# Feasibility of Extracting $V_{t d}$ from Radiative $B\left(B_{S}\right)$ Decays 

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#### Abstract

We use experimental information on $D^{-} \rightarrow K^{* 0} \rho^{-}$and $B \rightarrow \psi+K^{*}$, coupled with flavor independence of QCD, and with vector meson dominance to show that long distance contributions to $B \rightarrow \rho+\gamma$, especially to $B^{-} \rightarrow \rho^{-}+\gamma$, are potentially very serious. Estimates based on the annihilation graph are shown to lead to similar conclusions. We emphasize that long-distance (LD) contributions can be appreciably different in $B^{-} \rightarrow \rho^{-}+\gamma$ and $B^{0} \rightarrow \rho^{0}(\omega)+\gamma$. All radiative decays of $B, B_{S}$ are shown to be governed essentially by two LD and two short-distance (SD) hadronic entities. Despite the presence of considerable LD contributions, we show how separate measurements of $B^{-} \rightarrow \rho^{-}+\gamma, B^{0} \rightarrow \rho^{0}(\omega)+\gamma$, along with $B \rightarrow K^{*}+\gamma$, can be used for a systematic extraction of $V_{t d}$. Measurements of $B_{S} \rightarrow \phi+\gamma$ and $K^{* 0}+\gamma$ could also provide very useful consistency checks.


## I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) mixing angle $V_{t d}$ is a parameter of crucial importance to the Standard Model (SM), and it is still very poorly known [1]. Considerable experimental effort is directed towards its determination via the rare decay $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ [2]. This process is considered to be theoretically clean for extraction of $V_{t d}$ [3]. However, its branching ratio is extremely rare, being about a few times $10^{-10}$, rendering a precise determination of $V_{t d}$ very challenging. In $B$-physics, one well known method for determining $V_{t d}$ is via the experimentally measured $B-\bar{B}$ mixing. This requires a knowledge of the pseudoscalar decay constant $f_{B}$ and the "bag parameter" $B_{B}$. Neither of these quantities is directly accessible to experiment, at least not in the near future; $f_{B}$ could eventually be measured directly in $B$ decays, say via $B \rightarrow \tau+\nu_{\tau}$, but this will surely take a long time. The reliability of the theoretical calculations for $f_{B}$ and $B_{B}$ may therefore be a cause for concern. In any case, the importance of $V_{t d}$ demands that we determine it in many ways and with as much precision as possible.

One $B$-decay into which $V_{t d}$ enters is $B \rightarrow \rho+\gamma[4-6]$. Since the related decay $B \rightarrow K^{*}+\gamma$ has already been detected [7] it is useful to understand what we may learn about $V_{t d}$ through a measurement of $B \rightarrow \rho+\gamma$. Rough estimates indicate that LD contributions to $B \rightarrow \rho+\gamma$ are potentially very serious. Since it is very difficult to accurately estimate these LD contributions, a precise extraction of $V_{t d}$ from $B \rightarrow \rho+\gamma[8]$ also appears rather difficult, at least from a single determination of $B \rightarrow \rho+\gamma$. We show how this situation could be remedied systematically by measurements of several of the related radiative modes.

Our purpose is thus twofold. First, it is to caution that LD contributions to $B \rightarrow \rho+\gamma$ (especially $\left.B^{-} \rightarrow \rho^{-}+\gamma\right)$ are very serious; determinations of $V_{t d}$ from $B \rightarrow \rho+\gamma$ disregarding these problems are likely to lead to inconsistencies. We present estimates of the LD contribution and stress the difficulties in an accurate calculation of these contaminations. By identifying the sources of the LD effects, we are then able to show how they can be determined systematically from future experiments. This then leads to a possible method for extraction of $V_{t d}$ from radiative decays.

We identify various LD and SD sources for radiative decays of all of the $B\left(B_{S}\right)$ mesons; i.e., for

$$
\begin{align*}
& B^{-} \rightarrow \rho^{-}+\gamma  \tag{1}\\
& B^{0} \rightarrow \rho^{0}+\gamma  \tag{2}\\
& B^{0} \rightarrow \omega+\gamma  \tag{3}\\
& B^{-} \rightarrow K^{*-}+\gamma  \tag{4}\\
& B^{0} \rightarrow K^{* 0}+\gamma  \tag{5}\\
& B_{S} \rightarrow \phi+\gamma  \tag{6}\\
& B_{S} \rightarrow K^{* 0}+\gamma \tag{7}
\end{align*}
$$

We show that two types of LD and essentially two types of SD contributions determine all of these decays. So, although the theoretical estimates for these LD effects are extremely unreliable, we outline here how separate experimental measurements of as many of these reactions as possible could allow a model independent determination of the hadronic entities and provide useful self consistency checks. We are thus able to write down necessary conditions that can quantify the extent of the LD contaminations. Consequently, with the systematic approach suggested here, extraction of $V_{t d}$ to a meaningful level of accuracy, in the long run, may well become possible. Clearly the necessary effort is then many times more than what is needed for a single measurement of $B \rightarrow \rho+\gamma$. On the other hand, we anticipate intense experimental activity in the area. Improvements at existing $e^{+} e^{-}$facilities such as CESR and LEP as well as construction of new $e^{+} e^{-}$based $B$-factories at SLAC and KEK will lead to an increased sample of $B$ 's. Furthermore many dedicated $B$ experiments are being proposed or planned at hadron machines. Bearing all that in mind we give a general strategy for attempting to extract $V_{t d}$ precisely from radiative $B$-decays.

