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DECOHERENCE, DETERMINISM and CHAOS Revisited*

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ABSTRACT

We suggest that the derivation of the free space Maxwell Equations for classical electromagnetism, using a discrete ordered calculus developed by L.H.Kauffman and T.Etter, *necessarily* pushes the discussion of determinism in natural science down to the level of relativistic quantum mechanics and hence renders the *mathematical* phenomena studied in deterministic chaos research irrelevant to the question of whether the world investigated by *physics* is deterministic. We believe that this argument reinforces Suppes' contention that the issue of determinism versus indeterminism should be viewed as a Kantian antinomy incapable of investigation using currently available scientific tools.

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1. Introduction

I am delighted to have had the opportunity to bring to this Symposium the question of whether recent work connecting relativistic quantum mechanics to the classical relativistic theory of fields sets the interpretation of "deterministic chaos" in a rather different—and possibly illuminating—context. My title comes from the fact that I had already raised this point at the 15th annual international meeting of the Alternative Natural Philosophy Association.[1] I did not gain much enlightenment on the significant and difficult issues raised from the resulting discussion. I hope that, thanks to the passage of time, subsequent work with L.H.Kauffman[2] and the different types of expertise present at this Symposium, I will gain a broader perspective from your comments.

The first argument I mount against the relevance of "chaos research" to the issue of determinism rests on the fact that physics is a science of measurement. If one accepts the operational methodology implied by this statement, and recognizes that the smallest space interval Δx and time interval Δt which one can measure is *always* bounded from below by the current state of technology, then there is a limit to the accuracy to which the initial conditions for prediction using a classical, deterministic system of equations can be stated. What chaos research has demonstrated is that there are many non-linear classical systems which require as much input information to obtain a "prediction" as can be obtained from the result "determined" by solving the deterministic equations. Hence the issue becomes irresolvable from the point of view of *physics* once one is asked to make a "prediction" that requires more accuracy in the input than is available from current technology. The next section tries to spell this out by invoking NO-YES events, and in particular the not-firing or firing of a recording counter as the paradigm for measurement in physics.

So far, this states a point of view, and may not sound particularly compelling. But when one asks where the classical equations come from, the argument can be tightened. So far as I can see, the *only* classical systems of equations which do

not depend in detail on the structure of matter — and hence on quantum effects — are electromagnetism and gravitation. Here an ancient piece of work by Feynman, recently resurrected by Dyson[3] and extended by Tanimura,[4] comes to our aid. Dyson derives electromagnetism and Tanimura also derives gravitation from Newton's second law and the commutation relations of non-relativistic quantum mechanics! This paradoxical result is shown by our analysis to depend only on the assumption that measurement accuracy is *finite, fixed* and *bounded from below.* By an appropriate and significant extension of the calculus of finite differences to a non-commuting *discrete ordered calculus* (DOC), due to Etter and Kauffman,[5] this derivation becomes rigorous in a very general context. Accepting this derivation, the *classical* equations require finite and discrete measurement accuracy to ground them in *physics.* But then, to treat them as deterministic goes beyond the range of applicability of their foundation. This puts bite into the argument that classical, deterministic equations are *always* approximate, and hence that the context in which chaos research is usually set has no validity within the world of *physics* as I understand the term. This argument is presented in more detail in Section 3.

A second reason for taking the classical equations to be approximate is the underlying non-determinism of quantum mechanics. Strictly speaking classical equations apply only at large enough distances so that the particles which probe the fields are *decoherent* in the quantum mechanical sense. Hence, we argue that "deterministic chaos" is *always* an approximation, and that any fundamental discussion of *determinism* must be conducted at the quantum level. This pushes the discussion back to the level of Bell's Theorem, which is often interpreted as showing that demonstrable laboratory effects (e.g. Aspect's experiment) preclude the possibility of a local, deterministic description of natural science. The relationship between measurement accuracy and "decoherence" in our context is discussed in Section 4.

The conflict between quantum mechanics and Einstein locality raises a third issue about the approximate character of classical physics. This is the problem of how to construct a relativistic quantum mechanics which has classical field the-

ory as a well defined correspondence limit. The specific *measurement* limitation involved is clearly the fact that when one attempts to measure distances shorter than $\hbar/2m_ec$, either directly or indirectly, one must take proper account of the degrees of freedom corresponding to electron-positron pair creation. We note that going below these bounds requires a *relativistic* quantum mechanical analysis. This provides a *third* reason why the deterministic interpretation of classical physics can never be more than an approximation. We explore, briefly, in Section 5 how a novel theory based on *bit-strings* might meet this problem. Our concluding section returns to the philosophical issues.

2. NO-YES EVENTS AS A RELATIVISTIC MEASUREMENT PARADIGM

My approach to the questions of law and prediction in *physics*—rather than in the broader context of (Natural) Science used in the title of this Symposium—starts from the trite comment that *physics is a science of measurement.* I take this characterization of physics as a *methodological requirement.* Unfortunately, from my operational and pragmatic point of view, this dictum is much more often honored in the breach than in the observance. In my practice of physics I do not allow my fundamental paradigms for how theoretical physics should be connected to laboratory experience to rest on considerations that are not in some sense bounded by the actual experimental accuracy of current measurements.

This statement of methodological principle is unabashedly taken from Bridgman's heroic attempt[6] to rescue physics from the philosophers. It is usually assumed that his program failed to provide a proper conceptual foundation for the startling and enormously fruitful developments in relativistic cosmology and elementary particle physics which have provided contemporary scientists with such a rich picture of the physical world accessible to precise measurement. But the actual reconciliation of quantum mechanics with relativity, and in particular the creation of a theory of "quantum gravity" that commands consensus among the

specialists, still eludes us as this century draws to a close. I have argued in more detail elsewhere[7] why a return to Bridgman's principles might help resolve some of the thorny problems that still face us.

My approach is also informed by the S-Matrix program of Chew and Heisenberg, which—according to Schweber[8]—really started with Dirac. The basic point for me is that by going to large enough distances (and hence, necessarily, times) in the experimental setup, momentum and energy can always be measured to arbitrarily high accuracy using essentially classical physics techniques and concepts. In contrast, direct space-time measurement at short distance is always restricted by the uncertainty principle and looses direct operational meaning. Hence the *formal* symmetry between position and momentum measurement in quantum mechanics is destroyed *in practice*. As Chew used to put it, short distance space-time is an *artifact* of Fourier transformation and cannot have physical significance. Unfortunately, from my point of view, he did not take the next step and reject *continuum mathematics* as well.

This next step has, for me, a long history which is briefly explained in my contribution to *PhysComp'94*.[9] The fundamental mathematical position comes from a necessary aspect of the practice of *computer science*, namely that you must name a largest integer N and the fixed, finite memory size *in advance*. If you need or wish to introduce larger numbers into the calculation, or change the size of the memory, you *must* re-examine everything you have done up to that point. This obvious fact has been particularly emphasized by David McGoveran;[10] in effect, he makes it into a methodological principle. Note that this not only rules out the continuum, but also mathematical induction. Few theoretical physicists and almost no mathematicians are willing to take such a drastic step. In elementary particle physics, whenever a theory is examined empirically, the events analysed, the model of the apparatus used in the analysis, and the theories under consideration are *necessarily* reduced to a finite number of bits on magnetic tape or some other digital form of memory. That this procedure must be used in order to test any *empirical* aspect of any theory may, perhaps, make our methodological purity seem

less outrageous.

