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**Polarization as a Probe to the Production Mechanisms of Charmonium in $\pi N$ Collisions**[1]

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**Abstract**

Measurements of the polarization of $J/\psi$ produced in pion-nucleus collisions are in disagreement with leading twist QCD prediction where $J/\psi$ is observed to have negligible polarization whereas theory predicts substantial polarization. We argue that this discrepancy cannot be due to poorly known structure functions nor the relative production rates of $J/\psi$ and $\chi_J$. The disagreement between theory and experiment suggests important higher twist corrections, as has earlier been surmised from the anomalous non-factorized nuclear $A$-dependence of the $J/\psi$ cross section.

# 1 Introduction

One of the most sensitive tests of the QCD mechanisms for the production of heavy quarkonium is the polarization of the $J/\psi$ in hadron collisions. In fact, there are serious disagreements between leading twist QCD prediction [2] and experimental data [3, 4, 5, 6, 7] on the production cross section of 'direct' $J/\psi$ and $\chi_1$. We would like to advocate that polarization of $J/\psi$ provides strong constraints on the production mechanisms of $J/\psi$ and thus can pinpoint the origin of these disagreements.

In this paper we will present some preliminary results on the theoretical calculation of the polarization of $J/\psi$ in $\pi N$ collisions. The completed analysis will be published in a later paper[1]. We found that the polarization of $J/\psi$ provides important constraints on the nature of the production mechanisms and urge that polarization measurement of $J/\psi$ should be included in the design of future charm production experiment.

The paper is organized as follow. In section 2, we show that from the experimental data on the production cross sections and leptonic decay widths of direct $J/\psi$ and $\psi'$, the long distance physics of formation of bound states of $c\bar{c}$ can be separated from the short distance physics of production of the $c\bar{c}$ pair. Thus, the perturbative analysis is under control in

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calculating $J/\psi$ production even though the mass of charm quark is not much larger than $\Lambda_{QCD}$. Once the validity of perturbative method is established, we calculate the production cross sections of direct $J/\psi$, $\chi_1$ and $\chi_2$ in $\pi N$ collisions in PQCD. These results are presented in section 3 and discrepancies are observed. We show that, in comparison with the recent E705 and E672 data [8, 10], the predicted ratio of direct $J/\psi$ production compared to the $\chi_2$ production is too low by a factor of about 3. In addition the production ratio of production cross sections of $\chi_1$ to $\chi_2$ is too low by a factor of 10 compared to data. A similar conclusion has been reached in [11]. The polarization data of $J/\psi$ [12, 13, 14] allows us to make further conclusion of the origin of the disagreements. In section 4, we find that even if the relative production rates of the $J/\psi$, $\chi_1$ and $\chi_2$ are adjusted (using $K$-factors) to agree with the data, the $J/\psi$ polarization data is still not reproduced. Therefore, the discrepancies do not arise from an incorrect relative normalization of the various channels and new production mechanisms are needed. We will present our conclusion in the last section.

# 2 Can direct $J/\psi$ production be calculated in PQCD?

In leading twist QCD, the production of the $J/\psi$ at low transverse momentum occurs both 'directly' from the gluon fusion subprocess $gg \to J/\psi + g$ [Fig. 1a] and indirectly via the production and decay of $\chi_1$ and $\chi_2$ states. These states have sizable decay branching fractions $\chi_{1,2} \to J/\psi + \gamma$ of 27% and 13%, respectively.

Figure 1: Fig. 1a shows direct $J/\psi$ production through $gg$ scattering. The formation of bound state is described by the wavefunction $\Psi^*_{J/\psi}(0)$ at the origin. Fig. 1b shows leptonic decay of $J/\psi$ into $e^+e^-$ pair. The probability of finding the $c\bar{c}$ pair is given by the wavefunction $\Psi_{J/\psi}(0)$.

