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SEMI-EXCLUSIVE PION PRODUCTION IN DEEP-INELASTIC SCATTERING

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Abstract

We calculate azimuthal asymmetries and the Callan-Gross R-ratio for semi-exclusive pion production in deep inelastic scattering taking into account higher twist effects. Our results are qualitatively different from the QCD-improved parton model predictions for semi-inclusive deep inelastic scattering.

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1. The semi-inclusive deep inelastic process

$$\ell + p \to \ell' + h + X, \tag{1}$$

where ℓ and ℓ' are charged leptons and h is a detected hadron, has been recognized long ago [1] as an important testing ground for QCD. In particular, measurements of the azimuthal angle φ of the observed hadron provide additional information on the underlying QCD production mechanism. The angle φ is defined in the target rest frame as follows: The z axis is chosen in the direction opposite to the three-momentum transfer $\mathbf{q}$ of the leptons, and the x-z plane is the lepton scattering plane with the lepton three momenta having positive x components. The angle φ is the azimuthal angle of the observed hadron about the z axis.

Different mechanisms to generate azimuthal asymmetries – $\langle \cos\varphi \rangle$ and $\langle \cos 2\varphi \rangle$ – have been discussed in the literature. Georgi and Politzer [1] found a negative contribution to $\langle \cos\varphi \rangle$ in the QCD-improved parton model and proposed the measurement of this quantity as a clean test of perturbative QCD. Cahn [2] took into account the effects due to the intrinsic transverse momenta of partons bound inside the proton. This nonperturbative effect produces a negative contribution to $\langle \cos\varphi \rangle$ and a positive one to $\langle \cos 2\varphi \rangle$. More recently Chay, Ellis and Stirling [3] combined perturbative and nonperturbative mechanisms and were able to fit the measured values for $\langle \cos\varphi \rangle$ [4]. The nonperturbative effects were parameterized in Ref. [3] by Gaussian distributions for the intrinsic momenta of both the target (proton) and the observed hadron (pion). In fact, the hadronization effects cannot be neglected for the fit of Ref. [3] to the data. The importance of hadronization effects for the azimuthal asymmetry was stressed by König and Kroll [5] and also by Berger [6]. The calculation of Berger is made specifically for the case of single pion production and takes into account pion bound-state effects.

It is an intriguing feature of Berger's higher twist mechanism that it generates azimuthal asymmetries opposite in sign to those of [3,2,1,5]. However, the competing mechanisms make it difficult to give reliable theoretical predictions for the azimuthal asymmetries and their dependence on the whole set of kinematic variables of the process (1). In this paper we reconsider Berger's mechanism and discuss ways of isolating its effects from those of the competing mechanisms discussed above. Our motivation is twofold: first, this approach calculates and incorporates in a nontrivial way higher twist effects by relating them to the distribution amplitude of the produced hadron; second, a similar approach for Drell-Yan process was shown in [7] to be of importance in resolving a long standing problem in explaining the data for the angular distribution of the produced lepton.

2. First we discuss the kinematics of process (1). Let l and l' be the four-momenta of the leptons with $q = l - l'$ being the four-momentum transfer and P_N and P_h those of the proton and the observed hadron. We define the usual set of kinematic variables

$$x_B = \frac{Q^2}{2P_N \cdot q}, \qquad y = \frac{P_N \cdot q}{P_N \cdot l}, \qquad z = \frac{P_h \cdot P_N}{q \cdot P_N}, \tag{2}$$

where $Q^2 = -q^2$. We will also use $Q = \sqrt{Q^2}$ and $\nu = q^0$, $E = l^0$ defined in the proton rest frame. The transverse momentum of the observed hadron is given by $\mathbf{P}_h^T = p_T\ (\cos\varphi, \sin\varphi)$, where φ is the azimuthal angle introduced in the previous section. The differential cross section for semi-inclusive deep inelastic scattering is given by

$$\frac{2P_h^0\ d\sigma}{d^3P_h dx_B dy} = \frac{2\pi\alpha^2 My}{Q^4}\ L_{\mu\nu}W^{\mu\nu}, \tag{3}$$