This much discussion seems necessary to justify my *measurement paradigm* based on what I call NO-YES events. The model I have in mind is a laboratory counter and associated memory storage which records whether an event *did not* take place in a time interval Δt in a volume Δx^3 with relevant linear dimension Δx (a NO event) or *did* take place (a YES event). I emphasize that, when it comes to precise measurement, the *absence* of a counter firing is often more important (eg in measuring "background") than its presence. For our paradigm we assume that the temporal resolution of the measurement Δt and the spacial resolution Δx are the *best* that can be achieved with current technology either by direct measurement, or *indirectly* as when one uses a Michelson interferometer to measure relative positions. Note that in order to relate such relative measurements to macroscopic laboratory coordinates, we would have to discuss the measurement accuracy with which we can connect the different space-time scales.

Up to this point we have treated length and time measurement as distinct. But the *System International*, employed universally by physicists in reporting the results of measurement, defines the *ratio* of space to time units by the *integer*

$$c \equiv 299\ 792\ 458\ meter/second \tag{2.1}$$

Thus, following current practice, we are no longer allowed to define Δx and Δt separately when specifying our lowest bound on measurement accuracy. In fact, we must make the *scale invariant* statement that

$$\frac{\Delta x}{c\Delta t} = 1 \tag{2.2}$$

in *any* system of units which allows us to talk about NO-YES events in a precise way.

We can summarize the content of this section by the phrase:

PHYSICS IS COUNTING

3. Classical Relativistic Fields from DOC

In 1948 Richard Feynman showed Freeman Dyson a remarkable "proof" of the Maxwell Equations starting from the non-relativistic quantum mechanical commutation relations and Newton's second law.[11] Dyson no longer retains contemporary records of this conversation, but was able to reconstruct and publish the proof using notes he had made at a later date.[12] Although Dyson finds the proof paradoxical, we have claimed[13] that in fact it makes good sense in terms of the new, fundamental theory discussed in Ref. 9.

Briefly, the argument goes as follows. The Feynman postulates are that

$$[x_i, x_j] = 0; \quad [x_i, m\dot{x}_j] = \frac{\hbar}{i}\delta_{ij}; \quad F_i(x, \dot{x}; t) = m\ddot{x}_i; \quad i, j \in 1, 2, 3 \tag{3.1}$$

However, the use made in the proof of the second postulate (i.e. of the commutation relation between position and velocity) in no way requires the constant on the right hand side to be imaginary, or scaled by Planck's constant. The linearity in the mass parameter m allows us to divide through by m and replace it by the postulate

$$[x_i, \dot{x}_j] = \kappa\delta_{ij} \tag{3.2}$$

with κ any constant with dimensions of area per unit time. For a particle acting under any force which obeys Newton's third law with respect to a reference particle, we know that the area (measured in units of Δx^2) swept out by the line from some appropriate center to the particle in a constant time interval (measured in units of Δt) is constant. This observation fixes κ in an *scale invariant* manner. Note that this generalization of Kepler's second law is *kinematic* rather than dynamic. It leaves both the mass standard and the mass ratio between the particle of interest and the reference particle *arbitrary*. Similarly, since Newton's second law is linear in mass, we can replace it by the assumption that the acceleration ($\ddot{x}$) is a function only of position, velocity and time. Finally, for any single particle for which the charge per unit mass is a Lorentz invariant, we can *also* divide the mass out of

Maxwell's Equations, and find that the whole derivation is *scale invariant* because it depends only on fixing, *arbitrarily*, the units of length and time.

As is noted in Ref.2:

"…. this aspect of scale invariance had already been introduced into the subject by Bohr and Rosenfeld in 1933.[14] In their classic paper, they point out that because QED depends only on the universal constants $\hbar$ and c, the discussion of the measurability of the fields can to a large extent be separated from any discussion of the atomic structure of matter (involving m_e and e^2). Consequently, they are able to derive from the *non-relativistic* uncertainty relations the same restrictions on measurability (over finite space-time volumes) of the electromagnetic fields that one obtains directly from the second-quantized commutation relations of the fields themselves. Hence, to the extent that one could "reverse engineer" their argument, one might be able to get back to the classical field equations and provide an alternative to the Feynman derivation based on the same *physical* ideas."

This point of view is also discussed in more detail elsewhere.[15]

Unfortunately, this *physical* argument has not proved compelling for many people in the relevant professional communities. We have therefore been forced to invoke the aid of a first rate mathematician and to go deeper into the mathematical foundations of the calculus of finite differences (see Ref. 2) than might be expected. This suggested further developments to T. Etter, which are now being pursued (see Ref. 5).

The basic physical point from which the discussion of the impact of finite measurement accuracy on the relation between position and velocity starts is that velocity has to be defined as the ratio of a finite space interval to a finite time interval. We also restrict the problem to the "trajectory" of a single particle, and a finite shift along that trajectory. Then measurement of velocity must involve either first the specification of position and then the finite shift to a new position from which the velocity can be calculated, or first the shift from a previous position at some velocity and then the specification of the new position consistent with that velocity.

These two velocities will not, in general, coincide. Note that this operational definition of velocity *precludes* the possibility of specifying both position and velocity at the same time. Thus the possibility of non-commutativity arises, and careful investigation of the possibilities leads to the discrete ordered calculus (DOC) of Etter and Kauffman. This (non-commutative) calculus of finite differences does, indeed, provide a rigorous mathematical context for the Feynman-Dyson "proof", allowing us to drop the quotation marks.

Exploring the mathematical niceties of this generalization of the calculus of finite differences would distract us from the thrust of this paper. When I recently showed Ref. 2 to my colleague, M.Peskin, he noted that the "shift operator J" defined by Kauffman is, in our context of a single particle, isomorphic to the operator $U = exp(-iH\Delta t$ representing a finite time shift in the Heisenberg representation. Then the formal steps in Kauffman's rigorous version of Feynman-Dyson-Tanimura "proof"go through easily. The difficulty with adopting Peskin's approach is that what *operational* context the Heisenberg formalism fits into is by no means obvious. So, for mathematical and physical clarity, one needs to invoke the DOC and discuss the relationship between measurement accuracy and the DOC. I am indebted to Peskin[16] for allowing me to quote his rewritten proof below.

Define

$$\dot{X} = XU - UX = [X, U] \tag{3.3}$$

where U is the time shift operator from X to X' in time Δt (eg $U = e^{-iH\Delta t}$).

Notice that

$$(AB)^{\cdot} = [AB, U] = [A, U]B + A[B, U] = \dot{A}B + A\dot{B} \tag{3.4}$$

as required.

Postulate:

$$1. \quad [X_i, X_j] = 0$$

$$2. \; [X_i, \dot{X}_j] = \kappa \delta_{ij}$$

Rewrite 2 as

$$[X_i, [X_j, U]] = -[X_j, [U, X_i]] - [U, [X_i, X_j]] \tag{3.5}$$

and noting that $[U, [X_i, X_j]] = [U, 0] = 0$ we find that

$$\kappa \delta_{ij} = [X_i, [X_j, U]] \; symmetric \; in \; i, j \tag{3.6}$$

Now *define*

$$H_l = \frac{1}{2\kappa} \epsilon_{jkl} [\dot{X}_j, \dot{X}_k] \tag{3.7}$$

Then

$$\nabla_l H_l = \frac{1}{2\kappa} \epsilon_{jkl} [[\dot{X}_j, \dot{X}_k], \dot{X}_l] \tag{3.8}$$

But this cyclic sum vanishes by the Jacobi identity. Thus

$$\nabla_l H_l = 0 \tag{3.9}$$

which is one of the two Maxwell equations we set out to derive.

