# Beam-Beam Interaction Working Group Summary

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# WORKING GROUP MEMBERS

#### INTRODUCTION

For a moment consider a single beam in a storage ring. Individual particle motions are linear at small amplitudes with tunes, optical functions, and, in the case of electrons, synchrotron radiation properties determined by the dipoles, quadrupoles, and RF system. At larger amplitudes, intentional and unintentional field nonlinearities lead to amplitude dependent tunes, nonlinear resonances, and an effective or "dynamic" aperture. Beam induced wakefields modify distributions and cause instabilities and intensity limits.

This is modified significantly by the beam-beam interaction which occurs when two beams are brought into collision. The electromagnetic fields at the collision point are strong and nonlinear. This changes the small amplitude motion and the amplitude dependence of the tunes, and introduces new, strong nonlinear resonances. New collective modes and instabilities mediated by the beam-beam interaction are possible, and, in addition, plasma effects absent with a single beam, could occur because of the focusing during the collision.

There are pragmatic and commonly quoted approaches to the beam-beam interaction that are based on years of colliding beam operation and are strikingly simple compared to all this potential complexity. This experience is that:

1. The beam beam-beam interaction can be characterized by a single parameter, the beam-beam strength parameter  $\xi$ , defined in the Appendix.

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Paper Presented at the Workshop on Non Linear Dynamics in Particle Accelerators: Theory and Experiments Arcidosso, Italy, September 4 - 9, 1994 2. The beam-beam limit in hadron colliders is roughly  $N_{IP}\xi \leq 0.02$  where  $N_{IP}$  is the number of interaction points. The lifetime decreases when this limit is exceeded.

3. The e<sup>+</sup>e<sup>-</sup> collider beam-beam limit is  $\xi \le 0.03 - 0.05$ . The lifetime decreases when this limit is reached although the beam core blows-up well below it making the luminosity (Appendix) proportional to n, the number of particles per bunch, rather than n<sup>2</sup>.

The limit in hadron colliders is understood phenomenologically. The beam-beam interaction produces nonlinear resonances and makes the transverse tunes amplitude dependent. Tune spreads result from the latter, and as long as these tune spreads do not overlap low order resonances, the lifetime and performance is acceptable. Experience is that tenth and sometimes twelfth order resonances must be avoided, and the hadron collider limit corresponds roughly to the space available between resonances of that and lower order when operating near the coupling resonance.

The beam-beam interaction in  $e^+e^-$  colliders is not understood well. This affects the performance of existing colliders and could lead to surprises in new ones. For example, a substantial amount of operator tuning is usually required to reach the performance limit given above, and this tuning has to be repeated after each major shutdown. The usual interpretation is that colliding beam performance is sensitive to small lattice errors, and these are being reduced during tuning. It is natural to ask what these errors are, how can a lattice be characterized to minimize tuning time, and what aspects of a lattice should receive particular attention when a new collider is being designed. The answers to this type of question are not known, and developing ideas for calculations, simulations and experiments that could illuminate the details of the beam-beam interaction was the primary working group activity.

## UNDERLYING PHYSICS

Single particle motion and nonlinear resonances are the predominant theme of the beam-beam interaction literature, but there are two other possibilities that could be important in some circumstances. One of them is a coherent beambeam interaction that leads to correlated turn-by-turn variations in the beam sizes, and the other is disruption, focusing during the collision.

## **Coherent Beam-Beam Interaction And Disruption**

The theory of coherent beam-beam interactions has been developed for space charge compensated beams (1) and for the more common situation of two colliding beams (2, 3, 4). Most computer simulations exclude coherent effects by assuming that the beams have Gaussian distributions, but simulations without that restriction have been performed (5, 6). The results are in qualitative agreement with coherent beam-beam theories. The conclusions of these theories and simulations are: i) coherent beam-beam instabilities are probably Landau damped in the two-beam system for the values of  $\xi$  at or below the performance limits because of the incoherent beam-beam tune spread, and ii) they dominate the four-

beam configuration because there is no incoherent beam-beam tune spread and therefore no Landau damping.

