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CP Nonconservation in $e^+e^- \rightarrow t\bar{t}g$

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Abstract: CP violation effects in $e^+e^- \rightarrow t\bar{t}g$ are examined. CP-odd, T_N -odd and T_N -even observables can both be used to extract information on the real and imaginary parts of Feynman amplitudes. Two Higgs doublet model with CP violating phase from neutral Higgs exchange is used to estimate possible effects.

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The prospects for CP violation in top physics has been receiving considerable attention in the past few years.¹⁻⁷ In the Standard Model (SM), with its CKM phase, the effects are expected to be extremely small. However, additional CP violating phases occur rather naturally in extensions of the SM. Besides, non-standard sources of CP violation may well be a necessity to understand baryogenesis.⁸ Therefore, experimental searches for effects of CP violation are perhaps the best probes of new physics. Thus it is important to investigate signatures of the additional phases that may be present. In particular, due to its mass, the top quark can be a very sensitive probe for the phase(s) from an extended Higgs sector. Such a phase residing, say, in neutral Higgs exchanges endows the top quark with a large dipole moment which drives many interesting CP violation asymmetries in production and decays of the top quark in high energy reactions.^{2,3,9}

In this paper we examine the CP violation effects in the reaction:

$$e^{-}(p_{-}) + e^{+}(p_{+}) \to t(p_{t}) + \bar{t}(p_{\bar{t}}) + g(p_{g})$$
 (1)

(g is the gluon). The advantage of this simple reaction is that it allows the possibility of probing both categories of CP asymmetries that can occur. We recall that CP violating observables can be subdivided into T_N -even and T_N -odd type. Here T_N is the "naive time-reversal" operator (i.e. time $\rightarrow -$ time without switching of initial and final states). T_N -even observables are driven by imaginary parts of Feynman amplitudes whereas T_N -odd observables are proportional to the real parts of the amplitudes. Of course, being CP violating, both types of observables do need CP violating phase(s) from the underlying theory.

CP violation effects can manifest in the momentum distributions of the incoming and outgoing particles. To illustrate this, let us $define^{10}$

$$E = \epsilon(p_{-}, p_{+}, p_{t}, p_{\bar{t}}); \qquad s = 2p_{-} \cdot p_{+}; \qquad s_{t} = (p_{t} + p_{\bar{t}})^{2}$$
$$u = (p_{-} - p_{+}) \cdot (p_{t} - p_{\bar{t}}); \qquad F = (p_{-} - p_{+}) \cdot (p_{t} + p_{\bar{t}})$$
$$G = (p_{-} + p_{+}) \cdot (p_{t} - p_{\bar{t}})$$

Any term in the cross section can be expressed by the above kinematic functions: G, F and E are CP-odd, where G and F are T_N -even while E is T_N -odd. All the other terms are CP-even; thus all CP violating effects in the momentum distributions will be proportional to G, F or E.

Simple examples of T_N -even observables that can be studied with reaction (1) are:

$$O_{i1} \equiv \vec{p}_{-} \cdot (\vec{p}_{t} + \vec{p}_{\bar{t}})/s \qquad (a)$$

$$O_{i2} \equiv (E_t - E_{\bar{t}})/\sqrt{s} \qquad (b) \qquad (2)$$
$$O_{i3} \equiv \vec{p}_g \cdot (\vec{p}_t - \vec{p}_{\bar{t}})/s \qquad (c)$$

We can identify O_{i1} with the forward-backward asymmetry of the gluon jet while O_{i2} is the energy asymmetry between t and \bar{t} . The CP violating T_N -odd observables have to be proportional to $\epsilon(p_-, p_+, p_t, p_{\bar{t}})$; since these are the only independent 4-momenta that are available. This leads us to consider the following CP-odd, T_N -odd triple correlation product:

$$O_{r1} \equiv \vec{p}_{-} \cdot (\vec{p}_t \times \vec{p}_{\bar{t}}) / s^{3/2} \tag{3}$$