Finally, *define*

$$E_i = \ddot{X}_i - \epsilon_{ijk} H_k \tag{3.10}$$

We wish to prove that

$$\frac{\partial H_i}{\partial t} + \epsilon_{ijk} \nabla_j E_k = 0 \tag{3.11}$$

First we need to *define* $\partial/\partial t$ by

$$\dot{H} = \frac{d}{dt} H = \frac{\partial H}{\partial t} + (\dot{X} \cdot \nabla) H \tag{3.12}$$

Then

$$\frac{\partial H_i}{\partial t} = \dot{H}_i - \dot{X}_j \nabla_j H_i$$

$$= \frac{1}{2\kappa} \epsilon_{ikl} ([\dot{X}_k, \dot{X}_l])^{\cdot} - \dot{X}_j \frac{1}{\kappa} [\frac{\epsilon_{ikl}}{2\kappa} [\dot{X}_k, \dot{X}_l], \dot{X}_j]$$

$$= \frac{1}{\kappa} \epsilon_{ikl} [\dot{X}_k, \ddot{X}_l] - \frac{1}{2\kappa^2} \dot{X}_j \epsilon_{ikl} [[\dot{X}_k, \dot{X}_l], \dot{X}_j] \tag{3.13}$$

$$\begin{aligned}
\epsilon_{ijk} \nabla_j E_k &= \epsilon_{ijk} \frac{1}{\kappa} \left[\left(\ddot{X}_k - \epsilon_{klm} \dot{X}_l H_m \right), \dot{X}_j \right] \\
&= \frac{1}{\kappa} \epsilon_{ijk} \left[\dot{X}_j \ddot{X}_k \right] \cdot (-1) - \epsilon_{ijk} \epsilon_{klm} \epsilon_{mab} \frac{1}{2\kappa^2} \left[\dot{X}_l \left[\dot{X}_a, \dot{X}_b \right], \dot{X}_j \right] \\
&= -\frac{1}{\kappa} \epsilon_{ijk} \left[\dot{X}_j, \ddot{X}_k \right] \\
&\quad - \left(\delta^{il} \delta^{jm} - \delta^{im} \delta^{jl} \right) \epsilon_{mab} \frac{1}{2\kappa^2} \left(\left[\dot{X}_l, \dot{X}_j \right] \left[\dot{X}_a, \dot{X}_b \right] + \dot{X}_l \left[\left[\dot{X}_a, \dot{X}_b \right] \dot{X}_j \right] \right) \\
&= -\frac{1}{\kappa} \epsilon_{ijk} \left[\dot{X}_j \ddot{X}_k \right] + \frac{1}{2\kappa^2} \epsilon_{iab} X_j \left[\left[\dot{X}_a \dot{X}_b \right], \dot{X}_j \right] \\
&\quad - \epsilon_{jab} \frac{1}{2\kappa^2} \left[\dot{X}_i, \dot{X}_j \right] \left[\dot{X}_a, \dot{X}_b \right]
\end{aligned} \tag{3.14}$$

now

$$\epsilon_{jab}\left[\dot{X}_i,\dot{X}_j\right]\,\left[\dot{X}_a,\dot{X}_b\right]=\left[\dot{X}_i,X_1\right]\,[X_2,X_3]$$
$$+\left[\dot{X}_i,\dot{X}_2\right]\,\left[\dot{X}_3,\dot{X}_1\right]+\left[\dot{X}_i,\dot{X}_3\right]\,\left[\dot{X}_1,\dot{X}_2\right] \tag{3.15}$$

for $i=1$, eg

$$=\left[\dot{X}_1,\dot{X}_2\right]\,\left[\dot{X}_3,\dot{X}_1\right]+\left[\dot{X}_1,\dot{X}_3\right]\,\left[\dot{X}_1,\dot{X}_2\right]=0 \tag{3.16}$$

so

$$\epsilon_{ijk}\nabla_j E_k=-\frac{1}{\kappa}\,\epsilon_{ijk}\left[\dot{X}_j,\ddot{X}_k\right]+\frac{1}{2\kappa^2}\,\epsilon_{iab}\,X_j\,\left[\left[\dot{X}_a,\dot{X}_b\right]\,\dot{X}_j\right]$$
$$=-\frac{\partial H}{\partial t}\qquad QED\ . \tag{3.17}$$

We conclude that the free field Maxwell Equations are a formal consequence of assuming finite time shifts along a single particle trajectory and showing that the changes in velocity (accelerations) have the form of the Lorentz force law (i.e. eq. 3.10 or $mF=eE+ev\times H$) for electromagnetic fields acting on a particle. This formula allows us to separate the acceleration into a vector which is a function of position and time (electric field) and produces an acceleration in that direction, and a second vector — also a function of position and time — which acts at right angles to the velocity and is proportional to the magnitude of the velocity (magnetic field).

We emphasize that *given* the fields, we can calculate the motion of a single particle passing through them, or *given* the trajectory, we can calculate the fields which would produce that trajectory. Invoking Newton's third law, and treating the field as a carrier of both energy and momentum, we can treat this second calculation as either the absorption of the radiation by the particle producing its motion or as the emission of the field by the particle when its motion is known. This language then allows us to treat single particle trajectories as either the sources or sinks of the fields *but not both at once*. The (insoluble) "self energy"

problem cannot be met this way. One can achieve consistency at the classical level only by separating sources and sinks, as was done by Feynman and Wheeler in their "relativistic action at a distance" theory.[17] But then, in a closed system, the source and sink are made macroscopically (and non-locally) *coherent* by the energy-momentum conservation laws. Thus, treating the field as a locally defined and causally efficacious agent is only possible in the *decoherent approximation* in which we can ignore where the radiation is coming from and where it is going.

We will discuss this intricate question of coherence and decoherence further in the next section. For the moment, we emphasize that our *derivation* of the field equations from measurement accuracy *necessarily* limits their applicability as deterministic predictors to situations in which the boundary conditions and the predictions are made to less accuracy than the $\Delta x = c\Delta t$ restriction which allows us to derive the "differential" form of the field equations in the first place. Hence, *if our understanding of the classical electromagnetic field is accepted*, "deterministic chaos" cannot enter the system, and the distinction between determinism and indeterminism eludes us.

To complete the argument of this section, we need to extend the argument to the only remaining classical field, namely gravitation. At least within the framework of the Feynman-Dyson "proof", this has already been done by Tanimura in Ref. 4. Tentatively, at least, we accept this extension, but will not be sure of our conclusion until we have a rigorous equivalent using the DOC. The novelty here is that we must consider not only non-commutativity between position and velocity but the connectivity between oriented *areas*. This gives (at least formally) the usual tensor field in free space and the resulting *non-locality* of general relativity. Again, the field as a local, causal agent appropriate to think of as "deterministic" can only be a *decoherent approximation*. Thus, independent of details, we again find the phenomena of "deterministic chaos" irrelevant to what we can know *physically*.

