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# Constraining the Strongly-Coupled Standard Model Including a $W'$ Isotriplet

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## Abstract

We consider the Strongly-Coupled Standard Model (Abbott-Farhi model) including an isotriplet of  $W'$  vector bosons. First we calculate the corrections to the low-energy theory, which can be effectively summarized in terms of the parameters  $S$ ,  $T$  and  $U$ . Then we use high-precision electroweak measurements to constrain the mass and couplings of the  $W'$ . The  $W'$  couplings are restricted to be unnaturally small, and we conclude that this model is no longer compelling as a theory of the electroweak interactions.

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# 1 Introduction

With the ever-increasing precision of electroweak measurements, viable theories of physics beyond the Standard Model have become somewhat scarce. In this paper, we reexamine a candidate theory which promises a rich spectrum of resonances and other strong-interaction phenomena beyond the weak scale: the Strongly-Coupled Standard Model (SCSM), or Abbott-Farhi model [1].

The SCSM is based on an underlying lagrangian identical in form to the Standard-Model lagrangian. However, the parameters of the gauge-Higgs sector are adjusted so that the Higgs field does not spontaneously break the  $SU(2)_L$  gauge symmetry. Instead the  $SU(2)_L$  interactions become confining, and the observed particle spectrum consists of  $SU(2)_L$  singlets. Nevertheless, given dynamical assumptions such as unbroken chiral symmetry, the low-energy theory of the SCSM looks very much like the spontaneously-broken Standard Model. The striking similarity of the confining and spontaneously broken phases of the theory exemplifies the concept of “complementarity.” The exciting possibility that nature might in fact be described by a confining version of the Standard Model, which predicts the discovery of new particles and strong interactions at future colliders, motivates the study of the SCSM.

Of course, if the SCSM really is the theory of the weak interactions, evidence for particle compositeness must eventually emerge. The effective theory of the SCSM must deviate from the renormalizable Standard-Model lagrangian: resonances and higher-dimensional interactions should appear. In this work we ask: Are the deviations expected in the SCSM allowed by current experimental constraints? We will attempt to answer this question by studying a test case in which we introduce an isotriplet of  $W'$  vector bosons into the effective theory. We calculate the corrections to Standard-Model predictions which result from including the  $W'$  bosons. Then we use high-precision electroweak data to constrain these corrections, and thereby bound the allowed region of the  $W'$  mass and couplings.

In an earlier analysis of experimental constraints on the SCSM, Korpa and Ryzak [2] added both an  $W'$  isotriplet and an isoscalar vector boson to the usual Standard-Model particle content. They concluded that experiment could accommodate these new particles without severely restricting their masses and couplings. Since then many new electroweak observables have been measured, and the accuracy of earlier measurements has greatly increased. Here we exploit this new data to find much stronger constraints on the model. In particular, the precise measurement of the  $Z$  mass permits a new approach to parameterizing the corrections to Standard-Model predictions in the SCSM.

In the original formulation of the model [1], the  $W$  and  $Z$  were not expected to have the masses predicted by the Standard Model; however, it was known that the Standard-Model

values could be recovered by invoking vector dominance [3]. Specifically, they follow from the assumption that the  $W$ -pole graph saturates the isovector electromagnetic form factor of the composite fermions. When the  $W$  and  $Z$  were later discovered with masses near the Standard-Model values, it became necessary to add to the SCSM the assumption that vector dominance holds at least approximately [4]. In our analysis we find that the  $W$  must saturate this form factor to within a few percent. We consider this unnatural and conclude that, in its present form, the SCSM is no longer a candidate for a theory of the electroweak interactions.

In the next Section we review the effective theory for the SCSM in a limit where non-Standard particle content and higher-dimensional interactions are absent, and show that it reduces to the Standard Model in this limit. In Section 3 we introduce a  $W'$  isotriplet and discuss the resulting modification of vector-dominance relations. In Section 4 we calculate the  $W'$ -induced corrections to electroweak observables, which we summarize in terms of contributions to  $S$ ,  $T$  and  $U$  in Section 5. We then use high-precision electroweak measurements in Section 6 to determine the allowed region of the  $W'$  mass and couplings, and present our conclusions in the final Section.

## 2 Review of the SCSM

The fundamental insight which underlies the SCSM is that the particle spectrum and interactions in the strong-coupling version of the Standard Model could closely resemble those of the familiar spontaneously-broken Standard Model. This is an example of complementarity: there is no phase transition between confinement and spontaneous symmetry breaking in a gauge-Higgs theory with a Higgs in the fundamental representation [5]. In this Section we review how the effective theory of the SCSM approximates the ordinary Standard Model (given certain dynamical assumptions). We closely follow the presentation and notation of Claudson, Farhi and Jaffe [4]. In the next Section we will begin our discussion of the deviations from Standard-Model predictions which appear when a  $W'$  isotriplet is added.

The SCSM is based on an underlying lagrangian which has the same form as the Standard-Model lagrangian. However, the parameters of the theory are adjusted so that the  $SU(2)_L$  interactions are not spontaneously broken, and instead become confining at low energies. All the observed particles are then  $SU(2)_L$  singlets.

Consider the potential for the fundamental scalar field,

$$V(\Omega) = \frac{\lambda}{2}(\text{tr } \Omega^\dagger \Omega - 2v^2)^2, \quad (1)$$

where

$$\Omega = (\phi \quad \tilde{\phi}) = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix}. \quad (2)$$

Note that  $\Omega^\dagger \Omega = |\phi|^2 \mathbf{1}$ . By expressing the potential in terms of  $\Omega$ , we make explicit the invariance of the potential under the custodial  $SU(2)_W$  symmetry, defined by  $\Omega \rightarrow \Omega h$  for  $h \in SU(2)_W$ . This symmetry is an invariance of the full lagrangian when the hypercharge and Yukawa couplings of the fermions are turned off.

The scale dependence of the  $SU(2)_L$  gauge coupling is characterized by a scale parameter  $\Lambda_2$ , analogous to  $\Lambda_{\text{QCD}}$ . This scale parameter and the constant  $v^2$ , which appears in the scalar-field potential in Eq. (1), together control whether the  $SU(2)_L$  interactions are confining or spontaneously broken. The  $SU(2)_L$  gauge symmetry will not be spontaneously broken if  $v^2 < 0$  or if  $v^2 \ll \Lambda_2^2$ , in which case the gauge interactions get strong at energies well above  $v^2$  and prevent spontaneous symmetry breaking. The fundamental fields which carry  $SU(2)_L$  charge will then be confined into  $SU(2)_L$  singlets. These can be classified using the custodial symmetry,  $SU(2)_W$ .

For example, the elementary left-handed fermions  $\psi_L^a$  (where  $a = 1, \dots, 12$  labels the  $SU(2)_L$  doublet) bind with the scalar particles  $\phi$  to form composite left-handed fermions,

$$F_L^a = \Omega^\dagger \psi_L^a = \begin{pmatrix} \phi^{*\alpha} \psi_{L\alpha}^a \\ \phi_\alpha \epsilon^{\alpha\beta} \psi_{L\beta}^a \end{pmatrix}, \quad (3)$$

which transform as  $SU(2)_W$  doublets. Here  $\alpha$  and  $\beta$  are  $SU(2)_L$  indices, which are contracted so that the  $F_L^a$  are  $SU(2)_L$  singlets. The hypercharge of a composite fermion is the sum of the elementary fermion and scalar hypercharges,  $y^a + \tau^3/2$ . This is simply the electric charge  $Q^a$  of the fermion, which implies that the hypercharge  $U(1)$  in the SCSM is actually electromagnetism.

