# Measurement of $\alpha_{s}$ from Hadronic Event Observables at the $Z^{0}$ Resonance 

The SLD Collaboration<br>Stanford Linear Accelerator Center<br>Stanford University, Stanford, CA 94309


#### Abstract

The strong coupling $\alpha_{s}$ has been measured using hadronic decays of $Z^{0}$ bosons collected by the SLD experiment at SLAC. The data were compared with QCD predictions both at fixed order, $\mathcal{O}\left(\alpha_{s}^{2}\right)$, and including resummed analytic formulae based on the leading and next-toleading logarithm approximation. We studied event shapes, jet rates, particle correlations, and energy fraction and checked the consistency between $\alpha_{s}$ extracted from these different measures. Combining all results we obtain $\alpha_{s}\left(M_{Z^{\circ}}^{2}\right)=0.120 \pm 0.003$ (exp.) $\pm 0.009$ (theor.), where the dominant uncertainty in $\alpha_{s}$ is from uncalculated higher order contributions.


Presented at the 27th International Conference on High Energy Physics (ICHEP), Glasgow, Scotland, July 20-27, 1994

[^0]
## 1 Introduction

The performance of precision tests of the Standard Model of elementary particle interactions is one of the key aims of experimental high energy physics experiments. Some measurements in the electroweak sector have reached a precision of better than $1 \%$ [1]. However, measurements of strong interactions, and hence tests of the theory of Quantum Chromodynamics (QCD) [2], have not yet achieved the same level of precision, largely due to the difficulty of performing QCD calculations, both at high order in perturbation theory and in the non-perturbative regime, where effects due to the hadronization process are important. QCD is a theory with only one free parameter, the strong coupling $\alpha_{s}$, which can be written in terms of a scale parameter $\Lambda_{\overline{M S}}$. All tests of QCD can therefore be reduced to a comparison of measurements of $\alpha_{s}$, either in different hard processes, such as hadron-hadron collisions or $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation, or at different scales. In this paper we present measurements of $\alpha_{s}$ in hadronic decays of $Z^{0}$ bosons produced by $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation at the SLAC Linear Collider (SLC) and recorded in the SLC Large Detector (SLD).

Complications arise in making accurate QCD predictions. In practice, because of the large number of Feynman diagrams that must be considered, QCD calculations are only possible with present techniques up to low order in perturbation theory. Perturbative calculations are pérformed within a particular renormalization scheme [3], which also defines the strong coupling. Translation between different schemes is possible, without changing the final predictions, by appropriate redefinition of $\alpha_{s}$ and of the hard scale $Q$ [4]. This leads to a scheme-dependence of $\alpha_{s}$, which can be alleviated in practice by choosing one particular scheme as a standard, and translating all $\alpha_{s}$ measurements to it. The modified minimal subtraction scheme ( $\overline{\mathrm{MS}}$ scheme) [3] is presently used widely in this context.

An additional complication is the truncation of the perturbative series at finite order, which yields a residual dependence on an unphysical parameter known as the renormalization scale, often denoted by $\mu$ or equivalently by $f=\mu^{2} / Q^{2}$. In our previous studies of jet rates [5] and energy-energy correlations [6] it was shown that the dominant uncertainty in $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ measurements arises from this renormalization scale ambiguity. Given that infinite order perturbative QCD calculations would be independent of $\mu$, the scale uncertainty inherent in $\alpha_{s}$ measurements is a reflection of the uncalculated higher order terms.

Distributions of observables in the process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons have been calculated exactly up to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ in QCD perturbation theory [7]. One expects a priori that the size of the uncalculated $\mathcal{O}\left(\alpha_{s}^{3}\right)$ and higher order terms will in general be different for each observable, and hence that the scale dependence of $\alpha_{s}$ values measured using different observables will also be different. In order to make a realistic determination of $\alpha_{s}$ and its associated theoretical uncertainty using $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations it is therefore advantageous to employ as many different observables as possible. Our previous measurements of

- $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ were based on extensive studies of jet rates [8] and energy-energy correlations
and their asymmetry [9], using approximately 10,000 hadronic $Z^{0}$ events collected by the SLD experiment in 1992. In this comprehensive analysis we have used the combined 1992 and 1993 data samples, comprising 60,000 events, to make an improved determination of $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ using all fifteen observables presently calculated up to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ in perturbative QCD.

In addition, for six of these fifteen observables improved calculations are available, incorporating the resummation $[10,11,12,13,14,15]$ of leading and next-to-leading (NLL) logarithms matched to the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ results; these matched calculations are expected a priori both to describe the data in a larger region of phase space than the fixed order results, and to yield a reduced dependence of $\alpha_{s}$ on the renormalization scale. We have employed the matched calculations for all six observables to determine $\alpha_{s}$, and have studied the uncertainties involved in the matching procedure. We have compared our results with our previous measurements and with similar measurements from LEP.

We describe the detector and the event trigger and selection criteria applied to the data in Section 2. In Section 3 we define the observables used to determine $\alpha_{s}$ in this analysis. The QCD predictions are discussed in Section 4. The analysis of the data is described in Section 5, and a summary and conclusions are presented in Section 6.

