# MULTIPLICITY MOMENTS IN $e^{+} e^{-}$ANNIHILATION INTO HADRONS AT THE $Z^{0}$ RESONANCE * 

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#### Abstract

We present the ratio of cumulant to factorial moments of multiplicity distributions in hadronic events from $Z^{0}$ decays. Our preliminary result shows that this ratio, as a function of moment rank $q$, decreases sharply to a negative minimum at $q \sim 5$, followed by a sequence of quasi-oscillations. These observed features are in qualitative agreement with expectations from higher-order perturbative QCD.


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## 1. Multiplicity Distributions and QCD

Multiplicity distributions of particles produced in high energy $e^{+} e^{-}$collisions have been the subject of intense experimental and theoretical investigation. Quantum chromodynamics (QCD) offers natural explanations for such features as KNO-scaling. ${ }^{1,2}$ However, in lowest order perturbation theory, the predicted distributions are wider than the experimentally observed ones. ${ }^{3}$ Efforts to include higher-order effects ${ }^{4-6}$ show that the resulting distributions should be narrower than predicted at lowest order. A new quantity, the ratio of cumulant to factorial moments, $H_{q}=K_{q} / F_{q}$, has recently been proposed ${ }^{7}$ and shown to be very sensitive to higher-order effects in multiplicity distributions, ${ }^{8}$ and is studied here.

The factorial moment of rank $q, F_{q}$, is defined as

$$
\begin{equation*}
F_{q} \equiv \frac{<n(n-1) \ldots(n-q+1)>}{<n>^{q}}=\frac{\sum_{n} n(n-1) \ldots(n-q+1) P(n)}{\left(\sum_{n} n P(n)\right)^{q}} \tag{1}
\end{equation*}
$$

where $P(n)$ is the probability for production of $n$ particles in an event, and $<n>$ is the average multiplicity in the event sample. The cumulant moments $K_{q}$ are related to $F_{q}$ by the formula ${ }^{9}$

$$
\begin{equation*}
F_{q}=\sum_{m=0}^{q-1} C_{q-1}^{m} K_{q-m} F_{m} \tag{2}
\end{equation*}
$$

Here $C_{q-1}^{m}=\frac{(q-1)!}{m!(q-m-1)!}$ are the binomial coefficients, and $F_{0}=F_{1}=K_{1}=1$. Eq. 2 allows one to solve for the $K_{q}$. Thus, $F_{q}, K_{q}$, and $H_{q}$ can be determined from the multiplicity distribution $P(n)$.

Some phenomenological models ${ }^{8,9}$ of particle production have been examined to demonstrate the sensitivity of $H_{q}$. For instance, $H_{q}$ is identically equal to zero for a Poisson distribution while for the negative binomial distribution (NBD) it gives rise to $H_{q} \sim q^{-k}$, where $k$ is the NBD parameter. In perturbative QCD the moments have been calculated ${ }^{7-10}$ in next-to-next-to leading order, neglecting corrections involving quarks. While the leading double logarithmic approximation (DLA) predicts $H_{q}$ monotonically decreasing to zero as $H_{q} \sim q^{-2}$, including the higher order corrections introduces additional features. Next-to-leading corrections give a minimum in $H_{q}$ for $q \sim 5$, and next-to-next-to-leading corrections predict that this minimum is negative, followed by quasi-oscillatory behavior at larger $q$. Neglecting quarks apparently has little effect ${ }^{9,11}$ on these features.

## 2. Data Analysis and Results

The SLAC Linear Collider (SLC) produces $e^{+} e^{-}$annihilation events at the $Z^{0}$ resonance which are recorded by the SLC Large Detector (SLD). The detector is described in detail elsewhere. ${ }^{12}$ The present analysis relies primarily on information from the Central Drift Chamber in which the charged particles are tracked and momentumanalysed. In addition, a silicon vertex detector provides an accurate measure of particle


Fig. 1. Ratio of cumulant to factorial moments, $H_{q}$. The solid curve is to guide the eye.
trajectories close to the beam axis. A set of cuts was applied to select well-measured tracks and hadronic events well-contained within the detector acceptance. ${ }^{13}$ Approximately 14,000 events surviving these cuts are included in the analysis presented here.

The moments $F_{q}, K_{q}$, and $H_{q}$ were calculated up to rank $q=17$, and the resulting $H_{q}$ are shown in Fig. 1 for $q>2$. It is clear that $H_{q}$ falls rapidly at the lower ranks and reaches a negative minimum at $q \sim 5$. For increasing $q, H_{q}$ apparently exhibits a quasi-oscillatory behavior.

We have examined sources of experimental measurement error. Systematic effects were found to be small compared with statistical errors. For ranks greater than $q \sim 7$, the observed features in $H_{q}$, for example the phase of the oscillations, are sensitive to the statistics of our data set. However, the steep decrease at small $q$, the first negative minimum near $q=5$, and the existence of quasi-oscillatory behavior at larger $q$ are all features of the data which appear to be well established. The errors shown in Fig. 1 are the statistical errors estimated from Monte Carlo data sets of the same statistical size as the data. For this preliminary result, we have not explicitly applied corrections to the data to account for the effects of acceptance and resolution.

The observed qualitative features exhibited in Fig. 1 are in good agreement with the predictions from the higher-order perturbative QCD calculations discussed above. The DLA parameterization of multiplicity, based on leading order QCD, and the phenomenological NBD distribution are clearly inconsistent with our data. An analysis ${ }^{14}$ of existing LEP data yielded similar conclusions.

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