In this model, we assume that the non-perturbative physics, which is described by the wave function at the origin in cases of production of $J/\psi$ and $\psi'$, is separable from the perturbative hard subprocess, $i.e.$, factorization holds. As the wave function at the origin can be related to the leptonic decay amplitude [Fig. 1b], the ratio of $\psi'$ to direct $J/\psi$ production can be expressed in terms of the ratio of their leptonic decay width. More precisely, taking into account of the phase space factor,

$$\frac{\sigma(\psi')}{\sigma_{dir}(J/\psi)} \simeq \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \frac{M^3_{J/\psi}}{M^3_{\psi'}} \simeq 0.24 \pm 0.03 \tag{1}$$

where $\sigma_{dir}(J/\psi)$ is the cross section for direct production of the $J/\psi$. The ratio (1) should hold for all beams and targets, independently of the size of the higher twist corrections in producing the point-like $c\bar{c}$ state. The energy should be large enough for the bound state to form outside the target. The available data is indeed compatible with (1). In particular, the E705 value [8] is 0.24. In Table 1, the ratio of $\psi'$ to direct $J/\psi$ production with different projectiles is presented. They are all consistent with the value 0.24.

| | $\sigma(\psi')$ [nb] | $\sigma_{dir}(J/\psi)$ | $\sigma(\psi')/\sigma_{dir}(J/\psi)$ |
|---|---|---|---|
| $\pi^+$ | $22 \pm 5$ | $97 \pm 14$ | $0.23 \pm 0.07$ |
| $\pi^-$ | $25 \pm 4$ | $102 \pm 14$ | $0.25 \pm 0.05$ |
| $p$ | $20 \pm 3$ | $89 \pm 12$ | $0.23 \pm 0.05$ |

Table 1: Production cross sections for $\psi'$, direct $J/\psi$ and their ratio in $\pi^+N$, $\pi^-N$ and $pN$ collisions. The data is from Ref. [8].

The anomalous nuclear target $A$-dependence observed for the $J/\psi$ is also seen for the $\psi'$ [15], so that the ratio (1) is indeed independent of $A$. Therefore, at high energies, the quarkonium bound state forms long after the production of the $c\bar{c}$ pair and the formation process is well described by the non-relativistic wavefunction at the origin.

# 3 Production rates of $\psi$ and $\chi_J$ states at leading twist

In leading twist and to leading order in $\alpha_s$, $J/\psi$ production can be computed from the convolution of hard subprocess cross section $gg \to J/\psi g$, $gg \to \chi_j$, *etc.*, with the parton distribution functions in the beam and target. Higher order corrections in $\alpha_s$, and relativistic corrections to the charmonium bound states, are unlikely to change our qualitative conclusions at moderate $x_F$. Contributions from direct $J/\psi$ production, as well as from indirect production via $\chi_1$ and $\chi_2$ decays, will be included. Due to the small branching fraction $\chi_0 \to J/\psi + \gamma$ of 0.7%, the contribution from $\chi_0$ to $J/\psi$ production is expected (and observed) to be negligible. Decays from the radially excited $2^3S_1$ state, $\psi' \to J/\psi + X$, contribute to the total $J/\psi$ rate at the few per cent level and will be ignored here.

The $\pi N \to \chi_2 + X$ production cross section to lowest order is

$$\sigma(\pi N \to \chi_2 + X;\ x_F > 0) = \int_{\sqrt{\tau}}^1 \frac{dx_1}{x_1} F_{g/\pi}(x_1) F_{g/N}(\tau/x_1) \sigma_0(gg \to \chi_2) \tag{2}$$

where $\tau = M^2_{\chi_2}/s$ and the quantity $\sigma_0(gg \to \chi_2) = 16\pi^2 \alpha_s^2 |R'_P(0)|^2 / M^7_{\chi_2}$ [18]. We restrict the $\chi_2$ momentum range to the forward CM hemisphere ($x_F > 0$) in accordance with the available data, and use the structure functions of Ref. [16, 17] evaluated at $Q^2 = M_{\chi_2}^2$. We also take the renormalization scale to be $Q^2 = M_{\chi_2}^2$.