4. DECOHERENCE; PERIODICITY FROM MEASUREMENT ACCURACY

[Spelling out in operational terms just what we mean by "decoherence" requires some care. I have already done this in Ref.1. The next four sub-sections repeat these considerations with a few modifications.]

4.1 The Geometrical Paradigm for Decoherence

To give form to our discussion of coherence and decoherence, we use the devices schematically illustrated in figure 1. We assume, initially, that the "source" labeled by a question mark emits charged particles with a unique charge-to-mass ratio and a unique velocity v. Note that these particles, taken one at a time, fit into our understanding of "particle" and "field" as established in Section 3. Devices which we will use to insure that, to some finite accuracy, these assumptions are true are included in the figure, and will be discussed in more detail subsequently. For the moment we omit the "path extender".

We start from the case when the detection screen beyond the double slit[18] exhibits a double slit interference pattern whose envelope is the single slit diffraction pattern for a slit of width Δw and a distance D from the detector array. We set the parameters such that the spacing from the center of the pattern to the first interference fringe is s. Then the "wavelength" λ exhibited by this coherent interference between the beams from the two slits is measured and can be calculated from the equation

$$\lambda = \frac{ws}{D} \tag{4.1}$$

We note that w, s and D are length intervals that can be measured by conventional macroscopic methods such as rods calibrated against international standards. We take this as the paradigmatic case for specifying what we mean by "coherence". We emphasize that, so far, only *length* measurements are implied and hence that our diagram is *scale invariant*.

In order to measure the "coherence length" we insert into the hypothetical "path" of the particle coming from one of the slits a "path extender", schematically represented by a wedge whose sides are mirrors. One face of the wedge reflects the beam to a second mirror which returns it to the second face of the wedge, which in turn returns it to the direction it followed in the paradigmatic case. The distance C from the wedge to the mirror is adjustable. $C = 0$ corresponds to the simplest double slit paradigm (wedge omitted). We find experimentally that for a source of a particular type the (double slit) fringe system disappears when we reach a value C_{max} or larger. We can then define the *coherence length* C_{coh} by

$$C_{coh} \equiv 2C_{max} \tag{4.2}$$

Note that the definition still depends directly on geometrical measurements. Indirectly the specification depends on the *sensitivity* of the detector array, since the *intensity* of the pattern along the detector array and (if the array records individual particulate events) the *probability* of a particular region of the array being activated decreases as C increases. The disappearance of the interference pattern is our paradigm for *decoherence*.

To go further in our analysis, we must measure the velocity v, or if this velocity is close to the limiting velocity for information transfer — for which we use the conventional symbol c — the momentum. Then we can define a second critical parameter called the *coherence time* and symbolized by T_{coh} by the relationship

$$C_{coh} = vT_{coh} \tag{4.3}$$

Here we assume that the measurement of v using the recording counters in the first counter telescope and the time from the firing of the first counter telescope to the firing of one counter in the detector array are consistent with each other, and that all three clocks associated with the counters are synchronized using the Einstein convention.

In the situation where the interference fringes have disappeared, we can distinguish two paths emerging from the double slit by noting that all particles which follow the longer path arrive at the detector with a time delay greater by at least $T_{coh} = C_{coh}/v$ compared to the particles which traverse the shorter path. Then we *know* that the two trajectories are decoherent and (in the stated context) are *classical, decoherent* trajectories of classical particles (ignoring the single slit interference pattern which takes higher precision to see).

Various checks on the confidence with which we can make the above statements can depend on the measurement accuracy to which we can establish all the relevant parameters. Several such checks will occur to any experimental particle physicist. Since these checks are irrelevant to our main theme, we stop our articulation of the basic paradigm at this point, and focus on the accuracy to which we can measure velocity or momentum. The main point we wish to establish is simply that in a carefully specified context, *outside* of some coherence length or coherence time, particles can be said to follow two (or more) distinct trajectories for at least part of their history between production and detection. Inside that length, two coherent beams of the same type of particle can be made to interfere with a characteristic wavelength that can be measured geometrically. But asking where *within* that pattern of two coherent trajectories the "particle" is located cannot find an answer within the experimental setup. This is an example of the "complementarity" between the wave and the particle description in our discrete context.

4.2 Space-time velocity measurement

The "counter telescope" we have included in figure 1 consists of two devices which *record* the time of firing *or of* **not** *firing* during some time interval. This is the next step in bringing the measurement paradigm presented in Section 2 closer to laboratory practice. The distance between the two counters is L and the time delay between the two recordings is T. These two recordings are NO-YES *events* in that whether the individual counters do not fire ("NO") or do fire ("YES") is

recorded by two distinguishable symbols in two correlated records. These records can be repeatedly examined without destroying this distinction or the sequential ordering. In this context the velocity of a particle v is measured by a YES_1, YES_2 pair of events and is calculated by the ratio

$$v = \frac{L}{T} \tag{4.4}$$

The *accuracy* to which this constitutes — or can constitute — a *measurement* of this velocity cannot be adequately discussed in an article of this length. We simply note that what are called "particles" in high energy elementary particle physics have never been demonstrated to have velocities greater than the scale parameter $c \equiv 299\ 792\ 458\ m\ sec^{-1}$. Further, there is no accepted situation in which *information* in the physical or computer science sense has been transferred at a velocity greater than this value. Demonstrable exceptions to these statements would be of extreme interest to the physics and computer science communities.

4.3 Energy-Momentum velocity measurement

The "magnetic selector" using a magnetic field $\mathcal{H}$ perpendicular to the plane of figure 1 can also be considered to be a device capable of measuring velocity when it is properly calibrated. Its action is compatible with the Lorentz force law we explained in Section 3. The calibration procedures are more complicated than the direct calibration of rods and clocks which suffice for space-time velocity measurement. It is here that our restriction to a particular type of particle begins to become important.

If the particle is electromagnetically neutral, or if the space-time velocity is not distinguishable from c (up to the maximum value of $\mathcal{H}$ available to us), no deflection is observed and the inverse radius of curvature ρ^{-1} is indistinguishable from zero. We exclude these cases for the moment because the measuring device invoked gives no information not already provided by the counter telescope. However, when a

deflection (finite, non-null ρ) is observed, we find that for fixed $\mathcal{H}$ the radius of curvature ρ changes with velocity. To cut a long story short, we find that if we measure velocity in units of c by defining

$$v \equiv \beta(v)c \tag{4.5}$$

and keep the magnetic field fixed,

$$\rho^2(v) \propto \frac{\beta^2}{1-\beta^2}; \quad \rho^{-2}(v) \propto \frac{1-\beta^2}{\beta^2} \tag{4.6}$$

This clearly allows us to calibrate our magnetic field to space-time measurements and, for a particular class of particles, to specify higher and lower magnetic fields over some range by the velocity-independent (over that range) definition

$$\mathcal{H} = \frac{\rho(v)}{\rho_0(v)}\mathcal{H}_0 \tag{4.7}$$

leaving open the units in which we ultimately decide to measure magnetic fields.