where M is the proton mass and $L_{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic tensors respectively. It is convenient to introduce helicity structure functions,

$$W_L = \epsilon_L^\mu W_{\mu\nu}\epsilon_L^\nu,$$

$$\begin{aligned} W_T &= \frac{1}{2}\big(\epsilon_x^\mu W_{\mu\nu}\epsilon_x^\nu + \epsilon_y^\mu W_{\mu\nu}\epsilon_y^\nu\big), \\ W_{LT} &= -\big(\epsilon_x^\mu W_{\mu\nu}\epsilon_L^\nu + \epsilon_L^\mu W_{\mu\nu}\epsilon_x^\nu\big), \\ W_{TT} &= \frac{1}{2}\big(\epsilon_x^\mu W_{\mu\nu}\epsilon_x^\nu - \epsilon_y^\mu W_{\mu\nu}\epsilon_y^\nu\big), \end{aligned} \tag{4}$$

where ϵ_x, ϵ_y and ϵ_L are polarization vectors for the vector boson polarized respectively in the x, y and z directions. Following Ref. [8] we introduce the following hadronic structure functions,

$$\begin{aligned} H_1 &= \frac{M}{2z}\ W_T, \\ H_2 &= \frac{\nu Q^2}{2z|\mathbf{q}|^2}\ (W_L + W_T), \\ H_3 &= \frac{Q^3}{2z|\mathbf{q}|p_T}\ W_{LT}, \end{aligned}$$

$$H_4 \;=\; \frac{Q^4}{2zp_T^2\nu}\; W_{TT}. \tag{5}$$

The differential cross section is conveniently expressed in terms of these structure functions [8], namely in the deep inelastic limit we get

$$\frac{Q^2\; d\sigma}{dx_B dy dz dp_T^2 d\varphi} \;=\; \frac{4\pi\alpha^2 ME}{Q^2}\,\Big(x_B y^2\; H_1 \;+\; (1-y)\; H_2$$

$$+\;\frac{p_T}{Q}(2-y)\sqrt{1-y}\; H_3\;\;\cos\varphi \;+\; \frac{p_T^2}{Q^2}(1-y)\; H_4\;\;\cos 2\varphi\;\Big), \tag{6}$$

where $H_i = H_i(x_B, z, p_T^2, Q^2)$.

This decomposition exhauts the kinematical information for reaction (1). Apart from the overall normalization factor one may specify three independent observables. Two of them we take to be the angular asymmetries,

$$\langle\cos\varphi\rangle \;=\; \frac{1}{2}\,\frac{p_T}{Q}\,\frac{(2-y)\sqrt{1-y}\; H_3}{x_B y^2\; H_1 \;+\; (1-y)\; H_2}, \tag{7}$$

$$\langle\cos 2\varphi\rangle \;=\; \frac{1}{2}\,\frac{p_T^2}{Q^2}\,\frac{(1-y)\; H_4}{x_B y^2\; H_1 \;+\; (1-y)\; H_2}. \tag{8}$$

For the third observable we propose

$$R \;=\; \frac{H_2 \;-\; 2x_B\; H_1}{H_2}, \tag{9}$$

which is the analog of the Callan-Gross R-ratio [9] in inclusive deep inelastic scattering.

3. We will now calculate observables (7-9) for the specific case of *semi-exclusive* pion production. More precisely, we want to consider a process in which a direct pion with z close to 1 observed in charged lepton – proton scattering is not surrounded by other high-p_T hadrons. In other words, the pion is not a member of a jet. The events we are interested in thus form only a small part of the whole semi-inclusive sample. If none of the specifications above apply to the observed hadron it is expected that the usual QCD-improved parton model diagrams of Refs. [1,3] would give the dominant contribution. Most of the experiments performed [4,10] did not make these specifications*.

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* Note that in the first paper of Ref. [10] single pions were identified and a nonzero asymmetry was observed. However, the data were not fitted to the general form of the φ-dependence given in eq. (6). Thus it is difficult to draw conclusions about the asymmetries defined in eqs. (7), (8).