The direct $\pi N \to J/\psi + X$ cross section is similarly given by

$$\begin{aligned} \sigma(\pi N \to J/\psi + X;\ x_F > 0) &= \int_\tau^1 dx_1 \int_{\tau/x_1}^1 dx_2 \int_{\hat{t}_{\min}}^0 d\hat{t} F_{g/\pi}(x_1) F_{g/N}(x_2) \\ &\quad \times \frac{d\sigma}{d\hat{t}}(gg \to J/\psi + g) \end{aligned} \tag{3}$$

where $\hat{t}$ is the invariant momentum transfer in the subprocess, and

$$\hat{t}_{\min} = \max\left(\frac{x_2 M_{J/\psi}^2 - x_1 \hat{s}}{x_1 + x_2}, M_{J/\psi}^2 - \hat{s}\right). \tag{4}$$

Eq. (3) also applies to the $\pi N \to \chi_1 + X$ reaction, in which case a sum over the relevant subprocesses $gg \to \chi_1 g$, $gq \to \chi_1 q$, $g\bar{q} \to \chi_1 \bar{q}$ and $q\bar{q} \to \chi_1 g$ is necessary. The differential cross sections $d\sigma/d\hat{t}$ for all subprocesses are given in [18, 19].

In Table 2 we compare the $\chi_2$ production cross section, and the relative rates of direct $J/\psi$ and $\chi_1$ production, with the data of E705 and WA11 on $\pi^- N$ collisions at $E_{lab} = 300$ GeV and 185 GeV [8].

| | $\sigma(\chi_2)$ [nb] | $\sigma_{dir}(J/\psi)/\sigma(\chi_2)$ | $\sigma(\chi_1)/\sigma(\chi_2)$ |
|---|---|---|---|
| Experiment | $188 \pm 30 \pm 21$ | $0.54 \pm 0.11 \pm 0.10$ | $0.70 \pm 0.15 \pm 0.12$ |
| Theory | 72 | 0.19 | 0.069 |

Table 2: Production cross sections for $\chi_1$, $\chi_2$ and directly produced $J/\psi$ in $\pi^- N$ collisions. The data from Ref. [8, 9] include measurements at 185 and 300 GeV. The theoretical calculation is at 300 GeV.

The $\chi_2$ production rate in QCD agrees with the data within a '$K$-factor' of order 2 to 3. This is within the theoretical uncertainties arising from the $J/\psi$ and $\chi$ wavefunctions, higher order corrections, structure functions, and the renormalization scale. A similar factor is found between the lowest-order QCD calculation and the data on lepton pair production [20, 21]. On the other hand, Table 2 shows a considerable discrepancy between the calculated and measured relative production rates of direct $J/\psi$ and $\chi_1$, compared to $\chi_2$ production. A *priori* we would expect the $K$-factors to be roughly similar for all three processes. We conclude that leading twist QCD appears to be in conflict with the data on direct $J/\psi$ and $\chi_1$ production. Although in Table 2 we have only compared our calculation with the E705 and WA11 $\pi^- N$ data, this comparison is representative of the overall situation (for a recent comprehensive review see [11]).

# 4 Polarization of the $J/\psi$

The polarization of the $J/\psi$ is determined by the angular distribution of its decay muons in the $J/\psi$ rest frame. By rotational symmetry and parity, the angular distribution of massless muons, integrated over the azimuthal angle, has the form

$$\frac{d\sigma}{d\cos\theta} \propto 1 + \lambda \cos^2\theta \tag{5}$$

where we take $\theta$ to be the angle between the $\mu^+$ and the projectile direction (*i.e.*, we use the Gottfried–Jackson frame). The parameter $\lambda$ can be calculated from the $c\bar{c}$ production amplitude and the electric dipole approximation of radiative $\chi$ decays.