There is no experimental evidence to support that conclusion. Beam position monitors are sensitive to beam centroid motion and not shape variations, and almost all beam profile monitors measure the profile averaged over many turns. There is a profile monitor at LEP capable of measuring turn-by-turn variations in size (7). Data from it showing such variations has been shown in the *CERN Courier* (8), but the conditions under which these data were taken and whether these variations are common or unusual has not been published. Data on turn-by-turn size variations at the beam-beam limit are needed for an unequivocal conclusion about the role of coherent beam-beam instabilities.

The beam-beam interaction acts like a quadrupole with focal length  $f_y \sim \beta_y / 4\pi \xi$  in the vertical plane. This is approximately

$$f_v \sim 2 - 3\beta_v^* \sim 3 - 5\sigma_L$$

in e<sup>+</sup>e<sup>-</sup> colliders with bunch lengths comparable to  $\beta_y^*$ . This focusing during the collision could lead to effects such as those expected from disruption in linear colliders. There have been some speculations about the importance of disruption, but these need to be developed further before a connection between disruption and the beam-beam limit is established.

# Single Particle, Nonlinear Motion

The remainder of this report concentrates on single particle, nonlinear motion rather than these alternative possibilities. To first order in perturbation theory the motion is described by a Hamiltonian

$$H = H_0(\vec{I}) + \frac{m_c}{C\gamma} \langle V_{BB}(\vec{I}) \rangle + \langle V_L(\vec{I}) \rangle$$

$$+ \sum_{\vec{m}, p \neq 0} \left\{ \frac{m_c}{C\gamma} B_{\vec{m}}(\vec{I}) + L_{\vec{m}}(\vec{I}) \right\} \exp[i(\vec{m} \cdot \vec{\psi} - 2\pi ps / C)].$$
(1)

The Hamiltonian of the linear motion is H<sub>0</sub> and  $\{\vec{I},\vec{\psi}\}\)$  are the action-angle variables of H<sub>0</sub>. The average values of the beam-beam potential and lattice nonlinearities are given by  $\langle V_{BB} \rangle$  and  $\langle V_L \rangle$ , and  $B_{\bar{m}}$  and  $L_{\bar{m}}$  are the coefficients in Fourier expansions of beam-beam and lattice potentials for the resonance

$$\vec{\mathbf{m}} \cdot \vec{\mathbf{Q}} = \mathbf{m}_{\mathbf{x}} \mathbf{Q}_{\mathbf{x}} + \mathbf{m}_{\mathbf{y}} \mathbf{Q}_{\mathbf{y}} + \mathbf{m}_{\mathbf{s}} \mathbf{Q}_{\mathbf{s}} = \mathbf{p}.$$

The amplitude dependences of the tunes have contributions from both the beambeam and lattice nonlinearities

$$\vec{Q} = \frac{1}{2\pi} \nabla_{\vec{I}} H = \vec{Q}_{x0} + \frac{1}{2\pi} \nabla_{\vec{I}} \left\{ \frac{nr_c}{C\gamma} \langle V_{BB}(\vec{I}) \rangle + \langle V_L(\vec{I}) \rangle \right\}.$$

The beam-beam interaction dominates at small values of the transverse actions,  $I_x \sim \sigma_x^2 / \beta_x$  etc., but at large values the particle spends most of the time far away from the core and the lattice nonlinearities are important. The strength of an isolated resonance is given its width in action which is proportional to

$$\left(\Delta \vec{I}\right)^{2} \sim \frac{\left\{\frac{nr_{c}}{C\gamma}B_{\vec{m}}(\vec{I}) + L_{\vec{m}}(\vec{I})\right\}}{\left(\vec{m}\cdot\nabla_{\vec{I}}\right)^{2}\left\{\frac{nr_{c}}{C\gamma}\left\langle V_{BB}(\vec{I})\right\rangle + \left\langle V_{L}(\vec{I})\right\rangle\right\}}$$

Again, this is dominated by the beam-beam interaction at small values of transverse action and by lattice nonlinearities at large values.