## II. A CLOSE LOOK AT $B \rightarrow \rho+\gamma$.

## A. The Long Distance Contribution from $u \bar{u}$ States.

It has been known for a long time [9] that for $b \rightarrow d$ flavor-changing loop transitions (unlike for $b \rightarrow s$ ) the tree graphs (i.e. long-distance) become appreciably large and can easily dominate over the loop (i.e. the SD) process. A simple example is the process

$$
\begin{equation*}
B^{-} \rightarrow d \bar{u} \gamma \rightarrow \rho^{-} \gamma \tag{2}
\end{equation*}
$$

via the non-spectator (or the annihilation) mechanism shown in Fig. 1a. Notice that this graph goes via $V_{u b}$, i.e. another poorly known CKM parameter. So the reaction $B \rightarrow \rho+\gamma$ can occur, in principle, even if $V_{t d}$ is vanishingly small. Although it is very difficult to accurately calculate such a contribution there are several ways of estimating its size, i.e. within.a factor of two or three. We outline below two ways of estimating such contributions.

In the first method we invoke the correspondence of such annihilation graphs with spectator plus final state interactions (FSI) to note that Fig. 1(a) is exactly the same as Fig. 1(b). Fig. 1(b) shows the color allowed simple spectator contribution to $B^{-} \rightarrow \rho^{-}+\rho_{V}^{0}$ followed by $\rho_{V}^{0} \rightarrow \gamma$ (where the subscript $V$ stands for virtual). The first step of $B^{-} \rightarrow \rho^{-}+\rho^{0}$ can be estimated by normalizing with the observed analogous decay: $D^{-} \rightarrow \rho^{-}+K^{* 0}$ via Fig. 1(c). This correspondence between the two decays should hold because of the flavor ( $b \leftrightarrow c$ ) symmetry of QCD. To the extent that $m_{c}\left(m_{b}\right) \gg \Lambda_{Q C D}$ the effects of QCD do note care about the flavor-label charm or bottom [10]. Also $\mathrm{SU}(3)$ flavor symmetry ensures that the change from $K^{* 0}$ to $\rho^{0}$ in $D$ versus $B$ decay is mild apart from phase-space correction. The conversion from $\rho \rightarrow \gamma$ can be dealt with by using vector-meson dominance. Thus

$$
\begin{equation*}
\frac{B R\left(B^{-} \rightarrow \rho^{-} \rho^{0}\right)}{B R\left(D^{-} \rightarrow \rho^{-} K^{* 0}\right)}=\left(\frac{m_{b}}{m_{c}}\right)^{5}\left|\frac{V_{u b}}{V_{c s}}\right|^{2} \frac{\tau_{B^{+}}}{\tau_{D^{+}}} \chi_{p s} \chi_{c} \tag{3}
\end{equation*}
$$

where $\chi_{p s}$ is the phase space ratio and $\chi_{c}$ is the ratio of the linear combination of the usual [11] Wilson coefficients $c_{1}, c_{2}$ that are relevant for the 4-quark operators in $b \rightarrow c$ decays versus $c \rightarrow s$ decays.
$\therefore$ The VMD conversion factor [12] for $\rho^{0} \rightarrow \gamma$ is taken as $\left(e e_{Q} f_{\rho} / m_{\rho}\right)^{2}$ where $e_{Q}$ is the weighted quark charges for $\rho^{0}$ (i.e. $e_{Q}=1$ ) and $f_{\rho} \sim 220 \mathrm{MeV}$. For the purpose of numerical estimates, following suggestions of phenomenological models [14], we assume that only $30 \%$ of the $K^{*}$ are transversely produced from $D^{-} \rightarrow K^{*} \rho^{-}$. At present, there is also considerable uncertainty in the BR of this mode [15] (esp. for the charged $D$ as it is $B\left(D^{-} \rightarrow K^{*} \rho^{-}\right)=2.1 \pm 1.4 \%$; for the neutral one the situation is a bit better being $B\left(D^{0} \rightarrow K^{*-} \rho^{+}\right)=(3.9 \pm 1.6) \%$.) For our purpose, we will use a $B R$ of $1 \%$ to serve as a convenient normalization.

For simplicity, the phase space ratio is taken as $\left(1-m_{K^{*}}^{2} / m_{D}^{2}\right)^{3}$ to yield $\chi_{p s} \sim 2$. Also using $c_{1}\left(m_{b}\right)=1.13$, $c_{2}\left(m_{b}\right)=-.29, c_{1}\left(m_{c}\right)=1.25, c_{2}\left(m_{c}\right)=-.50$, we find $\chi_{c} \sim 1.3$ and using $[1]\left|\frac{V_{u b}}{V_{c b}}\right|=.08,\left|V_{c b}\right|=.037$ we get:

$$
\begin{equation*}
B R\left(B^{-} \rightarrow \rho^{-} \gamma\right)_{L_{u}} \sim 6 \times 10^{-8} \tag{4}
\end{equation*}
$$

where the subscript $L_{u}$ is to denote the LD contributions coming from $u \bar{u}$ state(s) such as $\rho^{0}$. While it is clear that there are large uncertainties in the estimate in (4), the resulting numbers are plausible and indicate a sizeable LD contribution that is proportional to $V_{u b}$ and consequently is present even if $V_{t d}$ was zero.

A second method for estimating the same contribution is to use bound state method of Ref. 16 for writing down. the amplitude for $B^{-} \rightarrow \bar{u} d \gamma$ :

$$
\begin{gather*}
A\left(B^{-} \rightarrow \bar{u} d \gamma\right)_{1 a} \simeq \frac{f_{B}}{16} e_{u} e \frac{g_{W}^{2}}{m_{W}^{2}} \frac{1}{m_{u}} V_{u b} \operatorname{Tr}\left[\left(\not p_{b}-m_{b}\right) \not k_{\gamma} \not q \gamma^{u} P_{L}\right]  \tag{5}\\
\bar{u}_{d} \gamma^{u} P_{L} v_{u}
\end{gather*}
$$

where $m_{u}$ is the constituent mass of the $u$ quark and $e_{u}$ is its charge (i.e. $2 / 3$ ). Now the light-quark current can be (vacuum) saturated by the $\rho^{-}$via use of:

$$
\begin{equation*}
\langle 0| \bar{u}_{d} \gamma_{\mu} \gamma_{5} v_{u}\left|\rho^{-}\left(p_{\rho}, \epsilon_{p}\right)\right\rangle=f_{\rho} m_{\rho} \epsilon_{\rho}^{u} \tag{6}
\end{equation*}
$$

where $f_{\rho}$, again, is the decay constant of $\rho$ i.e. about 220 MeV . Thus:

$$
\begin{equation*}
\frac{\Gamma\left(B^{-} \rightarrow \rho^{-} \gamma\right)_{1 a}}{\Gamma\left(B^{-} \rightarrow \bar{u} d \gamma\right)_{1 a}} \simeq 18 \pi^{2} \frac{f_{\rho}^{2} m_{\rho}^{2}}{m_{B}^{4}}\left(1-\frac{m_{\rho}^{2}}{m_{B}^{2}}\right) \simeq 7 \times 10^{-3} \tag{7}
\end{equation*}
$$

