

# ENHANCEMENT OF CP VIOLATION IN $B^\pm \rightarrow K_i^\pm D^0$ BY RESONANT EFFECTS

*David Atwood*

*Stanford Linear Accelerator Center, Stanford University  
Stanford, CA 94309, USA*

*Gad Eilam<sup>1</sup>*

*Department of Physics, Brookhaven National Laboratory  
Upton, NY 11973, USA*

*Michael Gronau<sup>1</sup>*

*Theoretical Physics Division, CERN  
CH-1211 Geneva 23, Switzerland*

and

*Amarjit Soni*

*Department of Physics, Brookhaven National Laboratory  
Upton, NY 11973, USA*

## ABSTRACT

Resonance width effects in charged  $B$  decays to neutral  $D$  mesons and excited kaon states  $K_i$  around 1400 MeV are shown to lead to large calculable final state phases.  $CP$  asymmetries are defined for any charged  $B$  decay to three pseudoscalar mesons involving intermediate overlapping resonance states. Asymmetries up to about 10% are found in  $B^+ \rightarrow K_i^+ D^0 \rightarrow (K\pi)^+ D^0$ . Decay distributions can be used to determine the weak phase  $\gamma$  of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. No separation of the contributions from individual resonances is required.

CERN-TH.7428/94  
September 1994

---

<sup>1</sup> Permanent address: Department of Physics, Technion – Israel Institute of Technology, Haifa 32000, Israel

In order to observe CP violating effects in a physical process both a CP violating weak phase and a CP conserving strong phase must be present [1]. In the standard model the weak phase is provided by phases of the CKM matrix elements [2] while the strong phase must be supplied by the physical system in question. In the case of neutral  $B$  decays the latter phase is due to the well known oscillation effects. In order to obtain similar CP violating effects in  $B^\pm$  decay, some other mechanism must be supplied to provide the strong phase. No evidence for final state phases in  $B$  decays has been found yet [3][4]. It is possible that these phases are small in two body decays because of the heavy  $B$  meson mass. In this Letter we shall examine a class of decays of charged  $B$  mesons to hadronic quasi two body final states where the strong phase is provided by resonant effects in the final state. In previous papers this approach has been considered in the case of radiative  $B$  decays [5].

The processes we wish to study here are of a class suggested some time ago for a measurement of the weak phase  $\gamma$  [6], one of the angles of the CKM matrix unitarity triangle [1]. It was shown that this angle may be determined through the rate measurements of the following processes and their charge-conjugates:

$$B^+ \rightarrow K^+ \bar{D}^0, \quad B^+ \rightarrow K^+ D^0, \quad B^+ \rightarrow K^+ D_{1,2}^0. \quad (1)$$

$\bar{D}^0, D^0$  are the two flavor states, identified for instance by the lepton charge of their semileptonic decays, and  $D_{1,2}^0 = (D^0 \pm \bar{D}^0)/\sqrt{2}$  are the two CP-eigenstates identified by decay modes such as  $K^+ K^-, K_s \pi^0$ . In the next paragraph we explain briefly the method, drawing attention to the role played by final state phases.

The amplitudes of the first two processes in (1) are governed by CKM factors  $V_{cb}^* V_{us}, V_{ub}^* V_{cs}$ . The weak phase difference between them is  $\gamma$ . When these amplitudes acquire different final state phases, the amplitude of the third process, which is their coherent sum or difference, is expected to show a CP asymmetry with respect to its charge-conjugate. The angle  $\gamma$  is determined from the shape of the two triangles formed by the magnitudes of the amplitudes of the three processes in (1) and their charge-conjugates. Although in principle  $\gamma$  can be determined even if the above final state phases were equal and no CP asymmetry were observed [6], it would be of great importance to measure a nonzero asymmetry. Also, the potential accuracy to which  $\gamma$  is determined in this way increases with a growing final state phases difference [7]. For instance, a small final state phase difference would inhibit a useful determination of  $\gamma$  if this angle were also small or near  $180^\circ$ . This would correspond to skinny triangles for which the determination of  $\gamma$  becomes quite challenging. The final state phases are basically unknown and could be too small for giving rise to an observable asymmetry in  $B \rightarrow K D_{1,2}^0$ . In this case it would be useful to find other related decay channels in which final state phases are enhanced. Here we will show that when the  $K$  meson is replaced by kaonic resonances, large calculable final state phases are expected

