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Resonant Photon-Graviton Conversion in EM Fields: From Earth to Heaven

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RESONANT PHOTON-GRAVITON CONVERSION IN EM FIELDS: FROM EARTH TO HEAVEN*

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-Abstract- Gravitational radiation from high energy particles in modern accelerators is reviewed. We point out that the most effective way for laboratory production and detection of gravitons is through resonant photon-graviton conversion in a strong external EM field. Specific example using crystal channels for the $\gamma \to g \to \gamma$ process is given, where the physical parameters needed for such a test appears to be reasonable. As another application of this effect in astrophysics, we show that the coupling between the cosmic microwave background radiation (CMBR) and the primordial magnetic field can induce a frequency-independent fluctuation in the photon flux. Using the observed CMBR fluctuation, we derive a bound on the primordial field strength. The effect can also convert the relic gravitons into photons. For the string cosmology it gives a new bound on the Hubble parameter at the Big Bang.

1. -Introduction-

At an energy scale much lower than the Planck scale, we can linearize the Einstein equation. With the convention $G = c = \hbar = 1$, we write

$$\Box\, \psi_{\mu\nu} = 16\pi T_{\mu\nu} \quad , \tag{1}$$

where $\psi_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu}h/2$ is the trace-reversed metric perturbation around the flat space-time $\eta_{\mu\nu}$ with the curved metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\eta = \mathrm{diag}(1, -1, -1, -1)$, and $T_{\mu\nu}$ is the energy-momentum stress tensor. Clearly this equation provides solutions as propagating waves, i.e., the gravitational waves (GWs), with $T_{\mu\nu}$ served as the source. We all know that the change of the quadrupole moment (in time) of a massive object can give rise to a GW. It was pointed out by Gertsenshtein[1)] that when a propagating EM wave traverses a transverse background EM field, there is a nontrivial stress tensor, $T^f_{\mu\nu}$, which can *resonantly excite* a GW at the same frequency as the initial propagating EM wave. In the case where the propagating EM wave is produced by a charged particle interacting with a EM background field, the stress tensor has contributions from both the particle and the field: $T_{\mu\nu} = T^p_{\mu\nu} + T^f_{\mu\nu}$. Then both direct massive radiation and the resonant excitation contribute to the emission of GW.

In this paper we investigate two aspects of this resonant excitation mechanism, one on earth and one in heaven. On earth we ask how feasible it is to produce *and* detect GWs or gravitons in the laboratory setting. In heaven we ask whether this effect has any implication on cosmology. For the first issue we begin by reviewing the direct massive gravitational radiation from a high energy particle in a storage ring, i.e., the *gravitational synchrotron radiation* (GSR) in Sec.2, and the resonant conversion

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of beamstrahlung into gravitons in linear colliders in Sec.3. These discussions lead naturally to the conclusion that the most effective way of laboratory production and detection of gravitons is to separate the production of photons from that of gravitons. In Sec.4 we propose a *graviton factory* using long crystals to resonantly convert a large flux of photons into gravitons, and then back-convert the gravitons into photons in the same process as a means to "detect" gravitons. The parameters required, though large, appear to be within reach of the available resource on earth. For the second issue, we point out that the coupling between the cosmic microwave background radiation (CMBR) and the primordial magnetic field resonantly converts the CMBR photons into gravitons, and gives rise to a fluctuation in the CMBR flux. Using the observed CMBR fluctuation as a constraint, we deduce a bound on the primordial field strength. Since the effect can also convert relic gravitons into photons, we derive a new bound on the Hubble parameter at the Big Bang in string cosmology.

2. -Gravitational Synchrotron Radiation-

The problem of gravitational radiation of a relativistic charged particle in a background EM field has been investigated by many authors[2]. We emphasize that, as discussed in the Introduction, the totality of the energy-momentum stress tensor of the system is responsible for the gravitational radiation, where the direct massive radiation constitutes only a subset of the contribution. With this understanding, we now treat the part of the GW in such a sub-system that is generated by $T^{p}_{\mu\nu}$. In this case the electromagnetic interaction serves only as a means to bend the particle trajectory and the mass of the particle acts just like a gravitational "charge". For this reason we shall call this subset of the GW the *gravitational synchrotron radiation* (GSR).

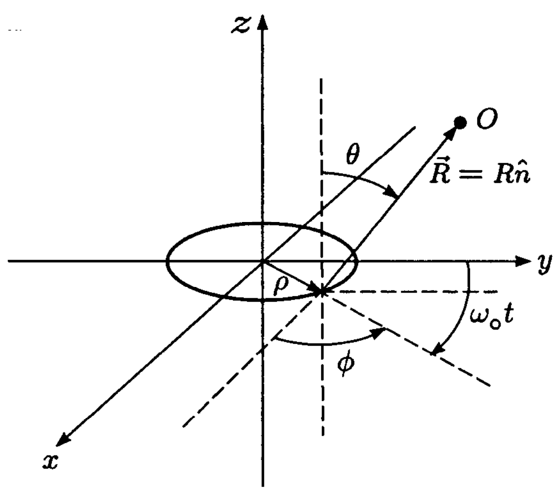

Fig.1 Coordinates involved in the gravitational synchrotron radiation.

As a general property of a wave equation, in the wave zone we have, from Eq.(1),

$$\psi_{\mu\nu}(\vec{R}) = -\frac{4}{R}e^{ikR}T_{\mu\nu}(\vec{k}) \quad , \tag{2}$$

where $\vec{R} = R\vec{n}$ and R is the distance to the observation point. The GW radiation power is[3)]

$$
\begin{aligned}
W_G(\omega) &= -\frac{R^2}{32\pi}\int d\Omega\Big[\partial_0\psi^{\mu\nu}\partial_i\psi_{\mu\nu} - \frac{1}{2}\partial_0\psi^{\mu}_{\mu}\partial_i\psi^{\nu}_{\nu}\Big]n^i \\
&= -\frac{\omega^2}{2\pi}\int d\Omega\; T^{\mu\nu}(\vec{k})T_{\mu\nu}(-\vec{k})
\end{aligned}
\quad , \tag{3}
$$

where n_i is the ith component of $\vec{n}$. For GSR it can be shown that the component which is doubly transverse to the tangent of the circular orbit,