From the scalar fields alone we can form a composite Higgs field,  $H = \frac{1}{2} \text{tr} (\Omega^\dagger \Omega)$ , which is an  $SU(2)_W$  singlet. We can also form an  $SU(2)_W$  triplet of vector bosons, with interpolating field  $\mathbf{W}_\mu = \text{tr} (\Omega^\dagger D_\mu \Omega \boldsymbol{\tau})$ . In these examples we can see the crucial role played by the custodial symmetry in organizing the composite particles into multiplets analogous to the familiar  $SU(2)_L$  multiplets of the Standard Model. (We will later see how this symmetry also ensures that the interactions of the composite particles have the standard form.)

Of course, in addition to the particles that are contained in the Standard Model, experience with the Strong Interactions leads us to expect in the SCSM a rich spectrum of bound states, including excited  $W'$  bosons, leptoquarks and so on. Since these particles have yet to be observed, we must assume that these exotic states are considerably more massive than the left-handed fermions and the  $W$  bosons.

Claudson, Farhi, and Jaffe enumerated three dynamical assumptions concerning the confining  $SU(2)_L$  sector of the theory, which must hold if the SCSM is to describe the observed electroweak phenomena [4]:

- (i) The approximate  $SU(12)$  chiral symmetry which relates the 12  $SU(2)_L$  fermion doublets is not spontaneously broken by a condensation of left-handed fermions (i.e.,  $\langle \psi_L^a \psi_L^b \rangle = 0$ ). This chiral symmetry then protects the composite left-handed fermions  $F_L^a$  from acquiring large masses. (If this chiral symmetry were broken, there would be light Goldstone bosons consisting of two left-handed fermions, and the composite fermions would be heavy, as their analogs are in QCD.)
- (ii) The  $W$  vector bosons are much lighter than the typical mass scale in the theory (e.g.,  $\Lambda_2$ ), and in particular, the  $W$  and  $Z$  are much lighter than their recurrences, the  $W'$  and  $Z'$ .
- (iii) The effective coupling of the  $W$  bosons to left-handed fermions is small ( $\bar{g} \approx 0.66$ ) even while the underlying theory is strongly-coupled.

With these assumptions we can write down the low-energy effective lagrangian for the SCSM. Interactions with dimension greater than four should be suppressed by the characteristic mass scale,  $\Lambda_2$ , which by assumption (ii) is much larger than  $M_W$ . As long as we work at energies no higher than the  $Z$  mass, we should be able to omit these higher-dimensional operators from the effective theory. Then the most general  $SU(2)_W$ -symmetric effective lagrangian involving the composite fermion and vector-boson fields is

$$\mathcal{L}_{\text{eff}}^0 = i\bar{F}_{La}\not{\partial}F_L^a - \frac{1}{4}\mathbf{W}^{\mu\nu} \cdot \mathbf{W}_{\mu\nu} + \frac{1}{2}M_W^2\mathbf{W}^\mu \cdot \mathbf{W}_\mu + \bar{g}\mathbf{W}_\mu \cdot \mathbf{j}_L^\mu + \dots, \quad (4)$$

where the  $W$  self-couplings have not been listed. Here  $\mathbf{j}_L^\mu = \frac{1}{2}\bar{F}_{La}\boldsymbol{\tau}\gamma^\mu F_L^a$ , and  $\mathbf{W}_{\mu\nu} = \partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu$ . Electromagnetism, which breaks the custodial symmetry, can then be added by minimal substitution of the vector potential,  $a_\mu$ , and insertion of the field-strength,  $\mathcal{F}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ :

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^0 + i\bar{\psi}_{Rb}\not{\partial}\psi_R^b + ea_\mu j_{\text{em}}^\mu - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} - \frac{k}{2}\mathcal{F}^{\mu\nu}W_{\mu\nu}^3 + \dots \quad (5)$$

Again cubic and quartic vector-boson interactions have not been listed. Here  $j_{\text{em}}^\mu$  is the contribution of the fermions to the electromagnetic current. If we now assume vector dominance, so that the isovector electromagnetic form factor of the  $F_L^a$  is saturated by the  $W$  boson,

then as will be discussed in the next Section, we find that the strength  $k$  of the photon- $W^3$  mixing<sup>1</sup> is given in terms of the U(1) coupling and the  $W\bar{F}_L F_L$  coupling as  $k = e/\bar{g}$ . Diagonalizing the quadratic terms in the lagrangian (5) which involve the neutral vector bosons, we find the propagating fields

$$\begin{aligned} A_\mu &= a_\mu + kW_\mu^3, \\ Z_\mu &= (1 - k^2)^{1/2} W_\mu^3, \end{aligned} \quad (6)$$

which couple to the neutral currents as

$$\mathcal{L}_{\text{NC}} = eA \cdot j_{\text{em}} + Z \cdot \frac{\bar{g}}{\sqrt{1 - k^2}} \left( j_L^3 - \frac{ek}{\bar{g}} j_{\text{em}} \right). \quad (7)$$

Hence the value of  $\sin^2 \theta$  that would be measured in low-energy neutrino scattering is  $\sin^2 \theta = ek/\bar{g} = k^2$ , where we have used the vector-dominance result  $k = e/\bar{g}$ . This implies that  $\bar{g} = e/\sin \theta$ , which leads to the standard prediction for the mass of the  $W$ :  $M_W^2 = \pi\alpha/(\sqrt{2}G_F \sin^2 \theta)$ . In the above diagonalization process one additionally finds a  $Z$  mass of  $M_Z = M_W/\sqrt{1 - k^2}$ . Applying the vector-dominance result again, we recover the Standard-Model relation

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta} = 1. \quad (8)$$

We see therefore how the additional assumption of vector dominance leads to the Standard-Model predictions for the masses of both the  $W$  and the  $Z$ . A vector-dominance analysis of the electromagnetic form factors of the  $W$  shows that the cubic and quartic self-couplings of the  $W$  are those of an SU(2) gauge theory with coupling  $\bar{g}$ , and that the corresponding couplings of the propagating fields,  $A$  and  $Z$ , are just those of the Standard Model [4].

### 3 Including a $W'$ Isotriplet

We have just seen that with certain dynamical assumptions, and invoking vector dominance, the effective theory of the SCSM approximates the Standard Model. We now begin our analysis of the corrections to the effective theory that result when we introduce an isotriplet of  $W'$  vector bosons. A  $W'$  should arise in this model as a radial excitation of the  $W$ , analogous to the  $\rho'$  in QCD. Because there is no evidence yet for deviations from the Standard Model,

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<sup>1</sup>This mixing is analogous to the familiar case of photon- $\rho$  mixing.

the  $W'$  must be considerably more massive than the  $W$  and/or less strongly coupled. We will therefore treat the inclusion of the  $W'$  as a perturbation of the Standard Model.

Of course, we could include other non-Standard particles in the theory. Alternatively, we could include in the lagrangian all operators up to some dimension which are consistent with the symmetries of the theory. However, as we will soon show, the  $W'$  is the degree of freedom which corresponds to relaxing the assumption that the  $W$  saturates the isovector electromagnetic form factor. Adding a  $W'$  thus allows us to consider corrections to the effective theory due to new particle content, and also to study deviations from exact vector dominance. At the same time, including a  $W'$  isotriplet adds only three new parameters to the low-energy effective theory, and therefore it is possible to significantly constrain the theory. The lagrangian terms for the  $W'$  are similar in form to those for the  $W$ , Eqs. (4) and (5):

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{W'} = & -\frac{1}{4}\mathbf{W}'^{\mu\nu} \cdot \mathbf{W}'_{\mu\nu} + \frac{1}{2}M_W'^2 \mathbf{W}'^\mu \cdot \mathbf{W}'_\mu + \bar{g}'\mathbf{W}'_\mu \cdot \mathbf{j}_L^\mu \\ & -\frac{k'}{2}\mathcal{F}^{\mu\nu}W'^3_{\mu\nu} + \dots\end{aligned}\quad (9)$$

In constructing  $\mathcal{L}_{\text{eff}}^{W'}$  we have proceeded much as before in arriving at Eqs. (4) and (5). We have first constructed the most general  $SU(2)_W$ -symmetric lagrangian. Next we have diagonalized the lagrangian to eliminate terms which mix the  $W$  and  $W'$  bosons. Finally we have included electromagnetism, which leads to mixing of the photon with  $W^3$  and  $W'^3$ . For later purposes, we note that by substituting  $\mathbf{W}' \rightarrow -\mathbf{W}'$  we can reverse the signs of both  $\bar{g}'$  and  $k'$ , showing that only the relative sign of these two couplings is meaningful.