to occur. This will not only improve the prospects of a precise measurement of  $\gamma$ , but can potentially also lead to sizable CP asymmetries.

We are thus led to study the process

$$B^+ \rightarrow K_i^+ \bar{D}^0, \quad B^+ \rightarrow K_i^+ D^0, \quad B^+ \rightarrow K_i^+ D_{1,2}^0, \quad (2)$$

in which  $K_i$  ( $i = a, b, \dots e$ ) are the five lowest lying resonances above the  $K^*(892)$ , the properties of which [8] are listed in Table 1. Since a strong phase difference is required to obtain an asymmetry, and since the dominant effect that we will be focussing on in this work results from resonance widths, at least two such resonances must decay to a common final state. The final states in Table 1 are  $f = K\pi, K^*\pi, K\rho$ .

Consider for illustration the final state  $f_1 = K\pi$ , to which three of the resonances decay, and let us study the process  $B \rightarrow K_i D \rightarrow K\pi D$  ( $i = c, d, e$ ). We define  $s = (p_K + p_\pi)^2 = (p_B - p_D)^2$ , and denote by  $\theta$  the angle between the  $B$  momentum and the  $K$  momentum in the  $K_i$  rest frame. The amplitudes of the two processes involving  $\bar{D}^0, D^0$  in the final state are  $\bar{A}_i^{f_1}(s, \theta), A_i^{f_1}(s, \theta)$ , respectively. They are proportional to the  $B^+ \rightarrow K_i^+ \bar{D}^0, B^+ \rightarrow K_i^+ D^0$  weak decay amplitudes:  $A(B^+ \rightarrow K_i^+ \bar{D}^0) = \bar{a}_i, A(B^+ \rightarrow K_i^+ D^0) = a_i e^{i\gamma}$ , respectively.  $\bar{a}_i$  and  $a_i$  are assumed to involve small final state phases which will be neglected. It should be noted that whereas large final state phases were measured in two body  $D$  Decays [9], and sizable phases are required to account for certain quasi two body  $D$  decays [10], such phases are expected to be smaller in decays of the heavier  $B$  meson. We will show later on how this assumption can be tested experimentally without having to separate the various resonance contributions. We note that such phases do not affect the considerations by which  $\gamma$  is determined. Their effect on the calculated CP asymmetries would be the appearance of cosines of these phase differences multiplying the respective interfering resonant amplitudes.

The two amplitudes,  $\bar{A}_i^{f_1}(s, \theta), A_i^{f_1}(s, \theta)$ , involve a common  $s$ -dependence, characterized by the resonance mass and width. We will assume a (normalized) Breit-Wigner form:

$$\Pi_i(s) = \frac{\sqrt{m_i \Gamma_i / \pi}}{s - m_i^2 + im_i \Gamma_i}. \quad (3)$$

The (normalized)  $K_i$  decay amplitudes are given by a spin- characteristic  $\theta$  dependence,  $\Theta_i^{f_1}(z)$  ( $z \equiv \cos \theta$ ), multiplied by the square root of the corresponding decay branching ratios appearing in Table 1,  $\sqrt{B_i^{f_1}}$ :

$$\begin{aligned} \Theta_c^{f_1} &= \sqrt{\frac{3}{2}}z, & \Theta_d^{f_1} &= \frac{1}{2}\sqrt{\frac{5}{2}}(3z^2 - 1), & \Theta_e^{f_1} &= \sqrt{\frac{1}{2}}; \\ B_c^{f_1} &= 0.07, & B_d^{f_1} &= 0.50, & B_e^{f_1} &= 0.93. \end{aligned} \quad (4)$$