$$
\begin{aligned}
T^p_{\perp\perp}(k) &= \frac{2\pi}{\omega_0}\gamma m e^{-i\nu(\pi/2-\phi)} \\
&\times\Big\{\sin^2\phi J_\nu(\xi) + 2i\sin\phi\cos\phi\Big[-\frac{\nu}{\xi^2}J_\nu(\xi)+\frac{\nu}{\xi}J'_\nu(\xi)\Big] - \cos 2\phi J''_\nu(\xi)\Big\},
\end{aligned}
\tag{4}
$$

dominates the contribution. Here ν is the harmonic number, $\xi = \nu\beta c\sin\theta, \omega_0 = c/\rho$ is the orbital frequency, θ, ϕ are the polar coordinates defined in Fig.1, and J_ν is the Bessel function. Inserting into Eq.(3) we obtain the GSR power spectrum

$$
\frac{dW_{GSR}}{dx} = \frac{3\sqrt{\pi}}{32}\frac{Gm^2\gamma^4\omega_0^2}{c}\Big[3x^{-1/3}\Phi(y) - 5x^{1/3}\Phi'(y) + 3x\Phi_2(y)\Big] \quad , \tag{5}
$$

where $x = \omega/\omega_c$, $y = x^{2/3}$, $\omega_c = \gamma^3\omega_0$ is the critical frequency of the synchrotron radiation, Φ is the Airy function, and

$$
\Phi_2(y) = \frac{y^{1/2}}{2^{2/3}\pi^{1/2}}\int_{-\infty}^{\infty} dz\Phi^2(y(1+z^2)/2^{2/3}) \quad . \tag{6}
$$

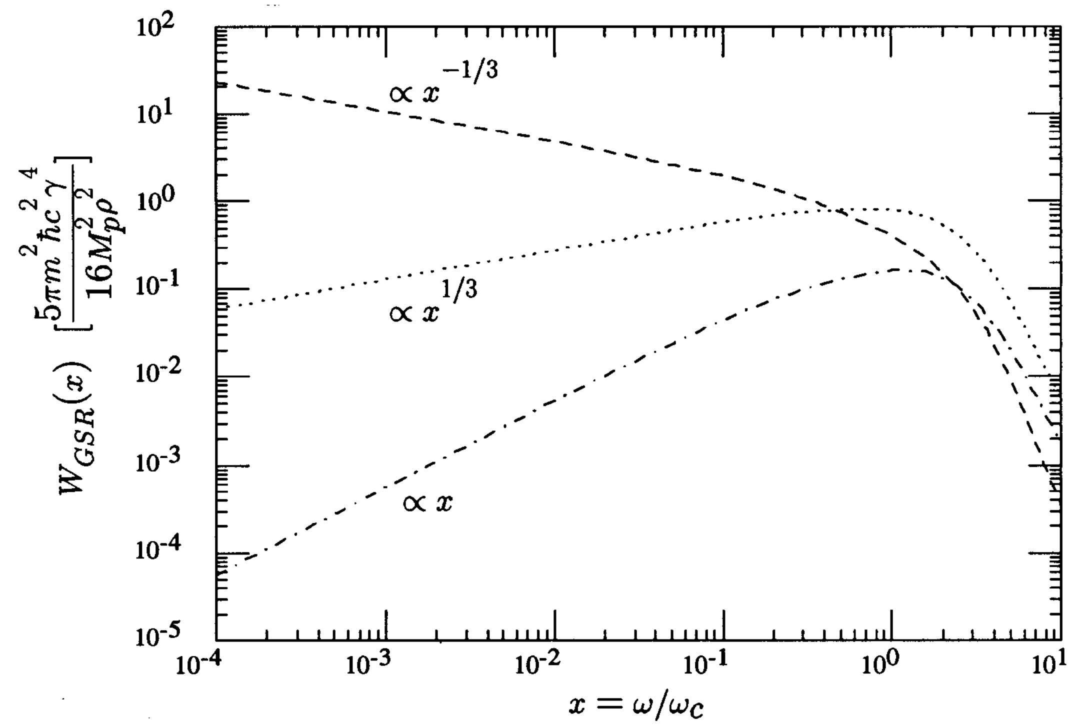

Fig. 2 Gravitational synchrotron radiation power spectrum.

Figure 2 shows the GSR spectrum with the contribution from the three terms in Eq.(5) plotted separately. At small x, the spectrum scales as $x^{-1/3}, x^{1/3}$, and x, respectively (See Fig. 2). Further integrating over the spectrum, we find the total pwer

$$W_{GSR} = \frac{5\pi}{16}\frac{Gm^2c\gamma^4}{\rho^2} = \frac{5\pi}{16}\frac{m^2}{M_P^2}\frac{\hbar c^2\gamma^4}{\rho^2} \quad , \tag{7}$$

where M_P is the Planck mass. Although the total power scales as γ^4, the same as in the electromagnetic synchrotron radiation (EMSR), the GSR power is dominated by the fundamental frequency due to its scaling law $x^{-1/3}$ (See Fig. 2). This is characteristically different from that in the EMSR, where the dominant frequency is ω_c. Therefore not only all N particles in a bunch in a storage ring radiate GSR coherently, all the n_b bunches in the ring can radiate coherently so long as the bunches are not distributed symmetrically around the ring. The total rate of graviton emmision is then

$$N_{GSR} \sim 5.6 n_b^2 N^2 \frac{m^2}{M_P^2}\frac{c\gamma^4}{\rho}. \tag{8}$$

Table 1 shows the estimated GSR graviton yields from various high energy storage rings. In the best case, i.e., the LHC, there will be of the order 10^{10} gravitons radiated per year. Note, however, that this is the total yield around the ring. The collectable signal is much reduced if concentrated at a single location with a finite solid angle. Furhtermore, at such low (fundamental) frequencies the notion of gravitons as discrete entities in the GW is questionable. We remind again that this is only a fraction of the total graviton yield from such an electromagnetic system where the EMSR can also convert into gravitons through resonant conversion. We will return to this issue at the end of the next section.