Using (5) and (7) we arrive at a second estimate for the LD correction due to $\bar{u} u$ states

$$
\begin{equation*}
B R\left(B^{-} \rightarrow \rho^{-} \gamma\right)_{L_{u 2}} \simeq 8 \times 10^{-8} \tag{8}
\end{equation*}
$$

$=$
where we have used $f_{B}=180 \mathrm{MeV}[18]$ and $m_{u}=330 \mathrm{MeV}$. In passing we note from eqns. (7) and (8) that the inclusive branching ratio for the reaction $B^{-} \rightarrow \bar{u} d \gamma$ via the annihilation graph is given by:

$$
\begin{equation*}
B R\left(B^{-} \rightarrow u \bar{d} \gamma\right) \approx 1.1 \times 10^{-5} \tag{9}
\end{equation*}
$$

Given the intrinsic uncertainties in each of the two methods outlined above the resulting numbers in eqns. (4) and (8) should be regarded as in rough agreement. Thus for one class of long distance contributions, namely those due to $u \bar{u}$ states we will take the mean of the two numbers from eqn. (4) and eqn. (8) and rather arbitrarily assign a factor of four uncertainty. Thus for the corresponding amplitude we get

$$
\begin{equation*}
A\left(B^{-} \rightarrow \rho^{-} \gamma\right)_{L_{u}}=(2-4) \times 10^{-4} \tag{10}
\end{equation*}
$$

Now let us address the case of the neutral $B$ i.e. the corresponding LD contributions from $\bar{u} u$ states to $\left(B^{0} \rightarrow \rho^{0} \gamma\right)$. Then Fig. 1(a) gets redrawn as Fig. 1(d) and Fig. 1(b) gets redrawn as Fig. 1(e). In each case we see that the graphs for $B^{0}$ are color suppressed. On the other hand, presence of the two anti-quarks ( $\bar{u}$ ) in the final state in $B^{-} \rightarrow \rho^{-} \gamma$ is expected to lead to a destructive interference resulting in a reduction in the amplitude by about a factor of $\left(N_{c}-1\right) / N_{c_{-}}$ ie. $\frac{2}{3}$. Thus

$$
\begin{align*}
A\left(B^{0} \rightarrow \rho^{0} \gamma\right)_{L_{u}} & =-\left(\frac{1}{2} \times \frac{3}{2} \times \frac{1}{\sqrt{2}}\right) \times(2-4) \times 10^{-4}  \tag{11}\\
A\left(B^{0} \rightarrow \omega \gamma\right)_{L_{u}} & =\frac{1}{4 \sqrt{2}}(2-4) \times 10^{-4} \tag{12}
\end{align*}
$$

Before ending this section, let us mention, in passing, that $\rho^{0} \rightarrow \gamma$ is not the only source of long range contributions due to $u \bar{u}$ states. These $u \bar{u}$ states can also manifest in terms of multiple pion states. Thus, for example, $B^{ \pm} \rightarrow$ $\rho^{ \pm} \pi^{+} \pi^{-}, \rho^{ \pm} \pi^{+} \pi^{-} \pi^{0}, \rho^{ \pm} \pi^{+} \pi^{-} \pi^{+} \pi^{-} \ldots$ followed by the annihilation of the (virtual) multiple pion state to $\gamma$ also contribute to LD effects. The $\rho^{0} \rightarrow \gamma$ discussed above is just a simple and perhaps the most important example of this class of contaminations.

## B. The Long Distance Contributions from $c \bar{c}$ States.

We next turn our attention to the LD contributions to $B^{-} \rightarrow \rho^{-}+\gamma$ from $c \bar{c}$ states. The most notable origin is the chain $B^{-} \rightarrow \rho^{-}+\psi_{V}$ followed by $\psi_{V} \rightarrow \gamma$. Using the measured rate [15]

$$
\begin{equation*}
B R\left(B^{0} \rightarrow K^{* 0} \psi\right)=(1.6 \pm .3) \times 10^{-3} \tag{13}
\end{equation*}
$$

we immediately get

$$
\begin{equation*}
B R\left(B^{-} \rightarrow \rho^{-} \psi\right)=2 B R\left(B^{0} \rightarrow \rho^{0} \psi\right)=\lambda^{2} B r\left(B^{0} \rightarrow K^{* 0} \psi\right) p_{s K^{*} \rho} \tag{14}
\end{equation*}
$$

where $\lambda \equiv \sin \theta_{c} \sim .23$ and $p_{s K^{*} \rho}$ is a phase space correction factor estimated to be about 1.4 due to the mass difference between $\rho$ and $K^{*}$ [19]. Following Ref. 12 conversion factor from $\psi \rightarrow \gamma$ is estimated at $5 \times 10^{-3}$. We emphasize that this conversion factor actually includes contribution from charmonium states other than $\psi(3097)$ as
well, i.e. $\psi^{\prime}(3685), \psi^{\prime \prime}(3770) \ldots$ (See Table 2 of Ref. 12 in this connection). Since these states contribute coherently to the formation of photons, this conversion factor ends up being about a factor of five larger than what it would be if only the $\psi$ had been kept. Now in this $\psi\left(\psi^{\prime}, \psi^{\prime \prime} \ldots\right) \rightarrow \gamma$ conversion we must include only the transversely polarized fraction of charmonia. These are estimated to be about $30 \%$ [20]. Thus, for the amplitude of LD contributions from $c \bar{c}$ states we get

