

A preliminary measurement of the average B hadron lifetime*

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The average B hadron lifetime was measured using data collected with the SLD detector at the SLC in 1993. From a sample of $\sim 50,000$ Z^0 events, a sample enriched in $Z^0 \rightarrow b\bar{b}$ was selected by applying an impact parameter tag. The lifetime was extracted from the decay length distribution of inclusive vertices reconstructed in three dimensions. A binned maximum likelihood method yielded an average B hadron lifetime of $\tau_B = 1.577 \pm 0.032(\text{stat.}) \pm 0.046(\text{syst.})$ ps.

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1. Introduction and Experimental Technique

Precision measurements of the average B hadron lifetime τ_B are important for the study of the b quark and its weak couplings to u and c quarks. Results of τ_B measurements presented in 1993[1] differed substantially from the 1992 world average[2]. Reports of additional independent precision measurements of τ_B are interesting and timely.

In this paper, results are presented on a measurement of τ_B using data taken with the SLD detector at the Stanford Linear Collider (SLC). This analysis was performed on $\sim 50,000$ Z^0 decays recorded during the 1993 run of SLC.

Charged particle tracking was done using the central drift chamber (CDC) and the vertex detector (VXD)[3]. The liquid argon calorimeter (LAC) was used in the event trigger and in the determination of event shape quantities[4]. The angular errors of the CDC combined with local $\sigma(r\phi)$ and $\sigma(rz)$ of $6\text{ }\mu\text{m}$ for the VXD clusters lead to an $r\phi$ (plane perpendicular to the beams) impact parameter resolution of $(\alpha, \beta)_{r\phi} = (11\text{ }\mu\text{m}, 70\text{ }\mu\text{m})^\dagger$. The rz (plane containing the beam axis) impact parameter resolution is $(\alpha, \beta)_{rz} = (38\text{ }\mu\text{m}, 70\text{ }\mu\text{m})$. During the 1993 SLD run the $\langle \text{rms} \rangle_{xyz}$ profile of the SLC beams was $2.4 \times 0.8 \times 700\text{ }\mu\text{m}^3$ at the interaction point (IP). The IP x and y positions were tracked by SLD using reconstructed tracks from hadronic Z^0 events. Muon pairs (not used in the average IP determination) were used to check the IP xy position, giving $\sigma_{xy}^{IP} = 7 \pm 2\text{ }\mu\text{m}$. The z position of the IP was measured on an event-by-event basis with $\sigma_z \simeq 35\text{ }\mu\text{m}$ as determined by simulation.

The Monte Carlo (MC) physics simulation modeled Z^0 and heavy flavor decays with the LUND JETSET (version 6.3) Monte Carlo generator[5], which was adjusted to reflect current knowledge of the B and D decay spectra. The MC detector simulation was based on GEANT (version 3.15)[6] and produced raw hits that were superimposed on randomly triggered events from the data to simulate SLC backgrounds.

Standard hadronic event and track selection cuts[7] were applied, resulting in a sample of 29,400 Z^0 events. The JADE jet finding algorithm[8] with $y_{cut} = 0.02$ was used to determine the jet axes in the

[†] The impact parameter resolution function is parametrized as $\alpha \oplus \beta/P\sqrt{\sin^3\theta}$.

event from calorimetry clusters. For each track passing selection criteria, an impact parameter relative to the interaction point was determined. It was signed with respect to the nearest jet axis using standard conventions[7]. A normalized impact parameter was formed by dividing the signed 2-D track impact parameter by the error on the extrapolated track added in quadrature with the beam position error. Events were tagged as potentially containing a B hadron by requiring at least three quality tracks in the event with positive normalized 2-D impact parameter greater than 3. With this tag a $Z^0 \rightarrow b\bar{b}$ efficiency of 69% and purity of 82% were obtained according to Monte Carlo studies. From the data sample passing the hadronic event selection described above, 4,294 events were tagged.

2. Vertex Selection

Candidate secondary vertices were formed from all pairs of charged tracks in the same hemisphere with at least one hit in the VXD. A vertex-constrained fit was performed on all such pairs that extrapolated to within three standard deviations of a common point having their common point in the same hemisphere as the two tracks. To reduce background from tracks originating from the interaction point, the distance from the interaction point to the secondary vertex was required to be at least 1 mm. Furthermore, two-prong vertices consistent with arising from γ conversions, K^0 , or Λ^0 decays were removed from the sample.