### 3.1 Vector Dominance

We now show how the  $W'$  parameterizes the deviation from vector-meson dominance: Consider the isovector electromagnetic form factor of the composite fermions,  $F_V(q^2)$ , defined through

$$e\langle \bar{F}_{La}, F_L^a | J_{\text{em}}^\mu | 0 \rangle = e\bar{U}_L\gamma^\mu y^a V_L + e F_V(q^2) \bar{U}_L\gamma^\mu \frac{\tau^3}{2} V_L. \quad (10)$$

Here  $J_{\text{em}}^\mu$  is the total electromagnetic current, which includes terms linear in  $W^3$  and  $W'^3$ . The sum of the contributions to the form factor (see Fig. 1) from direct coupling of the current to the left-handed fermions and from the  $W^3$ - and  $W'^3$ -pole diagrams is

$$eF_V(q^2) = e - k\bar{g}\frac{q^2}{q^2 - M_W^2} - k'\bar{g}'\frac{q^2}{q^2 - M_W'^2}. \quad (11)$$

Because the  $F_L$  are composite and the confining  $SU(2)_L$  interactions are asymptotically free,  $F_V(q^2) \rightarrow 0$  as  $|q^2| \rightarrow \infty$ . Then, assuming that  $F_V(q^2)$  is saturated by the  $W^3$  and  $W'^3$  poles (i.e., there are no other contributions to the form factor, such as from a  $W''$ ), we conclude that

$$e = k\bar{g} + k'\bar{g}' = k\bar{g}(1 + \kappa\gamma) , \quad (12)$$

where we have introduced the ratios of  $W$  and  $W'$  couplings

$$\kappa \equiv k'/k , \quad \gamma \equiv \bar{g}'/\bar{g} . \quad (13)$$

Had we assumed strict vector dominance so that only the  $W$ -pole diagram contributed, we would have found the result  $e = k\bar{g}$ , which leads to the Standard-Model lagrangian as was shown in Section 2. By including the  $W'$ , however, we depart from exact vector dominance. The  $W'$  contributes a fraction  $k'\bar{g}'/e \approx \kappa\gamma$  to the saturation of  $F_V(q^2)$ , so that  $\kappa\gamma$  measures the degree of the departure.

For the model to approximately reproduce the Standard Model, the  $W$  must nearly saturate this form factor. Here we have further assumed that the  $W'$  contribution to the saturation, though of necessity much smaller than that of the  $W$ , is nevertheless more important than contributions from higher-lying resonances, which have been ignored. In essence, we are claiming that the  $W'$  can be viewed as a stand-in for all the resonances beyond the  $W$  which contribute to  $F_V(q^2)$ . The quantity  $\kappa\gamma$  accordingly represents the combined contribution of these resonances to the saturation of this form factor.

### 3.2 The Physical Neutral Vector Bosons

The mixing of the photon with the neutral  $W^3$  and  $W'^3$  bosons introduces off-diagonal terms into the free (quadratic) part of the lagrangian. We need to diagonalize the free lagrangian to find the physical photon,  $Z$  and  $Z'$  fields. In the previous Section we gave expressions for the physical photon and  $Z$  fields that diagonalize the free part of lagrangian in the absence of the  $W'$  bosons. These expressions, and also the result for the  $Z$  mass, are modified by the mixing of the photon with  $W'^3$ . For the masses of the  $Z$  and  $Z'$  we find

$$\begin{aligned} M_Z^2 &\approx \frac{M_W^2}{c^2} \left[ 1 - \frac{s^4}{c^2(c^2 - \mu)} \mu \kappa^2 \right] , \\ M_{Z'}^2 &\approx M_{W'}^2 \left[ \frac{1 + \frac{s^4}{c^4} \mu \kappa^2}{1 - \frac{s^2}{c^2} \kappa^2} + \frac{s^4}{c^2(c^2 - \mu)} \mu \kappa^2 \right] . \end{aligned} \quad (14)$$



Here  $s \equiv \sin \theta \equiv k$ ,  $c \equiv \cos \theta$ , and  $\mu$  is the ratio of the  $W$  and  $W'$  squared masses:

$$\mu \equiv M_W^2/M_{W'}^2. \quad (15)$$

Terms containing extra factors of  $\mu\kappa^2$  have been omitted. Note that we must restrict  $\kappa$  to the interval  $|\kappa| < \cot \theta$ , since otherwise the  $Z'$  would have a negative squared mass and be a tachyon. The neutral vector-boson fields  $a$ ,  $W^3$  and  $W'^3$  are given in terms of the physical fields  $A$ ,  $Z$  and  $Z'$  as

$$\begin{aligned} a &= A - s(W^3 + \kappa W'^3), \\ W^3 &\approx \left[ 1 - \left( 1 - \frac{\mu}{2c^2} \right) \left( \frac{s^2}{c^2 - \mu} \right)^2 \mu\kappa^2 \right] \frac{Z}{c} + \frac{s^2}{c^2 - \mu} \mu\kappa \frac{Z'}{\sqrt{1 - \frac{s^2}{c^2} \kappa^2}}, \\ W'^3 &\approx -\frac{s^2}{c^2 - \mu} \mu\kappa \frac{Z}{c} + \frac{Z'}{\sqrt{1 - \frac{s^2}{c^2} \kappa^2}}. \end{aligned} \quad (16)$$

Corrections to the coefficients of  $Z$  and  $Z'$  are suppressed by additional factors of  $\mu\kappa^2$ . If we substitute these expressions into the interaction terms that couple  $a$ ,  $W$ , and  $W'$  to the fermions, we find the couplings of the  $Z$  and  $Z'$  to left-handed and electromagnetic currents.

These results for the  $Z$  and  $Z'$  masses (14) and the expressions for  $W^3$  and  $W'^3$  in terms of  $Z$  and  $Z'$  (16) were calculated by expanding in powers  $s^2\mu\kappa/(c^2 - \mu) \ll 1$ , which is assumed to be small. This assumption reflects our intuition that the  $W'$  should be heavier than the  $W$  ( $\mu \ll 1$ ) and should also mix more weakly with the photon ( $\kappa \ll 1$ ).

Note that all of the above corrections to Standard-Model relations — the vector dominance result for the electromagnetic coupling (12) and the mass and couplings of the  $W$  and  $Z$ , (14) and (16) — contain factors of at least two of the  $W'$  parameters  $\kappa$ ,  $\gamma$  and  $\mu$ . The same is true of corrections to four-fermi interactions mediated by  $W'$  exchange, which are of order  $\mu\gamma^2$ . Hence if any two of the  $W'$  parameters vanish, the effective low-energy theory reduces to that of the Standard Model, leaving the remaining  $W'$  parameter completely unconstrained. This means that we will be unable to obtain constraints on any individual parameter independent of the other parameters. We will either have to fix one of the parameters and then constrain the other two, or else constrain products of the parameters, e.g., the product  $\kappa\gamma$ .