$$
\begin{equation*}
A\left(B^{-} \rightarrow \rho^{-} \gamma\right)_{L_{c}}=(2-6) \times 10^{-4} \tag{15}
\end{equation*}
$$

where, in specifying the range, we are again estimating about a factor of two uncertainty (in the amplitude).
Once again we should remind that $\psi_{V} \rightarrow \gamma$ conversion is simply symptomatic of the LD contributions due to $c \bar{c}$ states. In principle, there are additional sources such as $B^{-} \rightarrow \rho^{-} D \bar{D}, \rho^{-} D^{*} \bar{D}, \rho^{-} D^{*} \bar{D}^{*} \ldots$ followed by the annihilation of the off-shell charm anticharm hadrons to $\gamma$. We suspect that the contribution of charmonia to $\gamma$ is the dominant one from $c \bar{c}$ states but we are unaware of any compelling reasons for this to be a guarantee.

## C. The Short-Distance Contributions to $B \rightarrow \rho+\gamma$

The SD (or penguin) contributions arise from loop graphs, such as Fig. 1(f) and $1(\mathrm{~g})$. It is known for a long time that QCD corrections play an important role here. We recall that this is due to the fact that in the pure electroweak penguin (Fig. 1(f)) there is an accidental cancellation of the coefficients of terms that maintain GIM unitarity with a legarithmic dependence on the internal quark mass (i.e. $m_{u}, m_{c}, m_{t}$ ). As a result the leading terms exhibit a power law dependence on that mass. On switching on QCD the coefficient of the log term becomes nonvanishing and results in enhanced $Q C D$ radiative effects.

By now there is an extensive literature describing the effects of QCD on radiative decays of $B$ 's. For our purpose it is useful to first discuss the $b \rightarrow s$ process namely the one relevant to $B^{-}\left(B^{0}\right) \rightarrow K^{*-}\left(K^{* 0}\right)+\gamma$ (or to $B_{S} \rightarrow \phi+\gamma$ ). Recall the CKM unitarity for this channel:

$$
\begin{equation*}
v_{u}^{q}+v_{c}^{q}+v_{t}^{q}=0 \tag{16}
\end{equation*}
$$

where $v_{j}^{q}=V_{j b} V_{j q}^{*}, j=u, c, t$ and $q=s$ or $d$. Recall also that [1]

$$
\begin{align*}
V_{u s} & =\lambda \simeq .23  \tag{17}\\
\frac{V_{u b}}{V_{c b}} & =.08 \pm .03  \tag{18}\\
V_{t b} & =.99 \pm .01 \tag{19}
\end{align*}
$$

Thus for $b \rightarrow s$ case the up quark term $\left(V_{u b} V_{u s}^{*}\right)$ is negligible in comparison to the other two terms in eqn. (16). This has two important consequences. First is that one gets the usual relation:

$$
\begin{equation*}
V_{t s} \simeq-V_{c b} \tag{20}
\end{equation*}
$$

to a very good approximation. The second important consequence of the smallness of $V_{u b} V_{u s}^{*}$ is that in the $b \rightarrow s$ penguin loop the $u$ quark contribution is forced to become so small that the precise dependence on $m_{u}$ is not at all important. Such is not the case for $b \rightarrow d$ penguins as we will soon elaborate.

The penguin (SD) contributions $a_{p}^{q}$ can be written as

$$
\begin{equation*}
a_{p}^{q}=\frac{G_{F}}{\sqrt{2}} \frac{e m_{b}}{8 \pi^{2}} A_{p}^{q} \bar{q}_{\alpha} \sigma^{\mu \nu}\left(1+\gamma_{5}\right) b_{\alpha} F_{\mu \nu} \tag{21}
\end{equation*}
$$

where

$$
\begin{equation*}
A_{p}^{q}=\sum_{j} f_{j} v_{j}^{q} \tag{22}
\end{equation*}
$$

For $q=s$, we can use eqn. (16) and rewrite

$$
\begin{equation*}
=\quad A_{p}^{s}=\left(f_{t}-f_{c}\right) v_{t}^{s}+\left(f_{u}-f_{c}\right) v_{u}^{s} \tag{23}
\end{equation*}
$$

Since $v_{u}^{s}$ is extremely small the second term is bound to make a negligibly small contribution and consequently the assumption that $f_{c}=f_{u}$ that one usually makes [21] becomes a very safe assumption. Then for $b \rightarrow s$ with a very good approximation one gets

$$
\begin{equation*}
A_{p}^{s}=\left(f_{t}-f_{c}\right) v_{t}^{s} \equiv\left(f_{t}-f_{c}\right) V_{t s} \tag{24}
\end{equation*}
$$

For the case of $b \rightarrow d$ transitions the $u$ quark in the loop no longer appears with the small parameter $V_{u s}(\equiv \lambda)$ multiplying its effects and the charm and the top quark both now have smaller CKM factors monitoring their contributions to the penguin amplitude. The $u$ quark contribution is no longer necessarily negligible in comparison to
the others and the assumption $f_{c}=f_{u}$ is no longer a good approximation since it forces a potentially important ( $u$ quark) contribution to unnaturally vanish. Any reasonable deviation of $f_{c} / f_{u}$ away from unity would have important corrections. To make the best use of the experimental information that one gets from measurement of $B \rightarrow K^{*} \gamma$, it is prudent now to use unitarity and rewrite the $b \rightarrow d$ penguin as:

$$
\begin{equation*}
A_{p}^{d}=\left(f_{t}-f_{c}\right) v_{t}^{d}+\left(f_{u}-f_{c}\right) v_{u}^{d} \tag{25}
\end{equation*}
$$

Taking ratios of equations (24) and (25):

$$
\begin{align*}
A_{p}^{d} / A_{p}^{s} & =\frac{V_{t d}}{V_{t s}}[1+\Delta] \\
\Delta & \equiv\left(\frac{f_{u}-f_{c}}{f_{t}-f_{c}}\right)\left(\frac{V_{u b}}{V_{t d}}\right) \tag{26}
\end{align*}
$$