Next, the two-prong vertices that shared common tracks were combined to form multi-prong vertices using a similar procedure. Tracks from multi-prong candidates were required to extrapolate to within ten standard deviations of a common point. A total vertex fit χ^2 less than 27(35) was required for three(four)-prong candidates. No candidates with more than four prongs were kept.

Events with more than 100 vertices remaining were removed. This was done to reduce the computer time consumed in the succeeding stages of the analysis. This cut removed 122 events. There were 22 events which had no secondary vertices. At this point, some vertex quality cuts were imposed on the sample. Vertices were removed if the vertex fit probability was less than 5% or if all tracks in the vertex had a normalized 2-D impact parameter to the IP less than 2.5. These cuts removed poor vertex fits and vertices with a high probability of containing a track originating from the IP, i.e., not from a secondary vertex. There were

Vertex type	Before	After	Final
	Partition Selection (%)	Partition Selection (%)	
b	16	19	22
cascade c	12	21	23
b+(cascade c)	44	33	35
b+other	12	7	4
(cascade c)+other	7	7	4
primary c	2	5	9
ip	1	3	1
Other	6	5	2

Table 1. Vertex type in sample according to Monte Carlo study.

approximately 84,000 vertices (not all independent) in 4,172 events remaining at this stage of the analysis.

The first column of Table 1 shows the percentage of remaining vertices broken down by vertex type according to a Monte Carlo study. A vertex is in the ‘b’ category if all its tracks originate from the weak decay of a b quark. The other categories are similarly defined.

In most events, the remaining vertices were not independent. Some tracks were shared by more than one vertex. In addition, finding a multi-pronged vertex meant that lower multiplicity vertices containing a subset of the tracks were found as well. An algorithm was developed to reduce these remaining vertices to a set of independent vertices. All possible unique sets of independent vertices (partitions) were found. The unique set used in the analysis was chosen by maximizing the joint fit probability of the event (product of the fit probabilities for each of the vertices in the partition, $P(\chi^2, d.o.f.)$). Table 1, column 2 shows the constitution of the remaining vertex sample according to a Monte Carlo study.

The primary background to real secondary vertices in the remaining sample was due to vertices made up entirely, or in part, of tracks originating from the IP. This background was substantially reduced by the decay length cut implemented earlier in the analysis. An additional cut was made to further reduce this background. Vertices were removed if the angle between the vertex line-of-flight and the nearest jet axis was

greater than 150 mrad. Two other cuts were made to enhance the track quality, and thus the vertex quality, as well as to reduce the number of vertices arising from IP tracks. Vertices were removed if any track in the vertex had momentum less than 0.7 GeV/c or if any track had a transverse momentum with respect to the vertex line-of-flight less than 0.07 GeV/c.

The final cut in the vertex selection was to demand no more than one vertex per event hemisphere by selecting the vertex closest to the IP. This simplified the statistical and systematic error calculations. Table 1, column 3 shows the constitution of the final vertex sample used in the analysis according to a Monte Carlo study. The final sample consisted of 5,427 vertices, made up of 4,104 two-prong vertices, 1,068 three-prong vertices, and 255 four-prong vertices. Note that 63% of the event hemispheres in selected heavy quark tagged events have at least one vertex at the end of vertex selection. Of these, 88% contain B hadron lifetime information according to a Monte Carlo study, i.e., they contain tracks associated with a weak b quark decay or the decay of the cascade c quark from the b quark.

3. Lifetime Analysis

The lifetime was extracted from the decay length distribution by using a binned likelihood function. The Monte Carlo was generated with a fixed value of τ_B . The τ_B dependence was introduced by a weighting procedure. Decay length distributions were produced for τ_B values ranging from 0.7 to 2.3 ps, in steps of 0.02 ps. The likelihood was then computed for each value of τ_B . The maximum likelihood fits to the decay length distribution yielded an average B hadron lifetime of $\tau_B = 1.577 \pm 0.032$ ps where the error is statistical only. Figure 1 shows the vertex decay length distribution compared to the Monte Carlo distribution giving the best lifetime fit. The $\chi^2/d.o.f.$ for this fit was ~ 2 .