Storage Rings	PEP-II	LEP-I	LEP-II	HERA	LHC
$\mathcal{E}$[GeV]	9	50	100	880	7000
$\gamma[10^3]$	18	100	200	7.5	0.88
$N[10^{10}]$	3.8	45	45	10	10
n_b	1700	4	4	210	2800
$l[cm]$	3.46	6.24	6.24	27.7	18.4
ρ[m]	500	4300	4300	1035	4300
Gravitational SR					
ω_0[kHz]	600	70	70	290	70
$N_{GSR}[10^{-7}\text{sec}^{-1}]$	1.3×10^3	38	150	6×10^6	1.8×10^{10}
Resonant Conversion					
$\omega_c[10^9\text{GHz}]$	3.5	70	560	0.12	4.8×10^{-5}
$N_{res}[10^{-7}\text{sec}^{-1}]$	0.1	0.1	0.3	10^3	2×10^5

3. -Gravitational Beamstrahlung-

For a radiation field (from a charged particle) F^b traversing a static background field F^0, the electromagnetic part of the stress tensor has the form $T^f \sim (F^b+F^0)(F^b+F^0)$. The square of the background field, F^0F^0, bears no relation to the motion of the particle, and we shall ignore it in the following. There is also no need to discuss the square of the radiation field, F^bF^b, since almost everywhere $F^b \ll F^0$ except at small distance from the particle. But this has been taken into account in the mass renormalization, and thus is already contained in T^p. So the contribution from T^f is simply $F^bF^0+F^0F^b$. It is clear that the more intense the radiation and the background field, the more effective the resonant conversion.

Earlier, this effect was included in the investigation of GW production from high energy storage rings[4)]. It happens that a very powerful laboratory EM radiation, called *beamstrahlung*, occurs in high energy linear colliders during the collision of e^+e^- beams. A substantial fraction of beam energy is lost through beamstrahlung when particles are bent by the strong collective macroscopic EM field of the oncoming beam. In the world's first linear collider, the SLC (Stanford Linear Collider), the typical beamstrahlung photon energy is $\sim 10^{-3}$ of the initial particle energy. For future linear colliders, it is found to be inevitable that the fractional photon energy is not negligible[5)], and the process is necessarily quantum mechanical. With its potential impacts on high energy experimentation and its challenge as a theoretical problem, the study of quantum beamstrahlung has been intensive in recent years[6)]. In these calculations the beam is often modeled as a uniform charge distribution inside a cylinder with length L. The collective fields clearly varies as a function of the cylinder radius. But it can be shown, by integrating over the impact parameter (i.e., the radius) of the test particle, that the average radiation power is well represented by a *mean field* B, where all particles from the opposing beam radiate as if the field was uniform.

Motivated by the very intense collective field intrinsic to such colliding beams ($B \sim 10^8$G for the next generation, 0.5 TeV linear colliders) and the very intense beamstrahlung that penetrates through such a field, Chen[7)] calculated the resonant excitation of *gravitational beamstrahlung.* With the end effects ignored and to the accuracy of the order $1/\gamma$, it was shown that[7)]

$$W_G(\omega) = \frac{\pi}{4}\frac{1}{\alpha}\frac{m^2}{M_P^2}\Big(\frac{L}{\lambda\!\!\!^{-}_c}\frac{B}{B_c}\Big)^2\Big[1-\frac{\sin(\omega L)}{\omega L}\Big]^2 W_{EM}(\omega) \quad , \tag{9}$$

where $\lambda\!\!\!^{-}_c$ is the Compton wavelength, $B_c \equiv m^2c^3/e\hbar \sim 4.4\times10^{13}$ Gauss is the Schwinger critical field, and W_{EM} the power spectrum of quantum beamstrahlung. The square bracket represents the form factor from the Fourier spectrum of the background field. We see that this form factor is essentially of the order unity for wavelengths $\lambda \lesssim 2L$, where the last zero at $\sin(2\pi L/\lambda) = \sin\pi$ occurs. Beyond this wavelength the GWs are largely suppressed.

On the other hand, as is well-known, coherent radiation occurs only for wavelengths longer than the length of the radiating beam. To take advantage of coherent radiation, we concieve a shorter, low energy beam as the radiating beam which collides with a linear collider beam as the target beam. In that case the graviton yield in gravitational beamstrahlung is

$$N_{GB} \simeq 0.15\Big(\frac{m}{M_P}\Big)^2\Big(\frac{B}{B_c}\Big)^{8/3}\Big(\frac{L}{\lambda_c}\Big)^3\Big(\frac{\lambda_c}{\gamma'^2 l}\Big)^{1/3} N'^2 \quad . \tag{10}$$

Table 2 shows the graviton yields using the design parameters of the next generation linear colliders currently pursuit by various institutions around the world as the target beams. As is clear from Eq.(9), the scaling is in favor of long target beams with high currents. The last column invokes a high current ($\sim$ 10kAm), long ($\sim$ 30m) beam from an induction linac, such as the ATA (Advanced Test Accelerator) at Lawrance Livermore Laboratory. We see that even in the best case the yield is not as good as that from GSR. Nevertheless, the gravitons so produced are much higher in frequency and are well confined to a $1/\gamma$ narrow cone.

Linear Colliders	CLIC	DLC	JLC	NLC	TESLA	VLEPP	IL
$\mathcal{E}$[GeV]	250	250	250	250	250	250	0.5
$n_b \cdot f_{\rm rep}$[Hz]	6800	8600	13500	16200	8000	300	300
$N[10^{10}]$	0.6	2.1	0.7	0.65	5.15	20	62500
$L[cm]$	0.059	0.173	0.028	0.035	0.35	0.26	3000
$B/B_c[10^{-7}]$	7.0	1.4	3.0	1.9	1.3	1.5	1.0
Incoherent GB							
ω_c[GeV]	87.5	17.8	37.5	24	16.3	18.5	
$N_{GB}[10^{-25}\text{sec}^{-1}]$	130	135	2.7	1.5	2000	190	
Coherent GB							
$\mathcal{E}'$[GeV]	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$N'[10^{11}]$	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$l[\mu m]$	10	10	10	10	10	10	10
ω_0[eV]	0.125	0.125	0.125	0.125	0.125	0.125	0.125
$N_{GB}[10^{-15}\text{sec}^{-1}]$	84	37	1.5	1.3	230	5.2	3×10^{12}

Since in this calculation beamstrahlung is treated as radiation in a effective uniform field, the result (Eq.(9)) can also be applied to resonant conversion of the conventional EMSR into GW's, so long as the subtlties arisen from the edges of a magnet is ignored. Returning to the previous section, we see that in addition to the GSR there is also a resonant conversion with the rate $N_{res} \sim (1/\alpha)(m/M_P)^2(B/B_c)^2(L/\lambda_c)^2 N_\gamma$, where N_γ is the EMSR photon number. Since EMSR is dominated by the critical frequency $\omega_c = \gamma^3\omega_0 \gg \omega_0$, this radiation is not coherent in the high energy storage rings that we considered. As a result, the relative yield is $N_{res}/N_{GSR} \sim (1/n_b N)(B/B_c)^2(L/\lambda_c)^2$. Take, for the sake of discussion, $B \sim$ 10Tesla and $L \sim$ 10m. Then since the number of particles per bunch is of the order 10^{11} in Table 1, the relative yield is reduced by roughly a factor $1/10n_b$. These estimates are listed at the end of Table 1 for comparison.