The resulting amplitudes, obtained in a narrow width approximation, are:

$$\begin{aligned}
\bar{A}_i^{f_1}(s, z) &\equiv A(B^+ \rightarrow K_i^+ \bar{D}^0 \rightarrow (K\pi)^+ \bar{D}^0) = \bar{a}_i \sqrt{B_i^{f_1}} \Pi_i(s) \Theta_i^{f_1}(z), \\
A_i^{f_1}(s, z) &\equiv A(B^+ \rightarrow K_i^+ D^0 \rightarrow (K\pi)^+ D^0) = e^{i\gamma} a_i \sqrt{B_i^{f_1}} \Pi_i(s) \Theta_i^{f_1}(z). \quad (5)
\end{aligned}$$

The amplitude involving the  $CP$ -eigenstate  $D_1^0$  in the final state is

$$A_{(1)i}^{f_1}(s, z) = \frac{1}{\sqrt{2}} [\bar{A}_i^{f_1}(s, z) + A_i^{f_1}(s, z)]. \quad (6)$$

(Of course, a similar expression applies to  $D_2^0$ ). The total amplitudes for  $K\pi$  with invariant mass in the region of the three overlapping resonances is given by a coherent sum of the amplitudes through the separate resonances:

$$\bar{A}^{f_1} = \sum_i \bar{A}_i^{f_1}, \quad A^{f_1} = \sum_i A_i^{f_1}, \quad A_{(1)}^{f_1} = \frac{1}{\sqrt{2}} (\bar{A}^{f_1} + A^{f_1}). \quad (7)$$

We are neglecting a possible nonresonant  $K\pi D$  term, the contribution of which in the relatively narrow resonance region is expected to be suppressed by the ratio of the resonance widths to  $m(B) - [m(D) + m(K) + m(\pi)]$ . Note that the first two amplitudes in (7) have well specified weak phases, their difference being the angle  $\gamma$ .

In principle, the measured differential decay distributions,  $d^2\bar{\Gamma}^{f_1}/dsdz$ ,  $d^2\Gamma^{f_1}/dsdz$ ,  $d^2\Gamma_{(1)}^{f_1}/dsdz$ , given by the square magnitudes of the amplitudes in (7), may be used to extract the three separate resonance contributions in a partial wave analysis. This would have simplified the theoretical study considerably. However, this scenario may be experimentally difficult due to the limitations of statistics and due to the large overlap among the resonances. We will therefore base our discussion on more realistic considerations, in which only the combined resonance decay distributions are assumed to be measurable.

The method of measuring  $\gamma$  as described in [6] can now be applied in a differential manner. The third relation of (7) can be described as a triangle in the complex plane, of which the three sides represent the three amplitudes. A similar triangle describes the amplitudes of the charge-conjugated  $B^-$  decays, in which only the sign of the weak phase  $\gamma$  has been changed. The lengths of the sides of the two triangles are given by the square roots of the differential rates,  $\sqrt{d^2\bar{\Gamma}^{f_1}/dsdz}$ ,  $\sqrt{d^2\Gamma^{f_1}/dsdz}$ ,  $\sqrt{d^2\Gamma_{(1)}^{f_1}/dsdz}$  and by the charge-conjugate distributions. This determines the shape of the two triangles for a given value of  $s$  and  $\theta$ . The angle between the sides representing  $A^{f_1}$  and its charge conjugate is  $2\gamma$ . The measured decay distributions for  $B^+$  and  $B^-$  decays provide a multitude of different pairs of triangles, all of which share the common angle  $2\gamma$ . The statistical power of this method of determining  $\gamma$  (leaving out questions of branching ratios to which we come later) is comparable to the one based on  $B \rightarrow KD$ , in which a single pair of triangles is determined from integrated rates. The advantage of excited kaon

resonances would be that large final state phase differences between  $\bar{A}^{f_1}$  and  $A^{f_1}$  occur in the resonance region. As mentioned in the introduction, this has the effect of improving the precision to which  $\gamma$  can be determined.

