CHANNELING ACCELERATION: A PATH TO ULTRAHIGH ENERGY COLLIDERS^{*}

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ABSTRACT

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Acceleration of charged particles along crystal channels has been proposed earlier in an attempt to achieve high acceleration gradient while at the same time to suppress excessive emittance growth. Recently we demonstrated that a particle in a generic focusing channel can in principle absolutely damp to its transverse ground state without any quantum excitation. This yields the minimum beam emittance that one can ever attain, $\gamma \epsilon_{min} = \hbar/2mc$, limited only by the uncertainty principle. In this paper we discuss sources of excitation when a more realistic channel is considered, including bremsstrahlung and multiple Coulomb scattering. We investigate the possibility of colliding ultrahigh energy particles in such strong focusing channels without the need of a final focusing system, where the concept of luminosity departs from the conventional approach. We show that a high luminosity can be attained with a rather modest beam power.

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1. INTRODUCTION-WHY ACCELERATION IN CRYSTALS?

It is known that in plasma accelerators^[1] the theoretically attainable particle acceleration gradient is $G \sim \sqrt{n_p} V/cm$, where n_p is the plasma ambient density. For a plasma density $n_p \sim 10^{18} \text{cm}^{-3}$, the corresponding gradient is $G \sim 1 \text{ GeV/cm}$, much larger than that in the conventional accelerators, and this is one of the main attractions for novel accelerator concepts, with the hope that the construction cost of future accelerators can be lowered.

However, a high gradient is not the only requirement for high energy linear colliders. Stability and emittance requirements for the accelerating system are very stringent. Since the beams from two independent accelerators must collide at an interaction point, excessive transverse motion and emittance growth of the beams induced during acceleration must be avoided. Motivated by these considerations, Chen and Noble^[2,3] proposed to accelerate particles along crystal channels by the plasma waves excited in the metallic crystal. It is known that the conduction electrons in a metal form a very uniform high-density plasma exhibiting longitudinal plasma oscillations.^[4] Typical conduction electron densities are of order 10^{22} cm⁻³ corresponding to a maximum gradient of order 100 GeV/cm! In an independent effort, Tajima and Cavenago^[5] suggest to power the crystal acceleration by an external source of x-ray. An extremely large acceleration gradient is also expected in this approach. But more importantly, it is hoped that the crystal channels can in addition help to stabilize the accelerated beam and to preserve its emittance.

Motivated by these crystal accelerator concepts and by the interest of the fundamental phenomena of radiation reaction, we have recently shown^[6,7] that in a generic continuous focusing channel the radiation reaction of the channeled particle is quantum excitation free, and this absolute damping effect leads to the existence of a transverse ground state which the particle inevitably decays to. In this paper we first review these novel characters in the channeling radiation reaction. We then further demonstrate that once at the ground state, other sources of excitation in a more realistic crystal channel, such as bremsstrahlung and multiple Coulomb scattering, are not deleterious enough to compete against the strong channeling radiation damping. We thus argue that the ground state of the channeled particle can in principle be preserved, at least in semiconducting crystals. Due to adiabatic invariance, one expects that the particle in the ground state can be accelerated without any radiation loss, and the theoretical minimum emittance can be preserved.

There are, however, further constraints in the conventional approach to linear colliders. Even if the beam emittance can be well-preserved during acceleration, the deliverable luminosity is constrained by the fundamental physics such as the Oide limit^[8] in the final focus and beamstrahlung and its backgrounds^[9] from beam-beam interaction. One naturally wonders how much further can the conventional approach be extended before these constraints severely limit the attainable luminosity. Earlier, the concept of *adiabatic focusing*^[10] was introduced as a means to circumvent the Oide limit. Nevertheless, the fact that it is still a focusing system, the adiabatic focusing helps only to relax the limit, but not to eliminate it. The existence of the channeling ground state gives us an entirely different prospect. We can envision the acceleration and collision of channeled particles in their transverse ground states, without the need of a final focusing system. Under this scenario the luminosity

is associated with the overlap of the quantum mechanical wave functions of the colliding particles. We investigate the luminosity attainable in such an approach.

