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SPIN STRUCTURE OF THE NUCLEON AND POLARIZATION

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Abstract

Recent experiments at CERN and SLAC have added new knowledge about the spin structure of the proton and the deuteron. A brief historical background is presented, the status of experiments is discussed, and progress in the understanding of the spin of the nucleon in the context of the quark parton model is summarized.

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1 Introduction

Polarized lepton beams on polarized targets at high energies are naturally suited for probing the spin structure of the nucleon. The original motivation for these experiments goes back to the early days of deep inelastic scattering at SLAC in the late 1960s. In the quark-parton picture, the incoming lepton beam (electron or muon) interacts with point-like quark constituents through the exchange of a virtual photon. When the lepton is longitudinally polarized, the exchanged virtual photon carries a longitudinal component of its spin. This longitudinal component of the spin-one photon cannot be absorbed by a spin-half quark except in the case where the quark spin is aligned opposite to the incoming photon and undergoes spin flip. In a quark-parton model where the proton spin consists largely of the constituent quark spins, one predicts a large and positive asymmetry,

$$A_1^p = \frac{\sigma_a - \sigma_p}{\sigma_a + \sigma_p} \ , \tag{1}$$

where σ_a (σ_p) is the cross section for virtual photon absorption when the beam and target spins are antiparallel (parallel). Measurements of A_1^p and the corresponding asymmetry for the neutron, A_1^n, for a range of kinematic variables x and Q^2 form the basis of the present experimental programs in nucleon spin structure.

Bjorken was the first to discuss spin structure in the context of deep inelastic scattering [1]. In 1966 he derived a fundamentally important sum rule

$$I_p - I_n = \int_0^1 (g_1^p - g_1^n)\, dx = \frac{1}{6}|g_A/g_V| \tag{2}$$

in the limit $Q^2 \to \infty$, based on current algebra, scaling behavior of the structure functions in deep inelastic scattering, and isospin symmetry in the nucleon. The factor $|g_A/g_V|$ is the ratio of the nucleon axial vector and vector couplings, and is accurately measured to be 1.2573 ± 0.0028 from the beta decay of the neutron. The structure functions g_1^p and g_1^n are related to the asymmetries A_1^p and A_1^n and the unpolarized structure functions F_1^p and F_1^n by $g_1(x) \approx A_1(x)F_1(x)$, where x is the Bjorken scaling variable. The relation (2) anticipated by several years the dramatic proof of scaling in deep inelastic scattering[2]. It also predated the ideas of asymptotic freedom embodied in the modern theory of strong interactions, quantum chromodynamics (QCD). Because experiments do not achieve the $Q^2 = \infty$ limit, but in fact are carried out in the range $Q^2 \approx 2 - 10(GeV/c)^2$, there are QCD corrections to equation 2. Today the Bjorken sum rule includes these α_s corrections (calculated in perturbative QCD) to be $I_{Bj} = (1/6)|g_A/g_V|(1 - \alpha_s/\pi - 3.58(\alpha_s/\pi)^2 - 20.2(\alpha_s/\pi)^3 - ...)$ [3]. Testing of this relation has become something of an industry in the past decade. It is important because of its relationship to the strong interactions and QCD, but also because the relationship is a fundamental part of understanding how spin in the nucleon arises and what the role of the quark constituents is.

The study of spin of the nucleon presently revolves around two sum rules: (i) The Bjorken sum rule, and (ii) the Ellis-Jaffe sum rules first written in 1973 [4]. The Ellis-Jaffe sum rules are

$$I_p = \int_0^1 g_1^p(x)\, dx = (1/12)|g_A/g_V|[1 + 5/3(3F - D)/(F + D)] \tag{3}$$

and

$$I_n = \int_0^1 g_1^n(x)\, dx = (1/12)|g_A/g_V|[-1 + 5/3(3F - D)/(F + D)] \tag{4}$$

where F and D are SU(3) parameters and are derived from experimental measurements in the beta decay of the neutron and hyperons in the baryon octet. The best current values are

$F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$ [5]. As in the case of the Bjorken sum rule, these sum rules are modified by QCD corrections[3]. The Ellis-Jaffe sum rules assume that strange quarks in the nucleon are unpolarized. The Bjorken sum rule does not depend on the model assumptions made by Ellis-Jaffe. The Ellis-Jaffe sum rules teach us about spin structure of the nucleon while the Bjorken sum rule is a test of QCD.