4. -Resonant Photon-Graviton Conversion-

From Eq.(9) we see that if the background field is much longer than the wavelength of the propagating wave, the form factor $[1 - \sin(\omega L)/\omega L]^2 \approx 1$. Then we can write

$$W_G(\omega) = P(\gamma \to g) W_{EM}(\omega) \quad , \tag{11}$$

where

$$P(\gamma \to g) \approx \frac{1}{\alpha}\Big(\frac{m}{M_P}\frac{B}{B_c}\frac{L}{\lambda_c}\Big)^2 \tag{12}$$

can be interpreted as the probability of exciting a graviton from a photon at the same frequency. To this end we really don't need to go through the $e \to \gamma \to g$ channel to produce gravitons directly from the charged particles. Indeed, it may be more advantageous if one separates the processes between the photon production and the graviton production, where the optimization of the photon yield may be rather different from that of gravitons. From now on we shall concentrate on the direct conversion from a photon to a graviton, and assume that the photons are provided by separate means. In our discussion, we shall adopt the matrix formalism developed by Raffelt and Stodolsky[8)].

For a mixed photon-graviton state traversing a magnetic field with strength B at an angle Θ, the wave equation can be linearized, using the expansion $\omega^2 + \partial_z^2 = (\omega + i\partial_z)(\omega - i\partial_z) = (\omega + k)(\omega - k) \approx 2\omega(\omega - i\partial_z)$, as

$$\left[\omega - i\partial_z + \begin{bmatrix} \Delta_\perp & \Delta_M & 0 & 0 \\ \Delta_M & 0 & 0 & 0 \\ 0 & 0 & \Delta_\parallel & \Delta_M \\ 0 & 0 & \Delta_M & 0 \end{bmatrix}\right] \begin{bmatrix} A_\perp \\ G_+ \\ A_\parallel \\ G_\times \end{bmatrix} = 0 \quad , \tag{13}$$

where $\Delta_M \approx (B \sin\Theta / M_P)$, M_P is the Planck mass; and $\Delta_j = (n_j - 1)\omega, n_j$ are the refractive indices. $A_\perp, A_\parallel$ and $G_+, G_\times$ are the amplitudes of the photon and graviton states, respectively. For a less than perfect vacuum imbedded in a strong external field, there are two major contributions to Δ_j. The Lagrangian for the Euler-Heisenberg nonlinear QED effect due to the presence of a strong magnetic field gives rise to[9)] $n_\perp^{\rm QED} = 1 + 2\xi, n_\parallel^{\rm QED} = 1 + 7\xi/2$ and $\xi = (\alpha/45\pi)(B\sin\Theta/B_c)^2$. In addition, the medium also introduces refractive index. So in principle we have $\Delta_j = \Delta_j^{\rm QED} + \Delta_j^{\rm m}$. For the plasma epoch prior to the decoupling, we have $\Delta_j^{\rm m} = -\omega_p^2/2\omega$, where ω_p is the plasma frequency. For the post recombination era when the Universe was essentially in gas form, $\Delta_j^{\rm m}$ is induced by the Cotton-Mouton effect[10)]: birefringence of the photon due to the presence of an external magnetic field in a medium. Note that $\Delta_j^{\rm QED} \propto \omega$, while $\Delta_j^{\rm m} \propto -1/\omega$.

Focused on the reduced 2×2 matrix, we can perform a rotation with angle θ for diagonalization. The strength of the mixing is characterized by the ratio of the off-diagonal term to the difference of the diagonal terms: $(1/2)\tan 2\theta = (\Delta_M/\Delta_\parallel)$. In the weak mixing case, $(1/2)\tan 2\theta \approx \theta \ll 1$, and the photon-graviton degeneracy is removed. In this case the transition probability is

$$P(\gamma_{\|} \to g_{\times}) = 4\theta^2 \sin^2(\Delta_{\|} z/2) \quad . \tag{14}$$

If the path is much longer than the "oscillation length", $l_{osc} = 2\pi/\Delta_{\|}$, then the probability $P \approx 4\theta^2 \ll 1$.

On the other hand, the maximum mixing occurs when $\theta = 45°$, corresponding to the situation where $\Delta_{\|} = 0$. Here the degeneracy between the photon and the graviton states is reinstated, and the two are in resonance. Then,

$$P(\gamma_{\|} \to g_{\times}) = \sin^2(\Delta_M z) \quad . \tag{15}$$

In this case a complete transition is possible. In the typical situation, however, the coupling is so weak that for any physically realistic distance the argument can never reach $\pi/2$. Then practically,

$$P(\gamma \to g) \approx \Delta_M^2 z^2 \quad , \tag{16}$$

which can be easily varified to be identical to Eq.(12), with z replaced by L.

Note also that if $\Delta_{\|} \neq 0$, yet $\Delta_{\|} z \ll 1$, then Eq.(14) reduces to the same form. This is to say that for a given external field and distance z, there is a resonance frequency window which satisfies the condition

$$\Delta_{\|}(\omega_{\mathbf{res}} \pm \Delta\omega) \lesssim \pi/z \quad , \tag{17}$$

and within this window the conversion probability is essentially $P(\gamma \to g) \approx \Delta_M^2 z^2$, independent of the photon frequency.

For the case of an inhomogeneous field, Raffelt and Stodolsky[8)] show that

$$P(\gamma_{\|} \to g_{\times}) = \Big| \int_0^z dz' \Delta_M(z') \exp\Big\{ -i \int_0^{z'} \Delta_{\|}(z'') dz'' \Big\} \Big|^2 \quad , \tag{18}$$

as long as the external field varies smoothly (in both strength and orientation) over the photon wavelength. In the case where the value of $\Delta_{\|}$ is so small that the phase factor in Eq.(18) for any frequency is entirely negligible, then the transition probability is identical for both $\|$ and $\perp$ modes.

