# Nonlinear Beam Dynamics Experimental Program at SPEAR* 

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#### Abstract

Since nonlinear effects can impose strict performance limitations on modern colliders and storage rings, future performance improvements depend on further understanding of nonlinear beam dynamics. Experimental studies of nonlinear beam motion in threedimensional space have begun in SPEAR using turn-by-turn transverse and longitudinal phase-space monitors. This paper presents preliminary results from an on-going experiment in SPEAR.


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## I. INTRODUCTION

Nonlinear effects have always demanded special consideration in the field of charged particle acceleration. For example, to achieve low-emittance beams, storage rings must be built with strong focusing fields. However, strong focusing fields produce more chromatic error that must be corrected using sextupoles placed in dispersive regions of the storage ring. The strong focusing forces also reduce the dispersion. Therefore, the effectiveness of sextupole fields in correcting chromatic errors decreases. When stronger sextupoles are used, they reduce the dynamic aperture of the storage ring, and limit its performance.

The dynamics of charged particles in nonlinear fields has been analyzed by various theoretical studies [1]. Although much progress has been made in recent experiments [2], the need to verify these theoretical results, especially those related to the 6-dimensional phase-space of a particle, still exists. This article presents the recent progress of the SPEAR nonlinear dynamics experimental program. The goal is to develop a 6-dimensional phase-space monitor to study turn-by-turn single-bunch dynamics. Such a monitor also is useful as an on-line diagnostic tool for SPEAR. The following sections discuss details of this program, including modeling work, phase-space monitor development, and typically obtained data. Analysis of the data is on-going.

## II. SPEAR MODELING

The racetrack configuration of the SPEAR storage ring has a modified FODO cell structure with a circumference of 234 m , operating energy of 3.0 GeV , and emittance of $0.127 \pi \mathrm{~mm}$-mrad. At the injection energy of 2.3 GeV , the smaller emittance beam closely approximates the single-particle motion. In addition, large
perturbations can be induced in all three dimensions. These features make SPEAR an ideal environment for investigating amplitude dependent effects.

Beam-control experiments require an accurate model (at least a linear model) of the accelerator. For the SPEAR storage ring, a procedure has been developed using the response matrix to calibrate the quadrupole strengths, the corrector strengths, and the beam position monitor (bpm) gains [3]. The orbit response matrix $\mathbf{M}$ of an accelerator satisfies the relation, $\mathbf{x}=\mathbf{M} \theta$, where $\mathbf{x}$ is a vector whose elements are the horizontal and vertical displacements at each bpm, and $\theta$ is a vector whose elements are the horizontal and vertical corrector strengths. One can measure $\mathbf{M}_{\text {meas }}$ of the as-built machine and calculate $\mathbf{M}_{\text {mod }}$ of the design lattice. Generally, $\mathbf{M}_{\text {meas }}$ and $\mathbf{M}_{\text {mod }}$ do not agree because the bpm gains may not be known accurately, the corrector strengths may not be calibrated, the quadrupole strengths may not be exactly at the design values, and so on. These factors can be formulated as parameters in a weighted $\chi$-square fitting between $\mathbf{M}_{\text {meas }}$ and $\mathbf{M}_{\text {mod }}$ to obtain a more faithful representation of the accelerator.

For example, let $\mathbf{V}$ be a vector whose elements represent the difference between $\mathbf{M}_{\text {meas }}$ and $\mathbf{M}_{\text {mod }}$ [4]. In the absence of $x-y$ coupling, the number of elements, m, in $\mathbf{V}$ equals the number of horizontal and vertical correctors times the number of bpms. Assuming that the main parameters are the bpm gains $\mathrm{G}_{\mathrm{j}}$, the corrector strengths $\theta_{\mathrm{j}}$, and the quadrupole strengths $\mathrm{K}_{\mathrm{j}}$, a first order equation for $\mathbf{V}$ can be written as

$$
\begin{equation*}
\mathbf{V}=\frac{\partial \mathbf{V}}{\partial \mathrm{K}_{\mathrm{j}}} \delta \mathrm{~K}_{\mathrm{j}}+\frac{\partial \mathbf{V}}{\partial \theta_{\mathrm{j}}} \delta \theta_{\mathrm{j}}+\frac{\partial \mathbf{V}}{\partial \mathrm{G}_{\mathrm{j}}} \delta \mathrm{G}_{\mathrm{j}}+\frac{\partial \mathbf{V}}{\partial(\Delta \mathrm{p} / \mathrm{p})_{\mathrm{j}}} \delta(\Delta \mathrm{p} / \mathrm{p})_{\mathrm{j}} \tag{1}
\end{equation*}
$$