If, as is often the case in high energy physics, it is more convenient to measure radius of curvature rather than space-time velocity, we can relate this approach to the space-component of the "four velocity" $(u_0, \vec{u}) = (\gamma, \gamma\vec{\beta})$ with $\gamma^2\beta^2 = \gamma^2 - 1$ and

$$\beta^2(u) = \frac{u^2}{1+u^2}; \quad \gamma^2(u) = 1 + u^2; \quad u = \pm|\vec{u}| \tag{4.8}$$

For a particular type of particle, this tells us that u^2 is proportional to ρ^2, and in a more articulated theory allows us to measure momentum by radius of curvature in a calibrated magnetic field. In this context we can ignore the (fixed) rest-mass of our "test particles" and keep our "momentum" measurements restricted to the "space-component of four velocity" or "momentum per unit mass".

Similarly, if we measure energy by the temperature rise in a calorimeter calibrated to the ideal gas law for particles of the same mass, i.e. measure pressure

per unit mass rather than pressure, we can verify that this is consistent with the usual relativistic single particle kinematics

$$\frac{E^2}{m^2} = 1 + u^2; \quad \frac{E^2}{m^2} - \frac{p^2}{m^2} = 1 \tag{4.9}$$

and so on.

4.4 Scale invariance

We have been at some pains to remove the mass scale from our basic paradigm for "coherence" and "decoherence" because the basic argument by which we gave meaning to classical electromagnetic fields Section 3. used only measurement of space and time with accuracy bounded from below. To *break* scale invariance requires us to model some *physical* phenomenon involving Planck's constant and the reconstruction of relativistic quantum mechanics consistent with our operational methodology. Quantum mechanics can be arrived at in a number of ways, eg historically by the analysis of black body radiation, photo-effect, line spectra of atoms, finite size and stability of atoms measured using deviations from the ideal gas law, and so on. This is possible because the whole idea of a "test-particle" is basic to the classical definition of "fields", and is consistent with the understanding of electromagnetic fields we developed in Section 3. But *why* the same constant $\hbar$ should appear in these diverse empirical contexts remains unanswered.

The cleanest breakpoint for the *relativistic* quantum mechanics which concerns us is the creation of electron-positron pairs or the less direct but predicted and confirmed effects (eg Lamb shift, vacuum polarization in p-p scattering,...) of these degrees of freedom (Ref. 8). Once the degrees of freedom due to the possibility of particle-antiparticle pair creation have to be included in the theory, even the concept of a "test particle" generates nonsense. This is obvious in the case of pair creation in a system containing electrons because, thanks to the *indistinguishability* of electrons, in any system which contains one or more electrons initially whether

the electron in the created pair and some initial electron are on the same or different trajectories becomes ambiguous and empirically irresolvable at distances less than $\hbar/2m_ec$. That this parameter occurs and can be measured even when there are *no* electrons in the system under examination is evidenced by the "vacuum polarization" contribution to both the energy and the angular distributions measured in proton-proton scattering below 3 Mev.

4.5 Velocity Resolution, Periodicity and "wavelength" in a discrete theory

As already noted, we assume that *information* cannot be transmitted from one distinct location to another at a velocity greater than $c = 299\ 792\ 458\ m/sec$. By information we mean anything which reduces the number of possibilities at the second location relative to a previously accepted, understood, finite and countable number of possibilities. This allows us to specify velocities v in units of c by rational fractions $\beta(N, n) = v/c = n/N$ with N a fixed, finite positive definite *integer* which can be context sensitive. We distinguish *massive particles* from other modes of communication by the requirement that n be an integer in the range $-N + 1 \leq n \leq N - 1$.

We can now define *velocity resolution* by $\Delta v = c/N$. This is, clearly, a context sensitive definition, which requires a careful investigation of the experimental tools at our disposal in that context, and can have unexpected consequences such as the connection between fixed measurement accuracy and the formal structure of the classical, relativistic field equations we discussed in Section 3.

The context which we wish to explore first is when velocity is measured by the distance between two counters at positions $x_1\Delta x$ and $x_2\Delta x$ which fire sequentially at times $t_1\Delta t$ and $t_2\Delta t$. We assume finite and fixed *measurement accuracy* to mean that x_1, x_2, t_1, t_2 are *integers*, as discussed in Section 2. Then these four integers can be related to our previous definition of velocity by

$$\beta(N, n) = \frac{n}{N} = \frac{x_2 - x_1}{t_2 - t_1} \tag{4.10}$$

Because we took $N > 0$ in our earlier definition, we will use the definitions

$$If\ t_2 - t_1 > 0\ then\ N = t_2 - t_1,\ n = x_2 - x_1;\ \ else\ N = t_1 - t_2,\ n = x_1 - x_2 \quad (4.11)$$

This convention specifies positive spacial directions to be $x_2 > x_1$ and positive time evolution to be $t_2 > t_1$ in a finite and discrete 1+1 "space-time" with origin $(x_0, t_0) = (0, 0)$.

It is important to realize that, provided $\Delta v(N) \equiv c/N$ is not the best velocity resolution we can achieve in the context of interest, and a resolution $\Delta v(N_X) \equiv c/2N_x$ is at least conceivably within our grasp, $\beta(N, n)$ defines a *periodic* function with up to $2N_T$ periods, provided $NN_T < N_x$. To see this, we need only note that

$$\beta(N, n) = \beta(n_t N, n_t n) = \beta(n_t N, n + (n_t - 1)n) = \frac{n + (n_t - 1)n}{n_t N} \quad (4.12)$$

But this "periodicity" can have some unexpected restrictions, if we take our physical restriction on Δv seriously. In particular, for the two counter firings specified in the last paragraph, and the $\Delta v(N_x) = c/2N_x$ just assumed, we are restricted to space and time intervals between the two firings which satisfy the constraint

$$|\frac{x_2 - x_1}{t_2 - t_1}| > \frac{c\Delta t}{2N_x \Delta x} \quad (4.13)$$

Otherwise the two counter firings would measure a velocity to a resolution better than $c/2N_x$, contrary to hypothesis. We also have the further restriction $\frac{\Delta x}{c\Delta t} = 1$ from the general argument given in Section 2 justifying Eq. 2.2. Then we can define an *event horizon* $R_x = N_x \Delta x$, and a *time boundary* $T_x = N_x \Delta t$ which restrict the 1+1 integer coordinate space-time points we consider to the integer square in 1+1 space-time

$$-N_x \leq t \leq +N_x;\ -N_x \leq x \leq +N_x \quad (4.14)$$

Thus any velocity measurement we consider restricts the "integer coordinate inter-

vals" we consider by the equations

$$|x_2 - x_1| = n_t N < |t_2 - t_1| = n_t N \tag{4.15}$$

Our next concern is to understand in more detail the "state" of a particle with "constant velocity" implied by the concept of fixed, finite velocity resolution we are developing. In a continuum theory the two "point events" (x_1, t_1), (x_2, t_2) determine a line in 1+1 space-time which, according to Newton's first law, can be extrapolated to include all points between $-\infty$ and $+\infty$ outside the interval so defined, and interpolated to include all points within this interval, so long as no "force" acts on the particle. In contrast, our assumption of fixed velocity resolution restricts the positions where a constant velocity particle can appear, once the two counter firings are measured, to a very small set of integers. Assume first that n and N have no common integer factor other than 1. Then *no* interpolated positions between the two counter firings are allowed for the velocity state $\beta(N, n)$. The only coordinate pairs we are allowed (by the construction developed so far) are the extrapolated event positions

$$(x(N, n; n_t), t(N, n; n_t)); \tag{4.16}$$

where

$$x(N, n; n_t) = x_1 + n(n_t - 1); \qquad t(N, n; n_t) = t_1 + n(n_t - 1) \tag{4.17}$$

Here we allow n_t as well as n to be negative, so long as the event horizon constraints

$$-N_x < x(N, n; n_t) < +N_x; \; -N_x < t(N, n; n_t)) < +N_x \tag{4.18}$$

are met.