The electric dipole approximation of the radiative decay $\chi_J \to \psi\gamma$ is exact in the heavy quark limit; *i.e.*, when terms of $\mathcal{O}(E_\gamma/m_c)$ are neglected. As a consequence, the heavy quark spins are conserved in the decay, while the orbital angular momentum changes.

The lowest order subprocess $g(\mu_1)g(\mu_2) \to c\bar{c} \to \chi_2(J_z)$ only produces $\chi_2$ with $J_z = \pm 2$ states asumming that the transverse momenta of the incoming gluons are neglected. In the $J_z = \pm 2$ polarization state the spin and orbital angular momenta of its constituent charm quarks are aligned, $S_z = L_z = \pm 1$. Since $S_z$ is conserved in the radiative decay $\chi_2 \to J/\psi + \gamma$, it follows that $J_z(J/\psi) = S_z = \pm 1$ ($L = 0$ for the $J/\psi$). Thus the $J/\psi$'s produced via $\chi_2$ decay are transversely polarized, *i.e.*, $\lambda = 1$ in (5). This result is exact if both the photon recoil and the intrinsic transverse momenta of the incoming partons are neglected. Smearing of the beam parton's transverse momentum distribution by a Gaussian function $\exp\left[-(k_\perp/500\text{ MeV})^2\right]$ would bring $\lambda$ down to $\lambda \simeq 0.85$.

From the $gg \to J/\psi + g$ amplitude we find for direct $J/\psi$ production, $\pi N \to J/\psi + X \to \mu^+\mu^- + X$,

$$\begin{aligned} \frac{1}{B_{\mu\mu}} \frac{d\sigma}{dx_F d\cos\theta} &= \frac{3}{64\pi} \int \frac{dx_1 dx_2}{(x_1+x_2)s} F_{g/\pi}(x_1) F_{g/N}(x_2) \\ &\quad \times \left[\varrho_{11} + \varrho_{00} + (\varrho_{11} - \varrho_{00})\cos^2\theta\right] \end{aligned} \tag{6}$$

where $B_{\mu\mu}$ is the $J/\psi \to \mu^+\mu^-$ branching fraction, $x_F = 2p^z_\psi/\sqrt{s}$ is the longitudinal-momentum fraction of the $J/\psi$, and $\theta$ is the muon decay angle of Eq. (5). The $\varrho_{11}, \varrho_{00}$ are the density matrix elements and can be found in [1].

For the $\pi N \to \chi_1 + X \to J/\psi + \gamma + X \to \mu^+\mu^- + \gamma + X$ production process we get similarly

$$\begin{aligned} \frac{1}{B_{\mu\mu}} \frac{d\sigma}{dx_F d\cos\theta} &= \frac{3}{128\pi} \mathrm{Br}(\chi_1 \to \psi\gamma) \Sigma_{ij} \int \frac{dx_1 dx_2}{(x_1+x_2)s} F_{i/\pi}(x_1) F_{j/N}(x_2) \\ &\quad \times \left[\varrho^{ij}_{00} + 3\varrho^{ij}_{11} + (\varrho^{ij}_{00} - \varrho^{ij}_{11})\cos^2\theta\right], \end{aligned} \tag{7}$$

where the density matrix elements for $ij = gg$, $gq$ $g\bar{q}$ and $q\bar{q}$ scattering are again given in [1].

In Fig. 2a we show the predicted value of the parameter $\lambda$ of Eq. (5) in the GJ-frame as a function of $x_F$, separately for the direct $J/\psi$ and the $\chi_{1,2} \to J/\psi + \gamma$ processes. Direct $J/\psi$ production gives $\lambda \simeq 0.25$, whereas the production via $\chi_1$ results in $\lambda \simeq -0.15$.