2. ABSOLUTE DAMPING IN CHANNELING RADIATION

Consider an electrostatic focusing channel that provides a transverse continuous potential $V(x) = Kx^2/2$ for a charged particle, say a positron, where K is the focusing strength. The parabolic potential could be, for example, an approximation of the Lindhard potential in the case of planar crystal channeling^[11,12] For simplicity, we take x as the single transverse dimension of the particle, which has relativistic energy $E = \gamma m$ and which moves freely (without acceleration) in the longitudinal z-direction with a constant momentum $p_z = \gamma m \beta_z$ in the absence of radiation. We set $e = \hbar = c = 1$ in most equations, but reinsert them when suitable. The effect of the additional transverse dimension will be discussed later. We consider the case in which the peak transverse momentum in one oscillation $p_{x,max} \ll p_z$. Defining $E_z = \sqrt{m^2 + p_z^2}$, we can approximate the total energy, $E = \sqrt{m^2 + p_z^2} + V(x)$, as $E_z + E_x$, where $E_x = p_x^2/2E_z + V(x)$ is the so-called transverse energy. The motion of the particle is now decoupled into two parts: a free relativistic longitudinal motion and a transverse harmonic oscillation with an effective mass E_z .

We now move straight to quantum mechanical analysis of the system because we want to calculate the full radiation reaction including damping and excitation due to discrete photon emissions. Work on relativistic crystal channeling has shown that the spin degree of freedom plays a negligible role^[13] Therefore, we use the Klein-Gordon equation to describe the general wavefunction $\Psi(x, z, t)$ of the channeled particle,

$$[(-i\vec{\nabla} - \vec{A})^2 + m^2]\Psi = (i\partial_t - V)^2\Psi \quad . \tag{1}$$

In the absence of radiation, we let $\vec{A} = 0$ and look for the energy levels E and the stationary states $\Psi(x, z, t) = e^{-iEt} |n, p_z\rangle$ of Eq.(1) by neglecting terms of the order $(E_x/E)^{2[14]}$. We find

$$E \simeq E_z + E_x = \sqrt{m^2 + p_z^2} + \omega_z (n + 1/2)$$
 , (2)

$$|n, p_z\rangle = (C_n/L)^{1/2} (E_z \omega_z)^{1/4} e^{ip_z z} e^{-E_z \omega_z x^2/2} H_n(\sqrt{E_z \omega_z} x) \quad , \tag{3}$$

where $C_n = (2^n n! \sqrt{\pi})^{-1}$, L is the length of the channel, $E_z = \sqrt{m^2 + p_z^2}$ as before, $\omega_z = \sqrt{K/E_z}$ is the transverse oscillation frequency, n is the transverse quantum number (n = 0, 1, 2...), and H_n is the n^{th} order Hermite polynomial. It is clear that the transverse energy level $E_x = (n + 1/2)\omega_z$ and the transverse state function are controlled by both n and p_z .

Coupling between the channeled particle and the radiation field, represented by the vector potential \vec{A} in Eq.(1), leads to spontaneous emission of photons. By choosing Coulomb gauge, $\vec{\nabla} \cdot \vec{A} = 0$, and ignoring the \vec{A}^2 term (double-photon emission), we arrive at

$$[-\nabla^2 + m^2 + i2\vec{A} \cdot \vec{\nabla}]\Psi(x, z, t) = (i\partial_t - V)^2\Psi(x, z, t) \quad .$$
(4)

Moving to the interaction representation we write $\Psi(x, z, t) = \exp(-i\mathcal{H}_0 t)\psi(x, z, t)$. Identifying $(\mathcal{H}_0 - V)^2 = (-\nabla^2 + m^2)$, and neglecting $\ddot{\psi}(t)$ in the expansion of $(i\partial_t - V)^2\Psi(t)$ in Eq.(4), we obtain

$$\dot{\psi}(t) = e^{i\mathcal{H}_0 t} [(\mathcal{H}_0 - V)^{-1} \vec{A} \cdot \vec{\nabla}] e^{-i\mathcal{H}_0 t} \psi(t) \quad .$$
(5)

Using first-order, time-dependent perturbation theory (Fermi's Golden Rule), we obtain the transition rate W_{fi} for the particle from an initial state $|n, p_z\rangle$ (with energy E) to a final state $|n', p'_z\rangle$ (with energy E'):

$$W_{fi} = 2\pi |M_{fi}|^2 \delta(E - E' - \omega_\gamma) \quad , \tag{6}$$

where the matrix element M_{fi} is defined by

$$|M_{fi}|^{2} = \left| \langle n', p'_{z}; k_{\gamma} | (\mathcal{H}_{0} - V)^{-1} \vec{A} \cdot \vec{\nabla} | n, p_{z}; 0 \rangle \right|^{2} \quad .$$
⁽⁷⁾