## 2 Apparatus and Hadronic Event Selection

The $e^{+} e^{-}$annihilation events produced at the $Z^{0}$ resonance by the SLAC Linear Collider (SLC) have been recorded using the SLC Large Detector (SLD). A General description of the SLD can be found elsewhere [16]. Charged tracks are measured in $=\quad$ the central drift chamber (CDC) and in the vertex detector (VXD) [17]. Momentum measurement is provided by a uniform axial magnetic field of 0.6 T . Particle energies are measured in the Liquid Argon Calorimeter (LAC) [18], which contains both electromagnetic and hadronic sections, and in the Warm Iron Calorimeter [19].

Three triggers were used for hadronic events, one requiring a total LAC electromagnetic energy greater than $15 \mathrm{GeV}(30 \mathrm{GeV})$, another requiring at least two wellseparated tracks in the CDC, and a third requiring at least $4 \mathrm{GeV}(8 \mathrm{GeV})$ in the LAC and one track in the CDC for 1993 data (1992 data). A selection of hadronic events was then made by two independent methods, one based on the topology of energy depositions in the calorimeters, the other on the number and topology of charged tracks measured in the CDC.

The analysis presented here used the charged tracks measured in the CDC and VXD. A set of cuts was applied to the data to select well-measured tracks and events well-contained within the detector acceptance region. The charged tracks were required to have: (i) a distance from the measured interaction point, at the point of closest approach, within 5 cm in the direction transverse to the beam axis and 10 cm along the beam axis; (ii) a polar. angle $\theta$ with respect to the beam axis within $|\cos \theta|<0.80$; and (iii) a momentum transverse to the beam axis of $P_{\perp}>0.15 \mathrm{GeV} / \mathrm{c}$. Events were
required to have: (i) a minimum of five such tracks; (ii) a thrust axis [20] direction within $\left|\cos \theta_{T}\right|<0.71$; (iii) a total visible energy $E_{v i s}$ of at least 20 GeV , which is calculated from the selected tracks assigned the charged pion mass. From our 1992 and 1993 data samples 37,226 events passed these cuts. The efficiency for selecting hadronic events satisfying the $\left|\cos \theta_{T}\right|$ cut was estimated to be above $96 \%$. The background in the selected event sample was estimated to be $0.3 \pm 0.1 \%$, dominated by $Z^{0} \rightarrow \tau^{+} \tau^{-}$ events. Distributions of single particle and event topology observables in the selected events were found to be well described by Monte Carlo models of hadronic $Z^{0}$ decays [21, 22] combined with a simulation of the SLD.

## 3 Definition of the Observables

In this section we present the definitions of the quantities used in our measurement of $\alpha_{s}$. We use all observables for which perturbative QCD calculations exist. These include six event shapes, jet rates defined by six algorithms, two particle correlations, and an energy fraction.

## Event Shapes

Various inclusive observables have been proposed to describe the shapes of hadronic events in $e^{+} e^{-}$annihilations. We consider those observables which are collinear and infrared-safe, and which can hence be calculated in perturbative QCD.

Thrust $T$ is defined by [20] :

$$
\begin{equation*}
T=\max \frac{\sum_{i}\left|\vec{p}_{i} \cdot \vec{n}_{T}\right|}{\sum_{i}\left|\vec{p}_{i}\right|} \tag{1}
\end{equation*}
$$

where $\vec{p}_{i}$ is the momentum vector of particle $i$ and $\vec{n}_{T}$ is the thrust axis to be determined. We define $\tau \equiv 1-T$. For back-to-back two-parton final states $\tau$ has a value of zero, while $0 \leq \tau \leq \frac{1}{3}$ for planar three-parton final states. Spherical events have $\tau=\frac{1}{2}$.

An axis $\vec{n}_{m a j}$ can be found to maximize the momentum sum transverse to $\vec{n}_{T}$. Finally, an axis $\vec{n}_{\text {min }}$ is defined to be perpendicular to the two axes $\vec{n}_{T}$ and $\vec{n}_{m a j}$. The variables thrust-major $T_{m a j}$, and thrust-minor $T_{m i n}$, are obtained by replacing $\vec{n}_{T}$ in Eq. (1) by $\vec{n}_{m a j}$ or $\vec{n}_{m i n}$ respectively. The oblateness $O$ is then defined by [23]:

$$
\begin{equation*}
O=T_{m a j}-T_{m i n} \tag{2}
\end{equation*}
$$

The value of $O$ is zero for collinear or cylindrically symmetric final states, and extends from zero to $\frac{1}{\sqrt{3}}$ for three-parton final states.