Thus there are two hadronic entities:

$$
\begin{align*}
f_{u}-f_{c} & \equiv S_{u c}  \tag{27}\\
f_{t}-f_{c} & \equiv S_{t c} \tag{28}
\end{align*}
$$

monitoring all the SD contributions in $b \rightarrow s$ and $b \rightarrow d$ penguins. $f_{t}$ and $f_{c}$ have recently been calculated in Ref. [21]:

$$
\begin{align*}
& f_{t} \simeq-.11  \tag{29}\\
& f_{c} \simeq .16 \tag{30}
\end{align*}
$$

giving

$$
\begin{equation*}
S_{t c} \approx-.27 \tag{31}
\end{equation*}
$$

For extraction of $V_{t d}$ from experiment the deviation from unity of the quantity in square parenthesis in equation (26) is important. First let us estimate the CKM ratio that enters there. We note that the use of [18]

$$
\begin{align*}
& f_{B}=180 \pm 40 \mathrm{MeV}  \tag{32}\\
& B_{B}=1 \pm .2 \tag{33}
\end{align*}
$$

emerging from lattice calculations along with the measured $B-\bar{B}$ mixing gives

$$
\begin{equation*}
\frac{V_{t d}}{V_{t s}} \simeq 0.22 \pm .08 \tag{34}
\end{equation*}
$$

Thus using (as $90 \%$ CL bounds)

$$
\begin{gather*}
\left|V_{u b} / V_{c b}\right|<.13  \tag{35}\\
\left|V_{t d} / V_{t s}\right| \geq .09 \tag{36}
\end{gather*}
$$

we get

$$
\begin{equation*}
\left|V_{u b} / V_{t d}\right| \lesssim 1.5 \tag{37}
\end{equation*}
$$

A precise value for $f_{u}-f_{c}$ is extremely difficult to calculate. Based on the reasoning given at the beginning of this section we will assume that $f_{u}$ and $f_{c}$ depend logarithmically on $m_{u}$ and $m_{c}$. Using constituent masses $m_{u} \simeq 0.3$ $\mathrm{GeV}, m_{c} \simeq 1.8 \mathrm{GeV}$ and the numerical result (30) of Ref. [21] we can estimate with logarithmic accuracy:

$$
\begin{equation*}
S_{u c} / S_{t c} \simeq-0.3 \tag{38}
\end{equation*}
$$

Note that the use of a constituent quark mass for the $u$-quark reflects the fact that we expect the logarithmic growth of the LD effects to approach a constant as the current quark mass goes to zero. One may also estimate the magnitude of this ratio using the perturbative calculation of the LD effects as in [5]. Numerically one obtains results that are reughly similar in magnitude to the above especially as they pertain to the correction term in eqn. (26).
¿From eqn. (38) we see that the ratio of the SD amplitudes for $b \rightarrow d$ and $b \rightarrow s$ may deviate appreciably from the CKM ratio $V_{t d} / Y_{t s}$. We note this deviation from the simple CKM scaling is controlled crucially by the ratio $V_{u b} / V_{t d}$ just as the relative importance of the LD contributions due to $u \bar{u}$ states (i.e $L_{u}$ ) to $B \rightarrow \rho+\gamma$ is controlled by $V_{u b} / V_{t d}$. If the mild indications from the current central values of $V_{u b} / V_{c b}$ and $V_{t d} / V_{t s}(.08$ versus .22$)$ is confirmed then the extraction of $V_{t d}$ from $B \rightarrow \rho+\gamma$ will clearly be easier than otherwise.

To gauge the relative importance of the LD and the SD contributions to $(B \rightarrow \rho+\gamma)$ we need to estimate $A_{p}^{s}$ (i.e. SD amplitude for $b \rightarrow s$ ) so as to be able to use eqn. (26) to get $A_{p}^{d}$ (i.e. the SD amplitude for $b \rightarrow d$ ). We can try to use the experimental result on $B \rightarrow K^{*}+\gamma$ for that purpose; so we turn our attention to that reaction now.

## D. The Long- and Short Distance Contributions to $B \rightarrow K^{*}+\gamma$

The LD contribution from $\bar{u} u$ states is easily estimated from eqn. (10)

$$
\begin{align*}
A\left(B^{-} \rightarrow K^{*-} \gamma\right)_{L_{u}} & \simeq(4-8) \times 10^{-5}  \tag{39}\\
A\left(B^{0} \rightarrow K^{* 0} \gamma\right)_{L_{u}} & \simeq(2-4) \times 10^{-5} \tag{40}
\end{align*}
$$

Similarly, from eqn. (13), with use of the $\psi \rightarrow \gamma$ conversion factor of $5 \times 10^{-3}$ and incorporating a factor of 0.3 for the fraction of transversely polarized $\psi$ 's we get

$$
\begin{equation*}
A\left(B \rightarrow K^{*}+\gamma\right)_{L_{c}}=(1-3) \times 10^{-3} \tag{41}
\end{equation*}
$$

So for $B \rightarrow K^{*} \gamma$ the LD contributions due to $c \bar{c}$ completely dominate over the $u \bar{u}$ ones [22].
Recall now the recent experimental result [7]

$$
\begin{equation*}
B R\left(B \rightarrow K^{*} \gamma\right)=(4.5 \pm 1.5 \pm 0.9) \times 10^{-5} \tag{42}
\end{equation*}
$$

For the amplitude we translate this as

$$
\begin{equation*}
\left.A\left(B \rightarrow K^{*} \gamma\right)\right|_{\mathrm{expt}} \simeq(6.7 \pm 1.7) \times 10^{-3} \tag{43}
\end{equation*}
$$

From equations (41) and (43) we see that there can be about 15-50\% LD contributions in the observed experimental result. Combining those two equations we arrive at the SD component $=$

$$
\begin{equation*}
A_{p}^{s} \equiv A\left(B \rightarrow K^{*}+\gamma\right)_{S D}=(4.7 \pm 2.7) \times 10^{-3} \tag{44}
\end{equation*}
$$