The vector potential \vec{A} acting on the radiation field creates a photon of momentum \vec{k}_{γ} and energy $\omega_{\gamma}(\omega_{\gamma} = |\vec{k}_{\gamma}|)$ with two possible polarizations \hat{e}_1 and \hat{e}_2 $(\hat{e}_1 \cdot \hat{e}_2 = 0 \text{ and } \hat{e}_{1,2} \cdot \vec{k}_{\gamma} = 0)$. The operator $(\mathcal{H}_0 - V)^{-1}$ can be approximated as \mathcal{H}_0^{-1} by neglecting terms of the order (E_x/E) . Therefore

$$|M_{fi}|^2 \simeq \frac{2\pi}{E'^2 \omega_{\gamma}} \sum_{j=1}^2 \left| \langle n', p_z' | e^{-i\vec{k}_{\gamma} \cdot \vec{x}} (\hat{e}_j \cdot \vec{\nabla}) | n, p_z \rangle \right|^2 \quad . \tag{8}$$

The integral over z in the above equation gives rise to $\delta(p_z - p'_z - k_{\gamma z})$, which expresses the conservation of longitudinal momentum. Together with the conservation of energy, this places a tight constraint on the radiation reaction of the particle. In order to conserve longitudinal momentum, we have $p'_z = p_z - \omega_\gamma \cos \theta$, where θ is the photon emission angle relative to the focusing axis. For the photon energy $\omega_\gamma \ll E$, the longitudinal energy, $E_z = \sqrt{m^2 + p_z^2}$, must accordingly decrease by an amount $\Delta E_z \simeq (p_z/E_z)\Delta p_z = \omega_\gamma\beta\cos\theta$. Since the total energy of the particle is reduced by an amount ω_γ , its transverse energy $E_x = E - E_z$ must decrease by $\Delta E_x = \omega_\gamma(1 - \beta\cos\theta) > 0$. It follows that $(n + 1/2)\omega_z - (n' + 1/2)\omega'_z = \omega_\gamma(1 - \beta\cos\theta) > 0$. For a small change in E_z , $\omega'_z = \sqrt{K/(E_z - \Delta E_z)} \simeq \omega_z(1 + \Delta E_z/2E_z)$. Substituting

 $\omega_{\gamma}\beta\cos\theta$ for ΔE_z , we obtain an equation that relates the change of the transverse quantum number to the photon energy and its emission angle,

$$(n - n')\omega_z = (1 - \beta\cos\theta)\omega_\gamma + (\omega_\gamma\beta\cos\theta)E_x/2E_z > 0 \quad , \tag{9}$$

which is always positive definite. We therefore conclude that both the transverse energy and the transverse quantum number always decrease after a photon emission process for all possible photon angles.

Introducing the harmonic number $\nu = n - n'$ and the pitch angle of the particle $\theta_p = p_{x,max}/p_z \simeq \sqrt{2E_x/E_z}$, we find from Eq.(9) a condition for the photon energy

$$\omega_{\gamma} \simeq \frac{\nu \omega_z}{1 - \beta \cos \theta + \theta_p^2/4} \simeq \frac{2\gamma^2 \nu \omega_z}{1 + \gamma^2 \theta^2 + \gamma^2 \theta_p^2/2} \quad . \tag{10}$$

Note that $\gamma \theta_p$ in the above equation plays the same role as the undulator strength parameter in undulator radiation^[15].

The exact form of the transition rate W_{fi} given by the integral over x in Eq.(8) is more complex than usual because the initial and the final transverse states have different effective masses. This issue is handled by expanding the final transverse wavefunction as a superposition of the initial ones. One can then express W_{fi} in terms of associated Legendre polynomials and Laguerre functions^[13,14]. However, in the "undulator" regime where $\gamma \theta_p \ll 1$, the effective mass difference can be ignored for $\omega_{\gamma} \ll E$, and Eq.(8) can be evaluated by the dipole approximation^[13] where terms beyond the linear order in x are neglected. Thus, the transition rate is nonzero only if n' = n - 1 (the dipole selection rule) and is simply given by

$$W_{fi} = \frac{2\pi^2 n\omega_z}{E_z \omega_\gamma} \left[\frac{\cos^2 \phi (\cos \theta - \beta)^2}{(1 - \beta \cos \theta)^2} + \sin^2 \phi \right] \delta[(1 - \beta \cos \theta)\omega_\gamma - \omega_z] \quad . \tag{11}$$

Therefore, in this regime the rate of change of the particle's total energy due to dipole radiation is

$$\frac{dE}{dt} = \sum_{f} \int \frac{d^3 \vec{k}_{\gamma}}{(2\pi)^3} (E' - E) W_{fi} = -\frac{2}{3} \frac{r_e K}{mc} \gamma^2 n \hbar \omega_z \quad , \tag{12}$$

where $r_e = e^2/mc^2$ is the classical electron radius. After identifying $n\hbar\omega_z$ with the *rms* amplitude of the oscillating particle in the large *n* limit $(n\hbar\omega_z \simeq E_x = K\langle x^2 \rangle)$, we see that dE/dt in the above expression is identical to the classical radiation power, which is proportional to $E^2 F_{\perp}^2$ (F_{\perp} being the transverse focusing field strength).