5. -Graviton Production and Detection using Crystals:

A Graviton Factory-

As is well-known, the faintness of the gravitational interaction makes the detection of gravity wave one of the main challenges in modern physics. In Sections 2 and 3 we saw that there is indeed a finite amount of gravitons radiated in storage rings and linear colliders. But in terms of its detection, the yield may appear to be too small[11]. This is really not very surprising, as these high energy accelerators are not designed for optimizing the GW emmissions in the first place. Since the resonant conversion process is actually an oscillation between the photon and the graviton states, one may conceive an experimental setting where the $\gamma \to g \to \gamma$ channel can be exploited for both graviton production and detection. For this purpose it is desirable to provide the largest possible photon flux that propagates through the strongest possible EM field for the longest possible distance.

Let us conceive two long straight cavities with length L_1 and L_2, where transverse magnetic fields with strength B_1 and B_2, respectively, are applied (See Fig. 3). The first cavity is used for $\gamma \to g$ conversion and the second for the $g \to \gamma$ back-conversion These two cavities are aligned but separated by a "wall", at which the unconverted photons are stopped while the converted gravitons penetrate into the second cavity[12]. This idea is very similar to the proposed axion experiment[13].

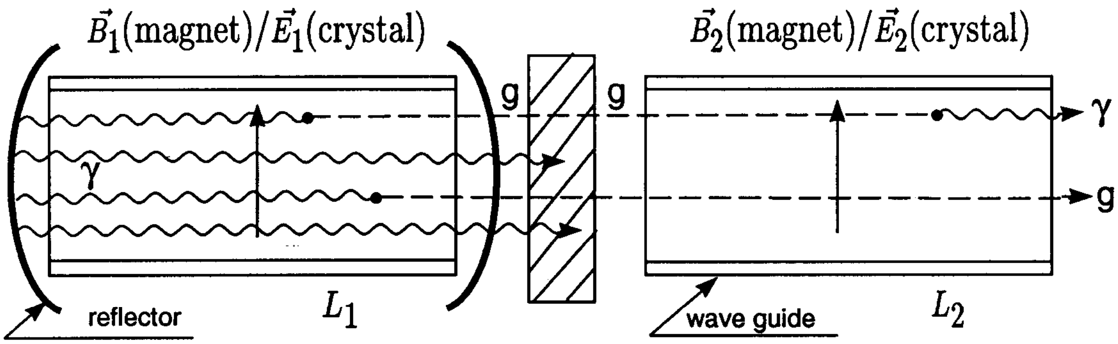

Fig. 3 A schematic diagram for a graviton factory

Since the first cavity is for $\gamma \to g$ conversion, one can in principle introduce a pair of reflectors at each end so that the photons can be reused. If the loss factor of the reflector is η, then the same photon pulse may rebounce for the order $1/\eta$ times inside the first cavity. The yield of the $\gamma \to g \to \gamma$ final state photons is then

$$N(\gamma \to g \to \gamma) = P_1(\gamma \to g)P_2(g \to \gamma)\frac{W_{EM}\Delta t}{\eta\hbar\omega} \quad , \tag{19}$$

for a given time Δt. In "practical" units, we can write

$$N(\gamma \to g \to \gamma) = 7.3\times 10^{-47}\Big(\frac{B_1}{100\mathrm{T}}\Big)^2\Big(\frac{L_1}{100\mathrm{km}}\Big)^2\Big(\frac{B_2}{100\mathrm{T}}\Big)^2\Big(\frac{L_2}{100\mathrm{km}}\Big)^2\frac{W_{EM}\Delta t}{\eta\hbar\omega} \quad . \tag{20}$$

Envision cavities with $L_1 = L_2 = 100$km and magnetic fields with $B_1 = B_2 = 100$Tesla. Assume further that the loss factor is $\eta \sim 10^{-5}$ and the photon energy is $\omega \sim 10^{-5}$eV. Then in order to generate one final photon in a year ($\Delta t \sim 10^7$sec), we would need a electromagnetic beam power of the order $W_{EM} \sim 50\text{GW}$. This is a rather formidable number. Furthermore, at such low frequency, whether one can really detect a single "photon" is questionable.

It is well-known that in crystal channels the electrostatic fields can be as large as 10^{16}V/m. This is equivalent to a magnetic field strength of $B \sim 10^4$ Tesla. Imagine two 100km transparent (non-conductive) crystals with a field in the channels equivalent to $B \sim 10^4$ T. Limited by the channel size, we are compelled to inject higher frequency photons, say $\hbar\omega \sim 10\text{eV}$. Then we find

$$W_{EM} \sim 0.5\text{GW} \quad , \tag{21}$$

in order to back-convert one final state photon in a year. Such a power is much more affordable.

Clearly, there is still a long way between such a simple minded conception and the experimental realization. Practical considerations may limit the preformance. For example, to ensure the resonance condition, from Eq.(18) we find that we need to have the conductivity in the crystal be low enough such that $\Delta_{\|} \lesssim \pi c/100\text{km}$. Another concern is that, since gravitons tend not to be bounded by the cavity walls, the natural divergence of the gravitons may result in a loss of back-conversion efficiency. It is doubtful that the 100 km structure can be made by a single crystal. Thus to ensure adiabatic variations of the background field along crystal channels, it is necessary that the alignment accuracy between successive crystal sections be better than the photon wavelength.

6. -Cosmic Microwave Background Fluctuations-

The cosmic microwave background radiation (CMBR) is one of the few windows from which we can look back into the early history of our Universe. The physical origin of the CMBR temperature fluctuation at large scales detected by COBE[14)] have been much discussed. These fluctuations are generally attributed to the well-known Sachs-Wolfe effect[15)]. Both density fluctuations (scalar modes) and relic gravitons (tensor modes) generated at earlier epochs, such as inflation[16)], can contribute to perturbations of the lightlike geodesics, causing a redshift in the CMBR spectrum, and therefore its temperature fluctuation and anisotropy[17)].