For definiteness we will focus on the effects of a two-Higgs doublet model (THDM).^{11,2,12} As is well known flavor changing neutral currents (FCNC) are avoided in such a model by imposing a discrete symmetry. This then restricts coupling of one Higgs doublet (Φ_2) with charge 2/3 quarks and the other doublet (Φ_1) couples only to the charge -1/3 quarks and the leptons. CP violation is induced in the model by softly breaking the discrete symmetry in the Higgs potential. This causes mixing between real and imaginary parts of Higgs fields and the Higgs mass eigenstates then do not have a definite CP property. The manifestation of such a CP violation is that the neutral Higgs couple to fermions with scalar as well as pseudoscalar couplings. Thus

$$\mathcal{L}_{Hff} = H\bar{f}(a_f + ib_f\gamma_5)f\tag{4}$$

where H is the lightest neutral Higgs. For simplicity we are assuming that the other Higgs particles are heavy enough that their effects can be ignored. The ZZH interaction also plays an important role in our calculation and is given by

$$\mathcal{L}_{ZZH} = \frac{2m_Z^2}{v} cg_{\mu\nu} Z^{\mu} Z^{\nu} \tag{5}$$

where $v = \sqrt{v_1^2 + v_2^2} \equiv (\sqrt{2}G_F)^{1/2}$, $v_{1,2}$ being the vacuum expectation values of the two Higgs, with the usual definition of $\tan \beta \equiv v_2/v_1$. In eqs. 4-5 the coefficients a, b, and c are functions of $\tan \beta$ and the mixing matrix elements between the three neutral scalar fields.^{2,12} Fig. 1 shows the tree-level Feynman graphs that contribute to the process $e^+e^- \rightarrow t\bar{t}g$. Fig. 2 shows the Feynman graphs that (to one loop order) are calculated to determine the CP asymmetry. From eqs. (4) and (5) we see that all the CP violating terms are proportional to either *ab* or *cb*.

Before presenting the numerical results we want to augment the list of CP violating observables. In eqs. (2) and (3) we gave examples of "naive" observables. A non-vanishing expectation value of any one of these would signal CP violation so that experimental searches for these can be done without recourse to any model. Of course in a theoretical discussion such as ours one can investigate the expected asymmetries based on specific models of CP violation. However, in the context of any given model one can also construct optimal observables i.e. those observables which will be the most sensitive to CP violation effects in that model. The recipe for such a construction is very simple. It can be shown¹² that the optimal observables of $(T_N$ -even and T_N -odd) CP violation are given by:

$$O_{\text{iopt}} \equiv \Sigma_1^{\text{Im}} / \Sigma_0 \quad , \quad O_{\text{ropt}} \equiv \Sigma_1^{\text{Re}} / \Sigma_0 \tag{6}$$

where the differential cross section (in the variable ϕ under consideration) is broken down as:

$$\Sigma(\phi) = \Sigma_0(\phi) + \Sigma_1^{\text{Re}}(\phi) + \Sigma_1^{\text{Im}}(\phi)$$
(7)

Here $\Sigma_0(\phi)$ is CP-even piece and Σ_1 is CP-odd piece which is further subdivided into a segment that depends on the real part of the amplitude and the one that depends on the imaginary part.

Let us define A_O to be:

$$A_O \equiv /\sqrt{} \tag{8}$$

where $\langle O^2 \rangle$ is the expected variance, then to observe a non-vanishing average value $\langle O \rangle$ with a statistical significance of one sigma one needs:

$$N_{t\bar{t}q} = 1/A_O^2 \tag{9}$$

 $N_{t\bar{t}g} = \mathcal{L}\sigma \ (e^+e^- \to t\bar{t}g)$ being the numbe of $t\bar{t}g$ events and \mathcal{L} is the collider luminosity.