The $C$-parameter is derived from the eigenvalues of the infrared-safe momentum tensor [24]:

$$
\begin{equation*}
-\theta_{\rho \sigma}=\frac{\sum_{i} p_{i}^{\rho} p_{i}^{\sigma} /\left|\vec{p}_{i}\right|}{\sum_{i}\left|\vec{p}_{i}\right|} \tag{3}
\end{equation*}
$$

where $p_{i}^{\rho}$ is the $\rho$-th component of the three momentum of particle $i$, and $i$ runs over all the final state particles. The tensor $\theta_{\rho \sigma}$ is normalized to have unit trace, and the $C$-parameter is defined by :

$$
\begin{equation*}
C=3\left(\lambda_{1} \lambda_{2}+\lambda_{2} \lambda_{3}+\lambda_{3} \lambda_{1}\right), \tag{4}
\end{equation*}
$$

where $\lambda_{i}, i=1,2,3$, are the eigenvalues of the tensor $\theta_{\rho \sigma}$. For back-to-back twoparton final states $C$ is zero, while for planar three-parton final states $C$ is in the range $0 \leq C \leq \frac{2}{3}$. For spherical events $C=1$.

Events can be divided into two hemispheres $a, b$ by a plane perpendicular to the thrust axis $\vec{n}_{T}$. The heavy jet mass $M_{H}$ is then defined as [25] :

$$
\begin{equation*}
M_{H}=\max \left(M_{a}, M_{b}\right) \tag{5}
\end{equation*}
$$

where $M_{a}$ and $M_{b}$ are the invariant masses of each hemisphere. Here we define the normalized quantity

$$
\begin{equation*}
\rho \equiv \frac{M_{H}^{2}}{E_{v i s}^{2}} \tag{6}
\end{equation*}
$$

where $E_{v i s}$ is the total visible energy measured in hadronic events. To first order in perturbative QCD, and for massless partons, the heavy jet mass and thrust are related by $\tau=\rho[7]$.

Jet broadening measures have been proposed in ref. [26]. In each hemisphere $a, b$ :

$$
\begin{equation*}
B_{a, b}=\frac{\sum_{i \in a, b}\left|\vec{p}_{i} \times \vec{n}_{T}\right|}{2 \sum_{i}\left|\vec{p}_{i}\right|} \tag{7}
\end{equation*}
$$

is calculated. The total jet broadening $B_{T}$ and wide jet broadening $B_{W}$ are defined by

$$
\begin{equation*}
B_{T}=B_{a}+B_{b} \quad \text { and } \quad B_{W}=\max \left(B_{a}, B_{b}\right) \tag{8}
\end{equation*}
$$

respectively. Both $B_{T}$ and $B_{W}$ are identically zero in two-parton final states, and are sensitive to the transverse structure of jets. To first order in perturbative QCD, $B_{T}=B_{W}=\frac{1}{2} O$.

## Jet Rates

Another useful method of classifying the structure of hadronic final states is in terms of jets. Jets may be reconstructed using iterative clustering algorithms [8] in which a measure $y_{i j}$, such as scaled invariant mass, is calculated for all pairs of particles $i$ and $j$, and the pair with the smallest $y_{i j}$ is combined into a single particle. This procedure is repeated until all pairs have $y_{i j}$ exceeding a value $y_{c u t}$, and the jet multiplicity of the event is defined as the number of particles remaining. The $n$-jet rate $R_{n}\left(y_{c u t}\right)$ is the fraction of events classified as $n$-jet, and the differential 2 -jet rate is defined as [27]:

$$
\begin{equation*}
\bar{D}_{2}\left(\bar{y}_{c u t}\right) \equiv \frac{R_{2}\left(y_{c u t}\right)-R_{2}\left(y_{c u t}-\Delta y_{c u t}\right)}{\Delta y_{c u t}} \tag{9}
\end{equation*}
$$

In contrast to $R_{n}$, each event contributes to $D_{2}$ at only one $y_{c u t}$.
Several algorithms have been proposed featuring different $y_{i j}$ definitions and recombination schemes. We have applied the E, E0, P, and P0 variations of the JADE algorithm [28] as well as the Durham (D) and Geneva (G) algorithms [8]. The six definitions of the jet resolution parameter $y_{i j}$ and recombination procedure are given below.

In the E-algorithm, $y_{i j}$ is defined as the square of the invariant mass of the pair of particles $i$ and $j$ scaled by the visible energy in the event,

$$
\begin{equation*}
y_{i j}=\frac{\left(p_{i}+p_{j}\right)^{2}}{E_{v i s}^{2}} \tag{10}
\end{equation*}
$$

with the recombination performed as

$$
\begin{equation*}
p_{k}=p_{i}+p_{j} \tag{11}
\end{equation*}
$$

where $p_{i}$ and $p_{j}$ are four-momenta of the particles and pion masses are assumed in calculating particle energies. Energy and momentum are explicitly conserved in this algorithm.

The $\mathrm{E} 0-\mathrm{P}$-, and P 0 -algorithms are variations of the E -algorithm. In the E 0 algorithm $y_{i j}$ is defined by Eq. (10), while the recombination scheme is defined by

$$
\begin{align*}
E_{k} & =E_{i}+E_{j}  \tag{12}\\
\vec{p}_{k} & =\frac{E_{k}}{\left|\vec{p}_{i}+\vec{p}_{j}\right|}\left(\vec{p}_{i}+\vec{p}_{j}\right) \tag{13}
\end{align*}
$$

$=\quad$ where $E_{i}$ and $E_{j}$ are the energies, and $\vec{p}_{i}$ and $\vec{p}_{j}$ are the three-momenta of the particles. The three-momentum $\vec{p}_{k}$ is rescaled so that particle $k$ has zero invariant mass. This algorithm does not conserve the total momentum sum of an event.