We have shown from Eq.(9) that the transverse quantum level n always decreases after a random photon emission. This conclusion is valid for all oscillation amplitudes, although we focus on the undulator regime where $\gamma \theta_p \ll 1$ to illustrate

the unique feature of radiation reaction in a focusing channel. With the dipole transition rate given by Eq.(11), we can calculate the rate of change of the transverse quantum level

$$\frac{dn}{dt} = \sum_{f} \int \frac{d^3 \vec{k}_{\gamma}}{(2\pi)^3} (n' - n) W_{fi} = -\frac{2}{3} \frac{r_e K}{mc} n \quad . \tag{13}$$

We see that n damps exponentially with an energy-independent damping constant, $\Gamma_c = 2r_e K/3mc$. Note that in the case of radiation in a bending magnet, there is an additional term of opposite sign independent of the quantum level in question that represents the excitation of transverse oscillations^[16]. That term is absent in Eq.(13) and the radiation damping is absolute because no quantum excitation is induced by random photon emissions. Since the action of the transverse oscillation is $J_n = E_x/\omega_z = (n+1/2)\hbar$, the decrement of the transverse energy level n leads to the radiation damping of this action given by $dJ_n/dt = -\Gamma_c(J_n - \hbar/2)$.

One can use classical radiation reaction to obtain a similar result for the radiation damping of the transverse oscillation amplitude^[17]. However, our treatment shows that it is the action that damps exponentially (the change of energy modifies the amplitude damping). It also clearly shows how to extend the results to the case where $\gamma \theta_p \gtrsim 1$. More importantly, the quantum mechanical calculation above automatically takes into account the full radiation reaction and shows the absence of excitation in this system (a surprising result viewed from the standpoint of electron synchrotrons and storage rings). It is difficult if not impossible to model the radiation reaction effect of discrete photon emissions *classically* for $\gamma \theta_p \ll 1$, because the time during which a typical photon is emitted is much longer than the oscillation period in the undulator regime^[18].

The excitation-free reaction of radiation comes from the fact that the transverse quantum level must *decrease* after each radiation process. In the longitudinal direction the particle recoils against the emitted photon in order to conserve the longitudinal momentum between the two particles. However in the transverse direction the existence of the focusing force destroys the momentum balance and suppresses the recoil effect. The external focusing environment absorbs the excess transverse momentum during the process of radiation. In this sense, the radiation reaction of a channeled particle in the transverse dimension is similar to that in the Mössbauer effect^[19].

3. GROUND STATE AND MINIMUM EMITTANCE

Because of the lack of recoil and excitation in the transverse dimension, the particle damps exponentially to its transverse ground state (n = 0), and this ground state is stable against further radiation (energy and momentum conservation forbid further radiation). In the ground state the particle reaches the minimum value of the action $J_0 = \hbar/2$. Relating this minimum action to a normalized emittance, we find _

$$\gamma \epsilon_{min} \equiv J_0/mc = \lambda_c/2 \quad , \tag{14}$$

where $\lambda_c = \hbar/mc$ is the Compton wavelength. This minimum is also the fundamental emittance limited by the uncertainty principle.

One can estimate the time needed for a particle to damp to its ground state. Suppose the particle enters the focusing channel with a transverse energy $(n_i + 1/2)\omega_z$ satisfying the undulator condition, it reaches the ground state in a time $t_g \sim \ell n(n_i)/\Gamma_c$. The channeling strength for a typical crystal channel is $K \sim 10^{11} \text{GeV/m}^2$, so $\Gamma_c \sim (10 \text{nsec})^{-1}$. When a 100MeV particle is initially barely captured by the crystal channel, the transverse energy of the particle is of the order of the maximum channeling potential energy 100eV, and the corresponding quantum number n_i is about 500. Thus, in the absence of any dechanneling effects the time it takes to damp to the ground state is $t_g \sim 60 \text{nsec}$.