Note that this periodic sequence of space-time positions where a third counter might (but need not) fire *also need not* include the "origin" $(0, 0)$. A naive interpretation of our formalism would allow us to include this origin as "physical"; in a more detailed discussion we would show why we must use caution in making this part of our construction. When we have shown how measure quantum interference phenomena using only counter firings as our paradigm, we will see that the inability to locate the origin "absolutely", but only with an uncertainty $|x_2 - x_1| n_t \Delta x$, and positions "relative" to some unique reference event only with an uncertainty $|x_2 - x_1| \Delta x$ is the analog in our theory of the inability to measure "absolute phase" in conventional quantum mechanics.

Here we can take only the preliminary step of relating this finite and discrete model of positions where counters can fire sequentially to the paradigmatic case of the measurement of coherence length illustrated in Figure 1. Suppose (x_1, t_1) and (x_2, t_2) are the space-time coordinates for the firing of the entrance and exit counter before the magnetic selector, and that the counters are thin enough and the clocks accurate enough so that all four numbers are integers in units of Δx or $\Delta t = c\Delta x$, making $N_{12} \equiv L/\Delta x$ and $D_{12} = T/\Delta t$ integers and $\beta(D_{12}, N_{12})$ a rational fraction. If N_{12} and D_{12} have a common factor N_T, so that $N_{12} = N_T n_{12}$, $D_{12} = N_T d_{12}$ and $\beta_{12} = n_{12}/d_{12}$, we could obviously postulate that the signal emerging from the counter telescope is a periodic phenomenon with N_T periods, spacial periodicity $\lambda = n_{12}\Delta x$ and temporal periodicity $\tau = d_{12}\Delta t$ and start articulating this model in such a way that the phenomena described in our paradigm defining coherence-decoherence can be reproduced.

We cannot flesh out this model in detail here, and stick to a few elementary points, confined to modeling the positions of the peaks in the double slit interference pattern. Two cases need to be distinguished. If the source contains a pseudo-random distribution of particle velocities which happens to include cases with v_{12}, the coherence time is $T_{coh} = N_T d_{12} \Delta t$. On the other hand, the source may be independently specified using some other part of the theory (eg. the decay of an excited atom). We must insure that our model properly includes both possibilities.

Another complication is that we must distinguish in our modeling the fact that there are *two* kinds of space and time periodicities corresponding to the group velocity (v_{12}) of the "wave packet" and the "phase velocity" defined by $v_{12}v_{ph} = c^2$. A third is that the interference pattern wave length is given by $h/p(\beta_{12})$ and must be computed using the proper relativistic formula given above relating β to 4-velocity u. Spelling all this out will take a textbook—which is being written.[19] We take a few steps in the next sub-section toward specifying what we mean by *finite and discrete Lorentz invariance* in a theory which takes measurement accuracy seriously.

4.6 Initial Steps Toward Constructing Finite and Discrete 1+1 Lorentz boosts in "space-time"

Keeping in mind the fact that we must eventually return to an examination of the experimental context in which our "origin" of coordinates is specified, we now develop "Lorentz boosts" between two velocity states $\beta_i(N_i, n_i)$, $\beta_f(N_f, n_f)$ under the assumption that the corresponding event coordinates are

$$t_i = N_i,\ x_i = \beta_i t_i = n_i; \quad t_f = N_f,\ x_i = \beta_f t_f = n_f \tag{4.19}$$

for a boost velocity $\beta(N, n) = n/N$. The obvious constraint we must satisfy is that

$$\beta_f = \frac{\beta + \beta_i}{1 + \beta\beta_i} \tag{4.20}$$

The less obvious constraint is, that is the minimum number of periods of each of the three velocities must allow us to insure that all three events are "physical" when referred (as we implicitly have) to a fourth "reference event" at $(0, 0)$ *and* that the counter firings which allow the velocity to be measured lie within the event horizon. This we start to work out here and will complete elsewhere.

By assigning (integer) coordinates (x_1, t_1) and (x_2, t_2) in a theory with a limiting velocity c, we have implicitly assumed that the clocks which record t_1 and

t_2 at these two distinct locations have been synchronized using the Einstein convention. We now include the possibility of this synchronization explicitly in our construction. We let $x_2 - x_1 = 2X > 0$, $t_2 - t_1 = 2T > 2X > 0$ and (formally) fix the space time coordinates of firings 1 and 2 at $(-X, -T)$ and (X, T) respectively. Then the two counter firing bracket our (formal) "origin" $(0, 0)$. To synchronize the clocks, we place a "mirror" at some position $(-T_X)$ with $T_X > T > 0$, and require that a light signal sent from $(-X, -T)$ to this mirror and reflected back along the same line will arrive at $(+X, +T)$. Then the time it takes for the signal light signal launched at the time of the first firing to reach the mirror is $T_X - T$, while the time interval from the reflection to the arrival in coincidence with the second counter firing is $T_X + T$. This insures that the time interval between the two firings is, in fact $2T$, consistent with our formal assignment of coordinates, *independent* of where along the line we place our reference "mirror" $(-T_X)$ and consistent with the Einstein synchronization convention. Note that the velocity measured by the two sequential counter firings is $\beta(T, X) = X/T$.

In a continuum, classical theory of space-time measurement, it is possible to specify both position and velocity simultaneously at any instant of time t. In our context, which we have constructed by paying careful attention to the constraints imposed by finite velocity resolution, this is no longer possible. If we use the times of the two counter firings and their previously measured positions (and clock calibrations) to measure the velocity, all we can say from the point of view of measurement is that the particle position and time (x, t) *during* the measurement of velocity is subject to the constraints $-X \leq x \leq +X$, $-T, \leq T \leq +T$ (with both x and t integer). If we are willing to assume that a third counter placed on the line *between* the first two does not interfere with the velocity measurement — an assumption that can only be checked "statistically" by repeated measurement — we can reduce this uncertainty considerably and check the assumption of "constant velocity" between the counters to limited accuracy. Place this counter at a position x which satisfies the position constraint, and assume it fires at time t, and hence with the time intervals $T^- = T + t$, $T^+ = T - t$. We have now made two, rather

than one, velocity measurements which give the values

$$\beta^- = \frac{X+x}{T+t}; \quad \beta^+- = \frac{X-x}{T-t} \tag{4.21}$$

Of course, if we can place our counter precisely at $x = 0$ and it always fires at $t = 0$, we will confirm the classical, continuum model. *But this assumption would violate our initial hypothesis of finite velocity resolution.* Clearly the detailed exploration of what we mean by *finite and discrete Lorentz invariance* would take us too far afield. We intend to develop it elsewhere (see Ref. 19).