The luminosity is inversely proportional to the product of the rms sizes of the core, and, since beam-beam nonlinearities are dominant in this region, the luminosity should be dominated by them. The motion of rare, large amplitude particles in the halo is determined by lattice nonlinearities which lead to the dynamic aperture. The intermediate region between the core and halo has a combination of lattice and beam-beam nonlinearities. Electrons can stream to large amplitudes if they are trapped in a resonance with sufficient strength and with (9)

$$\frac{\partial I_y}{\partial I_x}\Big|_{\bar{m}\cdot\bar{Q}=p} < 0.$$

Whether of not resonances stream depends on the combined action of beam-beam and lattice resonances, and this determines the flux to large amplitudes, the beam halo density, and the lifetime.

Beam-beam performance is determined by small errors. Observations during operations that show this are: i) sensitivity to orbits, coupling, and lattice nonlinearities, ii) difficulty in restoring conditions, and iii) a variable history of agreement between simulations and measured performance. The underlying reason is contained in the Hamiltonian, eq. 1.

The core can be described by  $H_0$  plus the beam-beam potential, and errors in the linear lattice affect resonance strengths. An example is symmetry breaking. In the one-dimensional beam-beam interaction with two interaction regions, the resonance condition is mQ = 2p if the interaction regions are identical and the phase advances between interaction regions is Q/2. If the interaction regions are not identical or the phase advances unequal, the resonance condition becomes (m/2)Q = p if m is an even integer. The resonance order has been reduced by a factor of two, and even though the Fourier expansion coefficient for this lower order resonance is proportional to the lattice error, it can be the dominant resonance.

The situation is more complicated when trying to understand the lifetime in collisions. The lattice nonlinearities and their interference with beam-beam resonances are important. Tune, orbit, and coupling changes can all affect lattice nonlinearities, and through them the lifetime.

# BEAM-BEAM PERFORMANCE & ACCELERATOR PROPERTIES

Discussions about the effects of the accelerator properties, especially the magnetic lattice, on the beam-beam interaction are summarized in Table 1. Coupling and the effects of lattice resonances deserve expansion beyond the table.

Accelerator	Beam-Beam		
Property	Core	Tail	
Tunes, $Q_{x0}$ , $Q_{y0}$ , $Q_s$	<b>**</b> Resonance locations	<b>**</b> Resonance locations	
Breaking of lattice symmetry	<b>**</b> Introduces low order resonances	<b>**</b> Introduces low order resonances	
Amplitude and energy depend- ence of tunes	- Weak at small amplitudes	<b>**</b> Determine resonance locations and widths at large amplitudes.	
Coupling	<b>**</b> Vertical beam size and luminosity	* Strengths of coupling resonances	
Dispersion at collision point	<b>** Beam</b> -beam driven synchrobetatron resonances	<b>**</b> Beam-beam driven synchrobetatron resonances	
Lattice resonances	- Weak at small amplitudes	<b>**</b> Locations and strengths of nonlinear resonances that determine the dynamic aperture	
Power supply ripple	<ul> <li>** Luminosity lifetime in hadron colliders</li> <li>? No known effect in e<sup>+</sup>e<sup>-</sup> colliders</li> </ul>	<b>**</b> Diffusion rates in hadron colliders	
Offsets at collision point	<b>**</b> Lowers luminosity, introduces odd order resonances	<b>**</b> Introduces odd order resonances	
Bunch length, $\sigma_l$	** Synchrobetatron resonance strength	** Synchrobetatron resonance strength	
Gas scattering	-	<b>**</b> Changes tail population	

TABLE 1: Accelerator Properties And The Beam-Beam Interaction<sup>†</sup>

† Symbols indicate consensus about importance (\*\* Important, \* Has an influence, - Not important)

Working group members had experience with CESR, LEP, PEP, SPEAR, the Tevatron, and VEPP-4. They agreed that coupling was important for single beam and beam-beam performance in these colliders, but had different experiences to report. In some cases it was sufficient to reduce the width of the coupling resonance below some value, and in others it was necessary to control the local coupling at the collision point. These differences are thought to be related to the details of the resonance structures, but since these details are not known, this remains a thought without quantitative confirmation.

