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Pure Leptonic Radiative Decays $B^{\pm}, D_s \to \ell\nu\gamma$ and the Annihilation Graph

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Abstract: Pure leptonic radiative decays of heavy-light mesons are calculated using a very simple non-relativistic model. Dominant contribution originates from photon emission from light initial quark. We find $BR(B^{\pm} \to \ell\nu\gamma) \sim 3.5 \times 10^{-6}$ and $BR(D_s \to \ell\nu\gamma) \sim 1.7 \times 10^{-4}$. The importance of these reactions to clarify the dynamics of the annihilation graph is emphasized.

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An important issue in weak decays of charm and bottom mesons that is not well understood quantitatively and needs clarification is the magnitude of the annihilation graph. This limits our ability to precisely calculate a variety of important quantities. For one thing it introduces an element of uncertainty in the calculation of the hadronic width and consequently the semi-leptonic branching ratio [1]. Extraction of the value of the CKM parameters from the experimental data also becomes problematic [2]. Furthermore, predictions of CP violating asymmetries often involve the annihilation graph [3]. Many tests of the Standard Model (SM) via rare decays also gets compromised [4]. For instance, in rare decays such as $B \to \rho\gamma$ or $B \to K^*\gamma$, the annihilation graph provides one source of long range contributions that need to be quantified before these simple decays can be reliably used to test the SM or to deduce precisely the value of the mixing angle [4–6]. The pure leptonic radiative decays such as:

$$
\begin{aligned}
B^{\pm} &\to \ell\nu\gamma && (1)\\
D_s &\to \ell\nu\gamma && (2)
\end{aligned}
$$

provide simple reactions that monitor the annihilation graph. Recently these reactions have been studied in one model [7]. Given the importance of the annihilation mechanism and the unique cleanliness of the above reactions as a monitor for it, it is clearly useful to calculate these in several different bound state models.

We will use a very simple non-relativistic model which has been previously used for exhibiting the importance of the closely related reactions [8, 9]:

$$
\begin{aligned}
D^0 &\to s\bar{d}g \\
B, D &\to u\bar{d}\gamma. && (3)
\end{aligned}
$$

The simplicity of the model has the advantage that it allows the rate and the differential spectra for the reactions to be readily calculated in terms of very few parameters. Thus detailed comparisons with the experiment can be made. Such comparisons should prove very valuable for illuminating our understanding of the dynamics of these reactions.

We begin with the amplitude for the pure leptonic reaction $B \to \ell\nu_\ell$:

$$M = \frac{1}{4} f_B Tr[\Theta(\not{p}_B + m_B)\gamma_5] \tag{4}$$

where f_B is normalized so that $f_\pi \simeq 130$ MeV and

$$\Theta = \sqrt{8} G_F (\bar{\ell}\gamma_\mu P_L \nu)\gamma^\mu P_L \tag{5}$$

with $P_L \equiv (1-\gamma_5)/2$. Thus

$$\Gamma(B \to \ell\nu) = \frac{m_B^3}{8\pi} G_F^2 f_B^2 x_\ell (1-x_\ell)^2 |V_{ub}|^2 \tag{6}$$

with $x_\ell \equiv m_\ell^2/m_B^2$.

The helicity suppression in eqn. (6), characterizing the pure leptonic decay, can be overcome by the emission of a photon (or gluon for the corresponding reaction into light quarks) [8, 9]. Amongst the Feynman graphs the most important contribution is the one arising from the photon emission from the initial light quark. Emission of the photon from the final fermion line is suppressed by powers of light fermion masses. Emission of the photon from the initial heavy quark is smaller (in the amplitude) by a factor of about m_u/m_b compared to the photon emission from the light initial quark. Thus, in this model, the amplitude for reaction (1) is given approximately by

$$M \simeq \frac{Q_u e g_W^2 f_B V_{ub}}{8 m_u (t+u) m_W^2}\ Tr[(\not{p}_B\ \not{\epsilon}^*\ \not{q}\gamma^u P_L)(\bar{\ell}\gamma_\mu P_L \nu)] \tag{7}$$

where $g_W^2 = 8 G_F m_W^2/\sqrt{2}$.

Thus the differential spectra are given by

$$d\Gamma_{\ell\nu\gamma} = \frac{Q_u^2 \alpha G_F^2}{16\pi^2 m_B} \left(\frac{f_B}{m_u}\right)^2 |V_{ub}|^2 \frac{t^2 s}{(m_B^2 - s)^2}\, ds dt \tag{8}$$

where $s = (p_\ell + p_\nu)^2$, $t = (p_\ell + q)^2$. Phase space integration gives:

$$\Gamma_{\ell\nu\gamma} = \frac{Q_u^2 \alpha m_B^5}{288\pi^2} G_F^2 |V_{ub}|^2 f_B^2/m_u^2 \tag{9}$$

Thus, in this simple model, the reaction is basically characterized by the ratio of the pseudoscalar decay constant (f_B) to an effective constituent mass parameter for the light quark (m_u). Intense efforts are underway using lattice gauge models to calculate f_B [10]. So far, these calculations have an accuracy of about 20% and in the next few years one should be able to pin f_B down to a precision of about 10%. The mass parameter in (9) is closely related to the constituent mass of the light quark. It is important to understand that this bound state picture makes sense only with constituent masses. These do not vanish in the limit as the current mass vanishes but rather they go over to a non-perturbative dimensional parameter in the theory akin to $\langle \bar{u}u \rangle$.