4.7 Conclusions about decoherence in a discrete theory

We hope that the discussion in this section at least gives the flavor of how we intend to develop a complete relativistic quantum mechanics of single particle phenomena which will give precision to question of where the limitations on the Feynman-Dyson-Tanimura- Kauffman derivation of the classical relativistic fields will arise due to quantum effects. In a more conventional vein, we could say that we can only apply classical considerations to systems where the "collapse of the wave function" has changed quantum states from a coherent superposition to a mixture. Lacking our own theory for this, and noting that there is considerable controversy in the literature both about the "correspondence limit" of relativistic quantum mechanics and whether there is such a thing as "quantum chaos", we again conclude that whatever the outcome of research pursued on current lines, it is bound to remove the question of the *physical* meaning of "deterministic chaos" still farther away from the practice of physics when examined operationally at the level of fundamental theory.

In the next section we describe a promising theory which *does* have correspondence limits in non-relativistic quantum mechanics, relativistic (classical) particle physics, and (if the derivation of classical relativistic field theory given in Section 3 is accepted) in the classical relativistic field theories of electromagnetism and gravitation.

5. Bit-String Physics: A novel relativistic quantum theory

[Since we have recently completed (in Ref. 9) a fairly complete and systematic presentation of our "theory of everything" we content ourselves here with quoting the introduction to that paper and the essential results, and refer the reader to the longer publication, and references therein, for details.]

"Although currently accepted relativistic quantum mechanical theories incorporate many discrete phenomena, they are embedded in an underlying space-time continuum in a way which guarantees the creation of infinities. Despite many phenomenological successes, they have as yet failed to achieve a consensus theory of 'quantum gravity'. We believe that these two difficulties are connected, and that both can be circumvented by basing fundamental physical theory directly on the computer tools of bit-strings and information theory based on bit-strings. This has the further advantage that we can base our model for space and time on finite intervals between events (eg. counter firings) measured to finite (and *fixed* in any particular context) accuracy. This operational methodology then allows us to avoid such metaphysical questions as whether the 'real world' is discrete or continuous (see Ref.7), or whether the 'act of observation' does or does not require 'consciousness'.[20]

"By a 'theory of everything' (ToE), we mean a systematic representation of the numerical results obtained in high energy particle physics *experiments* and by *observational* cosmology. The representation we use employs a growing but always finite assemblage of bit-strings of finite length constructed by a simple algorithm called *program universe* explained [in Ref. 9].

"More conventional ToE's are based on the mathematical continuum and the structures of second quantized relativistic field theories (QFT). They ignore the flaws of QFT (infinite answers to physically sensible questions, unobservable 'gauge potentials', and no well defined correspondence limit in either classical relativistic field theory, non-relativistic quantum mechanics or nuclear physics). The most ambitious of these theories assume that non-Abelian gauge theories in the form of

'string theory' succeed in explaining "quantum gravity". Comparison with practical metrology is made by identifying $\hbar$, c and G_{Newton} in their theoretical structures. It is then an act of faith that everything else is calculable. Less ambitious ToE's (eg. GUT's = grand unified theories) fix the third parameter as a universal coupling constant at an energy of about a thousandth of the Planck mass-energy and then 'run' it down in three different ways to energies a factor of 10^{15} smaller where these three distinct values are identified as the measurable fine structure constant ($\alpha = e^2/\hbar c$), weak interaction constant (G_{Fermi}) and strong coupling constant α_s; because the strong (QCD) coupling 'constant' is supposed to diverge at zero energy, models must include its energy dependence over a finite energy range. In practice, such theories contain a fairly large number of phenomenological parameters.

"In contrast, we employ a structure in which we need only identify $\hbar$, c and m_p (the proton mass) in order to make contact with standard MLT metrology, using the kilogram, meter and second as arbitrary but fixed dimensional units. α, G_{Fermi}, G_{Newton} and a number of other well measured parameters can be computed and the quality of the fit to experiment evaluated in a less problematic way. While these comparisons are very encouraging, with accuracies ranging from four to seven significant figures, they are not perfect. So far as we can see the discrepancies could arise from the concatenation of effects we know we have so far not included in the calculations, but we are prepared to encounter 'failure' as we extend the calculations. However, the quality of the results achieved to date lead us to expect that such 'failure' would point to where to look for 'new physics' in *our* sense. Since we leave no place for 'adjustable parameters', such a crisis should be more clear cut for us than in a conventional ToE. We do not believe that it is possible to make a 'final theory', and might even welcome a failure serious enough to allow us to abandon this whole approach and turn to more conventional activities.

"We start from a universe of bit-strings of the same length which grow in length by a random bit, randomly chosen for each string whenever XOR between two strings gives the null string; else the resulting non-null string is adjoined to

the universe. Then recurse. Because of closure under XOR,[21] and a mapping we present [in Ref. 9] of the quantum numbers of the 3-generation standard model of quarks and leptons onto the first 16 bits in these strings, we can model discrete quantum number conservation (lepton number, baryon number,charge, weak isospin and color) using a bit-string equivalent of 4-leg Feynman diagrams. Quarks and color are necessarily confined. All known elementary fermions and bosons are generated, and no unknown particles are predicted. The scheme implies reasonably accurate coupling constants and mass ratios, calculated assuming equal prior probabilities in the absence of further information. The combinatorics and the standard statistical method of assigning equal weights to each possibility provide an alternative interpretation of results previously obtained from the *combinatorial hierarchy*,[22−24] including the closure of these bit-string labels at length 256, and the prediction of the Newtonian gravitational constant. Baryon and lepton number conservation then gravitationally stabilizes the lightest charged (free) baryon (the proton) and lepton (the electron) as rotating black holes of spin 1/2 and unit charge.

"The growing portion of the bit-strings beyond the quantum number conserving labels can be interpreted as describing an expanding 3-space universe with a universal (cosmological) time parameter. Within this universe pairwise collisions produce products conserving relativistic 3-momentum (and, when on mass shell, energy) in terms of quantized Mandelstam parameters and masses. The baryon and lepton number, ratio of baryons to photons, fireball time, and ratio of dark to baryonic matter predicted by this cosmological model are in rough accord with observation. The model contains the free space Maxwell equations for electromagnetism and the free space Einstein equations for gravitation as appropriate macroscopic approximations for computing the motion of a single test particle."

Table I. **Coupling constants and mass ratios** predicted by the finite and discrete unification of quantum mechanics and relativity. Empirical Input: $c, \hbar$ and m_p as understood in the "Review of Particle Properties", Particle Data Group, *Physics Letters*, **B 239**, 12 April 1990.