Let us proceed to calculate a CP asymmetry. Such an asymmetry is expected to occur between  $d^2\Gamma_{(1)}^{f_1}/dsdz = |A_{(1)}^{f_1}|^2$  and its charge-conjugate,  $d^2\bar{\Gamma}_{(1)}^{f_1}/dsdz$ , due to the two interfering amplitudes,  $\bar{A}^{f_1}$  and  $A^{f_1}$ , which have different weak phases and different strong phases in the resonance region. The method developed below can be applied to calculate CP asymmetries in any three body hadronic  $B$  decay, in which two of the final particles are decay products of overlapping resonances.

From (5)(7) we obtain:

$$\Delta \equiv \frac{d^2(\Gamma_{(1)}^{f_1} - \bar{\Gamma}_{(1)}^{f_1})}{dsdz} = 2 \sum_{i,j} (\bar{a}_i a_j - \bar{a}_j a_i) (B_i^{f_1} B_j^{f_1})^{1/2} \Theta_i^{f_1} \Theta_j^{f_1} \text{Im}(\Pi_i \Pi_j^*) \sin \gamma. \quad (8)$$

For every pair of resonances this expression has the usual form,  $2A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$ , namely a product of decay amplitudes, a sine of final state phase difference (here given by  $\text{Im}(\Pi_i \Pi_j^*)$ ) and a sine of the weak phase difference ( $\gamma$ ). Allowing for final state phases in the weak amplitudes would have introduced cosines of these phase-differences multiplying the respective products of amplitudes. Another term, involving the sines of these phases, multiplies the separate Breit-Wigner rates and integrates to zero in the asymmetries as defined below.

Whereas the  $B^+ B^-$  partial rate-difference contains only the imaginary part of interfering resonances, the sum contains both the decay rates through resonances and the real part of their interference:

$$\begin{aligned} \Sigma \equiv \frac{d^2(\Gamma_{(1)}^{f_1} + \bar{\Gamma}_{(1)}^{f_1})}{dsdz} &= \sum_i (\bar{a}_i^2 + a_i^2 + 2\bar{a}_i a_i \cos \gamma) B_i^{f_1} (\Theta_i^{f_1})^2 |\Pi_i|^2 \\ &+ 2 \sum_{i,j} [\bar{a}_i \bar{a}_j + a_i a_j + (\bar{a}_i a_j + \bar{a}_j a_i) \cos \gamma] (B_i^{f_1} B_j^{f_1})^{1/2} \Theta_i^{f_1} \Theta_j^{f_1} \text{Re}(\Pi_i \Pi_j^*). \end{aligned} \quad (9)$$

The decay distributions (8)(9) have a clearly distinguishable  $s$ -dependence. For simplicity, let us neglect the small mass-differences among the three relevant resonances and denote the common mass by  $m$ . We then find:

$$\begin{aligned} \text{Im}(\Pi_i \Pi_j^*) &\approx \frac{m^2 \sqrt{\Gamma_i \Gamma_j} (\Gamma_j - \Gamma_i)}{\pi} \frac{s - m^2}{[(s - m^2)^2 + m^2 \Gamma_i^2][(s - m^2)^2 + m^2 \Gamma_j^2]}, \\ \text{Re}(\Pi_i \Pi_j^*) &\approx \frac{m \sqrt{\Gamma_i \Gamma_j}}{\pi} \frac{(s - m^2)^2 + m^2 \Gamma_i \Gamma_j}{[(s - m^2)^2 + m^2 \Gamma_i^2][(s - m^2)^2 + m^2 \Gamma_j^2]}. \end{aligned} \quad (10)$$

The difference in the particle-antiparticle differential width is seen to be an odd function of  $(s - m^2)$ , which changes sign at  $s = m^2$ . On the other hand, the sum is an even function and contains an interference term which does

not change sign at  $s = m^2$ . These expressions were obtained when neglecting final state phases in  $\bar{a}_i, a_i$ . It is easy to show that the sum of the rates picks up also an odd term if these amplitudes are allowed to involve final state phases. The absence of such a term in the measured decay distribution can be used to test the assumption of negligible non-resonant strong phases.