The Bjorken and Ellis-Jaffe sum rules are clearly closely related, and can be tested together in experiments using polarized electrons or muons on polarized targets of hydrogen, deuterium, or ^{3}He.

2 The EMC Experiment

In 1988 the European Muon Collaboration (EMC) reported results on $g_1^p(x)$ from high-energy muons scattering from a polarized ammonia (NH_3) target[6]. Using their results, plus earlier results from SLAC experiments E80 and E130, the EMC reported a value $I_p = 0.126 \pm 0.018$, which was considerably lower than expected for I_p as defined in (3) with QCD corrections added.

The experimental asymmetries were seen to be low, particularly for $x \leq 0.1$. Using the measured I_p, plus additional input from hyperon decay and the assumption that the Bjorken sum rule is valid, the EMC reported a total quark spin in the nucleon $\Delta q = 0.12 \pm 0.017$ (on a scale where the proton spin is normalized to 1), and a strange quark contribution opposite to the proton spin, $\Delta s = -0.19 \pm 0.06$. These results were dubbed the "spin crisis," and many papers explaining and/or discussing the meaning of these surprising results appeared.

By 1988, the early spin-dependent structure functions experiments clearly saw the large and positive asymmetries predicted by the quark-parton model. The 1988 results showed that the low x asymmetries were lower than expected, and concluded that the nucleon spin did not reside significantly in the quarks. However, the fundamentally important Bjorken sum rule had not been tested.

The EMC results in 1988 stimulated further experimental work. At CERN a new collaboration formed, the Spin Muon Collaboration (SMC), and at SLAC a collaboration formed, E142, to study a new target, ^{3}He, using a new technology for polarizing the ^{3}He nucleus. Both groups set out to test the Bjorken sum rule and the Ellis-Jaffe sum rules.

3 Recent Measurements

In March of 1993, SMC reported asymmetries for the deuteron. They reported $I_d = 0.049 \pm 0.054$ (where the Ellis-Jaffe sum rules predict 0.154) and extracted a value for the total quark spin $\Delta q = 0.06 \pm 0.25$ and $\Delta s = -0.21 \pm 0.08$, which were consistent with the earlier EMC results and with the Bjorken sum rule.

In May 1993, the E142 Collaboration reported the results for ^{3}He. From E142 $I_n = -0.022 \pm 0.011$ (where the Ellis-Jaffe sum rule predicts -0.018) and concluded that the Ellis-Jaffe sum rule for the neutron was satisfied. They extracted from the neutron data the quark spin content for the proton. The E142 collaboration reported $\Delta q = 0.57 \pm 0.11$ and $\Delta s = -0.01 \pm 0.06$, and combined their neutron results with the only existing proton results (EMC) to give the Bjorken sum rule $I_{Bj} = 0.146 \pm 0.21$, which was about two standard errors low.

The results of the 1993 deuteron and ^{3}He experiments were inconclusive, in part due to differences in the treatments of the extrapolations to x=0. But beyond those differences uncertainty was derived from the much different Q^2 values for the experiments and the related uncertainties in the higher-order QCD corrections.

More experiments were planned. The SMC planned a new proton run at CERN, and at SLAC a new collaboration, E143, formed to study the proton and the deuteron again.

Extraction of the physics asymmetries $A_1^p(x)$ and $A_1^n(x)$ and the spin-dependent structure functions $g_1^p(x)$ and $g_1^n(x)$ is somewhat involved experimentally. The targets are complex and contain unwanted materials that have to be accounted for. One must first measure raw asymmetries, and then make corrections for these materials in the target, for beam and target polarizations P_b and P_t, and for unwanted polarization seen in the nitrogen in NH_3 and ND_3. The experimental asymmetries $A_\parallel$ and $A_\perp$ are given by

$$A_\parallel \ (or A_\perp) = \frac{N_a - N_p}{N_a + N_p} \frac{C_N}{f P_b P_t} + A_{RC} \tag{5}$$

where N_a (N_p) are the event counts for beam and target spins antiparallel (parallel), C_N is a correction for polarized nitrogen nuclei (≈ 0.98), f is a dilution factor that corrects for the unpolarized target nucleons, and A_{RC} is the radiative correction factor. The radiative corrections are applied to account for events that radiate into the acceptance from other kinematic territory (such as tails of elastic scattering).