In the P-algorithm $y_{i j}$ is defined by Eq. (11) and the recombination scheme is defined by

$$
\begin{align*}
\vec{p}_{k} & =\vec{p}_{i}+\vec{p}_{j}  \tag{14}\\
E_{k} & =\left|\vec{p}_{k}\right| . \tag{15}
\end{align*}
$$

This algorithm conserves the total momentum of an event.
The P 0 -algorithm is similar to the P -algorithm, but the total energy $E_{v i s}$ in Eq. (10) is recalculated at each iteration according to

$$
\begin{equation*}
E_{v i s}=\sum_{k} E_{k} \tag{16}
\end{equation*}
$$

In the D-algorithm,

$$
\begin{equation*}
\overline{y_{i j}}=\frac{2 \min \left(E_{i}^{2}, E_{j}^{2}\right)\left(1-\cos \theta_{i j}\right)}{E_{v i s}^{2}} \tag{17}
\end{equation*}
$$

where $\theta_{i j}$ is the angle between the pair of particles $i$ and $j$. The recombination is defined by Eq. (11). With the D-algorithm, a soft particle will only be combined with another soft particle, instead of being combined with a high-energy particle, if the angle it makes with the other soft particle is smaller than the angle that it makes with the high-energy particle.

The definition of $y_{i j}$ for the G-algorithm is

$$
\begin{equation*}
y_{i j}=\frac{8 E_{i} E_{j}\left(1-\cos \theta_{i j}\right)}{9\left(E_{i}+E_{j}\right)^{2}} \tag{18}
\end{equation*}
$$

In this scheme soft particles are combined as the D -algorithm. In addition, $y_{i j}$ depends only on the energy of the particles to be combined, and not on the $E_{v i s}$ of the event.

## Particle correlations

Hadronic event observables can also be classified in terms of inclusive two-particle correlations. The energy-energy correlations (EEC) [9] is the normalized energyweighted cross section defined in terms of the angle $\chi_{i j}$ between two particles $i$ and $j$ in an event

$$
\begin{equation*}
E E C(\chi)=\frac{1}{N_{\text {events }} \Delta \chi} \sum_{e v e n t s} \int_{\chi-\frac{\Delta x}{2}}^{\chi+\frac{\Delta x}{2}} \sum_{i j} \frac{E_{i} E_{j}}{E_{v i s}^{2}} \delta\left(\chi^{\prime}-\chi_{i j}\right) \mathrm{d} \chi^{\prime} \tag{19}
\end{equation*}
$$

where $\chi$ is an opening angle to be studied for the correlations, $\Delta \chi$ is the angular bin width, $E_{i}$ and $E_{j}$ are the energies of particles $i$ and $j$, and $E_{v i s}$ is the sum of the energies of all particles in the event. The angle $\chi$ is taken from $\chi=0^{\circ}$ to $\chi=180^{\circ}$. The shape of the $E E C$ in the central region, $\chi \sim 90^{\circ}$, is determined by hard gluon emission. Hadronization contributions are expected to be large in the collinear and back-to-back regions, $\chi \sim 0^{\circ}$ and $180^{\circ}$ respectively. The asymmetries of the $E E C$ $(A E E C)$ are defined as $A E E C(\chi)=E E C\left(180^{\circ}-\chi\right)-E E C(\chi)$.

## Energy fraction

Another possibility, related to the angle of particle emission, is to integrate the energy within a conical shell of opening angle $\chi$ about the thrust axis. Here we studied the Jet Cone Energy Fraction (JCEF) defined by [29]

$$
\begin{equation*}
J C E F(\chi)=\frac{1}{N_{\text {events }} \Delta \chi} \sum_{\text {events }} \int_{\chi-\frac{\Delta x}{2}}^{\chi+\frac{\Delta x}{2}} \sum_{i} \frac{E_{i}}{E_{v i s}} \delta\left(\chi^{\prime}-\chi_{i}\right) \mathrm{d} \chi^{\prime} \tag{20}
\end{equation*}
$$

where

$$
\begin{equation*}
\chi_{i}=\frac{180^{\circ}}{\pi} \cdot \arccos \left(\frac{\vec{p}_{i} \cdot \vec{n}_{T}}{\left|\vec{p}_{i}\right|}\right) \tag{21}
\end{equation*}
$$

is the opening angle $\chi_{i}$ is the angle between a particle and the thrust axis vector, $\vec{n}_{T}$, whose direction is defined to point from the heavy jet mass hemisphere to the light jet . mass hemisphere, and $0^{\circ} \leq \chi \leq 180^{\circ}$. Hard gluon emissions contribute to the region corresponding to the heavy jet mass hemisphere, $90^{\circ} \leq \chi \leq 180^{\circ}$.