Since the rate difference changes sign at  $s = m^2$  and would vanish if integrated symmetrically around the resonance mass, it is useful to define an  $s$ -integrated asymmetry using the sign function,  $\theta(s - m^2) \equiv +1$  for  $s - m^2 > 0$  and  $-1$  for  $s - m^2 < 0$ . In the case under consideration this yields

$$\begin{aligned} \int ds \theta(s - m^2) \text{Im}(\Pi_i \Pi_j^*) &\approx \frac{2}{\pi} \frac{\sqrt{\Gamma_j/\Gamma_i}}{1 + \Gamma_j/\Gamma_i} \ln(\Gamma_j/\Gamma_i) \\ &= \begin{cases} 0.25 & i = c, j = d \\ 0.30 & i = e, j = d \\ 0.07 & i = e, j = c \end{cases} . \end{aligned} \quad (11)$$

We neglected terms of order  $(\Gamma_i/m)^2$  and used the width parameters of Table 1. The above numbers which describe the integrated imaginary part of the overlapping resonances could change somewhat with the resonance parameters but they tend not to be very sensitive to their precise values. *These “imaginary overlaps” and similar overlaps for other pairs of resonances are a key point in any discussion of resonance effects on final state phases.* In fact, as mentioned above, they represent the  $s$ -averaged sines of the final state phase difference corresponding to two interfering resonances. We see that in some cases these phases may be large. Also, since their sign is predicted, the sign of the resulting CP asymmetry will depend only on the relative magnitudes of certain weak amplitudes.

In the case that the two resonances have different masses,  $m_i \neq m_j$ , the corresponding rate difference changes sign at the  $s$ -value given by

$$s_0 = \frac{m_i^2 m_j \Gamma_j - m_j^2 m_i \Gamma_i}{m_j \Gamma_j - m_i \Gamma_i}, \quad (12)$$

then a useful  $s$ -integrated asymmetry will involve the sign function  $\theta(s - s_0)$ .

When integrating the particle-antiparticle asymmetry in angular distribution over all angles  $\theta$  one finds that the partial rate asymmetry vanishes, unless two intermediate resonances with identical quantum numbers contribute to the final state [5]. Thus, the usual  $z$ -integrated asymmetry will project out the interference of resonances with identical quantum numbers. To obtain a nonzero asymmetry when the intermediate resonance states are of different quantum numbers, one may use (similar to the  $s$ -integration) a suitable sign function of  $z$ . Two simple examples of such functions are:

1.  $\theta(z)$ , which projects out the interference of resonances with a unit spin difference.

2.  $\theta(|z| - 1/2)$ , which projects out the interference of spin 0 and spin 2 resonances.

In the case under discussion this yields

$$\begin{aligned}\int dz \Theta_e^{f_1} \Theta_d^{f_1} \theta(z) &= \sqrt{15}/8 = 0.48, \\ \int dz \Theta_e^{f_1} \Theta_d^{f_1} \theta(|z| - 1/2) &= 3\sqrt{5}/8 = 0.84, \\ \int dz \Theta_e^{f_1} \Theta_c^{f_1} \theta(z) &= \sqrt{3}/2 = 0.87.\end{aligned}\tag{13}$$

