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PROBING VIOLATION OF QUANTUM MECHANICS IN THE K_0-$\bar{K}_0$ SYSTEM

PATRICK HUET*

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

I present a recent study made in collaboration with M.E. Peskin, on the time dependence of a kaon beam propagating according to the "$\alpha\beta\gamma$" generalization of quantum mechanics due to Ellis, Hagelin, Nanopoulos and Srednicki, in which CP- and CPT-violating signatures arise from the evolution of pure states to mixed states. The magnitude of two of its parameters β and γ are constrained on the basis of existing experimental data. New facilities such as ϕ-Factories are shown to be particularly adequate to study this generalization from quantum mechanics and to disentangle its parameters from other CPT violating perturbations of the kaon system.

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In the early 70's, unitary evolution predicted by quantum mechanics had already been given an experimental scrutiny.[2,3] Theoretical motivations arose from developments in the quantum theory of gravity which led S. Hawking to propose a generalization of quantum mechanics which allows the evolution of pure states to mixed states.[4] This formulation was shown by D. Page to conflict with CPT conservation.[5] Subsequently, Ellis, Hagelin, Nanopoulos, and Srednicki (EHNS)[6] observed that systems which exhibit quantum coherence over a macroscopic distance are most appropriate to probe the violation of quantum mechanics of the type proposed by S. Hawking. One of the simplest system exhibiting this property is a beam of neutral kaons. EHNS set up a generalized evolution equation for the K_0–$\bar{K}_0$ system in the space of density matrices which contains three new CPT violating parameters α,β and γ. These parameters have dimension of mass and could be as large as $m_K^2/m_{\rm Pl} \sim 10^{-19}$ GeV. This equation was subsequently used by Ellis, Mavromatos, and Nanopoulos[7] who exploited experimental data on K_L and K_S to delineate an allowed region in the space of the parameters α, β, and γ. This region is compatible with the expected order of magnitude above and with the possibility that violation of quantum mechanics accounts for all CP violation observed in the K_0–$\bar{K}_0$ system.

The present talk is a brief outline of a recent work,[1] which develops a general parameterization to incorporate CPT violation from both within and outside quantum mechanics and uses it to analyze past, present and future experiments on the K_0–$\bar{K}_0$ system. Studies of the time dependence of the kaon system of the early 1970's are combined with recent results from CPLEAR to constrain the EHNS parameters β and γ, limiting the contribution of violation of quantum mechanics to no more than 10% of the CP violation observed in the K_0–$\bar{K}_0$ system.

ϕ factories are new facilities[8] dedicated to the study of the properties of the kaon system; they are expected to give particularly incisive tests of CPT violation.[9] It is shown that these future facilities are especially suitable to test violation of quantum mechanics and to disentangle the EHNS parameters from other CPT violating perturbations.

1. Violation of quantum mechanics: the formalism.

The composition of a beam of neutral kaons is uniquely characterized by the density matrix $\rho_K(\tau)$ as function of the proper time τ. The value of the density matrix $\rho_K(\tau)$ is obtained from its value at the source – given a certain production mechanism, an effective Hamiltonian $H = M - \frac{i}{2}\Gamma$, which incorporates the natural width of the system,* and the laws of generalized quantum mechanics

$$i\frac{d}{d\tau}\rho_K = H\,\rho_K - \rho_K\,H^\dagger + \delta\not{h}\,\rho_K. \tag{1}$$

The first term of the RHS of this equation accounts for the quantum mechanical evolution, while the second term accounts for the loss of coherence in the evolution of the

*as well as CP and CPT perturbations compatible with quantum mechanics.

beam and is written so to preserve the linearity of the time-evolution. This term was written by EHNS so to require that it does not break conservation of probability and does not decrease the entropy of the system; that makes $\delta\not{h}$ expressible in terms of six parameters. In order to lower this number to a more tractable one, EHNS further assumed that this term conserves strangeness, reducing $\delta\not{h}$ to three unknown parameters α, β and γ. These parameters have the dimensions of mass and might be as large as $m_K^2/m_{\rm Pl} \sim 10^{-19}$ GeV.