## 4 QCD Predictions

The fraction $R\left(\mathrm{y}, \alpha_{s}\right)$ for all observables defined in Section 3 is

$$
\begin{equation*}
R\left(\mathrm{y}, \alpha_{s}\right) \equiv \frac{1}{\sigma_{t}} \int_{0}^{\mathrm{y}} \frac{\mathrm{~d} \sigma}{\mathrm{dy}} \mathrm{dy} \tag{22}
\end{equation*}
$$

where y is the observable in question, $\sigma_{t}$ is the total hadronic cross section.
The predictions up to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ have the general form

$$
\begin{equation*}
\frac{1}{\sigma_{t}} \cdot \frac{\mathrm{~d} \sigma(\mathrm{y})}{\mathrm{dy}}=a(\mathrm{y}) \alpha_{s}+\left[b(\mathrm{y})+a(\mathrm{y}) 2 \pi b_{0} \ln f\right] \alpha_{s}^{2} \tag{23}
\end{equation*}
$$

where $f=\mu^{2} / s, b_{0}=\frac{33-2 n_{f}}{12 \pi}$, and $n_{f}$ is the number of active quark flavors; $n_{f}=5$ at $\sqrt{s}=M_{Z^{0}}$. We have computed the coefficients $a(\mathrm{y})$ and $b(\mathrm{y})$ using the EVENT program, which was developed by Kunszt and Nason [7]. Eq. (23) can also be cast into the integrated form

$$
\begin{equation*}
R^{\mathcal{O}\left(\alpha_{s}^{2}\right)}\left(\mathrm{y}, \alpha_{s}\right)=1+A(\mathrm{y}) \alpha_{s}+B(\mathrm{y}) \alpha_{s}^{2}, \tag{24}
\end{equation*}
$$

where $A(\mathrm{y})$ and $B(\mathrm{y})$ are the cumulative forms of $a(\mathrm{y})$ and $b(\mathrm{y})$ in Eq. (23). It should be noted that dependence on the QCD renormalization scale enters explicitly in the second order term in Eq. (23).

It has been found recently $[10,11,12,13,14,15]$ that several observables, namely $\tau, \rho, B_{T}, B_{W}, D_{2}(D$-algorithm $)$, and $E E C$, can be resummed, that is leading and next-to-leading logarithmic terms can be calculated to all orders in $\alpha_{s}$ using the exponentiation technique. This procedure is expected a priori to yield formulae which are less dependent on the renormalization scale. Using $L \equiv \ln (1 / \mathrm{y})$ the fraction $R\left(\mathrm{y}, \alpha_{s}\right)$ can be written in the general form

$$
\begin{equation*}
R\left(\mathrm{y}, \alpha_{s}\right)=C\left(\alpha_{s}\right) \exp \left\{\Sigma\left(\alpha_{s}, L\right)\right\}+F\left(\mathrm{y}, \alpha_{s}\right) \tag{25}
\end{equation*}
$$

where

$$
\begin{align*}
C\left(\alpha_{s}\right) & =1+\sum_{n=1}^{\infty} C_{n} \alpha_{s}^{n}  \tag{26}\\
\Sigma\left(\alpha_{s}, L\right) & =\sum_{n=1}^{\infty} \alpha_{s}^{n} \sum_{m=1}^{n+1} G_{n m} L^{m}  \tag{27}\\
F\left(\mathrm{y}, \alpha_{s}\right) & =\sum_{n=1}^{\infty} F_{n}(\mathrm{y}) \alpha_{s}^{n} . \tag{28}
\end{align*}
$$

The factor $\Sigma$ to be exponentiated can be reexpressed as a power series expansion in $L$ :

$$
\begin{equation*}
\Sigma\left(\alpha_{s}, L\right)^{-}=\stackrel{-}{L} \cdot f_{L L}\left(\alpha_{s} L\right)+f_{N L L}\left(\alpha_{s} L\right)+\mathcal{O}\left(\frac{1}{L} \cdot\left(\alpha_{s} L\right)^{n}\right) \tag{29}
\end{equation*}
$$

The $N L L$ calculations are thus given by an approximate expression for $R\left(\mathrm{y}, \alpha_{s}\right)$ in the form

$$
\begin{equation*}
R^{N L L}\left(\mathrm{y}, \alpha_{s}\right)=\left(1+C_{1} \alpha_{s}+C_{2} \alpha_{s}^{2}\right) \exp \left\{\Sigma^{N L L}\left(\alpha_{s}, L\right)\right\}, \tag{30}
\end{equation*}
$$

where

$$
\begin{equation*}
\Sigma^{N L L}\left(\alpha_{s}, L\right)=L \cdot f_{L L}\left(\alpha_{s} L\right)+f_{N L L}\left(\alpha_{s} L\right) \tag{31}
\end{equation*}
$$