We now define two CP asymmetries:

$$\begin{aligned}\mathcal{A}_1 &\equiv \frac{\int ds dz \theta(s - m^2) \theta(z) d^2(\Gamma_{(1)}^{f_1} - \bar{\Gamma}_{(1)}^{f_1})/ds dz}{\Gamma_{(1)}^{f_1} + \bar{\Gamma}_{(1)}^{f_1}}, \\ \mathcal{A}_2 &\equiv \frac{\int ds dz \theta(s - m^2) \theta(|z| - 1/2) d^2(\Gamma_{(1)}^{f_1} - \bar{\Gamma}_{(1)}^{f_1})/ds dz}{\Gamma_{(1)}^{f_1} + \bar{\Gamma}_{(1)}^{f_1}},\end{aligned}\tag{14}$$

and calculate them using (8)(9)(11)(13):

$$\begin{aligned}\mathcal{A}_1 &= \frac{[0.045(\bar{a}_c a_d - \bar{a}_d a_c) + 0.031(\bar{a}_e a_c - \bar{a}_c a_e)] \sin \gamma}{\sum_i (\bar{a}_i^2 + a_i^2 + 2\bar{a}_i a_i \cos \gamma) B_i^{f_1}}, \\ \mathcal{A}_2 &= \frac{0.35(\bar{a}_e a_d - \bar{a}_d a_e) \sin \gamma}{\sum_i (\bar{a}_i^2 + a_i^2 + 2\bar{a}_i a_i \cos \gamma) B_i^{f_1}}.\end{aligned}\tag{15}$$

The first asymmetry is suppressed largely due to the small decay branching ratio (7%) of the  $1^-$  resonances to  $K\pi$ . The large numerical coefficient in the second asymmetry demonstrates the power of this method in enhancing final state phases. Note that the numerical coefficients depend only on the  $K\pi$  decay branching ratios of the resonances and on the resonance mass-differences and widths. They do not change by much within the uncertainties in these experimental quantities. The sign of the asymmetries can be determined by which of the amplitudes in the parentheses of the numerator is dominant.

To obtain numerical estimates for these asymmetries one needs to know  $\bar{a}_i$  and  $a_i$ , the weak amplitudes into  $\bar{D}^0 K_i^+$  and  $D^0 K_i^+$ , respectively. In principle, these could be determined experimentally if the three resonance contributions could be separated. Some information is also obtained from measuring the total decay rates into  $\bar{D}^0 K\pi$  and  $D^0 K\pi$  final states in the resonance region:

$$\bar{\Gamma}^{f_1} = \sum_i \bar{a}_i^2 B_i^{f_1}, \quad \Gamma^{f_1} = \sum_i a_i^2 B_i^{f_1}.\tag{16}$$

Theoretical calculations of the exclusive weak amplitudes are of course model-dependent. This is particularly the case for the amplitudes  $a_i$ , which are generally expected to be color-suppressed. That is, they involve  $\bar{b} \rightarrow \bar{u} c \bar{s}$  quark transitions in which the  $\bar{u}c$  system is incorporated into a  $D^0$  while

the  $\bar{s}$  combines with the spectator quark to form the  $K_i^+$ . Although color suppression has been observed in  $\bar{D}^0\pi^0, \bar{D}^0\rho^0$  final states [3], the level of suppression might vary from process to process and is far from being understood. Also, to carry out such a calculation, one would have to assume factorization and to rely on quark model wave functions for the  $K_i$  resonances in order to evaluate their weak decay constants. Two of the kaon resonances under discussion are  $q\bar{q}$   $P$  wave states, for which weak decay constants have not been much investigated. Due to all these uncertainties, we will restrict our study to crude estimates based on an educated guess instead of attempting a model- dependent calculation. A model-calculation with similar results will be presented elsewhere [11].