The determination of g_1 is given by

$$g_1(x, Q^2) = \frac{F_1(x, Q^2)}{D} [A_\parallel + \tan(\theta/2) A_\perp] \tag{6}$$

where $D = (1-\epsilon)(2-y)/y(1+\epsilon R(x,Q^2))$, $R(x,Q^2) = \sigma_L/\sigma_T = (1+\gamma^2)F_2/(2xF_1) - 1$, $y = \nu/E$, $\gamma^2 = Q^2/\nu^2$, $\nu = E - E'$, $\epsilon^{-1} = 1 + 2[1 + (\nu^2/Q^2)]\tan^2(\theta/2)$, and θ is the scattering angle, which is typically small.

Measurement of $A_\perp$ is made in all of the recent experiments. This transverse asymmetry is generally small, but plays a role in the results when precise answers are sought. The measurement of $A_\perp$ is more important for the SLAC experiments because at lower energies it can be a bigger contribution.

Figure 1 shows the proton results from the EMC/SLAC experiment (1988), the SMC experiment (1994) and the E143 experiment (1994). Figure 2 shows the SMC deuteron results (1993) on the same scale as the proton in Figure 1. Figure 3 shows the E142 neutron results. Table I gives the results of the five experiments under these assumptions.

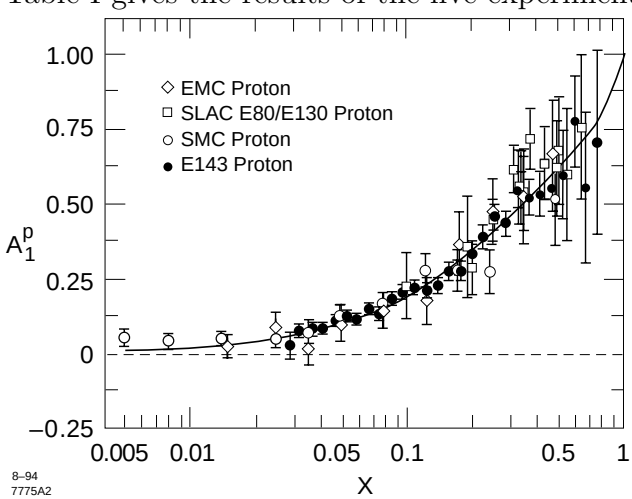

Figure 1. The proton physics asymmetry A_1^p for five experiments is plotted against x, showing the good agreement in the existing data. A solid line drawn through the data is a quartic polynomial fit, constrained to 0 at $x = 0$ and 1 at $x = 1$, to guide the eye.

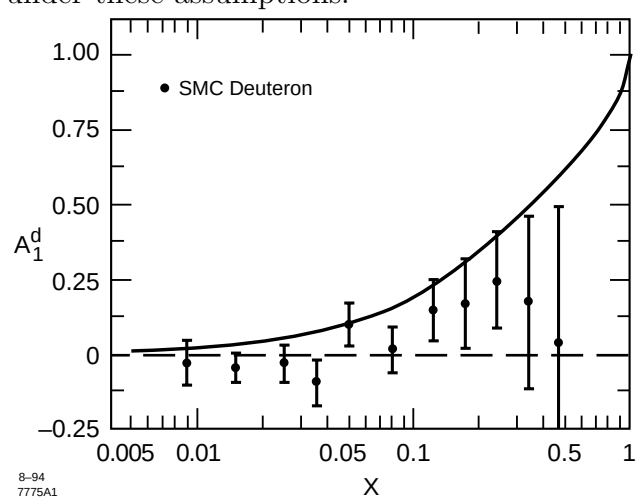

Figure 2. The SMC deuteron data is shown compared to the solid line, the same as in Figure 1, to represent the average of the proton data.