As mentioned above, Korpa and Ryzak in their earlier analysis of SCSM constraints considered the SCSM with not only an isotriplet of  $W'$  vector bosons but also with isoscalar vector bosons which are bound states of a left-handed fermion and a left-handed antifermion:  $(V_\mu)_a^b \sim \bar{\psi}_{La} \gamma_\mu \psi_L^b$ . Assuming that those  $V$  bosons which are color octets, and thus mix

with gluons, saturate the (isoscalar) color form factor of the composite, left-handed quarks, they were able to place a very stringent bound on the mass of the isoscalar bosons ( $m_V > 700 \text{ GeV}$ ), so that the  $V$  bosons would be just as massive as the rest of the non-Standard resonances. This called into question the assumption of vector dominance of the isoscalar form factors. Of course, vector dominance of the *isoscalar* channel is not a necessary ingredient in the SCSM. By contrast, vector dominance of the *isovector* channel must at least approximately hold in order to account for the  $W$  and  $Z$  masses. By including a  $W'$  we can determine to what accuracy vector dominance must be maintained in the isovector channel in order to retain agreement with electroweak data.

## 4 Corrections to Standard-Model Predictions

We have seen that the SCSM as formulated here reduces to the Standard Model if the  $W'$  is absent or if it has infinite mass and vanishing couplings. And, of course, the Standard Model is in impressive agreement with experiment. It is therefore logical to treat the SCSM with a  $W'$  as a perturbation of the Standard Model and calculate the corrections to SM predictions due to the  $W'$ .

### 4.1 Corrections to the Mass and Couplings of the $W$

The corrections induced by the  $W'$  are not simply given by the sum of the new graphs that include  $W'$ s. The three quantities  $\alpha$ ,  $G_F$  and  $M_Z$  are known to very high accuracy, and their values cannot change when the  $W'$  bosons are added to the theory. In the Standard Model, their values determine the masses and couplings of the vector bosons. However, as the  $W'$  parameters are turned on, the  $W$  parameters must deviate from their Standard-Model values if  $\alpha$ ,  $G_F$  and  $M_Z$  are to remain fixed. There are then two ways in which the  $W'$  modifies the effective theory: (1)  $W'$  exchange induces new effective (four-fermi) interactions, *and* (2) the  $W$  mass and couplings depart from their Standard-Model values in order to preserve the values of  $\alpha$ ,  $G_F$ , and  $M_Z$ .

Let us first compute the deviations of the  $W$  mass and couplings from their Standard-Model values by working at tree level. We define  $M_{W0}$ ,  $\bar{g}_0$  and  $k_0$  as the mass and couplings of the  $W$  when the  $W'$  is absent, and we define  $M_W$ ,  $\bar{g}$  and  $k$  as the mass and couplings when the  $W'$  is included. We compute  $e$ ,  $G_F$  and  $M_Z$  (at tree level) in the Standard Model when the  $W'$  is absent, and then in the SCSM, with a  $W'$  in the theory. Combining these

results we have

$$\begin{aligned}
e &= k_0 \bar{g}_0 = k \bar{g} (1 + \kappa \gamma) , \\
4\sqrt{2}G_F &= \frac{\bar{g}_0^2}{M_{W0}^2} = \frac{\bar{g}^2}{M_W^2} + \frac{\bar{g}'^2}{M_{W'}^2} = \frac{\bar{g}^2}{M_W^2} (1 + \mu \gamma^2) , \\
M_Z^2 &= \frac{M_{W0}^2}{c_0^2} \approx \frac{M_W^2}{c^2} \left[ 1 - \frac{s^4}{c^2(c^2 - \mu)} \mu \kappa^2 \right] .
\end{aligned} \tag{17}$$

Here  $c_0 \equiv \cos \theta_0 = \sqrt{1 - k_0^2}$ , and as before,  $s = k$  and  $c = \sqrt{1 - k^2}$ . The expressions for  $e = \sqrt{4\pi\alpha}$  and  $M_Z$  come from the previous Section, Eqs. (12) and (14). The result for  $G_F$  is simply the sum of  $W$  and  $W'$  exchange. These formulas can be used to express the deviations of the  $W$  mass and couplings from their Standard-Model values in terms of  $M_{W'}$ ,  $\bar{g}'$  and  $k'$  (or, equivalently, in terms of  $\mu$ ,  $\kappa$  and  $\gamma$ ). Let  $\delta M_W = M_W - M_{W0}$ ,  $\delta \bar{g} = \bar{g} - \bar{g}_0$  and  $\delta k = k - k_0$ . Then

$$\begin{aligned}
\frac{\delta M_W}{M_W} &\approx \frac{1}{2} \frac{1}{c^2 - s^2} \left[ 2s^2 \kappa \gamma - s^2 \mu \gamma^2 + \frac{s^4}{c^2 - \mu} \mu \kappa^2 \right] , \\
\frac{\delta \bar{g}}{\bar{g}} &\approx \frac{1}{2} \frac{1}{c^2 - s^2} \left[ 2s^2 \kappa \gamma - c^2 \mu \gamma^2 + \frac{s^4}{c^2 - \mu} \mu \kappa^2 \right] , \\
\frac{\delta k}{k} &\approx - \frac{1}{2} \frac{1}{c^2 - s^2} \left[ 2c^2 \kappa \gamma - c^2 \mu \gamma^2 + \frac{s^4}{c^2 - \mu} \mu \kappa^2 \right] .
\end{aligned} \tag{18}$$

Terms containing more powers of  $\kappa \gamma$ ,  $\mu \gamma^2$  or  $\mu \kappa^2$  have been omitted. The corrections to Standard-Model predictions we will find, such as all those found above, will in general be linear in these quantities, and so they will be forced to be small by our constraint analysis. In Eq. (18) where  $s$  and  $c$  multiply small quantities like  $\kappa \gamma$ , we could just as well use  $s_0$  and  $c_0$ , since the expressions would be unchanged within the accuracy to which we are working. Here and in the following, wherever the choice of  $\sin \theta_W$  is immaterial, we will use  $\sin \theta = k$  and write simply  $s$  (and  $c$ ) for brevity. It should be understood however, that another convention would do just as well.

These results have been obtained at tree level. Of course, in addition to the  $W'$ -induced corrections to the effective theory, there are also radiative corrections. Radiative corrections to the small  $W'$ -induced corrections are negligible, comparable to two-loop corrections in the Standard Model. Hence the effective theory is well approximated by adding the  $W'$ -induced corrections calculated here at tree level to the Standard-Model effective theory calculated to one-loop accuracy. In particular, the mass and couplings of the  $W$  in the SCSM are obtained

from their renormalized Standard-Model values by simply adding the deviations  $\delta M_W$ ,  $\delta \bar{g}$  and  $\delta k$  calculated above.