To estimate  $\bar{a}_c \equiv A(B^+ \rightarrow K_e^+(1^-)\bar{D}^0)$ , we use the measured branching ratio for  $B^+ \rightarrow \rho^+\bar{D}^0$  of 1.35% [3] multiplied by  $\sin^2\theta_c = 0.22^2$ , giving a value  $\bar{a}_c^2 = 6.5 \times 10^{-4}$ . The amplitude of the corresponding  $D^0$  mode,  $a_c$ , is expected to be suppressed both by a color factor, to be taken as 1/3 and by the ratio of CKM factors  $|V_{ub}V_{cs}/V_{cb}V_{us}| = 0.36$ . Correspondingly, we estimate  $a_c^2 = 9.4 \times 10^{-6}$ . The other amplitudes of the  $2^+$  and  $0^+$   $P$ -wave states are harder to estimate. Assuming factorization, the color- allowed contribution to  $K_d^+(2^+)\bar{D}^0$  is forbidden by angular momentum conservation. This state as well as the corresponding one involving a  $D^0$  obtain contributions from color-suppressed amplitudes. Thus, we will assume  $\bar{a}_d^2 \approx a_d^2 = 4 \times 10^{-6}$ , a somewhat smaller value than  $a_c^2$ . The same value will be taken for  $a_e^2$ . The color-allowed amplitude of the  $0^+$  state vanishes in the factorization approximation in a flavor SU(3) symmetry limit in which the  $0^+$  decay constant vanishes. With SU(3) breaking, this decay constant can be estimated to be given by 0.3 times the  $K$  meson decay constant. A corresponding branching ratio of  $\bar{a}_e^2 = 2.4 \times 10^{-5}$  is then obtained from the measured  $BR(B^+ \rightarrow \pi^+\bar{D}^0) = 5.5 \times 10^{-3}$  [3] multiplied by  $\sin^2\theta_c$  and by the SU(3)-breaking factor 0.3<sup>2</sup>.

Thus, we estimate:

$$\begin{aligned}\bar{a}_c^2 &= 6.5 \times 10^{-4}, & a_c^2 &= 9.4 \times 10^{-6}, \\ \bar{a}_d^2 &= 4 \times 10^{-6}, & a_d^2 &= 4 \times 10^{-6}, \\ \bar{a}_e^2 &= 2.4 \times 10^{-5}, & a_e^2 &= 4 \times 10^{-6}.\end{aligned}\tag{17}$$

With these values we find, for  $\gamma = \pi/2$  and for  $K\pi$  in the above resonance region:

$$BR(B^+ \rightarrow (K\pi)^+ D_1^0) = 3.8 \times 10^{-5}, \quad \mathcal{A}_1 = 1.2\%, \quad \mathcal{A}_2 = 2.7\%.\tag{18}$$

The two asymmetries are predicted to be positive. In the case of  $\mathcal{A}_2$  this follows from our estimate that  $\bar{a}_e$  is the dominant of the four weak amplitudes which are involved in the asymmetry. In the case of  $\mathcal{A}_1$ , in which  $\bar{a}_c$  is the dominant amplitude, destructive interferences occurs between the contributions of the two pairs of resonances. We note that the rather small values obtained for the asymmetries, in spite of the large final state phases found in



(11), are the result of having two interfering amplitudes which differ roughly by an order of magnitude.

By choosing the domain of energy integration more judiciously and/or by using more suitable weight functions, CP violating asymmetries of about 5 to 10% are readily attained [11]. For instance, weighing the asymmetry in decay distributions by  $\Delta/\Sigma$  (defined in (8)(9)) has been shown to define an optimal asymmetry [5]. This asymmetry obtains a value of about 10% (again, for  $\gamma = \pi/2$ ).

Similar asymmetry calculations were carried out for the two other resonance decay modes,  $f_2 = K^*\pi$  and  $f_3 = K\rho$  [11]. Asymmetries in the range of  $\sim 1$  to  $\sim 10\%$  are possible.