The solution of Eq. (1) is generally expressible in terms of two real eigenvectors ρ_L, ρ_S and one complex eigenvector ρ_I and its complex conjugate $\rho_{\bar{I}}$, as

$$\rho_K(\tau) = A_L\rho_L^{(\diamond)}e^{-\Gamma_L\tau} + A_S\rho_S^{(\diamond)}e^{-\Gamma_S\tau} + A_I\rho_I^{(\diamond)}e^{-\bar{\Gamma}\tau}e^{-i\Delta m\tau} + A^{\dagger}\rho_{\bar{I}}^{(\diamond)}e^{-\bar{\Gamma}\tau}e^{+i\Delta m\tau}. \quad (2)$$

In the absence of the quantum mechanics violating term $\delta\not{h}\,\rho_K$ in Eq. (1), the eigenmodes ρ_L, ρ_S and ρ_I are expressible in terms of the pure states $|K_L\rangle$ and $|K_S\rangle$ as† $\rho_L^{(\diamond)} = |K_L\rangle\langle K_L|$, $\rho_S^{(\diamond)} = |K_S\rangle\langle K_S|$ and $\rho_I^{(\diamond)} = |K_S\rangle\langle K_L|$, while $\bar{\Gamma} = (\Gamma_L + \Gamma_S)/2$, $\Delta\Gamma = \Gamma_S - \Gamma_L$ and $\Delta m = m_L - m_S$.

In presence of the quantum mechanics violating term $\delta\not{h}\,\rho_K$, the eigenmodes are changed to, in first order in small quantities ($d = \Delta m + i\Delta\Gamma/2$)

$$\rho_L = \rho_L^{(\diamond)} + \frac{\gamma}{\Delta\Gamma}\rho_S^{(\diamond)} + \frac{\beta}{d}\rho_I^{(\diamond)} + \frac{\beta}{d^\star}\rho_{\bar{I}}^{(\diamond)}\,, \quad \rho_S = \rho_S^{(\diamond)} - \frac{\gamma}{\Delta\Gamma}\rho_L^{(\diamond)} - \frac{\beta}{d^\star}\rho_I^{(\diamond)} - \frac{\beta}{d}\rho_{\bar{I}}^{(\diamond)} \quad (3)$$

$$\rho_I = \rho_I^{(\diamond)} - \frac{\beta}{d^\star}\rho_S^{(\diamond)} + \frac{\beta}{d}\rho_L^{(\diamond)} - \frac{i\alpha}{2\Delta m}\rho_{\bar{I}}^{(\diamond)} \qquad \rho_{\bar{I}} = \rho_I^{\dagger}\,. \quad (4)$$

The corresponding eigenvalues are corrected by the shifts $\Gamma_{L,S} \to \Gamma_{L,S} + \gamma$, $\bar{\Gamma} \to \bar{\Gamma} + \alpha$ and $\Delta m \to \Delta m\cdot(1 - \frac{1}{2}(\beta/\Delta\Gamma)^2)$. The experimental relevance of these shifts is discussed in Ref. 1.

Any observable of the kaon beam can be computed by tracing ρ_K with an appropriate operator $\mathcal{P}$, we write $\langle\mathcal{P}\rangle = {\rm Tr}[\rho_K \mathcal{O}_{\mathcal{P}}]$. The major effect of violation of quantum mechanics is embodied in the eigenmodes ρ_L, ρ_S, ρ_I. These density matrices are no longer pure density matrices in contrast to their quantum mechanical counterparts. This loss of purity alters the decay properties of the beam. For example, the properties of the beam at large time, $\tau \gg 1/\Gamma_S$, are dominated by the properties of ρ_L. The second term on the RHS of the equation for ρ_L as given in Eq. (3) is proportional to $\rho_S^{(\diamond)}$ and is even under CP conjugation. That results in an enhancement of the rate of decay into two pions at late time in the evolution of the beam, proportional to $\frac{\gamma}{\Delta\Gamma}$. A similar argument leads to expect an enhancement by an amount $\propto \beta/|d|\cos\phi_{SW}$ and $\beta/|d|\sin\phi_{SW}$ in the intermediate time region, $\tau \sim 1/\bar{\Gamma}$.

2. Experimental constraints on β and γ.

We can exploit these facts to establish some experimental bounds on violation of quantum mechanics. Two observables are used in the analysis. The time dependent

†We use a diamond superscript to label quantum mechanical quantities.

