# TRIPLE-PRODUCT SPIN-MOMENTUM CORRELATIONS IN POLARIZED Z DECAYS TO THREE JETS* 

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#### Abstract

We discuss hard rescattering effects that can be measured using CP-even, $\mathrm{T}_{\mathrm{N}^{-}}$ odd triple-product observables in polarized $Z$ decays to three jets. We show that the standard model contributions, from both QCD and electroweak rescattering, are very small. Thus these measurements are potentially sensitive to physics beyond the standard model. We investigate one such contribution which involves a new gauge boson coupling to baryon number.


In testing the standard model (SM) at higher orders, or in searching for new physics, one usually has to contend with the tree-level SM background. Three-jet decays of polarized $Z$ bosons produced in $e_{-e^{-}}^{-}$annihilation offer the possibility of measuring triple-product correlations such as ${ }^{\mathbf{1}^{1}}\left\langle\mathbf{S}_{\mathbf{Z}} \cdot\left(\mathbf{k}_{\mathbf{1}} \times \mathbf{k}_{\mathbf{2}}\right)\right\rangle$, where $\mathbf{k}_{\mathbf{1}}$ and $\mathbf{k}_{\mathbf{2}}$ are two of the three jet momentum vectors, and $\mathbf{S}_{\mathbf{Z}}$ is the $Z$ polarization vector, parallel to the beam axis. ${ }^{a}$ Such triple-products are odd under $\mathrm{T}_{\mathrm{N}}$, which reverses spatial momenta and spin vectors (but does not exchange initial and final states) so they arise from either CP violation or rescattering phases $\mathbf{s}^{2}$. Here we choose $\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}$ by energy-ordering the jets, $E_{1}>E_{2}>E_{3}$, so that $\left\langle\mathbf{S}_{\mathbf{Z}} \cdot\left(\mathbf{k}_{\mathbf{1}} \times \mathbf{k}_{\mathbf{2}}\right)\right\rangle$ is manifestly CPeven, and therefore only sensitive to rescattering phases originating from absorptive parts of loop amplitudes.

The triple-product correlation $\left\langle\mathbf{S}_{\mathbf{Z}} \cdot\left(\mathbf{k}_{\mathbf{1}} \times \mathbf{k}_{\mathbf{2}}\right)\right\rangle$ could also be termed "event handedness", by analogy to "jet handedness" observables ${ }^{\beta^{3}}$ in which $\mathbf{S}_{\mathbf{Z}}$ is replaced by a jet axis, and $\mathbf{k}_{\mathbf{i}}$ become momenta of particles inside a single jet, rather than jet momenta. Several different variations of event-handedness observables can be constructed. Here

[^0]we focus on
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\begin{equation*}
\left\langle\cos \theta_{n}\right\rangle=\left\langle\frac{\hat{\mathbf{S}}_{\mathbf{Z}} \cdot\left(\mathbf{k}_{\mathbf{1}} \times \mathbf{k}_{\mathbf{2}}\right)}{\left|\mathbf{k}_{\mathbf{1}} \times \mathbf{k}_{\mathbf{2}}\right|}\right\rangle . \tag{1}
\end{equation*}
$$

\]

In a covariant framework, a nonzero $\left\langle\cos \theta_{n}\right\rangle$ is produced by terms in the differential cross-section that are proportional to the Levi-Civita tensor $\varepsilon_{\mu \nu \sigma \rho}$ contracted with four of the five momentum vectors in $e^{+} e^{-} \rightarrow q g \bar{q}$. To contribute to the cross-section, this contraction must be multiplied by the imaginary part of some loop integral. There are several possible sources for imaginary parts in the standard model. The dominant one, a priori, is QCD rescattering (Fig. 1a). However, as argued below, this contribution vanishes for massless quarkst, and at the $Z$ pole is suppressed as $\mathcal{O}\left(m_{b}^{2} / M_{Z}^{2}\right)$. Electroweak rescattering could therefore give comparable effects.

In the following we discuss the four relevant SM contributions to (1): two types of QCD contributions, and $W$ - and $Z$ - exchange loops; as well as one non standard model contribution. For brevity, here we give only some numerical results on the $Z$ pole. (Our analytic results including the $\gamma^{*}$ contributions will be given elsewhereris. ) We do not discuss here long-distance, non-perturbative QCD effects. These are hard to estimate, although they are probably suppressed by $\Lambda_{Q C D}^{2} / M_{Z}^{2}$.