The dominant contribution to the *semi-exclusive pion* production described above is given by the diagrams of Fig. 1 a,b. Here the amplitude $\mathcal{A}$ for the reaction

$$u + e^- \to e^- + \pi^- + d, \tag{10}$$

is obtained [11,6,7] by convoluting the hard scattering partonic amplitude $T(u + e^- \to e^- + u\overline{d} + d)$ with the pion *distribution amplitude* $\phi(\xi)$,

$$\mathcal{A} \;=\; \int_0^1 d\xi \; \phi(\xi) \; T. \tag{11}$$

This factorized form of the amplitude should be valid for z close to 1.

For the hadronic cross section (6) we have to convolute the cross section obtained from the amplitude of eq. (11) with the parton distribution function of the proton (see [6,7] for more details). A typical contribution to the hadronic tensor $W_{\mu\nu}$, that is to the hadronic structure functions H_i, is depicted in Fig. 1 c.

We note that no *intrinsic* transverse momentum for the *pion* constituents is introduced in our model. The source of the observed p_T of the pion is the gluon exchange, see Fig 1. For the sake of simplicity of our presentation the intrinsic transverse momentum of the *proton* constituents will be neglected in the formulae below, but will be discussed later.

For the structure functions of eq. (5) we find

$$\begin{aligned}
H_1 \;&=\; \mathcal{N}\,\frac{1}{2x_B}\,\Big(\;[I_2(z,p_T/Q)]^2 \;+\; \frac{p_T^4}{Q^4}[I_1(z,p_T/Q)]^2\;\Big),\\
H_2 \;&=\; \mathcal{N}\,\Big(\;[I_2(z,p_T/Q)]^2 \;+\; 4\frac{p_T^2}{Q^2}z^2[I_1(z,p_T/Q)]^2 \;+\; \frac{p_T^4}{Q^4}[I_1(z,p_T/Q)]^2\;\Big),\\
H_3 \;&=\; \mathcal{N}\;2z\;I_1(z,p_T/Q)\;\Big(\;I_2(z,p_T/Q) \;-\; \frac{p_T^2}{Q^2}I_1(z,p_T/Q)\;\Big),\\
H_4 \;&=\; -\mathcal{N}\;2\;I_1(z,p_T/Q)\;I_2(z,p_T/Q),
\end{aligned} \tag{12}$$

where $I_1(z,p_T/Q)$ and $I_2(z,p_T/Q)$ are defined as follows,

$$\begin{aligned}
I_1(z,p_T/Q) \;&=\; z\int_0^1 d\xi\frac{\phi(\xi)}{z \;-\; \xi(z^2 - p_T^2/Q^2)},\\
I_2(z,p_T/Q) \;&=\; \int_0^1 d\xi\frac{\phi(\xi)}{1-\xi} \;-\; z^2 I_1(z,p_T/Q).
\end{aligned} \tag{13}$$

Since we will only be interested in the ratios (7)–(9), the overall factor $\mathcal{N}$ is irrelevant.

Using crossing invariance the results for H_i can also be obtained by analytic continuation from the formulae for the angular distribution coefficients for the Drell-Yan process given in [7]. The transformation formulae are given in the Appendix.

Substituting the expressions for H_i given in eq. (12) in eqs. (7)–(9) we obtain predictions for our observables for a specified distribution amplitude $\phi(\xi)$. Since the expression for H_3 turns out to be positive and the one for H_4 to be negative, we find $\langle\cos\varphi\rangle > 0$ and $\langle\cos 2\varphi\rangle < 0$.