COUPLING CONSTANTS

Coupling Constant	Calculated	Observed
$G^{-1}\frac{\hbar c}{m_p^2}$	$[2^{127}+136]\times[1-\frac{1}{3\cdot 7\cdot 10}] = 1.693\ 31\ldots\times 10^{38}$	$[1.69358(21)\times 10^{38}]$
$G_F m_p^2/\hbar c$	$[256^2\sqrt{2}]^{-1}\times[1-\frac{1}{3\cdot 7}] = 1.02\ 758\ldots\times 10^{-5}$	$[1.02\ 682(2)\times 10^{-5}]$
$sin^2\theta_{Weak}$	$0.25[1-\frac{1}{3\cdot 7}]^2 = 0.2267\ldots$	$[0.2259(46)]$
$\alpha^{-1}(m_e)$	$137\times[1-\frac{1}{30\times 127}]^{-1} = 137.0359\ 674\ldots$	$[137.0359\ 895(61)]$
$G^2_{\pi N\bar{N}}$	$[(\frac{2M_N}{m_\pi})^2-1]^{\frac{1}{2}} = [195]^{\frac{1}{2}} = 13.96..$	$[13, 3(3), > 13.9?]$

MASS RATIOS

Mass ratio	Calculated	Observed
m_p/m_e	$\frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\ \frac{4}{5}} = 1836.15\ 1497\ldots$	$[1836.15\ 2701(37)]$
$m_\pi^\pm/m_e$	$275[1-\frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.12\ 92\ldots$	$[273.12\ 67(4)]$
m_{π^0}/m_e	$274[1-\frac{3}{2\cdot 3\cdot 7\cdot 2}]= 264.2\ 143\ldots$	$[264.1\ 373(6)]$
m_μ/m_e	$3\cdot 7\cdot 10[1-\frac{3}{3\cdot 7\cdot 10}] = 207$	$[206.768\ 26(13)]$

COSMOLOGICAL PARAMETERS

Parameter	Calculated	Observed
N_B/N_γ	$\frac{1}{256^4} = 2.328....\times 10^{-10}$	$\approx 2\times 10^{-10}$
M_{dark}/M_{vis}	≈ 12.7	$M_{dark} > 10M_{vis}$
$N_B - N_{\bar{B}}$	$(2^{127}+136)^2 = 2.89...\times 10^{78}$	*compatible*
$\rho_/\rho_{crit}$	$\approx \frac{4\times 10^{79} m_p}{M_{crit}}$	$.05 < \rho_/\rho_{crit} < 4$

[This paper ends with the following caveat]

"We warn the reader that detailed and rigorous mathematical proof of some of the statements made above is still missing. We wish to thank David McGoveran for pointing out to us that this caveat is particularly relevant for the use we make of the corrections he derived in the context of the combinatorial hierarchy construction. For him, constructing our bit-strings using program universe and bringing in the identification of the labels from 'outside'—i.e. from known facts about quantum number conservation in particle physics—amounts to creating a *different* theory. While we have confidence that mixing up the two approaches in this way can, eventually, be justified in a compelling way, it may well turn out that our confidence in this outcome is overly optimistic.

"To summarize, by using a simple algorithm and detailed physical interpretation, we believe we have constructed a self-organizing universe which bears a close resemblance to the one in which physicists think we live. It is not 'self-generating'—unless one grants that the two postulates with which Parker-Rhodes begins his unpublished book on the 'inevitable universe', namely: '*Something exists!* ' and '*This statement conveys no information*' suffice to explain why our universe started up."

6. PHILOSOPHICAL IMPLICATIONS

We now return to the question of how this work in foundations of particle physics and physical cosmology relates to the question determinism versus indeterminism in *physics*. Since the theory we present is, to put it mildly, controversial, it is obvious is that any conclusions must be tentative. Nevertheless, we believe that the fact that a "theory of everything" (i.e. of particle physics and physical cosmology) using only finite and discrete observations and sticking to this methodology is at least possible is relevant to the issue of determinism versus indeterminism. Clearly the theory is computational, and in that sense "deterministic". Yet, be-

cause it rigorously excludes both the continuum and mathematical induction, it provides a *physical* theory in which "deterministic chaos" simply cannot arise.

Of course the operational methodology on which our approach to physics is based cannot be argued for to the exclusion of more conventional approaches. But even those approaches provide three reasons why "deterministic chaos" should not be considered a *fundamental* theory and hence relevant to metaphysical conclusions. The first is that the classical theories of electromagnetism and gravitation can be *derived* by accepting a lowest, finite and fixed bound on the accuracy to which space and time intervals can be measured. Hence assuming boundary conditions known to an accuracy needed to reach "deterministic chaos" is logically *inconsistent* with using these equations to establish it. The second is that the non-relativistic uncertainty principle removes "deterministic chaos" from consideration as a physical theory in any case. The third is that once we take into account the observed phenomenon of electron-positron pair creation, *any* theory which tries to specify distances to better than $\hbar/2m_e c$ is *ipso facto* operationally meaningless.

Patrick Suppes has argued on quite general grounds in his paper entitled "The Transcendental Character of Determinism" [25] that modern work on what are called "deterministic systems" has shown that "Deterministic metaphysicians can comfortably hold to their view knowing that they cannot be empirically refuted, but so can indeterministic ones as well." He then proposes a fundamental reinterpretation of Kant's Third Antinomy, claiming that "Both Thesis and Antithesis can be supported empirically, not just the Antithesis." We offer this paper as support to his claim, using a very different body of physical theory and experimentation as our context.

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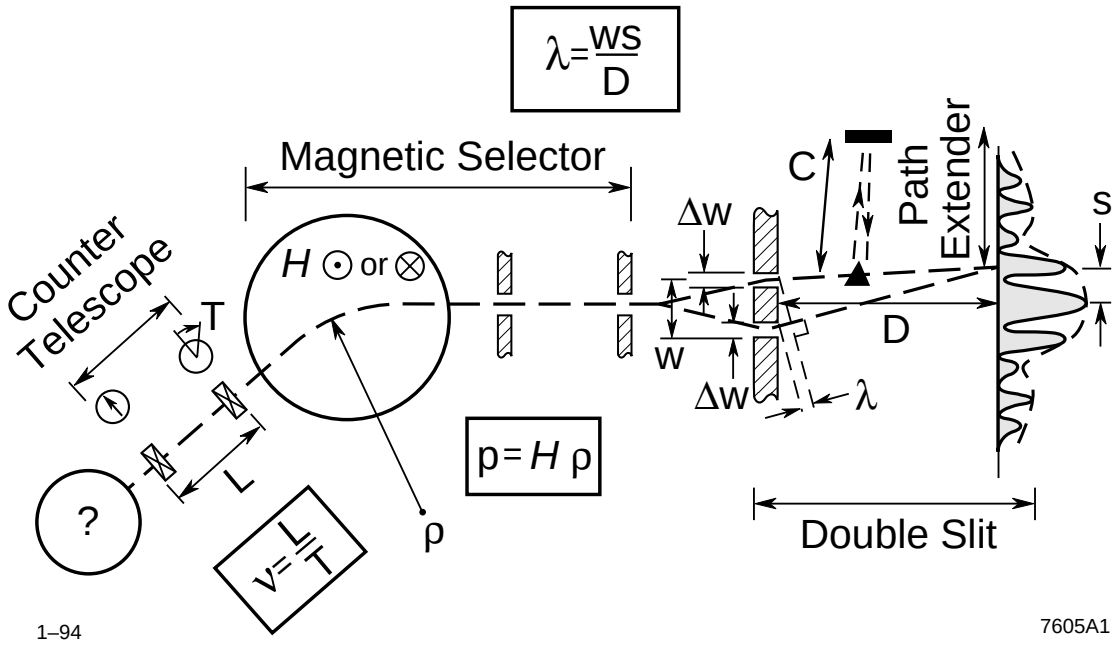
FIGURE CAPTIONS

1) Measurement of coherence and decoherence of de Broglie waves using a counter telescope, magnetic selector, and a double slit with a path extender in one arm.


λ=ws/D
Magnetic Selector
Counter Telescope
T
H ⊙ or ⊗
ρ
p=H ρ
L
?
v=L/T
Δw
w
Δw
λ
C
Path Extender
D
s
Double Slit

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