Table I. Summary of Experimental Sum Rules

Exp (year)	$< Q^2 > (GeV/c)^2$	Integral
EMC/SLAC proton (1988)	10.7	$I_p = \quad 0.126 \pm .010 \pm .015$
SMC deuteron (1993)	5	$I_d = \quad 0.023 \pm .020 \pm .015$
E142 neutron (1993)	2	$I_n = -0.022 \pm .007 \pm .009$
SMC proton (1994)	10	$I_p = \quad 0.136 \pm .011 \pm .011$
E143 proton (1994)	3	$I_p = \quad 0.129 \pm .004 \pm .009$

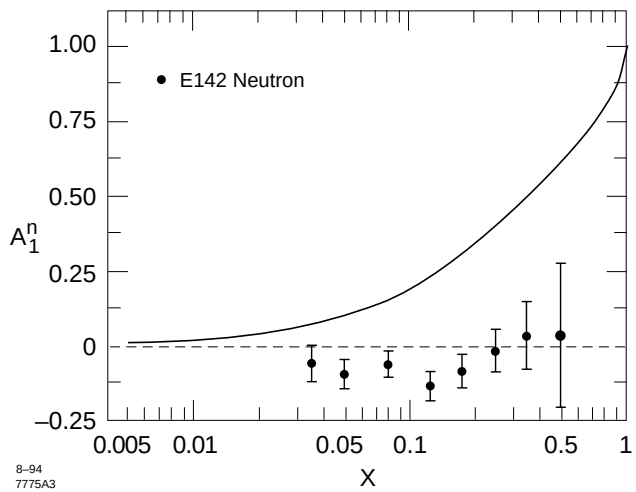

Figure 3. The E142 neutron data is shown compared to the solid line, the same as in Figure 1, to represent the average of the proton data.

4 Interpretation of the Results

Comparing these experimental results is not entirely trivial; they report integrals at different Q^2 values. In extracting the full integrals, extrapolations to $x = 0$ and to $x = 1$ have been done. The assumptions in how to do this extrapolation are not necessarily consistent among the experiments. Also, evaluation of g_1 from the measured A_1 values requires knowledge of F_1 from other experiments. These values are not fully agreed upon, and depending what is used, can influence the quoted integrals. Thus any comparison leaves some room for questions and adjustments. Nevertheless, one seems compelled to proceed with some comparisons.

For the following analysis, a fixed value of $Q^2 = 3$ was chosen. The predictions for the Bjorken sum rule and the Ellis-Jaffe sum rules have been calculated in perturbative QCD to $(\alpha_s/\pi)^3$ terms (for the flavor triplet and octet parts) and to $(\alpha_s/\pi)^2$ for the flavor singlet part[3]. The experimental values for the integrals are adjusted for the Q^2 dependence of α_s, assuming other factors remain constant. In all cases, the adjustments are small and considerably less than the quoted errors. For the deuteron, an additional correction must be made for the probability of the deuteron to be in a D state; $I_p + I_n = 2I_d/(1 - 1.5\omega_D)$, with $\omega_D = .058$.

Figure 4 displays the results on a two-dimensional plot of I_n versus I_p. The Bjorken sum rule is a line across the plane and the Ellis-Jaffe sum rules define a point on this plane, each

with a width given by the error in $\alpha_s = 0.35 \pm 0.02$. Three experiments on the proton are shown as vertical bands. The neutron result is a horizontal band, and the deuteron result is given by a diagonal band.

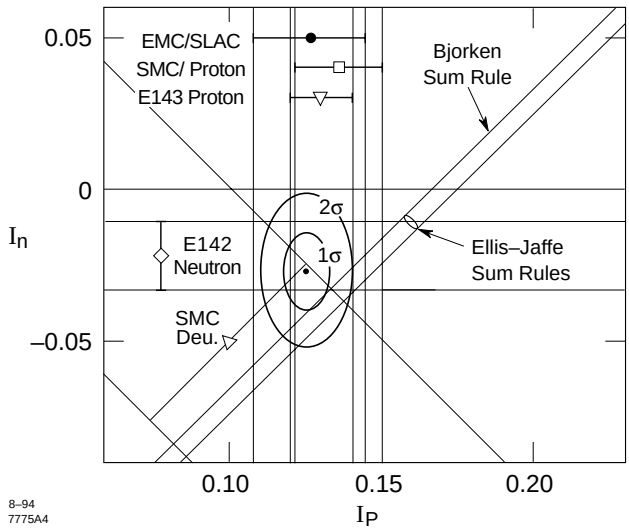

Figure 4. The world's data are combined in a plot of I_n versus I_p. The Ellis-Jaffe sum rules define a point in this plane, while the Bjorken sum rule is a narrow band across the plane at 45° to the horizontal. Three proton measurements, indicated by three data points with errors, give vertical bands in the plane, while the neutron measurement gives a horizontal band and the deuteron data gives a band at −45°. The overlap of these bands defines an ellipse centered at $(I_p, I_n) = (0.125,-0.027)$. The one- and two-sigma contours are indicated.