Now we want to discuss the effects of intrinsic transverse momenta $\mathbf{k_T}$ of partons bound inside the proton, since from the work of Cahn [2] we know that such effects alone produce asymmetries opposite in sign to our results. We introduce intrinsic $\mathbf{k}_T$ by modifying the distribution function of the proton,

$$G_{q/N}(x,Q^2) \rightarrow d^2k_T \frac{1}{4\,\langle k_T\rangle^2}\ \exp\left\{-\frac{\pi\ \mathbf{k}_T^2}{4\ \langle k_T\rangle^2}\right\} G_{q/N}(x,Q^2). \tag{14}$$

In the diagrams of Fig. 1 the transverse momentum $\mathbf{p}_q^T$ of the quark from the proton is now set equal to $\mathbf{k}_T$ and the quark is off-shell. More precisely we use the parameterization $p_q^+ = x\ P_N^+,\ p_q^- = 0,\ \mathbf{p}_q^T = \mathbf{k}_T$. The use of different prescriptions will modify our results in subleading orders of k_T/Q. In the calculation we use the mean value theorem for the integration over the absolute value of $\mathbf{k}_T$ and neglect subleading terms in k_T/Q.

We plot our results as a function of z for three different values of $\langle k_T\rangle$. In Fig. 2 we plot the observables of eqs. (7)–(9) for $Q = 2.5$ GeV, $p_T = 0.5$ GeV and $y = 0.4$. For the distribution amplitude of the pion we choose a two-humped form, $\phi(\xi) = 30(1-\xi)\xi(1-4(1-\xi)\xi)$ [12]. Here we neglect the evolution of $\phi(\xi)$ with Q. The solid line corresponds to $\langle k_T\rangle = 0$, the dashed line to $\langle k_T\rangle = 0.25$ GeV and the dash-dotted line to $\langle k_T\rangle = 0.5$ GeV. In Fig. 3 we plot the same observables for $Q = 5$ GeV and $p_T = 1$ GeV. The choices of $\langle k_T\rangle$ and y are the same as in Fig. 2. Figs. 2 and 3 depict our main results.

An important feature of our model is a steep increase of the R-ratio as $z \rightarrow 1$. This is analogous to the steep decrease of the λ parameter in the end point region in the Drell-Yan process predicted by the same higher twist model [6,7]. This behavior of λ was observed experimentally [13].

As expected, a nonzero $\langle k_T\rangle$ causes a change in the prediction of our model for the azimuthal asymmetries and the Callan-Gross R-ratio. This change becomes insignificiant for higher values of Q, see Fig. 3. For Q high enough, the analytic expressions (12), (7)–(9) can be used even for nonzero $\langle k_T \rangle$.

In Fig. 4 we show the dependence of the R-ratio and of the azimuthal asymmetry $\langle \cos\varphi \rangle$ on different distribution amplitudes assuming vanishing $\langle k_T \rangle$, $p_T/Q = 0.3$ and $y = 0.4$. To demonstrate that our predictions depend essentially only on the value of $\langle (1-\xi)^{-1} \rangle \equiv \int_0^1 d\xi \phi(\xi)(1-\xi)^{-1}$ and not on the specific shape of the distribution amplitude, we choose the two-humped distribution amplitude of Figs. 2, 3 (solid line in Fig. 4) and a convex amplitude $\phi(\xi) = [(1-\xi)\xi]^{1/3}[\mathrm{B}(4/3, 4/3)]^{-1}$ (dashed line) which both have $\langle (1-\xi)^{-1} \rangle = 5$. Here $\mathrm{B}(x, y)$ denotes the Beta function. For comparison we plot results also for the asymptotic amplitude $\phi(\xi) = 6(1-\xi)\xi$ (dash-dotted line) with $\langle (1-\xi)^{-1} \rangle = 3$. The magnitude of the R-ratio in the endpoint region and the value of $\langle \cos\varphi \rangle$ are decreasing with increasing $\langle (1-\xi)^{-1} \rangle$ and are not sensitive to the shape of the distribution amplitude.

As demonstrated, higher twist effects may be isolated in semi-exclusive pion production for moderate values of Q. We therefore look forward to future precision experiments.