## 4.2 Neutral-Current Interactions at the $Z$ Pole

Having calculated the correction to  $M_W$ , we need to compute the corrections to the neutral-current interactions in order to obtain the remaining constraints on the  $W'$  parameters. (Charged-current interactions are precisely constrained only at low energy, and there they are completely fixed by the value of  $G_F$ .) Above we presented expressions for the fields  $a$ ,  $W$  and  $W'$  in terms of the physical neutral vector bosons  $A$ ,  $Z$  and  $Z'$  (16). Using those results we can write down the coupling of the physical bosons to the fermion currents  $j_L^3$  and  $j_{\text{em}}$  (again, we only need to calculate the  $W'$ -induced corrections at tree-level):

$$\begin{aligned}\mathcal{L}_{\text{NC}} &= ea \cdot j_{\text{em}} + \bar{g} W^3 \cdot j_L^3 + \bar{g}' W'^3 \cdot j_L^3 \\ &\approx eA \cdot j_{\text{em}} \\ &\quad + \bar{g} \left[ 1 - \frac{1}{2} \frac{s^4}{c^2} \frac{2c^2 - \mu}{(c^2 - \mu)^2} \mu \kappa^2 - \frac{s^2}{c^2 - \mu} \mu \kappa \gamma \right] \\ &\quad \times \frac{Z}{c} \cdot \left[ j_L^3 - \frac{ek}{\bar{g}} \left( 1 + \frac{s^2}{c^2 - \mu} \mu \kappa (\gamma - \kappa) \right) j_{\text{em}} \right] \\ &\quad + \bar{g} Z' \cdot \left[ \left( \gamma + \frac{s^2}{c^2 - \mu} \kappa \right) j_L^3 - \frac{ek}{\bar{g}} \left( 1 + \frac{s^2}{c^2 - \mu} \right) \kappa j_{\text{em}} \right]\end{aligned}\tag{19}$$

$$\begin{aligned}&\approx eA \cdot j_{\text{em}} \\ &\quad + \bar{g}_0 (1 + \zeta) \frac{Z}{c_0} \cdot \left[ j_L^3 - s_0^2 (1 + \delta_Z) j_{\text{em}} \right] \\ &\quad + \bar{g} Z' \cdot \left[ \sqrt{\frac{-\zeta}{\mu}} (j_L^3 - s^2 j_{\text{em}}) - s^2 (\kappa - \gamma) j_{\text{em}} \right].\end{aligned}\tag{20}$$

From the coupling of the  $Z$  in (20), we see that corrections to observables measured at the  $Z$  pole are summarized by the quantities  $\delta_Z$  and  $\zeta$ , which are given by

$$\delta_Z = \left. \frac{\delta \sin^2 \theta}{\sin^2 \theta} \right|_{Z \text{ pole}} = -\frac{c^2}{c^2 - s^2} \left[ \frac{s^2}{c^2 - \mu} \mu \kappa^2 + \frac{1 - (1 + 2s^2)\mu}{c^2 - \mu} \kappa \gamma - \mu \gamma^2 \right],\tag{21}$$

$$\zeta = -\mu \left( \gamma + \frac{s^2}{c^2 - \mu} \kappa \right)^2.\tag{22}$$

$\delta_Z$  is the fractional deviation of  $\sin^2 \theta$  that is measured by the  $Z$ -pole asymmetries, and  $\zeta$  ( $\leq 0$ ) is the fractional deviation of the coupling of the  $Z$  to  $j_L^3$ . In deriving Eqs. (20)–(22) we have used the results for  $\delta M_W$ ,  $\delta \bar{g}$  and  $\delta k$  given in Eq. (18).

### 4.3 Neutral-Current Interactions at Low Energy

Experimental constraints on the effective value of  $\sin^2 \theta$  measured at low energy, via neutrino scattering and atomic parity violation, no longer match the precision of measurements at high energy which constrain  $\delta_Z$ ,  $\zeta$  and  $M_W$ , and thus will not be part of our constraint analysis, which will be the subject of the next Section. Nevertheless, such a low-energy measurement of  $\sin^2 \theta$  can in principle have different sensitivity to  $Z'$  bosons, and so we conclude this Section by presenting the correction to the low-energy value of  $\sin^2 \theta$  that would be measured in this model. We can use the results for  $\mathcal{L}_{\text{NC}}$  to calculate the neutral-current matrix element,  $\mathcal{M}_{\text{NC}}$ , at zero momentum transfer. To establish notation, we first mention that in the Standard Model,  $\mathcal{M}_{\text{NC}}$  is given (at tree level) by

$$\mathcal{M}_{\text{NC}}^0(q^2 \approx 0) = \frac{e^2}{q^2} Q \cdot Q' - 4\sqrt{2}G_F(I_3 - s_0^2 Q) \cdot (I'_3 - s_0^2 Q') , \quad (23)$$

where  $(I_3, Q)$  and  $(I'_3, Q')$  stand for the matrix elements of the neutral  $\text{SU}(2)_W$  and electromagnetic currents in the external fermionic states. The matrix element including corrections resulting from the  $W'$  isotriplet,  $\mathcal{M}_{\text{NC}}$ , is then given at zero momentum transfer by

$$\begin{aligned} \mathcal{M}_{\text{NC}}(q^2 \approx 0) \approx & \frac{e^2}{q^2} Q \cdot Q' \\ & - 4\sqrt{2}G_F \left[ (I_3 - s_0^2(1 + \delta_0)Q) \cdot (I'_3 - s_0^2(1 + \delta_0)Q') \right. \\ & \left. + (2\delta_0 + \mu(\kappa - \gamma)^2) s^2 Q \cdot s^2 Q' \right] . \end{aligned} \quad (24)$$

Here  $\delta_0$  is the  $q^2 = 0$  analog of  $\delta_Z$ :

$$\delta_0 = \left. \frac{\delta \sin^2 \theta}{\sin^2 \theta} \right|_{q^2 \approx 0} = \delta_Z + \mu(\kappa - \gamma) \left( \gamma + \frac{s^2}{c^2 - \mu} \kappa \right) . \quad (25)$$

The correction to the coefficient of  $Q \cdot Q'$  is unobservable in practice. Note the absence of a correction to the term proportional to  $I_3 \cdot I'_3$  in  $\mathcal{M}_{\text{NC}}$ , i.e.,  $\rho(q^2 = 0)$  is exactly 1 (apart from the usual Standard-Model radiative corrections), which is due to the custodial

symmetry and the constraint on  $G_F$  measured in low-energy charged-current interactions. This removes some of the possible sensitivity to  $Z'$  bosons.

We see that corrections to the low-energy theory due to the  $W'$  enter through 4 independent functions of the  $W'$  parameters:  $\delta M_W/M_W$ ,  $\zeta$ ,  $\delta_Z$ , and  $\delta_0$ . Again it should be noted that all corrections to Standard-Model predictions vanish if any two of the  $W'$  parameters  $\kappa$ ,  $\gamma$  and  $\mu$  vanish. As mentioned, the first three of these functions,  $\delta M_W/M_W$ ,  $\zeta$  and  $\delta_Z$ , are measured with much greater accuracy than the last,  $\delta_0$ . We therefore ignore  $\delta_0$  in our constraint analysis, which we now present.

## 5 Summary of Corrections in Terms of S, T and U

The above corrections to Standard-Model predictions which result from adding an isotriplet of  $W'$  bosons can be conveniently summarized by the  $S$ ,  $T$  and  $U$  parameters introduced by Peskin and Takeuchi [6, 7]. At a fundamental level,  $S$ ,  $T$  and  $U$  are defined to measure oblique corrections to Standard-Model predictions, i.e., corrections due to non-Standard particles appearing in vacuum polarization graphs for the photon,  $W$  and  $Z$ . However, at a practical level,  $S$ ,  $T$  and  $U$  simply parameterize corrections to the three Standard-Model quantities that are measured with high precision (putting aside  $\alpha$ ,  $G_F$  and  $M_Z$ , which are fixed):  $M_W$ , the coupling of the  $Z$  to  $j_L^3$ , and  $\sin^2 \theta$  measured at the  $Z$  pole. Therefore, although corrections due to the  $W'$  are in general nonoblique, by comparing the corrections to these three quantities due to the  $W'$  with their expressions in terms of  $S$ ,  $T$  and  $U$ , we can find the effective contributions of the  $W'$  to  $S$ ,  $T$  and  $U$ .