The $\lambda(x_F)$-distribution obtained when both the direct and indirect $J/\psi$ production processes are taken into account is shown in Fig. 2b and is compared with the Chicago–Iowa–Princeton [13] and E537 data [14] for 252 GeV $\pi W$ collisions and 150 GeV $\pi^- W$ collisions respectively. Our QCD calculation gives $\lambda \simeq 0.5$ for $x_F \lesssim 0.6$, significantly different from the measured value $\lambda \simeq 0$.

Figure 2: CIP (•) and E537 (○) data compared with theoretical prediction. Fig. 2a shows the parameter $\lambda$ from different contributions: direct $J/\psi$, $\chi_{1,2} \to J/\psi + \gamma$ processes. Solid curves shows the results with the intrinsic transverse momentum of the incoming partons neglected while the dashed curves have the beam parton's transverse momentum modeled by a Guassian function $\exp[-(k_\perp/500\mathrm{MeV})^2]$. Fig. 2b takes into account both the direct and indirect $J/\psi$ production: without $K$ factors correction (solid curve), and with $K$ factors correction (dashed curve).

The discrepancies between the calculated and measured values of $\lambda$ is one further indication that the standard leading twist processes considered here are not adequate for explaining charmonium production. The $J/\psi$ polarization is particularly sensitive to the production mechanisms and allows us to make further conclusions on the origin of the disagreements, including the above discrepancies in the relative production cross sections of $J/\psi$, $\chi_1$ and $\chi_2$. If these discrepancies arise from an incorrect relative normalization of the various subprocess contributions (*e.g.*, due to higher order effects), then we would expect the $J/\psi$ polarization to agree with data when the relative rates of the subprocesses are adjusted according to the measured cross sections of direct $J/\psi$, $\chi_1$ and $\chi_2$ production[3]. The dashed curve in Fig. 2b shows the effect of multiplying the partial $J/\psi$ cross sections with the required $K$-factors. The $\lambda$ parameter is still predicted incorrectly over most of the $x_F$ range.

A similar conclusion is reached (within somewhat larger experimental errors) if we compare our calculated value for the polarization of direct $J/\psi$ production, shown in Fig. 2a, with the measured value of $\lambda$ for $\psi'$ production. In analogy to Eq. (1), the $\psi'$ polarization data should agree with the polarization of directly produced $J/\psi$'s, regardless of the produc-

[3]In the case of Drell-Yan virtual photon production, it is known that higher-order corrections do not change the $\gamma^*$ polarization significantly [22], which makes it plausible to represent these corrections by a simple multiplicative factor, which does not affect the polarization of the photon.

tion mechanism. Based on the angular distribution of the muons from $\psi' \to \mu^+\mu^-$ decays in 253 GeV $\pi^- W$ collisions, Ref. [23] quotes $\lambda_{\psi'} = 0.02 \pm 0.14$ for $x_F > 0.25$, appreciably smaller than our QCD values for direct $J/\psi$'s in Fig. 2a.

# 5 Discussion

We have seen that the $J/\psi$ and $\chi_1$ hadroproduction cross sections in leading twist QCD are at considerable variance with the data, whereas the $\chi_2$ cross section agrees with measurements within a reasonable $K$-factor of 2 to 3. On the other hand, the inclusive decays of the charmonium states based on the minimal perturbative final states ($gg$ and $q\bar{q}g$) have been studied in detail using perturbation theory [24, 25, 11], and appear to work fairly well. It is therefore improbable that the treatment of the $c\bar{c}$ binding should require large corrections. This conclusion is supported by the fact that the relative rate of $\psi'$ and direct $J/\psi$ production (Eq. 1), which at high energies should be independent of the production mechanism, is in agreement with experiment.

In a leading twist description, an incorrect normalization of the charmonium production cross sections can arise from large higher order corrections or uncertainties in the parton distributions[11]. Taking into account that the normalization may be wrong by as much as a factor of 10 and that even such a $K-$factor does not explain the polarization data of $J/\psi$, a more likely explanation may be that there are important higher-twist contributions to the production of the $J/\psi$ and $\chi_1$ as suggested in large $x_F$ case [26, 27].