Acknowledgments

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Appendix

By using crossing invariance we can relate the hadronic structure functions H_i for semi-inclusive deep inelastic scattering defined in eq. (6) with the angular coefficients for the Drell-Yan process defined in the Gottfried-Jackson frame by

$$\frac{q^2 d\sigma}{dq^2 dq_T^2 dx d\Omega} \sim (1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi), \qquad (A.1)$$

where the angular distribution coefficients λ, μ and ν are functions of the kinematic variables $x = (q \cdot P_N)/(P_\pi \cdot P_N)$ and q_T^2/q^2 . Here, P_π and P_N are the momenta of the initial state hadrons and q is the four-vector of the produced photon with transverse momentum q_T.

We get the following expressions relating coefficients λ, μ and ν of the Drell-Yan process with the structure functions H_i of eq. (5):

$$\begin{aligned}
x_B\ H_1(z, p_T/Q) &= -N\ [1 + \lambda(x', \rho) - \Delta(x', \rho)]/2 \\
H_2(z, p_T/Q) &= -2\ N\ [\lambda(x', \rho) - \frac{3}{2}\Delta(x', \rho)] \\
p_T/Q\ H_3(z, p_T/Q) &= \sqrt{-\rho^2}\ N\ [-2\frac{\mu(x', \rho)}{\sqrt{\rho^2}} + \Delta'(x', \rho)] \\
p_T^2/Q^2\ H_4(z, p_T/Q) &= N\ [\nu(x', \rho) + \Delta(x', \rho)] \qquad (A.2)
\end{aligned}$$

with

$$\begin{aligned}
\Delta(x', \rho) &= \frac{4\rho^2}{(1+\rho^2)^2}[\lambda(x', \rho) - \nu(x', \rho)/2 - \frac{(1-\rho^2)}{\sqrt{\rho^2}}\mu(x', \rho)], \\
\Delta'(x', \rho) &= \frac{4}{(1+\rho^2)^2}[(1-\rho^2)(\lambda(x', \rho) - \nu(x', \rho)/2) + 4\sqrt{\rho^2}\mu(x', \rho)] \qquad (A.3)
\end{aligned}$$

and

$$x'(z) = \frac{1}{z}\ , \qquad \rho(z, p_T/Q) = \frac{i\ p_T/Q}{z}\ , \qquad (A.4)$$

and N is an overall normalization factor.

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Figure Captions

Fig. 1: Diagrams (a) and (b) give the leading contribution to the amplitude of reaction (10). Diagram (c) gives a typical (one out of four) contribution to the hadronic tensor $W^{\mu\nu}$.

Fig. 2: The observables R, $\langle\cos\varphi\rangle$ and $\langle\cos 2\varphi\rangle$ defined in eqs. (7)–(9) vs. z for $Q = 2.5$ GeV, $p_T = 0.5$ GeV, $y = 0.4$ and $\phi(\xi) = 30(1-\xi)\xi(1-4(1-\xi)\xi)$. The solid line corresponds to $\langle k_T\rangle = 0$, the dashed line to $\langle k_T\rangle = 0.25$ GeV and the dash-dotted line to $\langle k_T\rangle = 0.5$ GeV.

Fig. 3: The same quantities as in Fig.2 for $Q = 5$ GeV and $p_T = 1$ GeV. The choices for $\langle k_T\rangle$ and y are the same as in Fig. 2.

Fig. 4: The R-ratio and $\langle\cos\varphi\rangle$ as a function of z for $\langle k_T\rangle = 0$, $p_T/Q = 0.3$ and $y = 0.4$. The results are plotted for three different distribution amplitudes. The solid line corresponds to the two-humped distribution amplitude also used in Figs. 2, 3. The dashed line corresponds to the convex amplitude $\phi(\xi) = [(1-\xi)\xi]^{1/3}[B(4/3,4/3)]^{-1}$ and the dash-dotted line to the asymptotic one $\phi(\xi) = 6(1-\xi)\xi$.