For definiteness, we will set a = b = c = 1. Furthermore, we have imposed an invariant mass cut on the jet pairs so that $(p_q + p_t)^2$ and $(p_q + p_{\bar{t}})^2 \geq$ $(m_t + m_0)^2$ where we have taken $m_0 = 25$ GeV and $m_t = 174$ GeV. Also we will first focus on the case of left-polarized electrons. Fig. 3-4 show our main numerical results¹⁴ for $m_H = 100$ and 200 GeV respectively. The number of events needed to see a non-vanishing value (to one sigma) for some of the naive observables $(O_{i1} \text{ and } O_{r1})$ and the optimal observables are shown. We see that near threshold ($E_{CM} \sim 400$ GeV) the T_N -odd and T_N -even observables have comparable effectiveness; T_N -odd ones are a bit better. As the CM energy increases both types become worse rather rapidly. However as the CM energy is increased further the T_N -even ones improve and can become almost as effective as they are near threshold. The turn around in energy where they regain their effectiveness depends on m_H . Thus for $m_H = 100$ GeV, $E_{CM} \gtrsim 500$ GeV is needed and for $m_H = 200$ GeV, $E_{CM} \gtrsim 700$ GeV becomes necessary. Note, though, that the sensitivity of the observables near threshold does not depend too heavily on the precise value of m_H .

Table 1 gives a brief comparison of the left, right and un-polarized cases. We note that for the T_N -even (e.g. O_{i1} and O_{iopt}) cases the polarization makes a significant difference and improves their effectiveness by an order of magnitude or even more. For these it seems that the left-polarized case is marginally better over the right one. For the T_N -odd observables (e.g. O_{r1} and O_{ropt}) beam polarization does not make much of a difference.

We see that $10^5-10^6 t\bar{t}g$ events may be necessary to see an indication of these effects. We recall that a future e^+e^- collider could perhaps produce $\sim 10^5 t\bar{t}$ pairs. Table 2 gives the ratio of $t\bar{t}g$ to $t\bar{t}$ events. We note that even at $\sqrt{s} \gtrsim 800$ GeV the presence of the extra gluon could reduce the rate by about a factor of 3–4 for the polarized case. Bearing Figs. 3–4 and Table 2 in mind it would seem that for $e^+e^- \rightarrow t\bar{t}g$, study at higher energy (i.e. away from threshold) would be better, at least for T_N -even observables.

It must be emphasized, though, that many simplifying assumptions were made along the way (e.g. a = b = c = 1) so that the actual size of the effects could be a lot smaller or even bigger. For example, even a modest change from the values we used to say $ab \sim 3$ (which is equivalent to setting $\tan \beta = 0.5$) would increase CP asymmetries by the same amount (since the asymmetry is linear in ab or bc) and would tend to reduce the number of events from those given in Fig. 3–4 and Table 1 by about an order of magnitude. Furthermore, the CP violation may have other sources, say charged Higgs exchanges or supersymmetry. The key point is that the study of the simple reaction $e^+e^- \rightarrow t\bar{t}q$, with beam energy of several hundred GeV, via the observables discussed here, could be a very valuable probe for searching for non-standard sources of CP violation. It should also be noted that in this work we have not included the detection of the decay product of the top quark. In particular, as is well known, the decay of the top quark acts as an analyzer of the top spin.⁹ Inclusion of the t, \bar{t} spins is very likely also to help in the analysis of CP violation effects. We will return to some of these points in a future publication.