Further theoretical work is needed to establish that the data on direct $J/\psi$ and $\chi_1$ production indeed can be described from higher twist mechanisms. Experimentally, it is important to check whether the $J/\psi$'s produced indirectly via $\chi_2$ decay are transversely polarized. This would show that $\chi_2$ production is dominantly leading twist, as we have argued. Thus, the polarization of $J/\psi$ production from different channels provides a very sensitive discriminant of different production mechanisms.

# References


[1] M. Vänttinen, P. Hoyer, S. J. Brodsky and W.-K. Tang, in preparation.

[2] V. Barger and A. D. Martin, Phys. Rev. **D31**, 1051 (1985).

[3] A. G. Clark, *et al.*, Nucl. Phys. **B142**, 29 (1978).

[4] R806: C. Kourkoumelis *et al.*, Phys. Lett. **B81**, 405 (1979)

[5] WA11: Y. Lemoigne, *et al.*, Phys. Lett. **B113**, 509 (1982).

[6] E673: S. R. Hahn, *et al.*, Phys. Rev. **D30**, 671 (1984); D. A. Bauer, *et al.*, Phys. Rev. Lett. **54**, 753 (1985).

[7] F. Binon, *et al.*, Nucl. Phys. **B239**, 311 (1984).

[8] E705: L. Antoniazzi, *et al.*, Phys. Rev. Lett. **70**, 383 (1993)

[9] E705: L. Antoniazzi, *et al.*, Phys. Rev. **D46**, 4828 (1992)

[10] E672: A. Zieminski, *et al.*, Proc. XXVI Int. Conf. on High Energy Physics, Dallas, Texas, 1992, AIP Conf. Proc. No. 272, Ed. by J. R. Sanford, p. 1062.

[11] G. A. Schuler, preprint CERN-TH.7170/94.

[12] NA3: J. Badier, *et al.*, Z. Phys. **C20**, 101 (1983).

[13] C. Biino, *et al.*, Phys. Rev. Lett. **58**, 2523 (1987).

[14] E537: C. Akerlof, *et al.*, Phys. Rev. **D48**, 5067 (1993).

[15] E772: D. M. Alde, *et al.*, Phys. Rev. Lett. **66**, 133 (1991).

[16] J. F. Owens, Phys. Rev. **D30**, 943 (1984).

[17] J. F. Owens, Phys. Lett. **B266**, 126 (1991).

[18] R. Baier and R. Rückl, Z. Phys. **C19**, 251 (1983).

[19] R. Gastmans and T. T. Wu, *The Ubiquitous Photon: Helicity Method for QED and QCD*, Clarendon Press, Oxford, 1990

[20] J. Badier, *et al.*, Z. Phys. **C18**, 281 (1983).

[21] J. S. Conway, *et al.*, Phys. Rev. **D39**, 92 (1989).

[22] P. Chiappetta and M. Le Bellac, Z. Phys. **C32**, 521 (1986)

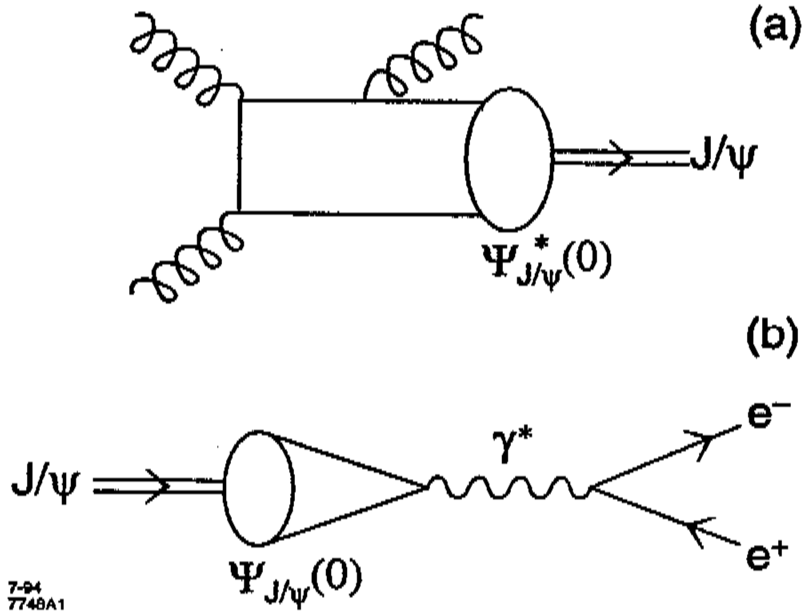
[23] J. G. Heinrich, *et al.*, Phys. Rev. **D44**, 1909 (1991).