A number of checks was made to increase confidence in this analysis. In one check, the Monte Carlo events were divided into five independent samples, each containing events scattered throughout the entire sample. Each of the five sets was successively analyzed as if it was the data and the other four subsets were Monte Carlo events. The five lifetime measurements yielded 1.528 ± 0.036 ps, 1.539 ± 0.037 ps, 1.449 ± 0.036 ps, 1.563 ± 0.039 ps, and 1.524 ± 0.037 ps, respectively, all in good agreement with the generated value. In

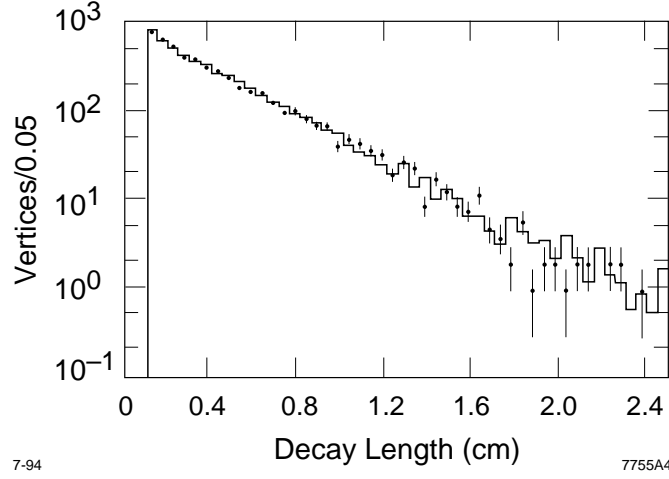


Figure 1. Decay length distribution for vertices passing all selection criteria in data (points) and Monte Carlo (histogram) events. The Monte Carlo distribution corresponds to that with the best-fit lifetime.

another check of the analysis method and weighting scheme, Monte Carlo events were generated with an average B hadron lifetime of 1.954 ps. If these events were used (instead of the nominal 1.515 ps sample) in the lifetime analysis, a measurement of 1.573 ± 0.030 ps resulted. A final check was performed to verify that no significant bias resulted from using events which passed the heavy quark tag. An impact parameter based heavy quark jet tag was used on the data. The lifetime analysis was done using only hemispheres opposite a tagged jet. The lifetime resulting from this study was 1.596 ± 0.039 ps, consistent with the result found above.

The systematic errors associated with this measurement are listed in Table 2. A detailed description of how the systematic errors were determined can be found elsewhere[7]. The dominant systematic error is that due to b quark fragmentation. Refinements in the analysis are expected to reduce this error. Other significant contributions arise from a lack of knowledge of the charm content of B hadron decays and from some uncertainty in the SLD charged particle tracking efficiency. Added in quadrature, the net systematic error is 0.046 ps. This yields a (preliminary) result of $\tau_B = 1.577 \pm 0.032(\text{stat.}) \pm 0.046(\text{syst.})$ ps for the average B hadron lifetime.

Systematic error	$\sigma\tau_B$ in ps
b fragmentation	0.038
c fragmentation	0.004
charm content	0.018
B baryon fraction	0.002
R_b	0.001
R_c	0.003
b multiplicity	0.006
detector effects	0.013
fit and binsize	0.010
TOTAL	0.046 ps

Table 2. Summary of systematic errors in this lifetime analysis.

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References

- [1] W. Venus, Proc. of the 16th Symp. on Lepton and Photon Interactions, Cornell University, 1993, p.274; Eds. P. Drell and D. Rubin (AIP 1993).
- [2] Particle Data Group, Phys. Rev. **D45**, Part II (1992).
- [3] G.Agnew *et al.*, SLAC-PUB-5906;
Also in the Proc. of the 26th International Conf. on High Energy Phys., Dallas, 1992, p. 1862; Ed. J. Sanford (AIP 1992).
- [4] D. Axen *et al.*, Nucl. Instr. and Meth. **A238** (1993) 472.
- [5] T. Sjöstrand, and M. Bengtsson, Comp. Phys. Comm. **43** (1987) 367.
- [6] R. Brun *et al.*, CERN DD/EE/84-1, 1987.
- [7] SLD Collaboration: K. Abe *et al.*, SLAC-PUB-6586.
- [8] JADE Collaboration: W. Bartel *et al.*, Z. Phys. **C33** (1986) 23.

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