If there indeed exists a primordial magnetic field, then the thermal CMBR photons can couple to this primordial magnetic field in the post-decoupling (or recombination) epoch and resonantly convert into gravitons. This effect can therefore cause a fluctuation in the number and energy flux in the CMBR. As we will see in the following, this resonant conversion probability is essentially the same for all frequencies that we consider. Using the observed CMBR fluctuation as a bound, we derive a constraint on the primordial field strength and show that, within the uncertainties and approximations, it is reasonably consistent with the bounds deduced from other astrophysical considerations.

First we derive the probability for a photon to convert into a graviton by traversing one large magnetic domain, or "bubble", with size L and a uniform field strength B at an angle Θ with respect to the photon propagation direction. Let t be the time when the photon enters the bubble. As the photon propagates through this domain both L and B will evolve. Assuming the conservation of magnetic flux, we find $B(t) \propto 1/L^2(t)$. As the post-decoupling era is matter dominated, we have $L \propto t^{2/3}$ and thus $B \propto t^{-4/3}$. Neglecting the phase factor, we find from Eq.(18)

$$P(t) \approx \begin{cases} L^2(t)B^2(t)\sin^2\Theta/M_P^2 \quad , & L(t) \lesssim H^{-1}(t) \quad , \\ 9t^2[1-t/L]B^2(t)\sin^2\Theta/M_P^2 \quad , & L(t) \gtrsim H^{-1}(t) \quad , \end{cases} \tag{22}$$

where $H(t)$ is the Hubble parameter at time t. The upper expression is strictly true for $L_* \ll H_*^{-1}$, but is $\sim$ 20% over-estimation for $L_* \sim H_*^{-1}$. Note also that $P(t)$ is asymptotically independent of the bubble size. Starting from the recombination time t_* to the present time t_1, a photon will have to cross N such bubbles with similar size L_* at t_*:

$$N \sim \frac{1}{L_*}\int_{t_*}^{t_1}\left(\frac{t_*}{t}\right)^{2/3} dt \sim 3\left(\frac{t_1}{t_*}\right)^{1/3}\frac{t_*}{L_*} \quad . \tag{23}$$

Let us first examine the case where $L_* \lesssim H_*^{-1}$. If the bubbles have sharp domain walls, i.e., the change of field strength and orientation across the boundary is not adiabatic, and if these changes are entirely random from bubble to bubble, then the mean total probability is

$$P = \sum_{i=1}^{N} P(t_i) \approx \frac{1}{\pi}\int_0^{\pi} d\Theta \int_{t_*}^{t_1}\frac{dt}{L(t)}P(t) \sim \frac{1}{2}\frac{t_*}{L_*}P_* \quad , \tag{24}$$

where $P_* \sim B_*^2 L_*^2/M_P^2$. The *rms* fluctuation around the mean is

$$P_{rms} \approx \left[\frac{1}{\pi}\int_0^{\pi} d\Theta \int_{t_*}^{t_1}\frac{dt}{L(t)}P^2(t) - \frac{1}{4N}\left(\frac{t_*}{L_*}\right)^2 P_*^2\right]^{1/2} \sim \frac{3}{2\sqrt{14}}\left(\frac{t_*}{L_*}\right)^{1/2} P_* \quad . \tag{25}$$

This "leakage" of photons into gravitons leads to a *frequency-independent* fluctuation in the CMBR flux, i.e.,

$$\langle \delta\rho_\gamma/\rho_\gamma \rangle \sim \frac{3}{2\sqrt{14}}\left(\frac{t_*}{L_*}\right)^{1/2} P_* \quad , \qquad L_* \lesssim H_*^{-1} \quad , \tag{26}$$

where $\rho_\gamma(x) = (T^4/\pi^2)x^3/(e^x - 1)$, and $x \equiv \omega/T$.

If, on the other hand, the coherence scales are much larger than H_*^{-1}, the mean total conversion probability is obtained by integrating the lower expression of Eq.(21) over the angle, and we find $P \sim (9/2)B_*^2 t_*^2/M_P^2$. In this limit, the *rms* fluctuation is primarily induced through the randomness of the field orientations in different bubbles, which gives a coefficient of $(3/8 - 1/4)^{1/2} = 1/2\sqrt{2}$. Thus the fluctuation reaches an asymptotic value

$$\langle \delta\rho_\gamma/\rho_\gamma \rangle \sim \frac{9}{2\sqrt{2}}\left(\frac{t_*}{L_*}\right)^2 P_* \quad , \qquad L_* \gg H_*^{-1} \quad , \tag{27}$$

independent of L_* (since $P_* \propto L_*^2$).

The anisotropy of such a fluctuation is associated with the only physical scale of the process, namely the bubble size L_* at t_*. Thermal photons arriving at our detector from different angles have crossed different sets of randomly oriented bubbles. So the flux varies at the scale of the bubble size across the sky. For an observer at present, this bubble size has been Hubble-expanded to $L_1 \sim (t_1/t_*)^{2/3} L_*$.

This fluctuation is different in character from that generated by the Sachs-Wolfe effect, which is frequency dependent. Since the number of photons per mode in blackbody radiation is an adiabatic invariant, a frequency variation is equivalent to a temperature variation: $\delta\omega/\omega = \delta T/T$. So for the Sachs-Wolfe effect we have

$$\langle \delta\rho_\gamma/\rho_\gamma \rangle_{sw} = \frac{x}{1-e^{-x}} \langle \delta T/T \rangle \quad . \tag{28}$$

Note that for $x \gg 1, \langle \delta\rho_\gamma/\rho_\gamma \rangle_{sw} \approx x\langle \delta T/T \rangle$; while for $x \ll 1$, $\langle \delta\rho_\gamma/\rho_\gamma \rangle_{sw} \approx \langle \delta T/T \rangle$, independent of frequency.