[24] W. Kwong, J. L. Rosner and C. Quigg, Ann. Rev. Nucl. Part. Sci. **37**, 325 (1987).

[25] L. Köpke and N. Wermes, Phys. Rep. **174**, 67 (1989).

[26] S. J. Brodsky, P. Hoyer, A. H. Mueller and W.-K. Tang, Nucl. Phys. **B369**, 519 (1992).

[27] P. Hoyer, M. Vänttinen and U. Sukhatme, Phys. Lett. **B246**, 217 (1990).

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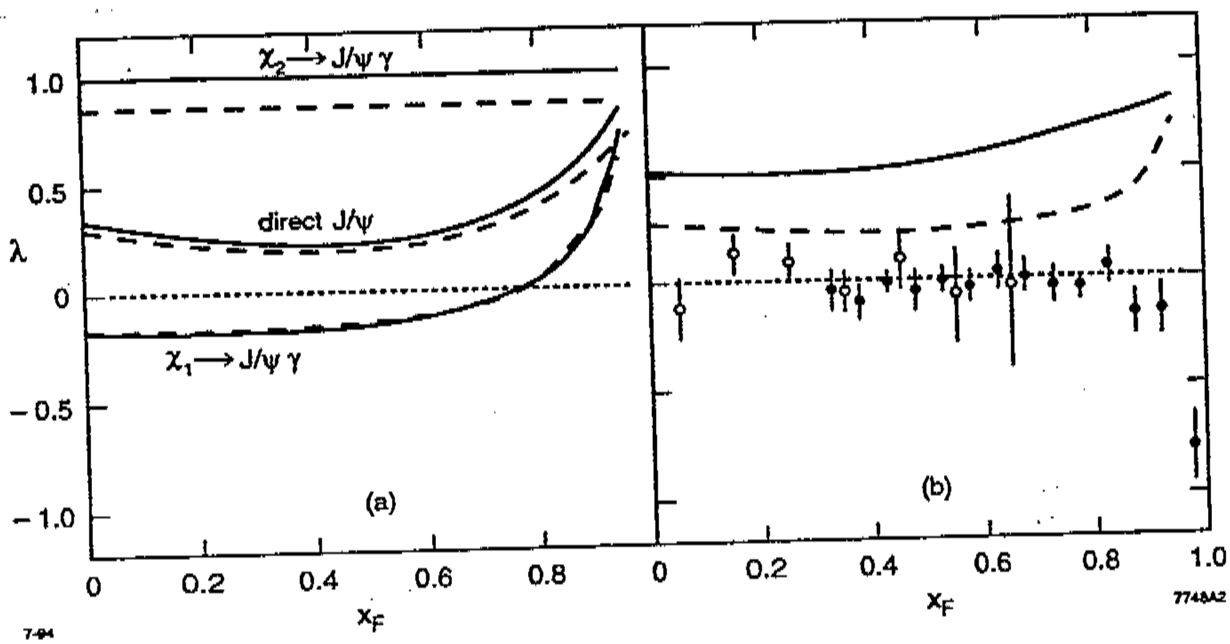


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