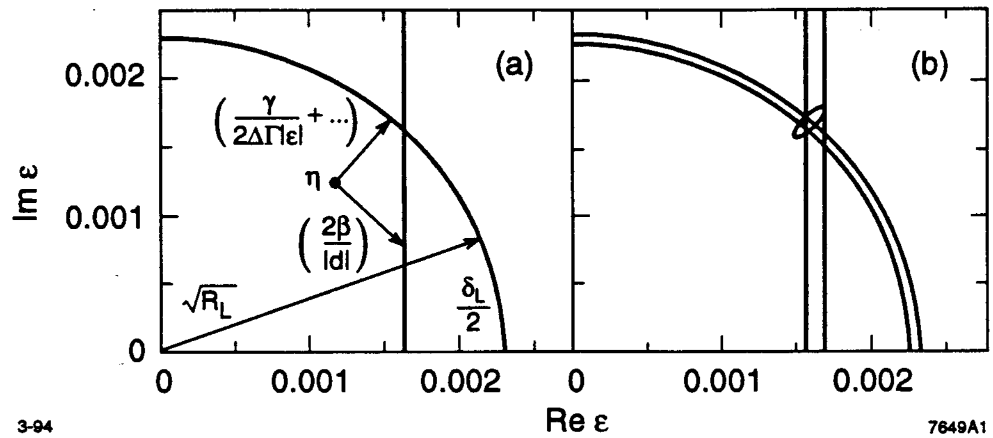
two-pion decay rate

$$\frac{\Gamma(K(\tau) \to \pi^+\pi^-)}{\Gamma(K(0) \to \pi^+\pi^-)} = \frac{\mathrm{Tr}\rho_K(\tau)\mathcal{O}_{+-}}{\mathrm{Tr}\rho_K(0)\mathcal{O}_{+-}} = e^{-\Gamma_S\tau} + 2|\eta_{+-}|^2 \cos(\Delta m\tau - \phi_{+-}) + R_L e^{-\Gamma_L\tau} \quad (5)$$

and the semileptonic decay rate at large time $\tau \gg 1/\Gamma_S$

$$\delta_L = \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\nu)}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\nu)} = \frac{\mathrm{Tr}\rho_K(\tau)(\mathcal{O}_{\ell^+} - \mathcal{O}_{\ell^-})}{\mathrm{Tr}\rho_K(\tau)(\mathcal{O}_{\ell^+} + \mathcal{O}_{\ell^-})}. \quad (6)$$

The relevant measurable quantities are R_L, δ_L and η_{+-}. R_L and δ_L reflect the large time properties of the beam; the complex number $\eta_{+-} = |\eta_{+-}|\exp(i\phi_{+-})$ is a property of the intermediate time region. In quantum mechanics, these quantities relate according to $R_L = |\eta_{+-}|^2$ and $\delta_L/2 = \mathrm{Re}\,\eta_{+-}$. After allowance has been made for violation of quantum mechanics, they relate according to $R_L \simeq |\eta_{+-}|^2 + \gamma/\Delta\Gamma + 4|\eta_{+-}|\beta/|d|$, $\delta_L/2 = \mathrm{Re}\,(\eta_{+-} - 2\beta/d)$. The geometry of these corrections is given in Fig. 1a . The current experimental situation is discussed in Ref. 1 and shown on Fig. 1b. The parameter β is proportional to the distance of the ellipse to the vertical band while the distance of the ellipse to the arc provides a measurement of γ. This comparison leads to the bounds $\beta = (0.12 \pm 0.44) \times 10^{-18}$ GeV and $\gamma = (-1.1 \pm 3.6) \times 10^{-21}$ GeV. In obtaining these bounds, we made allowance for CP and CPT violation in the Hamiltonian time evolution of the beam but we set to zero the CPT quantum mechanics perturbations of the decay amplitudes in the two-pion and semi-leptonic channels. In general,[1] the previous constraints on β and γ appear as constraints on combinations of CPT-violating parameters. Unless, unnatural cancellations occur among these parameters, they can be independently constrain, in which case, neither of them contributes more than 10% of the total CP violation observed in the K_0–$\bar{K}_0$ system.



-Fig.1-
Theoretical predictions (a) and experimental data (b).

3. Tests of quantum mechanics at a ϕ-factory.

At a ϕ factory, a spin-1 meson decays to an antisymmetric state of two kaons which propagates with opposite momentum. If the kaons are neutral, the resulting wavefunction, in the basis of CP eigenstates $|K_1\rangle$, $|K_2\rangle$, is $\phi \to (|K_1, p> \otimes |K_2, -p> - |K_2, p> \otimes |K_1, -p>)\sqrt{2}$. The two-kaon density matrix resulting from this decay is a 4×4 matrix P, which, in the context of generalized quantum mechanics, evolves according to Eq. (1). When expressed in terms of the eigenmodes ρ_L, ρ_S and ρ_I, it takes the form