Observations of CMBR fluctuations at large scales by COBE, at medium scales by ARGO[18)] and MSAM[19)], plus other measurements, at various frequency ranges fit reasonably well with the above scaling law. Nevertheless, due to uncertainties in the measurements and noise in the signals, the possibility of a frequency independent contribution to $\langle \delta\rho_\gamma/\rho_\gamma \rangle$ in addition to the frequency dependent one, cannot be ruled out. It is clear that the maximum allowed photon-graviton conversion induced fluctuation can never exceed the observed CMBR fluctuation. Since our effect is frequency independent, the constraint should be set by the measurements at low frequencies. From Eq.(26), this means

$$\frac{B_*}{B_c} \lesssim 0.14 \frac{M_p}{m} \frac{\lambda_c}{t_*^{1/4} L_*^{3/4}} \sqrt{\langle \delta T/T \rangle} \quad , \qquad L_* \lesssim H_*^{-1} \quad . \tag{29}$$

Note that the anisotropy scale $L_1 \sim (t_1/t_*)^{2/3} H_*^{-1} \sim$ 280Mpc, i.e., the Hubble-expanded horizon size at t_*, corresponds to a coherence angle $\theta_c \sim 1.5°$. From the observations at this scale[20)], which gives $\langle \delta T/T \rangle \sim 1 \times 10^{-5}$, we find $B_* \lesssim 0.03$G. To be sure, further measurements and analysis of the observed data with the inclusion of a frequency-independent contribution would help to refine this bound.

At the recombination time, the typical photon energy is $T_* \sim 0.3$ eV, and the gas density is $n_* \sim 10^3\text{cm}^{-3}$. With $B_* \sim 0.03$ G, the corresponding changes in the refractive index are $\Delta_{j*}^{\mathrm{QED}} \sim 10^{-38}\text{cm}^{-1}$ and $\Delta_{j*}^{\mathrm{m}} \sim -10^{-33}\text{cm}^{-1}$.[21)] These values are so small that the corresponding oscillation length $l_{osc}^*(\omega = T_*) = 2\pi/|\Delta_{\|,\perp}^*| \sim 10^{34}\text{cm} \gg H_1^{-1} \sim 10^{28}$ cm. It is clear that the resonance window covers all possible frequencies. This confirms our assumption that this fluctuation is essentially frequency independent.

Let us now check this constraint against the bounds on the primordial field derived from other astrophysical considerations. There are several arguments for the existence of an intergalactic magnetic field. For example, to obtain the observed high energy cosmic rays ($E > 10^{20}$eV), one would need an intergalactic magnetic field with strength of the order $\sim 10^{-7} - 10^{-9}$G at scales $L_1 \sim$100Mpc to confine the accelerated particles[22)]. There have been many proposals regarding the origin of this magnetic field[23,24,25)], as well as efforts to look for its constraints. In particular, Cheng, Schramm, and Truran[26] recently obtained constraints from the abundances of the light elements during Big Bang Nucleosynthesis (BBN).

In Ref.26 it was found that the maximum strength of the primordial magnetic field at the BBN epoch ($t \sim 1$ min.,$\sim 2\times 10^{12}$ cm) is $B \lesssim 10^{11}$ G on scales $H_{\rm BBN}^{-1} \gtrsim L \gtrsim 10^4$ cm. By assuming magnetic flux conservation, the authors of Ref.26 deduced that these bounds evolve into $B_* \lesssim 0.1$ G on scales 10^{18}cm $\gtrsim L_* \gtrsim 10^{11}$ cm at t_*. Note that although this field strength at t_* is an upper bound, it was argued[26], based on Hogan's theory[27], that it corresponds to an intergalactic field of $\lesssim 7\times 10^{-9}$ G at present. (Although by the argument of magnetic flux conservation one would have deduced that $B \sim 10^{-7}$G at present, about one order of magnitude larger.) On the other hand, the bounds on the coherence scales appear to be conservative. These are the Hubble-expanded values of the bounds at the BBN epoch, with the implicit assumption that the magnetic bubbles have been "frozen" in time without interactions. However, as demonstrated by Tajima et al.[25,28], during the plasma epoch magnetic bubbles, once in contact, tend to quickly "polymerize" into larger bubbles. For example, near the recombination time, it takes only $\sim 10^8$ sec ($\ll t_* \sim 10^{13}$ sec) before the polymer extends to the event horizon. Under this scenario of "polymerization", the bounds deduced from BBN can in principle be extended to the scale $L_* \lesssim H_*^{-1}$, the largest possible causally connected scale at t_*. Although this bound is larger than what we deduced from the CMBR fluctuation by about a factor of 3, with various uncertainties and approximations in mind, we should consider them to be reasonably consistent.

In the models where the magnetic field "seeds" are generated during inflation[23], the coherence scale can in principle be larger than H_*^{-1}. In this case, our fluctuation reaches an asymptotic value, yet the CMBR constraint scales as $L_1^{-2/3}$. At large scales, we deduce from the COBE result[29] a scaling law: $\langle \delta T/T\rangle \sim 1\times 10^{-5}(10^\circ/\theta_c)^{2/3}$. Combining with Eq.(11), we find

$$\frac{B_*}{B_c} \lesssim 2.9\times 10^{-4}\frac{M_p}{m}\frac{\lambda_c}{t_*}(H_*L_*)^{-1/3} \quad , \qquad L_* \gg H_*^{-1} \quad . \tag{30}$$

7. -Implications on Cosmology-

This effect can in principle also convert relic gravitons[30,31] into photons. It can be shown that prior to the decoupling, e.g., during the e-p plasma epoch, the magnetic field and the plasma density are both so high that the resonance window is very narrow around the resonance frequency at any given time: $\omega_{res}(t) = \sqrt{90\pi/7\alpha}[B_c/B(t)]\omega_p(t)$. In turn the time for a photon to remain in resonance, or the so-called "level crossing", $\Delta t \sim [\sqrt{90\pi/7\alpha}B_c/B(t)(\pi t/\omega_p(t))]^{1/2}$, is very short. As a result the resonant conversion is negligible. Thus the relic graviton spectrum is well preserved until the decoupling time.

Nonstring-based inflation theories predict a flat or decreasing graviton spectrum (in frequency). For scales $L_* \sim H_*^{-1}$, the lower limit of the resonant frequency set by $\Delta_j^{\rm m}(\omega_{*l}) = 2\pi H_*$ allows for resonant conversion for frequencies $\omega_* \gtrsim \omega_{*l} \sim 3\times 10^{-10}$eV, or $\lambda_* \lesssim 3\times 10^6$cm. In terms of the value at present, $\lambda_{res} \sim (t_1/t_*)^{2/3}\lambda_* \lesssim 3\times$

10^9cm. We see that the lower limit of the Harrison-Zel'dovich scale-invariant spectrum ($\lambda_{HZ}^{min} \sim 10^7$cm) lies inside the resonance window. Here the wavelength is $\sim$ 7-9 orders of magnitude larger than the CMBR wavelength, which is way out in the Planckian tail. Any measured EM wave at this wavelength and scale may be a signal of $g \to \gamma$ conversion. Constraint on the graviton density at the maximum wavelength ($\lambda_1 \sim H_1^{-1}$), gives the maximum possible energy density $\Omega_{HZ} \sim 10^{-14}$ at present[30]. This gives the density fluctuation $\sim$ 8 orders of magnitude above the CMBR spectrum at λ_{HZ}^{min}. A direct measurement of the EM waves with such wavelength at large scales would be a test of the inflation theories.