Contributions to  $\delta M_W/M_W$ ,  $\delta_Z$  and  $\zeta$  are given in terms of  $S$ ,  $T$  and  $U$  as [6],

$$\begin{aligned}\frac{\delta M_W}{M_W} &= \frac{1}{2} \frac{\alpha}{c^2 - s^2} \left[ -\frac{1}{2} S + c^2 T + \frac{c^2 - s^2}{4s^2} U \right] , \\ \delta_Z &= \frac{\alpha}{c^2 - s^2} \left[ \frac{1}{4s^2} S - c^2 T \right] , \\ \zeta &= \alpha T .\end{aligned}\tag{26}$$

Equating these expressions with the corresponding expressions for these quantities in terms of the  $W'$  parameters, Eqs. (18) and (22), we obtain the contributions of the  $W'$  to  $S$ ,  $T$  and  $U$ :

$$\begin{aligned}\frac{\alpha}{4c^2} S &= -(1 - \mu) \tilde{\kappa} (\mu \tilde{\kappa} + \gamma) , \\ \alpha T &= -\mu (\tilde{\kappa} + \gamma)^2 ,\end{aligned}$$

$$\frac{\alpha}{4s^2}U = \mu(\mu\tilde{\kappa}^2 + 2\tilde{\kappa}\gamma + \gamma^2) . \quad (27)$$

Here we have introduced  $\tilde{\kappa} \equiv s^2\kappa/(c^2 - \mu)$ , in terms of which the contributions of the  $W'$  to  $S$ ,  $T$  and  $U$  can be expressed very concisely. We denote the contribution of the  $W'$  to  $T$  as  $T'$  because there is another important contribution to  $T$ ,  $T_{\text{top}}$ , due to the top quark. Using these expressions, limits on  $S$ ,  $T$  and  $U$  can be converted into limits on the mass and couplings of the  $W'$ . However, the contribution to  $T$  from the top quark must first be removed, as we now describe.

## 5.1 The Likelihood Function

The electroweak constraints on  $S$ ,  $T$  and  $U$  are combined [6] by first constructing  $\chi_0^2$ :

$$\chi_0^2(S, T, U) = \sum_i \left[ \frac{x_i(S, T, U) - x_i^{\text{exp}}}{\sigma_i} \right]^2 . \quad (28)$$

Here the  $x_i$  are electroweak observables, namely  $M_W$ , the  $Z$  width,<sup>2</sup> and  $\sin^2 \theta(M_Z)$ , as shown in Table 1. The  $x_i(S, T, U)$  are the theoretical predictions for these observables obtained by adding the oblique corrections linear in  $S$ ,  $T$  and  $U$  to Standard-Model predictions;  $x_i^{\text{exp}}$  are the experimental values; and  $\sigma_i$  are the experimental errors. All these are shown in Table 1. The theoretical values are given for a 1000 GeV Higgs with Standard-Model couplings. Of course we do not know the mass of the Higgs in the SCSM, though it should be of order the weak scale. Further, unlike the Standard-Model Higgs, the couplings of the SCSM Higgs to the  $W$  bosons are unspecified. However, this represents a small uncertainty in the predictions of the SCSM which does not alter our basic conclusions.

As mentioned above, there is a contribution from the top quark to  $T$ , which is quadratic in  $m_t$  and can be sizable if the top is heavy. (There are also small contributions from the top which are only logarithmic in  $m_t$ , which can safely be neglected.) In the absence of information about the top mass, an arbitrarily negative value of  $T'$  could be canceled by an opposite, positive value of  $T_{\text{top}}$  due to a heavy top. In this case, bounds on  $T$  would tell us nothing about  $T'$ , and only the bounds on  $S$  and  $U$  would constrain the  $W'$  parameters.

Of course, we now have information about the top mass from CDF, and we use the result of their fit:  $m_t = 174 \pm 16$  GeV [8]. However, the error in this measurement is not negligible, which can be seen by noting that an  $S$ - $T$ - $U$  analysis of the Standard Model predicts the

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<sup>2</sup>In particular, we use the measurement of the leptonic width of the  $Z$ , which is free of the theoretical uncertainties from  $\alpha_s$  and  $\Gamma_b(Z)$  that plague the hadronic component of the  $Z$  width.

top mass with comparable uncertainty. To incorporate the CDF result for  $m_t$ , including the error, we convert it to a measurement of  $T_{\text{top}}$ :  $T_{\text{top}} = T_{\text{top}}^0 \pm \delta T_{\text{top}}$ . We then add a term to  $\chi_0^2$ ,

$$\chi_t^2(S, T', U; T_{\text{top}}) = \chi_0^2(S, T' + T_{\text{top}}, U) + \left( \frac{T_{\text{top}} - T_{\text{top}}^0}{\delta T_{\text{top}}} \right)^2. \quad (29)$$

This leads to a likelihood function  $L_t(S, T', U; T_{\text{top}}) \equiv N_t \exp[-\chi_t^2/2]$ . Since we are here interested in the  $W'$  parameters, and not  $m_t$ , we integrate over  $T_{\text{top}}$  to find a likelihood function of  $S$ ,  $T'$  and  $U$  alone:

$$L(S, T', U) = \int dT_{\text{top}} L_t(S, T', U; T_{\text{top}}) \equiv N \exp[-\chi^2(S, T', U)/2]. \quad (30)$$

In  $L(S, T', U)$  the only unknowns are  $\mu$ ,  $\kappa$ , and  $\gamma$ , i.e., the  $W'$  mass and couplings. We will now exploit this likelihood function to constrain these parameters.

## 6 Constraints on a $W'$ in the SCSM

### 6.1 Bounds on $\kappa\gamma$

From the expressions in Eq. (27) for  $S$ ,  $T'$  and  $U$  in terms of  $\kappa$ ,  $\gamma$  and  $\mu$ , we can derive bounds on the product  $\tilde{\kappa}\gamma$ . We first express the product  $\tilde{\kappa}\gamma$  as

$$\tilde{\kappa}\gamma = -\frac{1}{1-\mu} \frac{\alpha}{4c^2} S - \mu \tilde{\kappa}^2. \quad (31)$$

Using  $\alpha(T' + U/4s^2) = -\mu(1-\mu)\tilde{\kappa}^2$ , we find

$$\kappa\gamma = \frac{c^2 - \mu}{1 - \mu} \frac{\alpha}{s^2} \left( -\frac{S}{4c^2} + T' + \frac{U}{4s^2} \right). \quad (32)$$

Then because  $(c^2 - \mu)/(1 - \mu)$  is at most  $c^2$ ,  $\kappa\gamma$  is bounded as

$$\frac{\alpha c^2}{s^2} \left( -\frac{S}{4c^2} + T' + \frac{U}{4s^2} \right)_{\min} < \kappa\gamma < -\frac{\alpha c^2}{s^2} S_{\min}, \quad (33)$$

where in deriving the upper bound we used  $\alpha(T' + U/4s^2) = -\mu(1-\mu)\tilde{\kappa}^2 < 0$ . Here  $S_{\min}$  refers to the smallest (nonpositive) allowed value of  $S$ . From the 95% Confidence Level (CL) bounds on  $S$  and on  $(-S/4c^2 + T' + U/4s^2)$ , obtained from the likelihood function in Eq. (30), we find that

$$-0.049 < \kappa\gamma < 0.0055 \quad (95\% \text{ CL}). \quad (34)$$



## 6.2 Allowed Region of $W'$ Mass and Couplings

Our remaining results are obtained by exploring the volume of  $\kappa$ - $\gamma$ - $\mu$  space allowed by the likelihood function (30). Specifically, we consider points  $(\kappa, \gamma, \mu)$  for which  $(S, T', U)$  falls inside an  $S$ - $T'$ - $U$  ellipsoid defined by the value of  $\chi^2$  corresponding to 95% CL. This maximum allowed value,  $\chi_{\max}^2$ , depends on the number of degrees of freedom being constrained: for just one degree of freedom,  $\chi_{\max}^2 = 4$ , which corresponds to two standard deviations, while for two degrees of freedom,  $\chi_{\max}^2 \simeq 6.18$ .