To get a feel for the rates involved we will use $f_B = 175$ MeV [10], $\frac{V_{ub}}{V_{cb}} = .08$, $V_{cb} = 0.04$ [2] and $m_u = 350$ MeV. Then we find:

$$BR(B^{\pm} \to \ell\nu\gamma) \simeq 3.5 \times 10^{-6} \tag{10}$$

Comparing this radiative decay to the pure leptonic decay, say $B^{\pm} \to \mu^{+}\nu$ we get

$$\frac{BR(B^{\pm} \to \mu\nu\gamma)}{BR(B^{\pm} \to \mu\nu)} \simeq 16 \tag{11}$$

Of course the increase with respect to the electron mode is much more pronounced. It is also useful to compare the lepton (μ, e) arising from the decay sequence

$$\begin{array}{l} B \to \tau\,\nu \\ \qquad \hookrightarrow \mu\nu\nu,\, e\nu\nu \end{array} \tag{12}$$

with $B^{+} \to \mu(e)\nu\gamma$. From equation (6)

$$BR(B^{\pm} \to \tau\nu) \simeq 5.1 \times 10^{-5} \tag{13}$$

Thus, for example,

$$\frac{BR(B^{+} \to \mu^{+}\nu\gamma)}{BR(B^{+} \to \tau^{+}\nu, \tau^{+} \to \mu^{+}\nu\nu)} \sim 0.4 \tag{14}$$

For experimental purposes it is also useful to consider the differential spectra. Indeed the photon energy spectrum for the annihilation reaction is well known [8, 9]:

$$\frac{dN}{d\lambda_\gamma} = \frac{m_B}{\Gamma}\frac{d\Gamma_{\ell\nu\gamma}}{dE_\gamma} = 24\lambda_\gamma(1 - 2\lambda_\gamma) \tag{15}$$

where $\lambda_i = E_i/m_B$. This is clearly very distinct from the steeply falling bremsstrahlung photon spectrum. The invariant mass (t) of the charged lepton-photon combination is related directly to the energy carried by the ν and this along with the lepton energy distribution are given by:

$$\frac{dN}{d\lambda_\nu} = 36(1-2\lambda_\nu)[2\lambda_\nu + (1-2\lambda_\nu)\ell n(1-2\lambda_\nu)] \tag{16}$$

$$\frac{dN}{d\lambda_\ell} = 36[2\lambda_\ell(3-5\lambda_\ell) + (1-2\lambda_\ell)(3-2\lambda_\ell)\ell n(1-2\lambda_\ell)] \tag{17}$$

These normalized spectra are displayed in Fig. 2.

We can apply this formalism directly to the D_s case, i.e. for $D_s \to \mu\nu\gamma$ or $e\nu\gamma$. The decay constant is already determined quite well with an accuracy of $\lesssim 15\%$ to be $f_{D_s} = 230$ MeV [10]. Note also that the dominant contribution is now the emission of the photon from the strange quark and use of the simple non-relativistic picture should work better; we use $m_s = 500$ MeV and get

$$BR(D_s \to \ell\nu\gamma) \simeq 1.7 \times 10^{-4} \tag{18}$$

and

$$\frac{BR(D_s \to \mu\nu\gamma)}{BR(D_s \to \mu\nu)} \simeq 3.9 \times 10^{-2} \tag{19}$$

The spectra for $D_s \to \ell\nu\gamma$ may also be obtained from Figure 2 except the role of ℓ and ν are interchanged from the case for B decays.

If we consider inclusively the final state e or $\mu + 0$ *hadrons* in the decay of $B^\pm$ then $\ell\nu\gamma$ dominates over $\ell\nu$ though the later may be distinguished by the fixed energy of the lepton in the B frame. Another important source of this signal is the decay chain (12). The resultant energy spectrum is:

$$\begin{aligned}\frac{dN}{d\lambda_\ell}(\tau \to \ell) &= \frac{1+2x_\tau}{3x_\tau^2} f(\lambda_\ell)\theta(x_\tau - 2\lambda_\ell) \\ &+ \frac{3-2x_\tau}{3(1-x_\tau)^2} f(\frac{1}{2} - \lambda_\ell)\theta(2\lambda_\ell - x_\tau)\end{aligned} \tag{20}$$

where $x_\tau = m_\tau^2/m_B$, θ is the Heavyside function and

$$f(\lambda) = 16(3-4\lambda)\lambda^2 \tag{21}$$

This is displayed in Fig. 3 for B^+ and also for D_s^+. It is clear from this figure that the lepton spectrum from $B \to \ell\nu\gamma$ is much harder than the spectrum from (12) and may be experimentally distinguished in this way.

In the case of the decay of D_s the corresponding normalized spectra are also shown in Fig. 3. From (6)

$$BR(D_s \to \tau\nu) \sim 4.3\%. \tag{22}$$

It follows that

$$\frac{BR(D_s \to \mu^+\nu\gamma)}{BR(D_s \to \tau^+\nu, \tau^+ \to \mu^+\nu\nu)} \sim 0.023. \tag{23}$$

Thus, since the normalized spectra for the charged lepton, in Fig.3, have similar shapes for the D_s case, detection of the photon in $D_s \to \mu^+\nu\gamma$ would be necessary to observe it against the chain (12) for D_s decay.

As more data becomes available it will be very instructive to see how well this simple picture, outlined above for the important $\ell\nu\gamma$ modes, works and especially if the data can be accounted for with m_u of about 350 MeV and m_s of about 500 MeV. Let us briefly contrast this with the model of Ref. (7). That model certainly has interesting features of the heavy quark symmetry [11] built into it. However it also necessitates the introduction of several hadronic parameters which are not readily accessible, at least at present. Therefore it is difficult to quantify the rates and the detailed spectra. In particular it is useful to note that one of the form factors of the model of Ref. (7) also goes as $1/m_u$, characteristic of the photon emission from the light quark [12]. Needless to say given the importance of the reactions it would be very helpful if comparison of the data with both of these as well as other models [13] is pursued vigorously.

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