightarrow X_{s} \ell^{+} \ell^{-}$naturally avoids the problem of introducing cutoffs to regulate the divergent loop integrals due to cancellations provided by the GIM mechanism[15]. As expected, we find little sensitivity to modifications in $C_{7,8}\left(M_{W}\right)$ from anomalous $W W \gamma$ couplings, but a large variation due to the influence of anomalous $W W Z$ vertices in $C_{9,10}\left(M_{W}\right)$.

In conclusion, we have shown that measurement of the $\tau$ polarization in $B \rightarrow X_{s} \tau^{+} \tau^{-}$ is highly sensitive to new physics and hence provides a powerful probe of the SM. Together, measurement of the polarization asymmetry and the remaining kinematic distributions associated with $B \rightarrow X_{s} \ell^{+} \ell^{-}$, will provide enough information to completely determine the parameters of the FCNC effective Hamiltonian. We find that the values of the polarization can be precisely determined with the large data samples that will be available at the BFactories presently under construction. We eagerly await the completion of these machines!

## ACKNOWLEDGEMENTS

The author thanks T.G. Rizzo for invaluable discussions, and the Phenomenology Institute at the University of Wisconsin for their hospitality while this work was completed.

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Figure 1: (a) Differential branching fraction and (b) tau polarization asymmetry as a function of $\hat{s}$ for $\ell=\tau$ (solid and dashed curves) and $\ell=e$ (dotted and dash-dotted curves), with and without the long distance resonance contribution.




Figure 2: Tau polarization asymmetry (a) with changes in the sign of the Wilson coefficients at the electroweak scale, corresponding to $C_{10}, C_{9,10}, C_{9}$, SM $C_{7,8}$ from top to bottom; (b) in two-Higgs-doublet models as a function of $\tan \beta$ with $m_{H^{ \pm}}=50,100,250,500$ corresponding to the solid, dashed, dotted, and dash-dotted curves, respectively. The SM value is denoted by the solid horizontal line. (c) with anomalous couplings $W W \gamma$ and $W W Z$ couplings as described in the text.


[^0]:    *Work supported by the Department of Energy, Contract DE-AC03-76SF00515