A numerical search of the boundary of the allowed region of  $(\kappa, \gamma, \mu)$ , defined by  $\chi^2 \leq 4$ , shows that  $\kappa\gamma$  is bounded as

$$-0.028 < \kappa\gamma < 0.0052 \quad (95\% \text{ CL}) . \quad (35)$$

Hence we can state, with a confidence level of 95%, that the  $W$  boson must saturate the isovector electromagnetic form factor to within 3%.

Figures 2 and 3 correspond to slices of the allowed region of  $(\kappa, \gamma, \mu)$  at fixed  $\mu$  and  $\gamma$  respectively. In Fig. 2 we show the allowed regions ( $\chi_{\max}^2 \leq 6.18$ ) of  $(\kappa, \gamma)$  for  $M_{W'} = 150$  and 400 GeV. These regions necessarily lie inside the hyperbolic bounds (solid lines) which correspond to the extreme values of  $\kappa\gamma$  allowed for this value of  $\chi_{\max}^2$ . Note that most of each allowed region corresponds to both  $\kappa$  and  $\gamma$  much smaller than one. However, when either  $\kappa$  or  $\gamma$  is extremely small, the other can become large, particularly when the  $W'$  is heavy. This is expected since, as was pointed out in Section 3.2, if any two of  $\kappa$ ,  $\gamma$  and  $\mu$  vanish, the remaining quantity is unconstrained.

In Figure 3 we show the allowed range of  $\kappa$  ( $\chi^2 \leq 4$ ) as a function of  $M_W/M_{W'}$  for  $\gamma = 1, 1/2, 1/3$  and  $1/4$ . Here we exploit the symmetry noted after Eq. (9) — namely invariance under simultaneous change of sign of  $\gamma$  and  $\kappa$  — in order to restrict attention to positive  $\gamma$ . From this Figure we conclude that for reasonable values of  $\gamma$ ,  $\kappa$  must be extremely small; in other words the  $W'$  must mix much more weakly with the photon than the  $W$ . In particular, for  $\bar{g}'/\bar{g} > 1/4$ ,  $\kappa < 0.025$ .

In Figure 4 we show the maximum value of  $\bar{g}'$  allowed by our constraints ( $\chi^2 \leq 4$ ) as a function of  $M_{W'}$ , and we compare our constraints with those obtained in the direct  $W'$  search at CDF [14]. Note that our bounds are more restrictive than the CDF bounds. Further, the CDF analysis assumes that the  $W'$  decays entirely into left-handed fermions, whereas in the SCSM a  $W'$  will primarily decay into  $WZ$  for  $M_{W'}$  above the decay threshold,  $M_W + M_Z$ . Hence the CDF bounds do not help us bound  $\gamma$  in the SCSM. Our constraint analysis indicates that at a moderate value of  $M_{W'}$  such as 300 GeV, the  $W'$  coupling to fermions must be less than a quarter of the  $W$  coupling.

## 7 Conclusion

Comparing our results with the earlier analysis by Korpa and Ryzak [2], we can see how the continually improving electroweak measurements have drastically pared away the allowed parameter space in this model. In their Figure 4 they found  $\kappa\gamma$  allowed to be as large as 0.2, compared to our upper bound of 0.0052. Similarly, for  $\gamma = 1$  they found that the  $W'$  could be as light as 170 GeV and  $\kappa$  could be as large as 0.13, while for  $\gamma = 1$  we now find that the  $W'$  must be heavier than 1075 GeV and  $\kappa$  can be at most 0.006. Moreover, our much more restrictive bounds hold at 95% CL, while the earlier bounds held only at 68% CL.

Korpa and Ryzak concluded from their analysis that there was plenty of room for the non-Standard particle content predicted by the SCSM. From our analysis of the SCSM exploiting recent electroweak data, we conclude that the currently allowed parameter space is so small as to strongly argue against the model. There is no reason to expect vector dominance to hold at a level of 3%. Nor can we understand how the  $W'$  could mix with the photon only 1/40 as much as the  $W$  mixes; yet we have found that this would have to be the case even if the coupling of the  $W'$  to the left-handed fermions is allowed to be as small as 1/4 the  $W$  coupling (itself already small for a strongly-coupled theory).

It is possible that by including more resonances in our analysis we could find regions in the enlarged parameter space where the various corrections to Standard-Model predictions cancel, without forcing the masses and couplings of the resonances to be unnaturally small. But from our analysis it is clear that these cancellations would have to be rather delicate, and the agreement of the SCSM with experiment would be just as inexplicable.

Of course, we can never completely exclude the SCSM solely on the basis of experimental constraints. The strongly-coupled dynamics underlying the effective theory do not allow us to find predictions for masses and couplings which could be contradicted by experiment. However, the model offers no natural understanding of how it could continue to evade detection, disguised as the spontaneously-broken Standard Model. For this reason, we conclude that unless there emerges from a study of the strong dynamics an explanation of how it could be so nearly indistinguishable from the Standard Model, the Strongly-Coupled Standard Model can no longer be viewed as a possible theory of the electroweak interactions.

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## Figure Captions

- Figure 1. Diagrammatic expansion of the isovector electromagnetic form factor including the direct-coupling graph and the  $W$ - and  $W'$ -pole graphs.
- Figure 2. Contours bounding the regions of the  $\kappa$ - $\gamma$  plane allowed at 95% CL for  $M_{W'} = 150$  GeV (dashes) and 400 GeV (dots). The solid lines are hyperbolas defined by the bound on  $\kappa\gamma$  at the same value of  $\chi^2$ .
- Figure 3. Contours bounding the allowed region of the  $\kappa$ - $M_W/M_{W'}$  plane for  $\gamma = 1$  (solid),  $\gamma = 1/2$  (dashes),  $\gamma = 1/3$  (dotdash) and  $\gamma = 1/4$  (dots).
- Figure 4. Bounds on  $\gamma$  (95% CL) for a range of  $M_{W'}$ . The dotted curve interpolates through the bounds obtained in the direct  $W'$  search at CDF [14] while the solid curve shows the bounds obtained in our analysis of electroweak constraints applied to the SCSM.

## Table Caption

- Table 1. Theoretical and measured values of the electroweak observables used to constrain the  $W'$  couplings. The theoretical values correspond to vanishing  $W'$  couplings, a top mass of 174 GeV, and a Higgs of mass 1000 GeV (with Standard-Model couplings).

**Table 1**

| Observable                             | Theoretical Value | Measured Value       | Experiment               |
|--|-------------------|----------------------|--------------------------|
| $M_W$                                  | 80.23 GeV         | $80.23 \pm 0.18$ GeV | CDF and D0 [9]; UA2 [10] |
| $\Gamma(Z \rightarrow \text{leptons})$ | 83.68 MeV         | $83.96 \pm 0.18$ MeV | LEP [11]                 |
| $\sin^2 \theta(M_Z)$                   | 0.2331            | $0.2317 \pm 0.0004$  | LEP [12]; SLD [13]       |

Figure 1.

The diagram shows an equation between four Feynman diagrams. The leftmost diagram consists of a vertical line with an upward arrow, a black dot, and a wavy line labeled  $a$  extending to the right. This is followed by an equals sign. The right side of the equation has three terms separated by plus signs. The first term is a vertical line with an upward arrow and a wavy line labeled  $a$  extending to the right. The second term is a vertical line with an upward arrow, a wavy line labeled  $W^3$  extending to the right, a black dot labeled  $k$ , and another wavy line labeled  $a$  extending to the right. The third term is a vertical line with an upward arrow, a wavy line labeled  $W'^3$  extending to the right, a black dot labeled  $k'$ , and another wavy line labeled  $a$  extending to the right.

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}$$

Figure 2.