Another novel characteristic of this radiation reaction is that the relative damping rate of the transverse action can be much faster than the relative damping rate of longitudinal momentum, i.e., the radiation reaction is asymmetric in these two dimensions. The rate of change of the longitudinal momentum can be obtained from the energy loss equation, Eq.(12), with the approximation $p_z \simeq E_z \simeq E$. We obtain

$$\left|\frac{1}{p_z}\frac{dp_z}{dt}\right| \simeq \frac{1}{E} \left|\frac{dE}{dt}\right| \simeq \frac{\Gamma_c}{2} \gamma^2 \theta_p^2 \quad , \tag{15}$$

which is less than Γ_c for $\gamma^2 \theta_p^2 < 2$. In the undulator regime we have the condition $\gamma \theta_p \ll 1$, thus

$$\left|\frac{1}{J_n}\frac{dJ_n}{dt}\right| \simeq \Gamma_c \gg \left|\frac{1}{p_z}\frac{dp_z}{dt}\right| \quad . \tag{16}$$

One major consequence of the above inequality is that a particle may lose only a negligible amount of total energy when it is damped to the transverse ground state. By replacing $n = n_i \exp(-\Gamma_c t)$ and $\omega_z \simeq \sqrt{Kc^2/E}$ in Eq.(12) and integrating over time, we find the final energy retained in the ground state $n_f = 0$ is

$$E_f = E_i / [1 + (\gamma \theta_p)_i^2 / 4]^2 \quad . \tag{17}$$

Note that Eq.(17) is derived in the undulator regime where $\gamma \theta_p \ll 1$. Thus particles that enter the focusing channel with the same initial energy but different initial pitch angles will all end up in the transverse ground state with a very small relative longitudinal energy spread of $(\gamma \theta_p)_i^2/2$.

We have shown that the radiation reaction in a straight, continuous focusing channel is fundamentally different from that in a bending magnet. In a uniform magnetic field, the radiating particle recoils against the emitted photon by both reducing its orbital quantum number and by shifting the center of its circular orbit^[18]. This latter change is allowed due to the translational invariance of the system in the plane perpendicular to the magnetic field, i.e., the system is degenerate with regard to the orbiting centers. The center shift is even necessary in order that the tangent of the particle trajectory be continuous before and after the emission. Therefore, the photon emission yields a random recoil of the electron due to variations in both angle and magnitude of the photon's momentum. The resulting random shifts in the orbit center give rise to the random excitations of radial betatron oscillations. On the other hand, the existence of a focusing axis in a straight, continuous focusing environment removes such a degeneracy and therefore eliminates any quantum excitation to the particle from random photon emissions. In a conventional storage ring, the stored particles are confined by both bending and focusing fields. However, the focusing field is typically so much weaker than the bending field that its radiation effect is negligible. On the average, radiation damping in a conventional storage ring shrinks the momentum vector of the particle proportionally^[20,21].

4. RANDOM EXCITATIONS IN A CRYSTAL CHANNEL

When realistic crystal channels are considered, however, one immediate concern is that the novel characteristics of radiation reaction found above may be prone to various sources of excitation such as bremsstrahlung, multiple Coulomb scattering, imperfections, etc. The issue of such excitation in crystal channels has been studied by many authors^[13,17,22]. The standard calculation employs the Born approximation, in which the final results were obtained by integrating over all possible impact parameters^[23]. This approach is in fact improper in the case of small-angle channeling phenomena.

In channeling one basic notion is that the trajectory of the channeled particle can be well described by classical oscillatory motion in the potential well of the channel. The problem of channeling is therefore a semiclassical one in which the impact parameter between the channeled particle and the crystal atomic chain is assumed to be known. To this end the cross section of any physical process that involve the channeled particle and the atomic chain should not be freely integrated over all impact parameters.

Let us first look at bremsstrahlung. As a model we assume a rectilinear motion of the particle and ignore its transverse oscillation. In the channeling oscillation, especially for the undulator regime, the oscillation takes place over a chain of tens of thousands of atoms, while the amplitude is much smaller than the channel size. So locally for a particular target atom the particle moves by with a fixed impact parameter b. At the instant when the channeled particle is a distance R from the target atom, their longitudinal separation is z where $R = \sqrt{b^2 + z^2}$. The screened potential can be represented by the following function^[13]:

$$U(R) = \frac{A_1 Z e^2}{R} e^{-A_2 R/a} \quad , \tag{18}$$

where $a = a_0 Z^{-1/3}$ is the screening radius. $a_0 \simeq 0.52$ Å is the Bohr radius and Z is the atomic number. A_1 and A_2 are numerical coefficients and $A_1 \sim A_2 \sim 0.3$ for typical crystals. In the rest frame of the channeled particle the nucleus interact with the particle via its virtual photons with the spectral density

$$I(\omega_0',b) = \frac{c}{\pi^2} \frac{Z^2 e^2}{c^2 b^2} A_1^2 (\xi'^2 + A_2^2 b^2 / a^2) K_1^2 (\sqrt{\xi'^2 + A_2^2 b^2 / a^2}) \quad , \tag{19}$$

where $\xi' = \omega'_0 b/\gamma c$ and K_1 is the modified Bessel function. These virtual photons Compton-scatter against the channeled particle. When transformed back to the laboratory frame, the Compton backscattered photons emerge as the bremsstrahlung. Applying the Klein-Nishina formula in the rest frame and carrying out the proper transformation back to the Lab frame, we find the energy loss per nuclear encounter:

$$\Delta E(b) = \frac{2Z^2 \alpha r_e^2}{3\pi a^2} A_1^2 A_2^2 K_1^2 (A_2 b/a) [4\epsilon_* - 2\epsilon_*^2 + \epsilon_*^3] \gamma mc^2 \quad , \tag{20}$$

where

$$\epsilon_* = \frac{2\gamma\lambda_c}{b+2\gamma\lambda_c} \quad . \tag{21}$$

It can be straight-forwardly verified that integration of $\Delta E(b)$ in Eq(20) over all impact parameters (i.e., $\int \Delta E(b) 2\pi b db$) recovers the standard Bethe-Heitler formula of bremsstrahlung for screened atoms (with a mild difference in the logarithm).

To look for the transverse random excitations, we note that only a part of the above energy loss formula contributes. The longitudinal momentum of the virtual photon is

$$q_z = p_z - p'_z - k_z \sim \frac{m^2 E_{\gamma}}{2E(E - E_{\gamma})} = \frac{m}{2\gamma} \frac{\epsilon}{1 - \epsilon} \quad , \tag{22}$$

where $\epsilon \equiv \hbar \omega / \gamma mc^2$. From the uncertainty principle, there is a formation length during which the bremsstrahlung process takes place:

$$l_f \sim \frac{\hbar}{q_z} = 2\gamma \lambda_c \frac{1-\epsilon}{\epsilon} \quad . \tag{23}$$

In the regime where the formation length is larger than the atomic spacing, d, the bremsstrahlung becomes coherent, i.e., the neighboring atoms consortedly induce the radiation. However, the coherent limit of bremsstrahlung is precisely the channeling radiation which has already been treated in the previous sections. For a rough estimate of the random element, we restrict the bremsstrahlung to the regime where $l_f \leq d$, or

$$\epsilon \gtrsim \frac{2\gamma\lambda_c}{d+2\gamma\lambda_c} \equiv \epsilon_d \quad . \tag{24}$$

Thus the only part of the bremsstrahlung spectrum that contributes to the random excitation is $\epsilon_d \leq \epsilon \leq \epsilon_*$, and we find, in the limit of $b, d \ll \gamma \lambda_c$,

$$\Delta E(b) = \frac{4Z^2 \alpha^2}{3\pi} \frac{(d-b)r_e}{a^2} A_1^2 A_2^2 K_1^2 (A_2 b/a) mc^2 \quad . \tag{25}$$

It can be seen that since the above expression is controlled by the Bessel function, $\Delta E(b)$ is largely suppressed if $\bar{b} \gg a$.

Now we look for the excitation of the transverse momentum of the channeled particle upon emitting a bremsstrahlung photon. The typical opening angle of the emitted photon is $1/\gamma$. Thus the transverse momentum induced is $\Delta p_x \sim \Delta E(b)/\gamma$. The *rms* increase in the transverse energy is then

$$\langle \Delta E_x \rangle = \frac{\langle \Delta p_x^2 \rangle}{2E_z} \sim \frac{[\Delta E(b)]^2}{2\gamma^3 mc^2} \quad , \tag{26}$$

per nuclear encounter. Consider an axial channel where there are four atomic chains surrounding the channel axis. If we ignore the small channeling oscillation amplitude for a particle in a low-lying transverse eigenstate, we can put $b \simeq d/2$. The rate of bremsstrahlung induced excitation is then

$$\frac{dE_x^b}{dt} = \frac{4c\langle \Delta E_x \rangle}{d} = \left[\frac{2Z^2 \alpha^2}{3\pi} \frac{r_e d}{a^2} A_1^2 A_2^2 K_1^2 (A_2 d/2a) \right]^2 \frac{2mc^3}{\gamma^3 d} \quad . \tag{27}$$