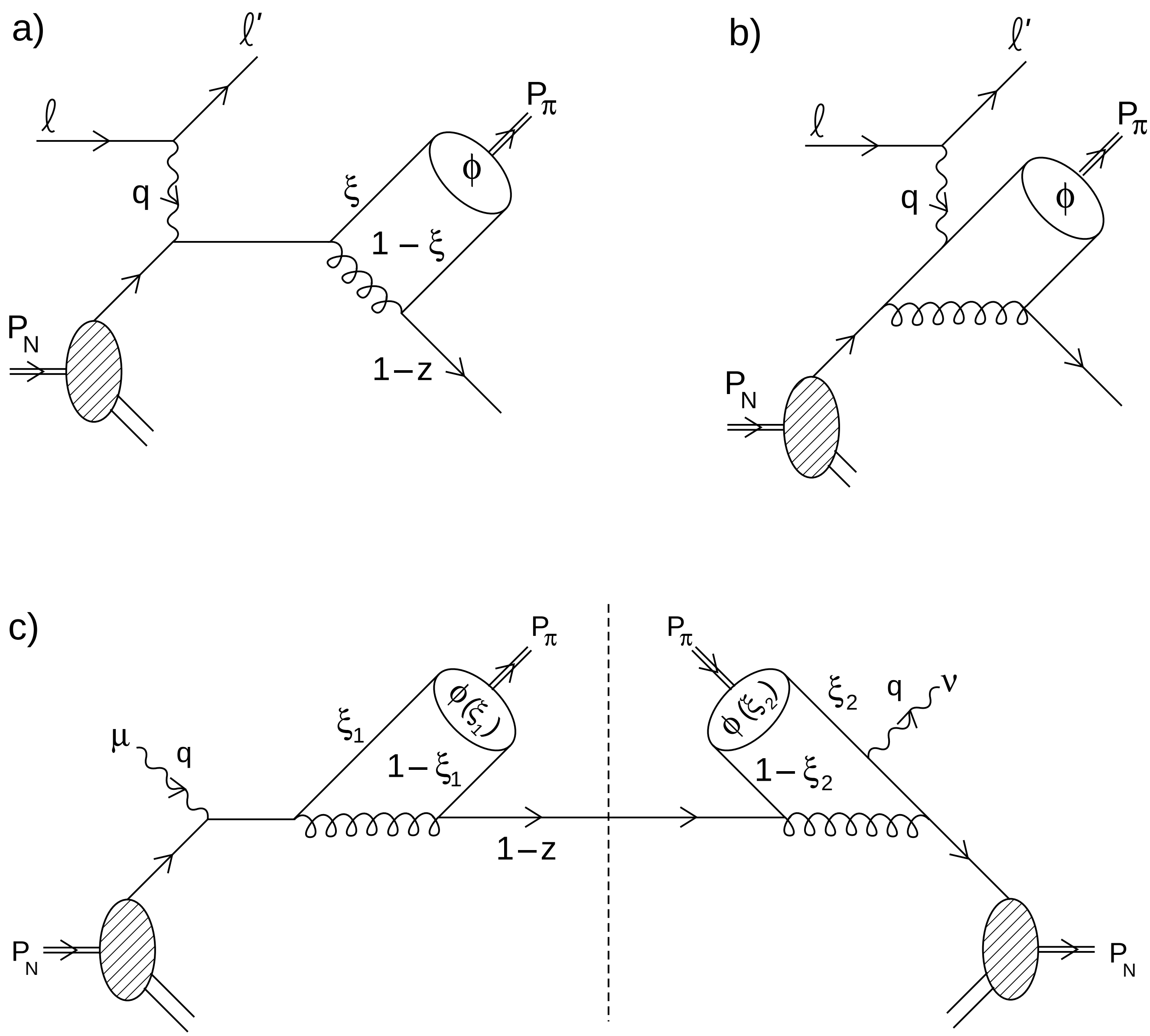

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Fig. 1