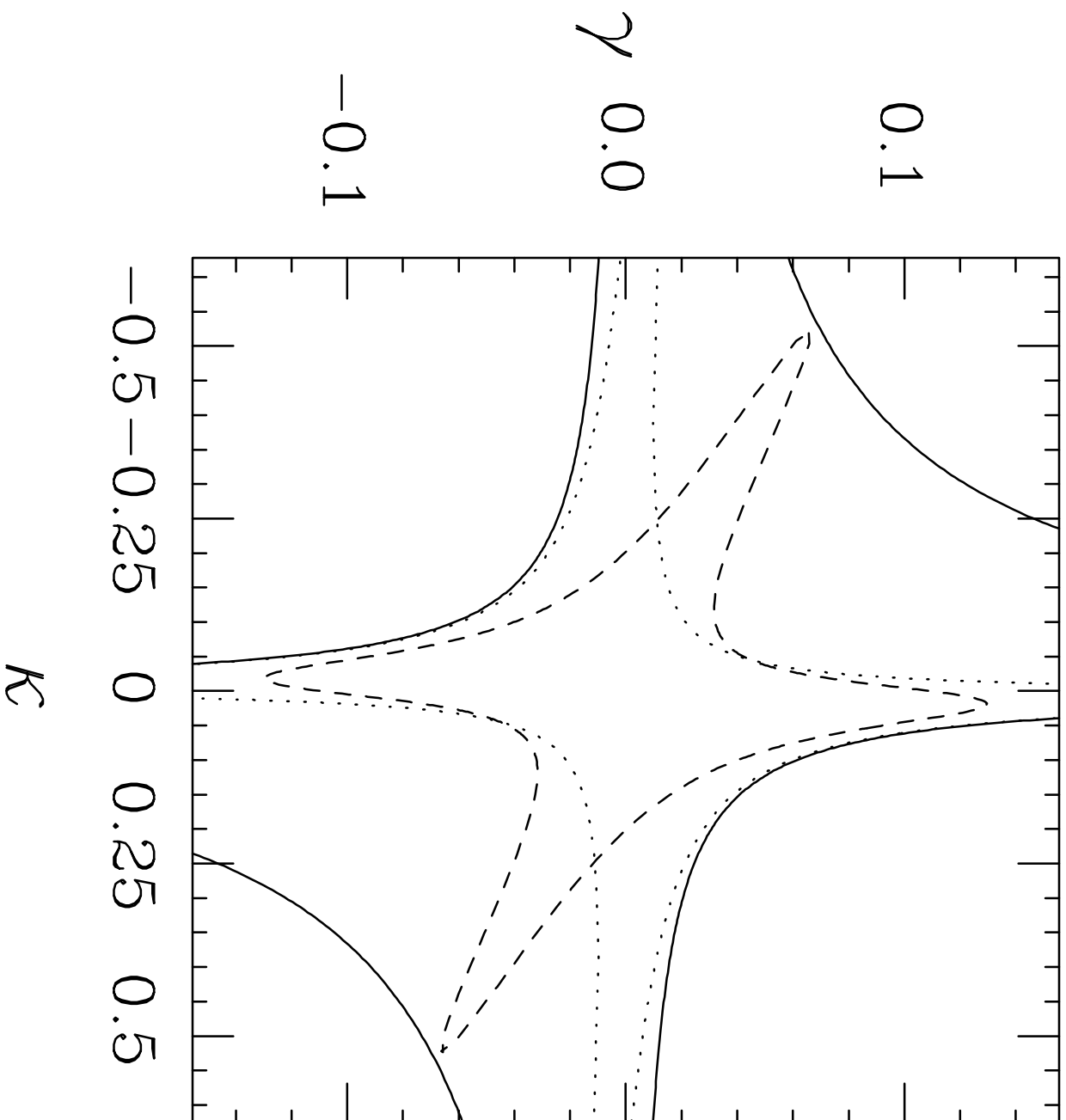


Figure 3.

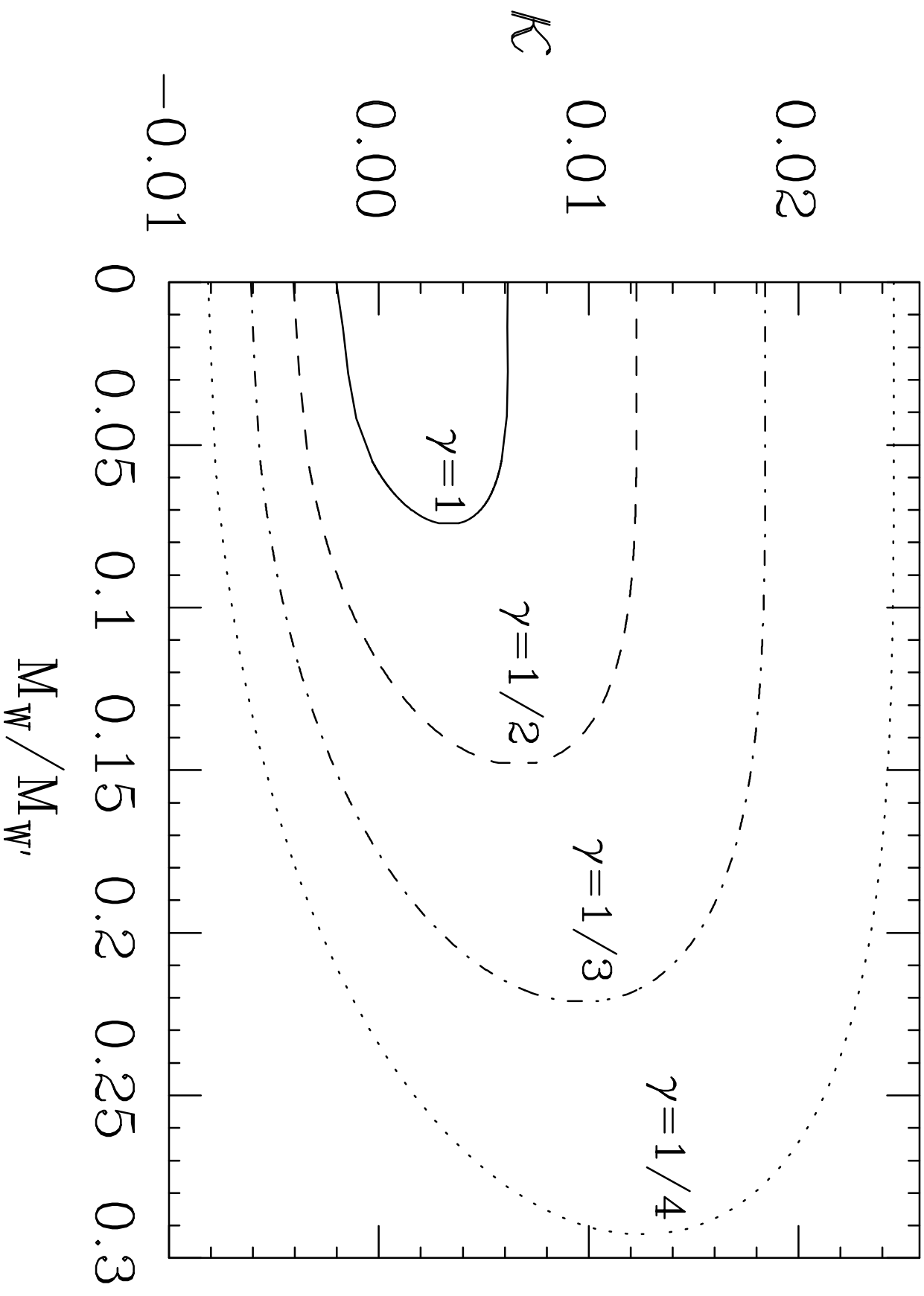




Figure 4.