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References

- C.R. Schmidt and M. Peskin, Phys. Rev. Lett. **69**, 410 (1992); W. Bernreuther and A. Brandenburg, Phys. Lett. B**314**, 104 (1993); C.R. Schmidt, Phys. Lett. B**293**, 111 (1992).
- W. Bernreuthor, T. Schroder and T.N. Pham, Phys. Lett. B279, 389 (1992).
- 3. A. Soni and R.M. Xu, Phys. Rev. Lett. **69**, 33 (1992).
- 4. G.L. Kane, G. Ladinski and C.P. Yuan, Phys. Rev. D45, 124 (1992).
- B. Grzadkowski, Phys. Lett. B305, 384 (1993); B. Grzadkowski and W-Y. Keung, Phys. Lett. B319, 526 (1993); E. Christova and M. Fabbrichese, Phys. Lett. B320, 299 (1994).
- 6. D. Atwood, G. Eilam and A. Soni, Phys. Rev. Lett. 71, 492 (1993).
- See e.g. N. Deshpande *et al.*, Phys. Rev. D45, 178 (1992); G. Eilam, J. Hewett and A. Soni, Phys. Rev. Lett. 67, 1979 (1991); B. Grzadkowski and W-Y. Keung, *ibid*.
- 8. For a review see: A.G. Cohen, D.B. Kaplan and A.E. Nelson, Ann. Rev. Nucl. Part. Phys. 43, 27 (1993).
- 9. D. Atwood, A. Aeppli, and A. Soni, Phys. Rev. Lett. **69**, 2754 (1992).
- D. Atwood, S. Bar-Shalom, and A. Soni, SLAC-PUB-6435 to appear in Phys. Rev. D.
- G.C. Branco and M.N. Rebelo, Phys. Lett. B160, 117 (1985); J. Liu and L. Wolfenstein, Nucl. Phys. B289, 1 (1987).
- C.D. Froggett, R.G. Moorhouse, and I.G. Knowles, Nucl. Phys. B386, 63 (1992).
- 13. D. Atwood and A. Soni, Phys. Rev. D45, 45 (1992).
- For the FF-package that was used for numerical evaluation of loop integrals see: G.J. van Oldenborgh, Comput. Phys. Commun. 66, 1 (1991).
 For the algorithms used in the FF-package see: G.J. van Oldenborgh and J.A.M. Vermaseren, Z. Phys. C46, 425 (1990).

Figure Captions

- Fig. 1 Tree-level Feynman diagrams contributing to $e^+e^- \rightarrow t\bar{t}g$.
- Fig. 2 CP violating Feynman diagrams contributing to $e^+e^- \rightarrow t\bar{t}g$ to one loop order in a two Higgs doublet model. Diagrams with permuted vertices (e.g. $t \rightarrow \bar{t}$) are not drawn.
- Fig. 3 Number of events (in units of 10^5) needed to detect CP violation via $\langle O_{i1} \rangle$, $\langle O_{r1} \rangle$, $\langle O_{iopt} \rangle$ and $\langle O_{ropt} \rangle$ as a function of total beam energy, $m_H = 100$ GeV is used.
- Fig. 4 Same as Fig. 2 except $m_H = 200$ GeV.

Table 1: The unpolarized case (pol = 0) is compared with left polarization (pol = -1) and the right polarization (pol = +1). The number of events in units of 10⁵ needed, for detection of asymmetries, to one sigma are given. The values of \sqrt{s} and m_H are given in GeV.

		O_{i1}		O_{r1}		$O_{i \text{opt}}$		O_{ropt}				
\sqrt{s}	pol.	$m_{H} = 100$	$m_H = 200$	$m_{H} = 100$	$m_H = 200$	$m_H = 100$	$m_H = 200$	$m_{H} = 100$	$m_H = 200$			
400	-1	1.8	11.5	1.0	0.9	1.0	6.0	0.8	0.7			
	0	22.5	134.8	1.8	1.5	6.5	37.0	0.7	0.6			
	1	2.3	17.1	0.7	0.6	1.3	8.4	0.6	0.5			
700	-1	3.4	20.0	66.7	37.8	1.7	5.1	57.2	32.9			
	0	48.6	263.9	124.6	70.2	12.2	38.6	52.8	30.3			
	1	4.5	30.8	55.1	30.7	2.0	5.9	45.0	25.9			
1000	-1	4.0	10.5	625.6	363.8	2.6	4.5	496.9	320.4			
	0	63.4	158.2	1193.9	699.8	20.5	35.3	461.9	301.2			
	1	5.0	14.0	547.9	325.7	3.1	5.4	399.2	264.6			

Table 2: The numbers for the ratio: $\left[\frac{\sigma(e^+e^- \to t\bar{t}g)}{\sigma(e^+e^- \to t\bar{t})}\right]$ with the cut: $E_{\text{glue}} \ge 25$ [GeV] and for different polarizations. The value of \sqrt{s} is given in GeV.

$\sqrt{s} \Rightarrow$					
↓ Pol.	400	550	700	850	1000
Left (-1)	0.003	0.08	0.19	0.29	0.49
Unpol. (0)	0.007	0.19	0.42	0.63	0.83
Right (1)	0.006	0.13	0.27	0.40	0.51





Fig. 1





Fig. 2