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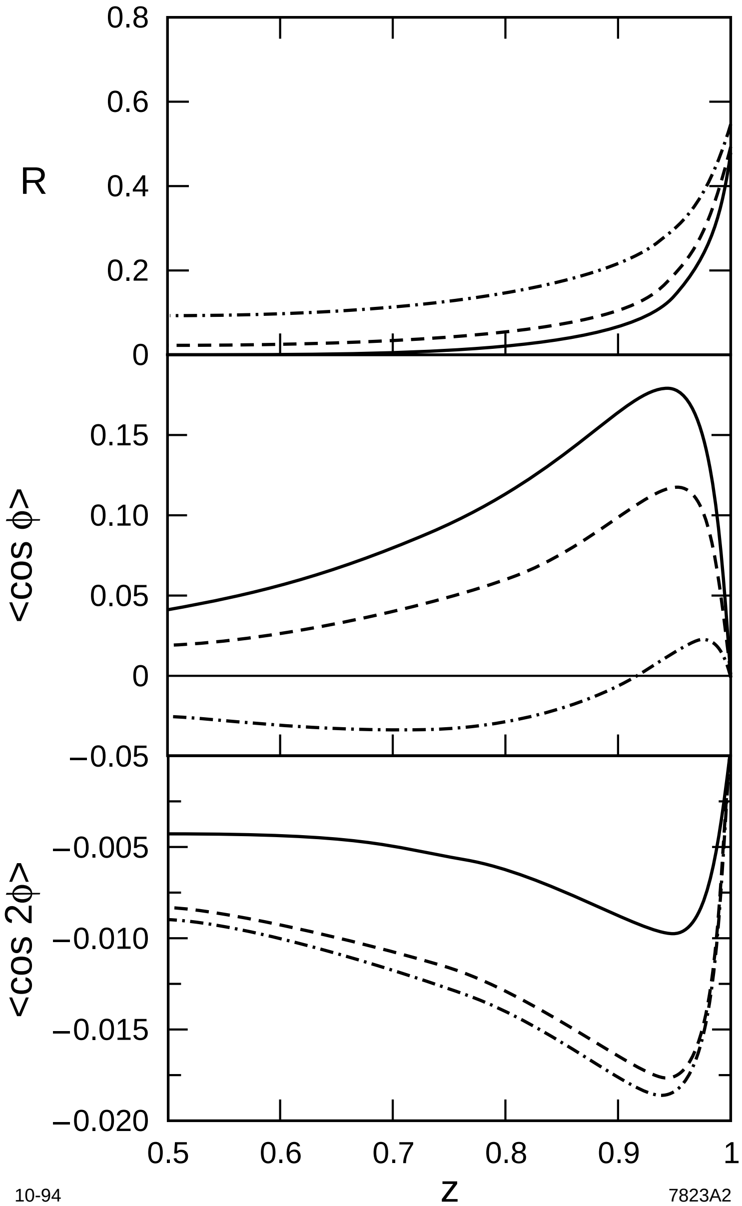
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Fig. 2