The leading logarithmic $\left(L \cdot f_{L L}\right)$ and next-to-leading logarithmic $\left(f_{N L L}\right)$ terms in $\Sigma(m \geq$ $n$ ) have been calculated, while the subleading terms in this formula have not been completely computed. However, some terms included in $\Sigma(m<n)$, as well as $C$ and $F$, are also included to the second order calculation. In order to make reliable predictions including hard gluon emission in the $N L L$ calculations, it is necessary to combine them with the second order calculation, taking overlapping terms into account. This procedure is called matching, and four matching schemes have been proposed in the literature. Taking the logarithm of the $N L L$ formula (Eq. (30)) and the exact calculation up to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ (Eq. 24),

$$
\begin{equation*}
\ln R^{N L L}\left(\mathrm{y}, \alpha_{s}\right)=\Sigma^{N L L}\left(\alpha_{s}, L\right)+C_{1} \alpha_{s}+\left(C_{2}-\frac{C_{1}^{2}}{2}\right) \alpha_{s}^{2}+\mathcal{O}\left(\alpha_{s}^{3}\right) \tag{32}
\end{equation*}
$$

and

$$
\begin{equation*}
\ln R^{\mathcal{O}\left(\alpha_{s}^{2}\right)}\left(\mathrm{y}, \alpha_{s}\right)=A(\mathrm{y}) \alpha_{s}+\left(B(\mathrm{y})-\frac{A^{2}(\mathrm{y})}{2}\right) \alpha_{s}^{2} \tag{33}
\end{equation*}
$$

Adding Eq. (32) and Eq. (33), and subtracting the overlapping first and second order terms from Eq. (32), yields [10, 11].

$$
\begin{align*}
\ln R^{N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)}\left(\mathrm{y}, \alpha_{s}\right)=\Sigma^{N L L}\left(\alpha_{s}, L\right)- & \Sigma^{N L L(1)}\left(\alpha_{s}, L\right)-\Sigma^{N L L(2)}\left(\alpha_{s}, L\right) \\
& +A(\mathrm{y}) \alpha_{s}+\left(B(\mathrm{y})-\frac{A^{2}(\mathrm{y})}{2}\right) \alpha_{s}^{2} \tag{34}
\end{align*}
$$

where

$$
\begin{align*}
& \Sigma^{N L L(1)}\left(\alpha_{s}, L\right)=G_{12} \alpha_{s} L^{2}+G_{11} \alpha_{s} L  \tag{35}\\
& \Sigma^{N L L(2)}\left(\alpha_{s}, L\right)=G_{23} \alpha_{s}^{2} L^{3}+G_{22} \alpha_{s}^{2} L^{2} . \tag{36}
\end{align*}
$$

Finally, one can derive $R\left(\mathrm{y}, \alpha_{s}\right)$ by taking the exponential of Eq. (34). This procedure is called $\ln R$-matching.

In an alternative approach, the overlapping terms $\Sigma^{N L L(1)}\left(\alpha_{s}, L\right)$ and $\Sigma^{N L L(2)}\left(\alpha_{s}, L\right)$ are subtracted from $\Sigma^{N L L}\left(\alpha_{s}, L\right)$ in the form of an exponential. The exact formula up
to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ is then added as follows [15]

$$
\begin{align*}
R^{N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)}\left(\mathrm{y}, \alpha_{s}\right)= & \left(1+C_{1} \alpha_{s}+C_{2} \alpha_{s}^{2}\right)\left[\exp \left\{\Sigma^{N L L}\left(\alpha_{s}, L\right)\right\}\right. \\
& \left.-\exp \left\{\Sigma^{N L L(1)}\left(\alpha_{s}, L\right)+\Sigma^{N L L(2)}\left(\alpha_{s}, L\right)\right\}\right] \\
& +1+A(\mathrm{y}) \alpha_{s}+B(\mathrm{y}) \alpha_{s}^{2} \\
= & \left(1+C_{1} \alpha_{s}+C_{2} \alpha_{s}^{2}\right) \exp \left\{\Sigma^{N L L}\left(\alpha_{s}, L\right)\right\} \\
& -\left(C_{1} \alpha_{s}+\Sigma^{N L L(1)}\left(\alpha_{s}, L\right)\right)-\left[C_{2} \alpha_{s}^{2}+C_{1} \alpha_{s} \Sigma^{N L L(1)}\left(\alpha_{s}, L\right)\right. \\
& \left.+\frac{1}{2}\left\{\Sigma^{N L L(1)}\left(\alpha_{s}, L\right)\right\}^{2}+\Sigma^{N L L(2)}\left(\alpha_{s}, L\right)\right] \\
& +A(\mathrm{y}) \alpha_{s}+B(\mathrm{y}) \alpha_{s}^{2} \tag{37}
\end{align*}
$$

This is called $R$-matching, and differs in that the subleading term $G_{21} \alpha_{s}^{2} L$ is not exponentiated. In order to raise this procedure to the same level as the lnR-matching scheme, Eq. (37) may be modified by replacing $\Sigma^{N L L}\left(\alpha_{s}, L\right)$ and $\Sigma^{N L L(2)}\left(\alpha_{s}, L\right)$ with $\Sigma\left(\alpha_{s}, L\right)$ and $\Sigma^{(2)}\left(\alpha_{s}, L\right)=G_{23} \alpha_{s}^{2} L^{3}+G_{22} \alpha_{s}^{2} L^{2}+G_{21} \alpha_{s}^{2} L$, respectively. This procedure is called modified $R$-matching* [14].