Resonance structure is determined by the lattice and beam-beam interaction acting in concert. However, there have been only a few studies of beam-beam performance with lattice nonlinearities, and these have been limited to changing the amplitude dependence of tunes. Temnykh used octupoles for this in VEPP-4 and found that the loss rate increased considerably with a positive cubic nonlinearity (10). This result is understood qualitatively because the amplitude dependence of tunes affects the propensity to stream.

Lattice resonances and/or the interference between beam-beam and lattice resonances has not been considered. Recently, a tracking algorithm based on a Poisson Bracket expansion of the one-turn map has been developed (11). It is fast enough to be used in a beam-beam simulation, and individual resonances can be included or excluded. It promises progress in understanding the role of the lattice in the beam-beam interaction.

Accelerator characterization, measuring and restoring accelerator properties, was discussed often and was considered a critical issue by the working group. It is the key to good peak and average luminosities and to interpretation of experiments. The discussions are summarized below.

The closed orbit is measured often in routine operation, and keeping the beam on the *gold orbit* can be the single most important action for good performance. The *gold orbit* is found by empirically tuning to the orbit that gives good luminosity. Whether it is gold because of coupling or resonance effects or for another reason is usually poorly understood as is the method to produce a new gold orbit after a major shutdown.

Lattice functions,  $\beta^*$ 's, chromaticites, coupling resonance width, emittances, collision point dispersions, energy spread and bunch length are measured as part of routine operation with frequency of measurement ranging from once or twice per week to once or twice per operating cycle.

Other properties are rarely, if ever, measured. These include phase advances between interaction points, local coupling, amplitude dependence of tunes, apertures, and lattice resonances. They are difficult to measure, but lack of knowledge about them is the likely reason for lack of reproducibility and long tuning times. Developing techniques for making these measurements and employing them in times of good and bad performance was felt to be an important step in improving accelerator characterization.

## **BEAM-BEAM EXPERIMENTS**

Theory, simulation, and experiment are all essential. The underlying physics and the framework for interpreting simulations and experiments come from theory. Simulations are numerical experiments that can guide and test theoretical developments. They are also used to predict the performance of future colliders, but one must be cautious about that given the mixed record of success. There are innumerable real and potential complications with the beam-beam interaction, and experiments are the only way to insure a focus on the most important issues.

Accelerator time is precious, and experiments can be difficult to design, perform, and interpret so the working group spent a substantial amount of time discussing beam-beam experiments. There are different types of experiments:

1. Experiments parasitic to operations are limited because there is no control of accelerator conditions, but their virtue is that performance will be typical during the experiment. An example is measurement of turn-by-turn size variations when operating at the beam-beam limit to understand the importance of coherent beam-beam effects.

2. Experiments requiring the accelerator be configured in a non-standard way to test a hypotheses or measure parametric dependences. Close in spirit to a numerical simulation, it is natural to perform such experiments in conjunction

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Instrument/Apparatus	Comments			
Beam profile monitors that produce an image averaged over many turns	Synchrotron light monitors in e <sup>+</sup> e <sup>-</sup> colliders and wire scanners in hadron colliders are essential for routine operation and experiments			
Turn-by-turn beam profile measurements	Such a monitor needs to be exploited at one or two colliders to clarify the role of coherent beam-beam effect			
Beam profile measurement at the interaction point	The Compton scattering techniques being explored for linear colliders could make it possible to measure local coupling at the collision point			
Current measurement	Individual bunch currents should be measured. Lifetimes are derived from current changes. Several lifetime measurements per second would be useful for $e^+e^-$ colliders			
Loss monitors	At long lifetimes the quantity of interest can be measured directly instead of small differences in current			
Adjustable collimators or scrapers	Allow variation of lifetime, beam halo measurement, and loss monitor calibration			
Fast luminosity monitors	$e^+e^ e^+e^-\gamma$ or $e^+e^ e^+e^-\gamma\gamma$ has been used for fast luminosity measurement in VEPP-4			
Horizontal and vertical kickers	No direct beam-beam experimental use, but needed for aperture and nonlinearity measurements that characterize the lattice			
"1000 turn" orbit recording	Also useful for lattice characterization			
Interaction point orbit feedback and monitoring	This is becoming important with electrostatically separated orbits and two ring colliders			
Automated tune measurement and history buffering	Scans measuring luminosity, lifetime, and losses versus tune require automated measurement and recording			
Octupoles	Control of amplitude dependence of tunes			

**TABLE 2:** Instruments and Accelerator Apparatus

with simulations or theories as tests of them. The VEPP-2M measurement of the beam-beam limit versus the phase advance errors between interaction points is an example (12).