As an example, consider a positron with energy E = 100MeV channeling in a Silicon crystal where $Z = 14, d \simeq 2$ Å, and $A_1 \sim A_2 \sim 0.3$. Note that in this case $a \simeq 0.2$ Å $\ll b \sim d/2 \simeq 1$ Å, i.e., a large fraction of the channel cross section is outside the effective size of the screened atoms. This helps to greatly reduce the bremsstrahlung cross section. Inserting these numbers, we find that the excitation rate due to bremsstrahlung is much smaller than the damping rate from the first excited state to the ground state due to channeling radiation:

$$\frac{dE_x^b}{dt} \sim 5 \times 10^{-7} mc^2 \mathrm{sec}^{-1} \ll -\frac{dE_x^c}{dt} \sim \hbar \omega_z \Gamma_c \sim 42 mc^2 \mathrm{sec}^{-1} \quad . \tag{28}$$

We thus see that near the channel axis any mild excitation due to bremsstrahlung will be quickly damped by the channeling radiation. Using the same methodology, it can be shown that the excitation due to ionization is also negligibly small.

Another source of excitation is due to multiple Coulomb scattering of the conduction electrons. A calculation was done earlier by Montague and Schnell^[24] on the multiple scattering in a plasma. The formula derived there is directly applicable if the minimum impact parameter associated with protons is replaced by that for the conduction electrons: $b_{min} \simeq 2\alpha\lambda_c$. With this replacement the *rms* angular divergence of the channeled particle is

$$\frac{d\langle\Delta\theta^2\rangle}{dz} = \frac{4\pi n_p r_e^2}{\gamma^2} \ell n\left(\frac{\lambda_D}{\lambda_c}\right) \quad , \tag{29}$$

where $\lambda_D = (kT/mc^2)^{1/2} (1/4\pi n_p r_e)^{1/2}$ is the Debye length. The angular divergence is related to the transverse energy. By definition, $\langle \Delta \theta^2 \rangle = \langle \Delta p_x^2 \rangle / E_z^2$. Thus

transverse energy induced by multiple scattering is

$$\frac{dE_x^m}{dt} = \frac{cE_z}{2} \frac{d\langle \Delta \theta^2 \rangle}{dz} = \frac{2\pi n_p r_e^2 mc^3}{\gamma} \ell n \left(\frac{\lambda_D}{\lambda_c}\right) \quad , \tag{30}$$

For Si, the conduction electron density is $n_p \sim 10^{15} \text{cm}^{-3}$. At 1 degree Kelvin, the Debye length is $\lambda_p \sim 2 \times 10^{-7}$ cm, and we find, for E = 100 MeV,

$$\frac{dE_x^m}{dt} \simeq 1.5mc^2 \mathrm{sec}^{-1} \quad , \tag{31}$$

which is again much smaller than the channeling radiation damping rate. We conclude that particles can indeed be damped to their ground states in (at least semiconductor) crystal channels.

5. QUANTUM LUMINOSITY IN A CRYSTAL COLLIDER

We note that all the results obtained here are not affected by adiabatic acceleration along the longitudinal direction, since both the action and the stationary states in our system are adiabatic invariants. The condition for adiabatic acceleration is given by

$$G = \frac{dE_{accel}}{dt} \ll \omega_z E \simeq \sqrt{KE} \quad . \tag{32}$$

Using the previous examples, we get $\omega_z E \sim 10^5 \text{GeV/m}$ for a crystal channel when the energy of the particle is only 100MeV. This is to be compared with the acceleration gradient $G \sim 10^4$ Gev/m attainable in metallic crystals and ~ 10 GeV/m in semiconductor crystals. We conclude that the particle, once damped to its transverse ground state in a continuous focusing channel, can be accelerated adiabatically along the channel. The occasional excitations due to bremsstrahlung, ionization or multiple Coulomb scattering will be rapidly damped by the channeling radiation. Therefore, the theoretical minimum transverse emittance can be retained at a much higher accelerated particle energy, and the relative longitudinal energy spread can be reduced through acceleration.