String cosmology allows for an increasing relic graviton spectrum[31]. In this case the constraint is fixed at the maximum frequency: $\omega_0 \sim 10^{29}(H_0/M_P)^{1/2}\omega_1$, where H_0, the Hubble parameter at $t=0$, is a free parameter in the theory. $\omega_1 \sim H_1 \sim 10^{-18}$Hz is the minimal frequency inside the present Hubble radius. With the bounds $10^2 \gtrsim H_0/M_P \gtrsim 10^{-4}$ for an increasing spectrum, we see that 0.03cm $\lesssim \lambda_0 \lesssim$ 30cm at present, which covers the range of CMBR.

Let us introduce the "magnetic energy density" in units of the critical energy density, ρ_c^*, at t_*:

$$\delta\Omega_{EM}^* = \frac{B_*^2}{8\pi}\frac{1}{\rho_c^*} \quad . \tag{31}$$

For the curvature signature $k=0$ and the isotropic pressure $p=0$ we have, from the Friedmann equation, $H_*^2 = (8\pi/3)G\rho_c^*$. Inserting this and Eq.(31) into Eq.(25), we get

$$P_{rms}(g \to \gamma) \sim \frac{9}{4\sqrt{7}}\delta\Omega_{EM}^*(H_*L_*)^{3/2} \quad , \quad L_* \lesssim H_*^{-1} \quad . \tag{32}$$

Here the relation $H_*^{-1} \simeq 2t_*$ has been used.

Using Eq.(32) and the graviton spectrum from Gasperini and Veneziano [31], we find a graviton-induced CMBR fluctuation at present:

$$\frac{\delta\rho_{GV}(x)}{\rho_\gamma(x)} \sim \delta\Omega_{EM}^*(H_*L_*)^{3/2}\left(\frac{H_0}{M_P}\right)^2 \frac{\rho_\gamma}{\rho_\gamma(x)}\frac{x^2}{x_0^3} \quad , \tag{33}$$

where $x_0 = \omega_0/T \sim (10^{29}H_1/T)(H_0/M_P)^{1/2}, T = 2.7°$K, and $\rho_\gamma = \int_0^\infty \rho_\gamma(x)dx$. Note that this fluctuation is frequency independent at small x. Since x_0 is not *a priori* determined in the string cosmology (because of H_0), we apply the general expression in Eq.(27) for the bound: $\delta\rho_{GV}(x_0)/\rho_\gamma(x_0) \lesssim x_0/(1-e^{-x_0})\langle\delta T/T\rangle$. After some algebra, we obtain the following constraint:

$$\frac{\sinh^2(x_0/2)}{x_0} \lesssim \frac{15}{4\pi^4}\left(10^{29}\frac{H_1}{T}\right)^4 (H_*L_*)^{-3/2}\frac{\langle\delta T/T\rangle}{\delta\Omega_{EM}^*} \quad . \tag{34}$$

If the primordial field strength can be independently determined, then x_0, and therefore H_0, is constrained by the CMBR fluctuation. Within our scenario, however, $\delta\Omega_{EM}^*$ is itself bounded by the CMBR fluctuation. As we discussed earlier, the primordial field so deduced, though an upper bound, is consistent with the field necessary to explain the high energy cosmic rays. We thus assume (cf. Eq.(32)) that $\delta\Omega^*_{EM} \sim \langle\delta T/T\rangle$, or $B_* \sim 0.03$G, at $L_* \sim H_*^{-1}$. Inserting into Eq.(34), we find an order-of-magnitude estimate for a bound on H_0:

$$H_0/M_P \lesssim 1 \quad . \tag{35}$$

This lies inside the previously deduced bounds[31].

In our consideration, the resonant conversion mediated by the primordial magnetic field was treated as unrelated to the Sachs-Wolfe effect. This may not necessarily be so. Prior to the decoupling time the Universe was in a plasma state. It is known in plasma physics that a local concentration of plasma density tends to expel the magnetic flux. In this regard the matter perturbation and the primordial magnetic bubbles may complement each other spatially. Indeed, we know that it takes $\delta\Omega^*_m = \delta\rho^*_m/\rho^*_c \sim 10^{-5}$ matter perturbation to give rise to a temperature fluctuation $\delta T/T \sim 10^{-5}$. Miraculously, from Eq.(32) we find that to attain the same level of fluctuation it also requires $\delta\Omega^*_{EM} \sim 10^{-5}$ at the scale $L_* \sim H_*^{-1}$. This suggests that certain balance between the density pressure and the magnetic pressure may have been attained at this scale prior to the decoupling. This may even provide a physical basis for the isothermal picture of the Universe. More details of the discussion on cosmology can be found in Ref. 32.

8. -Discussion-

In this paper we have review the GW production from high energy charged particles in modern accelerators. We then suggest that the best approach to a possible laboratory production and detection of gravitons is through resonant photon-graviton conversion in long crystals. With various idealizations invoked, the calculation indicates that the power consumption for such an experimental test of the existence of the graviton appears to be quite affordable. Evidently, the scales involved in such an experiment is gigantic, and the actual power consumption when more realistic conditions are introduced should be much more than what we estimated. But the prospect of a laboratory test of quantum gravity, or more specifically, of the existence of gravity quanta, is even concievable should be encouraging enough.

When we apply this effect to cosmology, we demonstrated that the CMBR photons can resonantly convert into gravitons by coupling with the primordial magnetic field. Using the observed CMBR fluctuations as a bound, we derived a bound on the strength of the primordial field. Since the effect can also convert gravitons to photons, we suggest that this effect can help to verify different models of cosmology. In particular, we found a new bound on the Hubble parameter at the Big Bang in string cosmology. It is hoped that further measurements and analysis of the CMBR fluctuation will help to tighten the constraint on this effect and the primordial field, which in turn will help to refine the bounds we found on cosmology.

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