where $(\Delta \mathrm{p} / \mathrm{p})_{\mathrm{j}}$ is the electron energy shift due to the j th corrector strength $\theta_{j}$. For a general set of $n$ parameters $a_{j}$, Eq. (1) can be written

$$
\begin{equation*}
\mathrm{V}_{\mathrm{i}}=\sum_{\mathrm{j}=1}^{\mathrm{n}}\left(\frac{\partial \mathrm{~V}_{\mathrm{i}}}{\partial \mathrm{a}_{\mathrm{j}}}\right) \delta \mathrm{a}_{\mathrm{j}} \tag{2}
\end{equation*}
$$

and the weighted $\chi$-square fitting equation for this problem has the form

$$
\begin{equation*}
\chi^{2}=\sum_{i=1}^{m}\left(\frac{V_{i}}{\sigma_{i}}-\sum_{j=1}^{n}\left(\frac{\partial V_{i}}{\partial a_{j}}\right) \delta a_{j}\right)^{2} \tag{3}
\end{equation*}
$$

where $\sigma_{\mathrm{i}}$ is the rms measurement noise of the $\mathrm{i}^{\mathrm{t}} \mathrm{bpm}$. The least $\chi$-square solution of Eq. (3) can be obtained conveniently using the singular value decomposition (SVD) method of linear algebra [5]. This technique has been used on SPEAR and on the NSLS x-ray ring [6] to calibrate the linear optics model.

## III. TRANSVERSE PHASE-SPACE MONITOR

Once we had calibrated the SPEAR model, the next step was to develop a 6-dimensional phase-space monitor. This monitor consisted of two principal units:
(1) the transverse phase-space unit described here, and
(2) the longitudinal phase-space unit that will be discussed in the next section.

The transverse phase-space monitor is an expanded version of a device first used in SPEAR to measure betatron phase advance [7]. Given the horizontal beam displacements $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ measured at $\mathrm{bpm}_{1}$ and $\mathrm{bpm}_{2}$, respectively, and assuming that there are only quadrupoles and/or bends between these two bpms, then $x_{1}$ and $x_{2}$ are related by [8]

$$
\begin{equation*}
\mathrm{x}_{2}=\sqrt{\beta_{2} / \beta_{1}}\left(\cos \mu_{12}+\alpha_{1} \sin \mu_{12}\right) \mathrm{x}_{1}+\sqrt{\beta_{1} \beta_{2}}\left(\sin \mu_{12}\right) \mathrm{x}^{\prime}{ }_{1}, \tag{4}
\end{equation*}
$$

where $\mathrm{x}^{\prime}{ }_{1}$ is the angle the beam made with respect to the design orbit at $\mathrm{bpm}_{1}, \beta_{\mathrm{i}}$ is the value of the betatron amplitude function at the $\mathrm{i}^{\text {th }} \mathrm{bpm}, \alpha_{\mathrm{i}}=-\beta_{\mathrm{i}}^{\prime} / 2$, and $\mu_{12}$ is the betatron phase advance between $\mathrm{bpm}_{1}$ and $\mathrm{bpm}_{2}$. Equation (4) can be solved for $\mathrm{x}^{\prime}{ }_{1}$,

$$
\begin{equation*}
x_{1}^{\prime}=\frac{x_{2}-\sqrt{\beta_{2} / \beta_{1}}\left(\cos \mu_{12}+\alpha_{1} \sin \mu_{12}\right) x_{1}}{\sqrt{\beta_{1} \beta_{2}\left(\sin \mu_{12}\right)}} \tag{5}
\end{equation*}
$$

However, analysis of phase-space data is more convenient using the CourantSnyder normalized coordinates $\left(\mathrm{x}, \mathrm{p}_{\mathrm{X}}\right)$, where $\mathrm{p}_{\mathrm{X}}=\alpha_{\mathrm{X}} \mathrm{x}+\beta_{\mathrm{X}} \mathrm{x}^{\prime}$. In the normalized coordinate system, linear motions are characterized by the equation of a circle,

$$
\begin{equation*}
\mathrm{p}_{\mathrm{x}}^{2}+\mathrm{x}^{2}=2 \beta_{\mathrm{x}} \mathrm{~J} \tag{6}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{J}=\frac{1}{2}\left(\gamma_{\mathrm{x}} \mathrm{x}^{2}+2 \alpha_{\mathrm{x}} \mathrm{x} \mathrm{x}^{\prime}+\beta_{\mathrm{x}} \mathrm{x}^{\prime 2}\right) \tag{7}
\end{equation*}
$$