In arriving at eqn. (44) we have made a strong assumption that the SD and $\mathrm{LD}(c \bar{c})$ amplitudes for $B \rightarrow K^{*}+\gamma$ have the same relative sign. This assumption is based on the belief that an opposite choice of signs would make the exclusive to inclusive ratio for the short distance component alone, i.e.

$$
\begin{equation*}
H_{K^{*}}=\frac{\Gamma\left(B \rightarrow K^{*}+\gamma\right)}{\Gamma(b \rightarrow s+\gamma)} \tag{45}
\end{equation*}
$$

become uncomfortably large. The point is that lattice methods have been used to calculate this hadronization ratio for the single operator

$$
\begin{equation*}
\bar{s}_{L} \sigma_{\mu \nu} b_{R} F^{\mu \nu} \tag{46}
\end{equation*}
$$

that emerges from the short distance expansion. The results of the lattice calculation are [23]:

$$
\begin{equation*}
H_{K^{*}}=6.0 \pm 1.2 \pm 3.4 \% \tag{47}
\end{equation*}
$$

For our purpose we will adopt a very conservative interpertation of the lattice results, namely

$$
\begin{equation*}
H_{K^{*}}<12 \% \tag{48}
\end{equation*}
$$

Recall now the recent CLEO result [24]:

$$
\begin{equation*}
B R(b \rightarrow \gamma+s)=(2.32 \pm .51 \pm .29 \pm .32) \times 10^{-4} \tag{49}
\end{equation*}
$$

Combining equations (42) and (49) indicates that the experimental value of the exlusive to inclusive ratio is around $20 \%$ which tends to be on the high side compared to the lattice expectation. By attributing a fraction of the observed exclusive signal to come from LD sources as in equation (44) brings the hadronization ratio for the SD piece i.e.

$$
\begin{equation*}
\frac{\left(4.7 \times 10^{-3}\right)^{2}}{2.3 \times 10^{-4}} \sim 9.6 \% \tag{50}
\end{equation*}
$$

to be more in the ball park of the lattice results. If, on the other hand, we take the LD and SD contributions to $B \rightarrow K^{*} \gamma$ to have a relative negative sign then the SD fraction would have to be

$$
=\quad \frac{\left(8.7 \times 10^{-3}\right)^{2}}{2.3 \times 10^{-4}} \sim 33 \%
$$

which is too large from the lattice persepective.
E. Estimates for the Relative Importance of LD Contribution to $B \rightarrow \rho+\gamma$

Using eqn. (26) and (44) and invoking $\mathrm{SU}(3)$ gives us the SD contribution to ( $B \rightarrow \rho+\gamma$ )

$$
\begin{align*}
A(B \rightarrow \rho+\gamma)_{S D} & =\frac{V_{t d}}{V_{t s}}[1+\Delta] \times(4.7 \pm 2.7) \times 10^{-3}  \tag{51}\\
& \sim(5-15) \times 10^{-4} \tag{52}
\end{align*}
$$

From eqn. (15) and (52) we see that for $B^{-} \rightarrow \rho^{-}+\gamma$ the LD $c \bar{c}$ states are at least $15 \%$ of (and could even dominate over) the SD ones. Indeed even that minimum value of $15 \%$ implies a contamination of these LD effects on the rate for $B^{-} \rightarrow \rho^{-} \gamma$ to approach $30 \%$. From eqn. (10) we see that the $u \bar{u}$ states seem to be somewhat less important than the $c \bar{c}$ but are roughly comparable. We emphasize again that the numbers given for $L_{u}$ in eqn. 10 assume $\left|\frac{V_{u b}}{V_{c b}}\right|=.08$. Given the intrinsic difficulties of the LD estimates it appears doubtful that $B^{-} \rightarrow \rho^{-}+\gamma$ alone in conjunction with $B \rightarrow K^{*}+\gamma$ can be used to deduce reliable information on $V_{t d}$ before a lot more experimental data on radiative decays becomes available. In this regard a precise value of $V_{u b}$ as well as the relative sign of $V_{u b} V_{u d}^{*}$ with $V_{c b} V_{c d}^{*}$ is very important since a relative negative sign between these two CKM elements would result in (at least) a partial cancellation of the long distance $L_{u}$ and $L_{c}$ terms.

The LD contribution to $B^{0} \rightarrow \rho^{0}+\gamma$ from $u \bar{u}$ states are substantially less (see eqn. (11)) than for $B^{-} \rightarrow \rho^{-}+\gamma$. The SD contributions are the same for $B^{0}$ and $B^{-}$(i.e. eqns. (51) and (52)). Thus $B^{0} \rightarrow \rho^{0}+\gamma$ may have appreciable advantages over $B^{-} \rightarrow \rho^{-}+\gamma$ for learning about $V_{t d}$. In any event, it seems clear from the preceding estimates that the rates for $B^{-} \rightarrow \rho+\gamma$ may be quite different from that of $B^{0} \rightarrow \rho+\gamma$. Since the SD contributions (which scale with $V_{t d}$ ) are the same for $B^{-}$and $B^{0}$ and the LD ones are not, separate measurements of $B^{-}$and $B^{0}$ radiative decays are important to understanding the dynamics of these decays and they are essential for facilitating any reliable determination of $V_{t d}$.