Figure 4 shows one-sigma and two-sigma contours where the data overlap. The best value falls at $(I_p, I_n) = (0.125 \pm .007, -0.027 \pm .012)$ with a $\chi^2 = 0.97$ for 3 degrees of freedom. The Bjorken sum rule is seen to approach these values to within one- and two-sigma contours. The Bjorken sum rule misses the best value by an amount δ, where $\delta/I_{Bj} = 0.08$ (or 1.4 sigma). One concludes that the Bjorken sum rule is satisfied within the experimental error of about 8%. On the other hand, the Ellis-Jaffe sum rules fall considerably outside the experimental errors.

The spin content of the proton is not fully understood. It can be extracted from the above experimental integrals under some assumptions. The assumptions involve three equations relating the spin content Δu, Δd, and Δs, where $\Delta u = \int_0^1 (u^\uparrow(x) - u^\downarrow(x) + \bar{u}^\uparrow(x) - \bar{u}^\downarrow(x))\, dx$, etc. From the flavor octet term a_8

$$a_8 = \Delta u + \Delta d - 2\Delta s = 3F - D. \tag{7}$$

From the Bjorken sum rule

$$\Delta u - \Delta d = |g_A/g_V| = 1.2573 \pm 0.0028 \ , \tag{8}$$

which is measured in neutron beta decay. From the paper of S. A. Larin listed in reference (3).

$$\begin{aligned} I_{p(n)} = {} & [+(-)\frac{1}{12}\,|g_A/g_V| + \frac{1}{36}a_8](1 - \alpha_s/\pi - 3.25(\alpha_s/\pi)^2 - 20.215(\alpha_s/\pi)^3) \\ & + \frac{1}{9}\Delta q(1 - \alpha_s/\pi - 1.096(\alpha_s/\pi)^2)(1 - 0.667\alpha_s/\pi - 1.213(\alpha_s/\pi)^2) \ , \end{aligned} \tag{9}$$

where $\Delta q = \Delta u + \Delta d + \Delta s$. Table II gives the results for Δq and Δs from the five experiments under these assumptions.

5 Summary

The Bjorken sum rule, first derived in 1966, has very recently seen results that are in good agreement with the experiments when higher-order QCD corrections are included. The present combined experiments fall between one and two σ of the predicted values. This result is in fact an agreement at about the 8% level, which is very good compared to experimental evaluation of other sum rules in deep inelastic scattering. The Ellis-Jaffe sum rules on the other hand appear to be violated. The source of the discrepancy may be due to polarization of the strange quark and antiquark sea which appears negative in all quark-parton model evaluations from the experiments.

Recent extensions of the higher-order corrections to the sum rules have been modifying the basic conclusions to some extent and seem to have helped by improving agreement between the various measurements. The effect of these higher-order corrections has been to bring the quark parton model interpretations of the low-Q^2 and high-Q^2 data closer together.

Future results are in process or are being planned. The E143 collaboration has more data on proton, deuteron, and transverse structure functions yet to be published. The SMC collaboration is presently running on deuterium, and should report new results in the future. At SLAC, future runs at 50 GeV are planned, and DESY now has an improved run with HERMES at HERA, which is scheduled to begin next year. The electron ring at HERA has recently operated successfully with high transverse polarization and longitudinally polarized electrons at the HERMES target point. We can look forward to several more years of improving data on the nucleon spin structure.

Table II. Quark Parton Model Interpretation of the Experimental Results

Exp (year)	$< Q^2 >$	α_s	Δ q	Δ s
EMC/SLAC proton (1988)	10.7	0.27	$0.19 \pm .17$	$-0.13 \pm .06$
SMC deuteron (1993)	5	0.26	$0.10 \pm .25$	$-0.16 \pm .09$
E142 neutron (1993)	2	0.39	$0.45 \pm .10$	$-0.04 \pm .04$
SMC proton (1994)	10	0.23	$0.25 \pm .13$	$-0.11 \pm .04$
E143 proton (1994)	3	0.35	$0.29 \pm .09$	$-0.10 \pm .03$
Average	—	—	$0.31 \pm .06$	$-0.09 \pm .02$
Ellis-Jaffe prediction	—	—	0.58	0.0

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