The predictions of these matching schemes have some troublesome features near the upper kinematic limit $y_{m a x}^{*}$, because terms of third and higher order generated by resummation do not vanish at the kinematic limit. This situation can be overcome by invoking a replacement of $L=\ln (1 / y)$ in Eq. (34) with $L^{\prime}=\ln \left(1 / y-1 / y_{\max }+1\right)^{\dagger}$ This procedure is called modified lnR-matching [35].

Finally, in order to account for the scale dependence, $f_{N L L}\left(\alpha_{s} L\right)$ should be modified to $f_{N L L}\left(\alpha_{s} L\right)+\left(\alpha_{s} L\right)^{2} \frac{\mathrm{~d} f_{L L}\left(\alpha_{s} L\right)}{\mathrm{d}\left(\alpha_{s} L\right)} b_{0} \ln f$, and $B(y)$ and $G_{22}$ should be modified as $B(\mathrm{y})+$ $A(\mathrm{y}) 2 \pi b_{0} \ln f$ and $G_{22}+G_{12} 2 \pi b_{0} \ln f$ respectively.

## 5 Measurement of $\alpha_{s}$

### 5.1 Data Analysis

The fifteen variables defined in section 3 were calculated from selected charged tracks in selected hadronic events defined in section 2. The experimental data must be corrected to the parton level in order to be compared with the QCD-calculated distributions. We used both JETSET [21, 36] and HERWIG [22] simulations to correct our data. For JETSET we used parameter values tuned to hadronic $e^{+} e^{-}$annihilation data $[37] \ddagger$. We first corrected for detector acceptance, efficiency and resolution, decays,

[^1]and initial state photon radiation to obtain hadron level data for direct comparison with results from other experiments. We then corrected for the hadronization effects.

We applied bin-by-bin correction factors to the experimental distribution $D_{\text {exp. }}^{\text {data }}(y)_{i}$ to obtain the parton-level distribution $D_{\text {parton }}^{\text {data }}(y)_{i}$ :

$$
\begin{equation*}
D_{\text {parton }}^{\text {data }}(y)_{i}=C_{H}(y)_{i} \cdot C_{D}(y)_{i} \cdot D_{\text {exp. }}^{\text {data }}(y)_{i} \quad i=\text { bin index }, \tag{38}
\end{equation*}
$$

where the detector and hadronization correction factors,

$$
\begin{equation*}
C_{D}(y)_{i}=\frac{D_{\text {hadron }}^{M C}(y)_{i}}{D_{S L D \text { sim. }}^{M M C}(y)_{i}} \quad \text { and } \quad C_{H}(y)_{i}=\frac{D_{\text {parton }}^{M C}(y)_{i}}{D_{\text {hadron }}^{M C}(y)_{i}} \tag{39}
\end{equation*}
$$

were evaluated using Monte Carlo simulations. The bin widths were chosen to minimize the effects of bin-to-bin migration. Here $D_{S L D \text { sim. }}^{M C}(y)_{i}$ represents the content of bin $i$ of the distribution obtained from reconstructed charged particles in Monte Carlo events after simulation of the SLD, $D_{\text {hadron }}^{M C}(y)_{i}$ represents that of all generated particles with lifetimes greater than $3 \times 10^{-11}$ seconds in Monte Carlo events with no initial state photon radiation, and $D_{\text {parton }}^{M C}(y)_{i}$ is generated partons in Monte Carlo events with no initial state photon radiation. We used JETSET 6.3 PS for $D_{S L D \text { sim. }}^{M C}(y)_{i}$ and JETSET 7.3 $\mathrm{PS}^{\S}$ and HERWIG 5.5 for $D_{\text {hadron }}^{M C}(y)_{i}$ and $D_{\text {parton }}^{M C}(y)_{i}$.

Figures 1.1-1.15 show comparisons of the data and Monte Carlo simulations at the hadron level. Table 1 shows the corrected data with statistical errors and experimental systematic errors. Multinomial statistical errors are shown except for $E E C, A E E C$, and $J C E F$. Due to bin-to-bin correlations and multiple entries per event, the statistical error of each bin for $E E C, A E E C$, and $J C E F$ were estimated by taking the rms of $=$ the contents of that bin over 50 Monte Carlo samples, each with the same number of events as the data sample. The experimental systematic errors, which arise from uncertainties in modeling the acceptance, efficiency, and resolution of the detector, are evaluated by varying the event selection cuts over wide ranges, and by varying the tracking efficiency and resolution by large amounts in our Monte Carlo simulations. Effects due to limited Monte Carlo statistics are also added but are small compared with the other errors. In all distributions good agreement between data and Monte Carlo simulations is found.