## III. FOUR HADRONIC ENTITIES ESSENTIALLY DETERMINE ALL THE RADIATIVE B-DECAYS.

## $=$

In the preceding sections we have discussed the long and short distance contributions to charged and neutral $B$ decays to $\rho+\bar{\gamma}$ and $K^{*}+\gamma$. During the course of that discussion we had to introduce two LD (namely $L_{u}$ and $L_{c}$ ) and two short distance (namely $S_{t c}$ and $S_{u c}$ ) entities. Indeed all the radiative $B, B_{S}$ decays to the seven final states given in eqn. (1) are governed by the same four hadronic entities [25]. Of course the dependence on CKM angles are not the same (also there are obvious differences in $N_{c}$ dependence and on flavor $\mathrm{SU}(3)$ ) that have to be taken into account. Thus we are led to the model independent parameterization of $B\left(B_{S}\right)$ decays:

$$
\begin{equation*}
A\left(B^{-} \rightarrow \rho^{-}+\gamma\right)=e_{u}\left[\left(N_{C}-1\right) v_{u}^{d} L_{u}+v_{c}^{d} L_{c}+k_{b} c_{B \rho} T_{1_{B \rho}}\left(v_{t}^{d} S_{t c}+v_{u}^{d} S_{u c}\right)\right] \tag{53}
\end{equation*}
$$

$$
\begin{equation*}
A\left(B^{0} \rightarrow \rho^{0}+\gamma\right)=-\frac{1}{\sqrt{2}}\left[\left(e_{u}-e_{d}\right) v_{u}^{d} L_{u}+e_{u} v_{c}^{d} L_{c}+e_{u} k_{b} c_{B \rho} T_{1_{B \rho}}\left(v_{t}^{d} S_{t c}+v_{u}^{d} S_{u c}\right)\right] \tag{54}
\end{equation*}
$$

Also

$$
\begin{align*}
& A\left(B^{0} \rightarrow \omega+\gamma\right)=\frac{1}{\sqrt{2}}\left[\left(e_{u}+e_{d}\right) v_{u}^{d} L_{u}+e_{u} v_{c}^{d} L_{c}+e_{u} k_{b} c_{B \rho} T_{1_{B \rho}}\left(v_{t}^{d} S_{t c}+v_{u}^{d} S_{u c}\right)\right]  \tag{55}\\
& A\left(B^{-} \rightarrow K^{*-}+\gamma\right)=e_{u}\left[v_{u}^{s} N_{c} L_{u}+v_{c}^{s} L_{c}+k_{b} c_{B K^{*}} T_{1_{B K}}\left(v_{t}^{s} S_{t c}+v_{u}^{s} S_{u c}\right)\right]+ \\
& \simeq e_{u}\left[v_{c}^{s} L_{c}+k_{b} c_{B K} \cdot T_{1_{B K} *} v_{t}^{s} S_{t c}\right]  \tag{56}\\
& A\left(B^{0} \rightarrow K^{* 0}+\gamma\right)=e_{u}\left[v_{u}^{s} L_{u}+v_{c}^{s} L_{c}+k_{b} c_{B K^{*}} \cdot T_{1_{B K} \cdot}\left(v_{t}^{s} S_{t c}+v_{u}^{s} S_{u c}\right)\right] \\
& \simeq e_{u}\left[v_{c}^{s} L_{c}+k_{b} c_{B K} \cdot T_{1_{B K} *} v_{t}^{s} S_{t c}\right] \tag{57}
\end{align*}
$$

Similarly for related decays of $B_{S}$ :

$$
\begin{align*}
A\left(B_{S} \rightarrow \phi+\gamma\right) & =e_{u}\left[v_{u}^{s} L_{u}+v_{c}^{s} L_{c}+k_{b} c_{B_{s} \phi} T_{1_{B_{s} \phi}}\left(v_{t}^{s} S_{t c}+v_{u}^{s} S_{u c}\right)\right] \\
& \simeq e_{u}\left[v_{c}^{s} L_{c}+k_{b} c_{B_{s} \phi} T_{1_{B_{s} \phi}} v_{t}^{s} S_{t c}\right]  \tag{58}\\
A\left(B_{S} \rightarrow K^{*}+\gamma\right) & =e_{u}\left[v_{u}^{d} L_{u}+v_{c}^{d} L_{c}+k_{b} c_{B_{s} K^{*}} T_{1_{B_{s} K^{*}}}\left(v_{t}^{d} S_{t c}+v_{u}^{d} S_{u c}\right)\right] \tag{59}
\end{align*}
$$

Here $k_{b}$ is a normalization constant designed so that the width for the flavor-changing transition coming from the short distance piece alone is related properly to the factors $S_{t c}$ and $S_{u c}$. Thus

$$
\begin{equation*}
\therefore \quad . \quad \Gamma(b \rightarrow d \gamma)_{S D} \equiv \Gamma(b \rightarrow d \gamma)_{\text {penguin }}=\left[e_{u} k_{b}\left(v_{t}^{d} S_{t c}+v_{u}^{d} S_{u c}\right)\right]^{2} \tag{60}
\end{equation*}
$$

$T_{1}$ is the only form factor (at $q^{2}=0$ ) that determines the exclusive to inclusive ratio from the short-distance penguin part [23]. Thus

$$
\begin{equation*}
\frac{\Gamma(B \rightarrow \gamma \rho)_{S D}}{\Gamma(b \rightarrow \gamma d)_{S D}}=c_{B \rho}^{2} T_{1 B \rho}^{2} \tag{61}
\end{equation*}
$$

where

$$
\begin{equation*}
C_{B \rho}^{2}=4\left(\frac{m_{B}}{m_{b}}\right)^{3}\left[1-\frac{m_{\rho}^{2}}{m_{B}^{2}}\right]^{3} \tag{62}
\end{equation*}
$$

## IV. DISCUSSION

In Table 1 we give rough estimates for the radiative modes [26]. For simplicity we have assumed that $T_{1}\left(q^{2}=0\right)$ is the same for $B \rightarrow K^{*} \rho$ and $B_{s} \rightarrow K^{*} \phi$. There could easily be differences between these form factors amounting to 10 or even $20 \%$. Future lattice and QCD sum rule calculations should be able to determine these quite reliably.