3. Historical records of routine operation show aspects of performance that cannot be seen in a short, dedicated experiment. Information such as the energy dependence of the beam-beam limit in SPEAR has potentially great value. A problem with these historical records is that adequate information about the accelerator may not be available either because it is not published or because of inadequate characterization.

Group discussions about instruments and accelerator apparatus for experiments and routine operation are summarized in Table 2. In addition, to these specifics there was a strong emphasis on automating experiments and data acquisition to have the control and data logging needed for complex experiments.

#### FUTURE MODES OF OPERATION

Plans for future colliders rely on one or more significant departures from previous experience with the beam-beam interaction: a large number of parasitic crossings, two rings, unequal energies, and crossing angles. All but unequal energies are motivated by the need to increase the number of bunches either to reach high luminosity while staying within the constraints of the beam-beam interaction or to limit the number of interactions per crossing.

#### **Parasitic Collisions In The Tevatron**

The discussion of parasitic collisions concentrated on the Tevatron. At the present time the Tevatron is running with six bunches per beam, approximately 50 center-to-center separation at parasitic crossing points, and a total shift  $N_{IP}\xi_{\overline{p}} = 0.015$ . Plans call for as many as 99 bunches and  $N_{IP}\xi_{\overline{p}} = 0.024$  in the Main Injector era. Performance is sensitive to the amplitude dependence of tunes, apertures, lattice and beam-beam resonances, etc. When these are not known it is impossible to generalize and extend the present experience with parasitic crossings at the Tevatron, CESR, and LEP to new Tevatron conditions.

It seemed best to identify some experiments that would explore parasitic crossings at the Tevatron directly. Three experiments were suggested:

1 Collide one  $\overline{p}$  bunch with 99 proton bunches in the planned configuration. Measure luminosity, lifetime and losses. This would approximate the new conditions since the  $\overline{p}$ 's are lower intensity and will have less effect on the protons.

2 Study lifetime and losses in the 6 bunch per beam configuration with smaller separations. This would show if the present situation were marginal. These first two experiments together would provide data on the tolerable strength of resonances produced by parasitic crossings.

3 Compare lifetimes and losses caused by a scraper positioned different distances from the beam center with those from parasitic crossings with different separations. If the effect of parasitic crossings was to scrape the beam removing all particles beyond a certain amplitude, there would be a relation between the results and a clear way to interpret parasitic beam-beam crossings as effectively a scraper of large amplitude particles.

#### **Two Ring Colliders**

Electron-positron heavy quark factories are being designed as two ring colliders to allow a large number of bunches without a large number of parasitic collisions. Coupling and coupling correction, discussed above, are likely to be more stringent than in the past since the constraints that come from the beams sharing common magnetic elements are not present. In addition, the coupling will be strong because of the relatively low beam energy and strong detector solenoids and high tunes for small emittances.

B-factories have the additional complication of unequal beam energies. The design criterion has been to make the beam-beam properties of the two beams the same. There are two difficulties with this: i) there are different opinions about the critical parameters and the meaning of equal beam-beam properties (13, 14), and ii) beam-beam performance depends on small errors and effects that will not be the same in two different energy rings. Therefore, it is unlikely that it is possible to make the beam-beam properties of the two beams equal.

Flexibility will be the key to obtaining good performance. The lattice must work for a wide range of operating points and different interaction point parameters. With flexibility one can respond to improved understanding during design and construction and perform beam-beam experiments during commissioning. As examples, one expectation that is not universally accepted is that the  $\beta$ \*'s of the two beams must be equal. Only time will tell whether this is correct, and the best thing to do at present is to be sure that the lattice can accommodate equal  $\beta$ \*'s. There is concern about "flip-flop" modes where the equilibrium condition is that the beams have grossly unequal sizes. Controlling the flip-flop requires vertical and horizontal emittance control. While the working group could not agree on the importance of the flip-flop effect, there was universal agreement that emittance control was typical of the lattice flexibility needed and that it should be available at the time the collider is commissioned.