Let us first turn to the $\mathcal{O}\left(m_{q}^{2} / M_{Z}^{2}\right)$ vanishing of the QCD contribution. The loop amplitude can be written as a sum of two parts: a divergent part, which is proportional to the tree amplitude, and a finite part. The former cannot contribute to (1), because no Levi-Civita tensor appears in the interference of the relevant tree amplitudes. Thus, the only contribution may come from imaginary parts of loop integrals that appear in the finite part. These integrals depend on dimensionless ratios of kinematic invariants, of the type $\left(-s_{i j}\right) /\left(-s_{k l}\right)$ and $M_{I}^{2} /\left(-s_{i j}\right)$, where $s_{i j}=$ $\left(k_{i}+k_{j}\right)^{2}$, and $M_{I}$ is the mass of a particle propagating in the loop. In the Euclidean region (all $s_{i j}<0$ ), the integrals are real. But the process we are considering has all $s_{i j}>0$, so the only ratios that change sign upon going from the Euclidean to the physical region are those involving internal masses. Therefore, the integrals develop imaginary parts only in the presence of internal masses. Since this is a kinematic effect, the contribution of any loop amplitude involving an internal mass $M_{I}$ to the triple-product correlations vanishes as $\mathcal{O}\left(M_{I}^{2} / M_{Z}^{2}\right)$ for small $M_{I}$.

Thus, at the $Z$, the dominant QCD contribution comes from $b$ rescattering, in diagrams such as Fig. 1a. This contribution was first calculated by Fabricius et al. $\mathrm{I}^{\prime \prime}$ in the case of virtual photon exchange (no axial couplings); they presented numerical results for two choices of $m_{q} / \sqrt{s}$. The contribution has the expected $m_{q}^{2} / M_{Z}^{2}$ suppression for small quark mass. A further suppression occurs at the $Z$ due to a cancellation between the vector and axial components of the signal. The obtained contribution (A) to $\left\langle\cos \theta_{n}\right\rangle$ is shown in Table 1 assuming $100 \% Z$ polarization, for several values of the three-jet cut, $y_{i j} \geq y_{c u t}$, where $y_{i j} \equiv\left(k_{i}+k_{j}\right)^{2} / M_{Z}^{2}$.

The _second type of QCD contribution arises from a massive $b$ quark triangle diagram ${ }^{6 i 6}$ (Fig. 1b), which, because of Furry's theorem, is proportional to the $Z$ -

Table 1. SM Contributions to $\left\langle\cos \theta_{n}\right\rangle$.
(For $P_{z}=\underline{P_{e}=100 \%, m_{b}=4.5 \mathrm{GeV}, \alpha=1 / 129, \alpha_{s}=.12, \sin ^{2} \theta_{W}}=.23$.)

| $y_{c u t}$ | $\mathrm{QCD}(\mathrm{A})$ | $W$-exchange | $Z$-exchange |
| :---: | :---: | :---: | :---: |
| .08 | $-1.5 \times 10^{-6}$ | $-4.3 \times 10^{-7}$ | $-1.7 \times 10^{-7}$ |
| .04 | $-2.6 \times 10^{-6}$ | $-5.1 \times 10^{-7}$ | $-2.0 \times 10^{-7}$ |
| .02 | $-3.4 \times 10^{-6}$ | $-4.9 \times 10^{-7}$ | $-2.0 \times 10^{-7}$ |

quark axial coupling. Diagrams with up-type and down-type final-state quarks generate triple-product correlations of opposite signs and equal (up to mass-splittings) magnitudes, so that here too, only $b$ quark final states contribute. This contribution turns out to be 2-3 orders of magnitude smaller than the one discussed above.

The other possible source for event-handedness in the SM is an electroweak loop, where the produced quark pair exchanges a $W$ or a $Z$ (Fig. 1c). As the latter are massive, no quark mass is required in order to get a non-vanishing effect. Hence, all quark flavors contribute here, except that the $b$ quark cannot appear in the final state after $W$ exchange below the $t \bar{t}$ threshold (we neglect off-diagonal CKM elements). The last two columns of Table 1 show typical values of these contributions.

Finally, we investigate the sensitivity of $\left\langle\cos \theta_{n}\right\rangle$ to a recently proposed $U(1)$ gauge boson $B$, coupling to baryon number only. This can be easily done by replacing the $W$ or $Z$ in Fig. 1c by the new boson. The resulting contribution is biggest for a $B$ mass of about 30 GeV , for which, taking the $B$ coupling to be $\alpha_{B} / 9$, with $\alpha_{B}=0.2$, $\left\langle\cos \theta_{n}\right\rangle \sim 3 \times 10^{-5}$. Although this can be an order of magnitude larger than the SM contribution, it will still be very difficult to find an event-handedness signature of this boson at present and future colliders.

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    ${ }^{a}$ The $Z$ polarization may be produced either with longitudinally polarized electron beams, or via the left-right asymmetry $A_{L R}^{(e)} \approx 16 \%$ ("natural" $Z$ polarization).