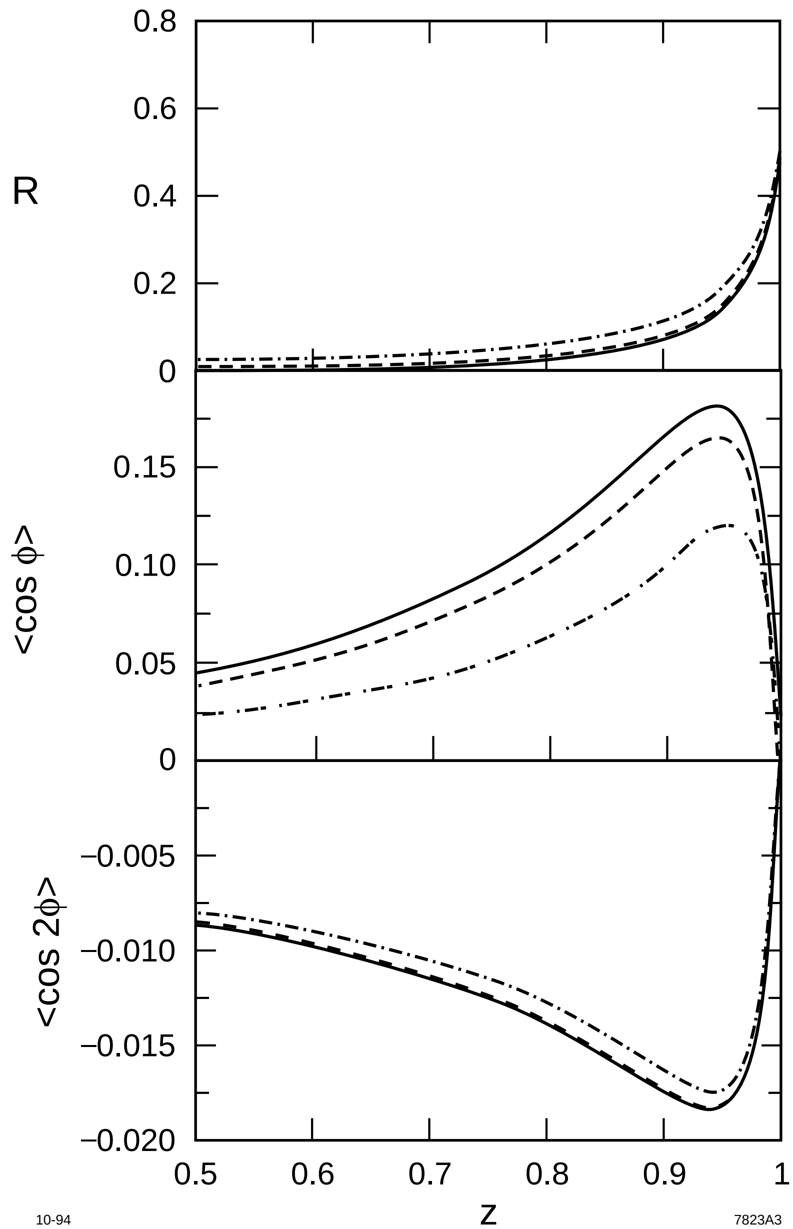
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Fig. 3

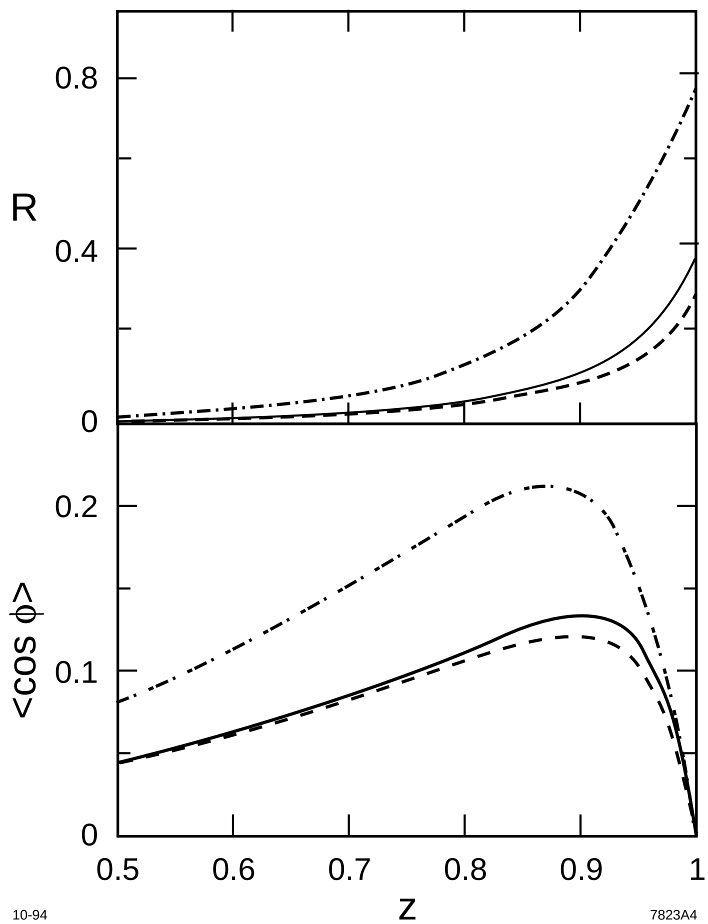

Fig. 4