### **Crossing Angles**

Colliding at an angle is extremely attractive way to avoid parasitic collisions with closely spaced bunches. This was attempted in DORIS in the early 1970's but was given up because of the beam-beam interaction excited strong synchrobetatron resonances (15). Interest has revived recently because of experiments at CESR (16, 17). Good beam-beam performance,  $\xi \sim 0.04$ , has been obtained with a crossing angle of  $\phi = \pm 2$  mrad. The ratio of offset due to the crossing angle and the bunch size was

$$a_{\phi} = \frac{\sigma_L \phi}{2\sigma_x} \sim \text{few} \times 10^{-2},$$

and performance was limited by the field quality of the interaction region quadrupoles.

Crossing angles are now features of TRISTAN B and DA $\Phi$ NE. The crossing angles are  $\pm 10$  mrad in both cases, and  $a_{\phi} \sim 0.25$  in TRISTAN B and  $a_{\phi} \sim 0.08$  in DA $\Phi$ NE. These are more aggressive than in the CESR case, and the designers may have to fall back to smaller crossing angles or crab-crossing to get good beam-beam performance.

#### **CONCLUDING REMARKS**

The major challenge for physicist studying the beam-beam interaction is that of identifying and measuring critical parameters. Such characterization of the collider is the key to good peak and average luminosity. This study cannot be performed with theory and simulation alone. Beam-beam experiments are essential.

#### **APPENDIX: BASIC DEFINITIONS AND EQUATIONS**

The luminosity is given by

$$L = \frac{1}{4\pi} \frac{n^2 f_c}{\sigma_x \sigma_y}$$

where n is the number of particles per bunch,  $f_c$  is the collision frequency, and  $\sigma_x$ ,  $\sigma_y$  are the horizontal and vertical rms beams sizes, respectively. Flat beams,  $\sigma_x >> \sigma_y$ , are assumed throughout.

The beam-beam strength parameter is

$$\xi = \frac{r_c}{2\pi} \frac{n\beta_y^*}{\gamma \sigma_y (\sigma_x + \sigma_y)}$$

where  $\beta_y^*$  is the vertical  $\beta^*$ ,  $\gamma$  is the beam energy in units of rest energy, and  $r_c$  is the classical particle radius;  $r_c = r_e = 2.82 \times 10^{-15}$  m for e<sup>+</sup>e<sup>-</sup> colliders and  $r_c = r_p = 1.54 \times 10^{-18}$  m for proton and proton-antiproton colliders. When the luminosity is limited by the beam-beam interaction it is given by

$$L = \xi n f_c \frac{\gamma}{r_c \beta_y^*} = \xi \frac{I_T}{e} \frac{\gamma}{r_c \beta_y^*}.$$

where  $e = 1.6 \times 10^{-19}$  C is the electronic charge and I<sub>T</sub> is the total current.

The beam-beam tune spreads are

$$\xi_{y} = \frac{r_{c}}{2\pi} \frac{n\beta_{y}}{\gamma \sigma_{y}(\sigma_{x} + \sigma_{y})}; \xi_{x} = \frac{r_{c}}{2\pi} \frac{n\beta_{x}^{*}}{\gamma \sigma_{x}(\sigma_{x} + \sigma_{y})}.$$

where  $\beta_x^*$  is the horizontal  $\beta^*$ . Small amplitude particles are shifted from the lattice tunes,  $Q_{x0}$ ,  $Q_{y0}$ , by  $\xi_x$  and  $\xi_y$  in the horizontal and vertical, respectively, and  $\xi = \xi_y$ .

Other symbols used in the paper are given in Table 3.

Symbol	Definition	
σL	RMS bunch length	
$Q_x, Q_y$	Horizontal, and vertical tunes including any amplitude dependent tune shift from the beam-beam interaction	
Qs	Synchrotron tune	
C, s	Collider circumference and the coordinate giving location along the perimeter	

#### **TABLE 3**: Definitions of symbols.

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