$$\begin{aligned} P &= \frac{1}{2}\Big[\rho_S \otimes \rho_L + \rho_L \otimes \rho_S - \rho_I \otimes \rho_{\bar{I}} - \rho_{\bar{I}} \otimes \rho_I\Big] \\ &- 2\frac{\beta}{d}(\rho_S \otimes \rho_I + \rho_I \otimes \rho_S) - 2\frac{\beta}{d^*}(\rho_S \otimes \rho_{\bar{I}} + \rho_{\bar{I}} \otimes \rho_S) \\ &+ 2\frac{\beta}{d^*}(\rho_L \otimes \rho_I + \rho_I \otimes \rho_L) + 2\frac{\beta}{d}(\rho_L \otimes \rho_{\bar{I}} + \rho_{\bar{I}} \otimes \rho_L) \\ &+ i\frac{\alpha}{\Delta m}(\rho_I \otimes \rho_I - \rho_{\bar{I}} \otimes \rho_{\bar{I}}) + \frac{2\gamma}{\Delta\Gamma}(\rho_L \otimes \rho_L - \rho_S \otimes \rho_S). \end{aligned} \quad (7)$$

The time dependence of each term is obtained from the substitutions $\rho_i \otimes \rho_j \to \rho_i \otimes \rho_j exp(-\lambda_i \tau_1 - \lambda_j \tau_2)$ with $\lambda_L = \Gamma_L$, $\lambda_S = \Gamma_S$ and $\lambda_i = \bar{\Gamma} + i\Delta m$.

The first term in the brackets has the canonical form predicted by quantum mechanics after the replacement $\rho_K^{(\diamond)} \to \rho_K$, while the remaining terms give systematic corrections to this result. These new terms signal the breakdown of the antisymmetry of the final state wave function, that is, the breakdown of angular momentum conservation. This is expected in the framework of density matrix evolution equations, as was explained in Ref. 10.

The above peculiar dependence on τ_1 and τ_2 is a unique signature of violation of quantum mechanics and provides an unambiguous method to isolate the EHNS parameters from the quantum mechanics CPT violating perturbations of the decay rates. As in the one kaon system, any observable is obtained by tracing the density matrix with a suitable hermitian operator. The basic observables computed from P are double differential decay rates, the probabilities that the kaon with momentum p decays into the final state f_1 at proper time τ_1 while the kaon with momentum $(-p)$ decays to the final state f_2 at proper time τ_2. We denote this quantity as $\mathcal{P}(f_1,\tau_1;f_2,\tau_2)$. A situation of particular importance is the decay into two identical final states $f_1 = f_2$. This quantity has no dependence on the CP and CPT parameters and depends on the two times in a manner completely fixed by quantum mechanics irrespective of the properties of the decay amplitudes. This characteristic is lost when violation of quantum mechanics is incorporated as in Eq. (7). One can, for instance, interpolate the double decay rates into identical final states $\mathcal{P}(f,\tau_1;f,\tau_2)$ on the line of equal time $\tau_1 = \tau_2$. This quantity vanishes identically according to the principles of quantum mechanics and thus is of order α, β and γ. As an illustration, the semileptonic double decay rate at equal time yields($\ell^\pm \equiv \pi^\mp \ell^\pm \nu$)

$$\mathcal{P}(\ell^\pm,\tau;\ell^\pm,\tau)/\mathcal{P}(\ell^\pm,\tau;\ell^\mp,\tau) = \frac{1}{2}[1-e^{-2(\alpha-\gamma)\tau}(1-\frac{\alpha}{\Delta m}\sin 2\Delta m\tau)]+\frac{1}{2}\frac{\gamma}{\Delta\Gamma}[e^{+\Delta\Gamma\tau}-e^{-\Delta\Gamma\tau}] \tag{8}$$

$$\pm 4\frac{\beta}{|d|}[\sin(\Delta m\tau - \phi_{SW})e^{-\Delta\Gamma\tau/2} + \sin(\Delta m\tau + \phi_{SW})e^{+\Delta\Gamma\tau/2}]. \tag{9}$$

The three coefficients α, β, and γ are selected by terms which are monotonic in τ, oscillatory with frequency Δm,and oscillatory with frequency $2\Delta m$.

There seems to be no difficulty in constraining CPT violation from outside quantum mechanics in a ϕ factory independently of other CPT violating perturbation of quantum mechanics . The reverse is not true: any observable at a ϕ factory is expected to receive α, β and γ corrections. These corrections are, however, easily computed and can be systematically taken into account.[1]

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