To summarize, we have shown that in quasi two body  $B$  decays to excited kaon resonances and neutral  $D$  mesons (or any other mesons) large final state phases are generated by the overlapping resonances. We explained how to use decay distributions to determine the weak phase  $\gamma$ . A general definition of (spin-dependent) CP asymmetries was given, which applies to any quasi two body decay, in which the dominant final state phases are accounted for in a calculable manner. Our method does not require resonance separation. Having a good control over final state phases, the difficult part in CP asymmetry calculations remains the evaluation of hadronic matrix elements. These difficulties, though, can get drastically reduced as the branching ratios for the relevant modes become experimentally known, which is clearly a much easier task than measurements involving CP violation. Crude estimates were shown to lead to asymmetries up to about ten percent. These asymmetries are the result of the different orders of magnitudes associated with the two interfering weak amplitudes. Clearly, larger asymmetries may be possible in decays to resonance states in which amplitudes of comparable magnitudes interfere. Work along these lines is in progress.

## Acknowledgements

We thank A. Ali, D. Cassel and J. Rosner for fruitful discussions. G. E. wishes to thank the Department of Physics at Brookhaven National Laboratory and M. G. wishes to thank the CERN Theory Division for their warm hospitality. This work was supported in part by the United States – Israel Binational Science Foundation under Research Grant Agreement 90-00483/3, by the German-Israeli Foundation of Scientific Research and Development, by an SSC fellowship and DOE contract DE-AC03-76SF00515 and by DOE contract DE-AC02-76CH0016. Partial support (for G. E. and M. G.) was also obtained by the Fund for Promotion of Research at the Technion.

## References

- [1] See, e.g. Y. Nir and H. Quinn, *Ann. Rev. Nucl. Part. Sci.* **42** (1992) 211.
- [2] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.
- [3] D. Besson, *International Symposium on Lepton and Photon Interactions at High Energies*, Cornell University, August 1993; M. S. Alam *et al.* (CLEO Collaboration), *Phys. Rev.* **D50** (1994) 43.
- [4] For upper limits on final state phases in  $B$  decays, see H. Yamamoto, Harvard University Report HUTP-94/A006 (1994).
- [5] D. Atwood and A. Soni, SLAC report SLAC-PUB-6425 (1994) to appear in *Zeit. für Physik*; SLAC report SLAC-PUB-6524.
- [6] M. Gronau and D. Wyler, *Phys. Lett.* **B265** (1991) 172. See also M. Gronau and D. London, *Phys. Lett.* **B253** (1991) 483; I. Dunietz, *Phys. Lett.* **B279** (1991) 75.
- [7] S. L. Stone, in *Beauty 93, Proceedings of the First International Workshop on B Physics at Hadron Machines*, Liblice Castle, Melnik, Czech Republic, January 1993, ed. P.E. Schlein, *Nucl. Instrum. Meth.* **33** (1993) 15.
- [8] Particle Data Group, *Review of Particle Properties*, *Phys. Rev.* **D50** (1994) 1173.
- [9] S. L. Stone, in *Heavy Flavours*, edited by A. J. Buras and M. Lindner (Singapore, World Scientific, 1992), p. 334.
- [10] J. Adler *et al.*, *Phys. Lett.* **B196** (1987) 107; J. C. Anjos *et al.*, *Phys. Rev.* **D48** (1993) 56; P. L. Frabetti *et al.*, *Phys. Lett.* **331B** (1994) 217.
- [11] D. Atwood, G. Eilam, M. Gronau and A. Soni, work in progress.

Label	Standard Notation	$J^P$	$m_i$	$\Gamma_i$	$f_1 = K\pi$	$f_2 = K^*\pi$	$f_3 = K\rho$
$K_a$	$K_1(1270)$	$1^+$	1273	90	–	16%	42%
$K_b$	$K_1(1400)$	$1^+$	1402	174	–	94%	3%
$K_c$	$K^*(1410)$	$1^-$	1412	227	7%	$> 40\%$	$< 7\%$
$K_d$	$K_2(1430)$	$2^+$	1425	98	50%	25%	9%
$K_e$	$K_0(1430)$	$0^+$	1429	287	93%	–	–

Table 1: Properties and branching ratios of  $K_i$  resonances. Masses and widths are given in MeV.