TABLE I. Numerical Estimates

| Reaction | \|Amplitudes $/ / 10^{-4}$ |  |  | Branching Ratio/ $10^{-7}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(u \bar{u})_{L D}$ | $(c \bar{c})_{L D}$ | SD | SD only | Total |
| $B^{+} \rightarrow \rho^{+} \gamma$ | $3 \pm 1$ | $4 \pm 2$ | $10 \pm 6$ | 2-25 | .4-68 |
| $B^{0} \rightarrow \rho^{0} \gamma$ | $1.6 \pm .5$ | $2.8 \pm 1.4$ | $7 \pm 4$ | 1-12 | 1-32 |
| $B^{0} \rightarrow \omega^{0} \gamma$ | . $5 \pm .2$ | $2.8 \pm 1.4$ | $7 \pm 4$ | 1-12 | 2-23 |
| $B \rightarrow K^{*} \gamma$ | . $6 \pm .3$ | $20 \pm 10$ | $50 \pm 30$ | $40-640$ | 90-1200 |
| $B_{S} \rightarrow K^{*} \gamma$ | $1.5 \pm .5$ | $4 \pm 2$ | $10 \pm 6$ | 2-26 | 2-58 |
| $B_{S} \rightarrow \phi \gamma$ | . $6 \pm .2$ | $20 \pm 10$ | $50 \pm 30$ | $40-640$ | 90-1200 |

Notice that the spread in the range due to the SD piece alone is less than the spread after the LD contributions are included. This is in part because the relative signs are not known at this time. Thus typically the SD piece alone has a range of about one order of magnitude and that increases appreciably to the extent that in one case it becomes as much as two orders of magnitude when the LD pieces are also included.

We must also emphasize that the entries in the table are highly correlated so that as better experimental information on any mode(s) becomes available then it will effect the estimates for all of the modes. This is of course another way of saying that all of the decays involve only a few (i.e. four) hadronic quantities. In the case of $B \rightarrow K^{*} \gamma$ the recently obtained experimental measurement has been used to fix the relative sign between the SD piece and the LD $(c \bar{c})$ piece. There still remains an uncertainty in the theoretical expectation for the total $B R$ of about one order of magnitude. Measurements of $B \rightarrow \rho(\omega)+\gamma$, especially separate ones for charged and neutral, will significantly aid such an analysis in the future. Differences in the $B R$ s for $\rho^{-}+\gamma, \rho^{0}+\gamma$ and $\omega+\gamma$ would be an excellent indicator of the extent of the LD contamination. If the LD contributions are small then the $B R$ sor these modes should follow the expected factor of two difference due to the difference in their naive quark content.

Indeed from eqns. (53-55) one finds:

$$
\begin{align*}
& \left|A\left(B^{-} \rightarrow \rho^{-}+\gamma\right)\right|-\sqrt{2}\left|A\left(B^{0} \rightarrow \rho^{0}+\gamma\right)\right|=V_{u}^{d} L_{u}\left[e_{u}+e_{d}\right]  \tag{63}\\
& \left|A\left(B^{-} \rightarrow \rho^{-}+\gamma\right)\right|-\sqrt{2}\left|A\left(B^{0} \rightarrow \omega+\gamma\right)\right|=V_{u}^{d} L_{u}\left[e_{u}-e_{d}\right] \tag{64}
\end{align*}
$$

Thus experimental determination of the differences in the BR's can be used to quantitatively deduce the long distance piece due to $u \bar{u}$. In fact, eqn. (63) or (64) can be viewed as an important necessary condition that must experimentally be demonstrated ( $n a m e l y$ the $B R$ for $B^{-} \rightarrow \rho^{-}+\gamma$ must equal twice the $B R$ for $B^{0} \rightarrow \rho^{0}+\gamma$ ) before assuming that the LD ( $\bar{u} u)$ contributions are ignorable.

Lattice calculations of $B \rightarrow K^{*}+\gamma$ could also play a very useful role. If improved lattice calculations for $B \rightarrow K^{*} \gamma$ also do not agree in their determination of the ratio $H_{K^{*}} \equiv\left[B R\left(B \rightarrow K^{*}+\gamma\right) / B R(b \rightarrow s+\gamma)\right]$ with improved experimental measurements then the difference between the two must be at tributed to long distance pieces (presumably due to $c \bar{c}$ states) that the lattice calculations do not include. This then forms the second important necessary condition that must be verified before one can assume that $\mathrm{LD}(c \bar{c})$ contributions are ignorable.

## Note Added:

After this manuscript was sent out for publication we received a preprint by Golowich and Pakvasa [27] in which long distance contributions from $B \rightarrow K^{*} \psi$ to $B \rightarrow K^{*} \gamma$ are assessed. In our notation this corresponds to LD effects from $c \bar{c}$ states. Their numerical results for this effect are appreciably smaller than ours. The most important reason for this difference appears to be due to the fact that they have retained only $\psi \rightarrow \gamma$ conversion whereas we believe contribution from other charmonium states is also important especially as it adds coherently to this conversion (see Sec. 2.2). As we have stressed throughout this paper, theoretical estimates of these LD effects are extremely unreliable. In the last paragraph of Sec. 4 we have given what we believe is the most reliable quantitative measure of such LD contributions.

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## Figure Captions

Fig. 1 a-e A partial set of long distance contributions due to $u \bar{u}$ states. Those due to $c \bar{c}$ states typically result by replacing $u \rightarrow c$ in Fig. 1e.

Fig. $1 \mathbf{f}-\mathrm{g}$ Show typical penguin (short-distance) contributions.
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[5] J.M. Soares, Phys. Rev. D49, 283 (1994); see also Nucl. Phys. B367, 579 (1991).
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[9] See e.g. G. Hou, A. Soni, and H. Steger, Phys. Rev. Lett. 59, 1521 (1987).
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[18] C. Bernard, J. Labrenz, and A. Soni, Phys. Rev. D49, 2536 (1994); see also: R. M. Baxter et al. Phys. Rev. D49, 1594 (1994).
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[26] Although the LD contributions from $\bar{u} u$ states are expected to be different for $B^{-} \rightarrow K^{*-}+\gamma$ with that in $B^{0} \rightarrow K^{* 0}+\gamma$ (see eqns. (39) and (40)) they are both much too small compared to LD effects from $\bar{c} c$ states. Therefore in Table 1 the difference between equations (39) and (40) is being disregarded.
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(b)

(e)

(f) $\bar{d}(\bar{u})$


Fig. 1