is the Courant-Snyder invariant with $\gamma_{\mathrm{X}}=\left(\alpha_{\mathrm{X}}+1\right) / \beta_{\mathrm{X}}$. Thus, by measuring the singleturn horizontal displacements $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ at two bpms, the normalized momentum $\mathrm{p}_{\mathrm{x} 1}$ at $\mathrm{bpm}_{1}$ can be derived using Eq. (5) and the definition of $\mathrm{p}_{\mathrm{X}}$. Data from multi-turn measurements can then be used to map out the horizontal phase space of a singlebunch. For small amplitude oscillations, one can fit the phase-space data to Eq. (6) to obtain the ratio of the betatron amplitude function $\left(\beta_{2} / \beta_{1}\right)$ and the betatron phase advance ( $\mu_{12}$ ). The equations for the vertical case are similar.

In practice, a signal generated by one of the four serially triggered 8 -bit, 2-channel waveform digitizers (LeCroy 6840) initiates the transverse data acquisition sequence. This signal is used to trigger either an injection kicker for horizontal excitation of the beam, a pair of electrostatic separation plates for vertical excitation, or both. The injection kicker pulse has a width of approximately $2 \mu \mathrm{~s}$ FWHM, and that


FIGURE 1. Block diagram of the transverse phase-space system.
of the separation plates (whose power supply circuit is currently being redesigned) will be roughly 800 ns . Once perturbed, the single-bunch beam executes coherent betatron oscillation. The transverse position signals of the beam detected by each bpm are stretched by passive filters and processed by hybrid junctions to produce four signals: the horizontal difference $\Delta x$, the vertical difference $\Delta y$, the sum, and the trigger. The latter is used to clock the LeCroy 6840 waveform digitizers to record the amplitudes of the other signals. The ratios ( $\Delta \mathrm{x} /$ sum) and ( $\Delta \mathrm{y} /$ sum) give single-turn current-independent horizontal and vertical displacements. Figure 1 shows a block diagram of the turn-by-turn transverse phase-space data acquisition system.

## IV. LONGITUDINAL PHASE-SPACE MONITOR

The longitudinal phase-space can be studied by analyzing relative oscillations in the longitudinal phase and momentum error. These variables are acquired by a technique similar to the one used at the IUCF [9].

Under ideal conditions, a particle (to be approximated by a bunch centroid) with the design momentum arriving at a reference point along the storage ring will have a constant phase, called the synchronous phase, relative to the rf. When a particle is excited longitudinally (as is the case when the rf frequency is phase shifted momentarily), it executes longitudinal phase oscillation. To measure the relative phase, a stripline signal is applied to a band-pass filter (BPF) with a center frequency identical to that of the rf [10]. To avoid the problem of current-dependent stripline signal, the BPF outputs are amplified to saturation by limiting amplifiers. The difference between the amplified BPF output and the reference rf is then measured by a phase detector. Figure 2 shows a schematic layout of the longitudinal phase detection system.


FIGURE 2. Block diagram of the SPEAR relative longitudinal phase detector.

To obtain turn-by-turn momentum error data, note that the total horizontal displacement $x_{\text {total }}$ consists of a betatron component $x \beta$ and a dispersive component $\eta \delta$,

$$
\begin{equation*}
x_{\text {total }}=x_{\beta}+\eta \delta \tag{8}
\end{equation*}
$$

The momentum error of a longitudinally excited particle is obtained by bandpass filtering the horizontal turn-by-turn data at the synchrotron frequency. This process is accomplished using off-line software.

## V. DATA SAMPLES

Samples of turn-by-turn data of the single-bunch motion in SPEAR are presented in this section. The data were obtained using the phase-space monitor discussed above. Figure 3 shows the phase oscillation of a longitudinally excited
electron bunch in SPEAR. The bunch can be simultaneously excited in all three dimensions. Figure 4 illustrates a typical horizontal betatron motion, and the derived transverse phase-space tracking.


FIGURE 3. The measured longitudinal phase as a function of time.


FIGURE 4. Typical turn-by-turn data measured by the transverse phase-space monitor: (a) turn-by-turn horizontal betatron motion data, and (b) horizontal phase-space tracking.

## VI. CONCLUSION

We have initiated a nonlinear dynamics experimental program at SSRL. In the process, we have calibrated the SPEAR model using a $\chi$-square fitting procedure based on the response matrix and have developed a 6-dimensional phase-space monitor.

Turn-by-turn data collected using the phase-space monitor will be used to study fully coupled nonlinear particle motion.

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