We can therefore envision the acceleration and collision of channeled particles in their transverse ground states, without the need of a final focusing system. Under this scenario the concept of luminosity turns into a quantum mechanical one, which involves the overlapping of the transverse wave functions of the colliding particles. Let there be N_c particles captured in every channel during each injection. Since there is no depth-of-focus problem, all N_c particles within each channel will pass through the N_c particles from the opposite side. Therefore, by definition, the luminosity per collision per channel is

$$\mathcal{L}_{c} = 2N_{c}^{2} \int dx dy ds dt \ n_{1}(x, y, z_{1}, t) \cdot n_{2}(x, y, z_{2}, t) \quad , \tag{33}$$

where $s = z_1 + t = -z_2 - t$, and n_i is the probability density of the colliding particle

in its transverse ground state:

$$n(x, y, z) = n_L(z) |\psi_{x0}(x)|^2 |\psi_{y0}(y)|^2 \quad , \tag{34}$$

where n_L is the longitudinal distribution of N_c particles and

$$\psi_{x0}(x) = \frac{1}{\pi^{1/4}\sqrt{x_0}} e^{-x^2/2x_0^2} \quad , \quad x_0 = \sqrt{\frac{\hbar}{\gamma m\omega_z}} = \left(\frac{\hbar c}{\sqrt{KE}}\right)^{1/2} \quad , \tag{35}$$

are the graound state wave function and amplitude, respectively. Similar expression applies to the y-dimension. Carrying out the integration, we find

$$\mathcal{L}_{c} = \frac{N_{c}^{2}}{2\pi x_{0} y_{0}} = \frac{N_{c}^{2} \sqrt{KE}}{2\pi \hbar c} \quad . \tag{36}$$

Consider a 5TeV + 5TeV crystal collider with $N = 10^9$ particles in a bunch, $n_b = 10$ bunches in a bunch train, and the repetition rate is $f_{\rm rep} = 180 {\rm sec}^{-1}$. For the sake of discussion, let us assume the beams are injected into the crystal at $E_0 = 1 {\rm GeV}$, with normalized emittance $\epsilon_n = 1 \times 10^{-8} {\rm mrad}$. Since the typical crystal critical angle is $\theta_c = \sqrt{2V_c/E}$, where V_c is the channel potential height, to match the injection optics, we choose the beta-function to be

$$\beta = \frac{\epsilon_n / \gamma}{\theta_c^2} = \frac{mc^2 \epsilon_n}{2V_c} \quad . \tag{37}$$

Note that it is independent of the injected beam energy. The typical crystal potential height is $V_c \sim 100$ eV. This means $\beta \sim 25\mu$ m, and in turn it gives the size of the beam upon injection: $\sigma = \sqrt{\beta\epsilon_n/\gamma} \sim 100$ Å. This corresponds to an area that occupies about $n_c = 10^4$ channels. The number of particles captured in each channel is then $N_c = 10^5$. With the typical crystal strength $K \sim 10^{11} \text{GeV/m}^2$, we find $x_0 \sim 3 \times 10^{-10}$ cm at 5 TeV, and the total luminosity is

$$\mathcal{L} = f_{\rm rep} n_c n_b^2 \mathcal{L}_c \sim 3 \times 10^{36} {\rm cm}^{-2} {\rm sec}^{-1} \quad . \tag{38}$$

This is a reasonable luminosity for colliders at such a center of mass energy, yet achieved without using any final focusing system! Note that although this luminosity may still be attainable with the conventional approach, the total beam power needed in our scenario is only 2×1.5 MW, at the same level of that for the 0.5 TeV next generation linear colliders, which is 20 times smaller in machine energy. We emphasize that the above numerical example is only for the sake of demonstrating the concept, and is not at all optimized.

6. DISCUSSION

As mentioned in the Introduction, the stability and the emittance requirements are very stringent in future linear colliders. In addition there are also constraints set by Nature such as the Oide limit and the beamstrahlung backgrounds. In this paper we explore the possibility of acceleration and collision in crystal channels. The discussion is evidently highly idealized. We have not looked into other deleterious effects such as that induced by potential crystal channel imperfections. In addition, while the mean value of the Coulomb potential from the screened atoms constitutes the channeling Lindhard potential, there can be an *rms* fluctuation of this potential which will induce random excitations. Furthermore, at finite temperature the channeled particle can interact with the collective motions of the lattice, namely the phonons. All these need to be further investigated.

The calculations in the paper are essentially based on single particle dynamics. When dealing with a bunch of particles, even in different channels, the equivalent of the conventional wakefield effects, here in the form of beam-solid interaction, should be addressed. Needless to say, our study is just the beginning. But we hope to have demonstrated in this paper that channeling acceleration and collision in crystals may indeed be a path that would lead to future ultrahigh energy colliders.

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