### 5.2 Measurement of $\alpha_{s}$ using $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations

We first measured $\alpha_{s}$ by comparing the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ QCD calculations with the corrected data for each observable at the parton level. Each distribution was fitted by minimizing $\chi^{2}$ with respect to $\alpha_{s}$ at selected values of the scale $f$ over the range for which the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculation provides a good description of the corrected data. For the jet rates analysis, the lower bound on $y_{c u t}$ was chosen such that the 4 -jet rate $R_{4}$ for the

[^2]algorithm in question is less than $1 \%$, since $R_{4}$ is only calculated to leading order in $\alpha_{s}$ at tree level. The upper bound is the kinematical limit, $y_{c u t}=0.33$. For the event shapes, particle correlations and jet cone energy fraction, the bounds are set by the kinematic limit for $\mathcal{O}\left(\alpha_{s}\right)$, i.e., $a(y) \neq 0$, since beyond this bound calculation is effectively leading order in $\alpha_{s}$ for 4-parton production. At back-to-back region higher order terms become important and the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations become unreliable. We take the empirical approach of extending the fit range toward the back-to-back region as long as $\chi_{\text {dof }}^{2}$ of a fit with $f=1$ remains less than five. In addition, we required high confidence in the correction factors; (1) the sizes of the hadronization correction factors are smaller than $40 \%$ and (2) the uncertainties of the detector and hadronization correction factors are smaller than $30 \%$. This last requirement had a small effect.

Figures 2.1-2.15 show (a) the distribution of each variable at the parton level with the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ QCD fit at $f=1$ overlaid. The hadronization (b) and detector correction factors (c) are shown in Figs. 2.1-2.15. The fit ranges for each observable are indicated in the figures and listed in Table 2.

Figures 3.1-3.3 show (a) $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ and (b) the corresponding $\chi_{d o f}^{2}$ respectively, derived from fits at different values of $f$ for each observable. Several features are common to each distribution: $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ depends strongly on $f$; the fit quality is good over a wide range in $f$, typically $f \geq 2 \times 10^{-3}$, and there is no preference for a particular scale for most of the observables. At low $f$ the fit quality deteriorates rapidly, and neither $\alpha_{s}$ nor its error can be interpreted meaningfully. Similar features were reported in our earlier $\alpha_{s}$ measurements from jet rates [5] and EEC [6]. For the oblateness the good fit region is $f \geq 10^{-1}$, which is much higher than for the other observables. In the E-algorithm the minimized $\chi_{d o f}^{2}$ is found in the small $f$ region around the value of $10^{-5}$ and the fit quality is good in that $f$ region.

The poor fit quality at low $f$ has been shown to be due to poor convergence of the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ cālculations [39]. We therefore consider for each observable the $f$ range bounded below by the criterion $\chi_{d o f}^{2}<5$. This is arbitrary but ensures that the minimum $\alpha_{s}$ is considered for all variables except $B_{T}$. We placed an upper bound of $f=4$ corresponding to a reasonable physical limit. We took the extrema of the $\alpha_{s}$ values in these $f$ ranges to define the scale uncertainty for each observable. Table 2 summarizes the $f$ ranges, the central values of $\alpha_{s}$ as defined by the midpoint between the extrema, and the scale uncertainties.

The statistical errors were defined by an increase of 1.0 in $\chi^{2}$ from the minimized value except for $E E C, A E E C$, and $J C E F$. Because of bin-to-bin correlations for these three observables, we estimated the statistical errors in $\alpha_{s}$ from ten sets of Monte Carlo events. We performed the same fitting procedure to the $E E C, A E E C$, and $J C E F$ for each of these sets and took the $r m s$ deviation of the ten $\alpha_{s}$ values thus determined to be the statistical error of the fitted $\alpha_{s}$. The statistical errors are less than $1 \%$ in all cases.

Experimental systematic errors are estimated by varying the cuts applied to the - data and changing paraméters in the simulation of the detector. The bin-by-bin exper-
imental systematic errors for the observables are shown in Table 1. These errors in the bins are all correlated. The band width of the correction factors in Figs. 2.1(c)-2.15(c) indicates the uncertainty of the detector correction factors. In each case, detector correction factors were reevaluated and the correction and fitting procedures were repeated. The estimated systematic error is found to be 2-3 $\%$ on $\alpha_{s}$ for each observable.

Hadronization uncertainties were studied by recalculating the hadronization correction factors using JETSET 7.3 PS with values of the parton virtuality cutoff $Q_{0}[21,36]$ in the range 0.5 to 2.0 GeV , and by using HERWIG 5.5 [22], which contains a different hadronization model. The band width of the correction factors in Figs. 2.1(b)-2.15(b) indicates the uncertainty of the hadronization correction factors. In order to determine a value of $\alpha_{s}$ we took the average value from the results using JETSET 7.3 PS and HERWIG 5.5 for hadronization corrections. The hadronization uncertainties on $\alpha_{s}$ were obtained by adding in quadrature the uncertainties from the $Q_{0}$ cutoff value and half of the difference between the $\alpha_{s}$ values using JETSET 7.3 PS and HERWIG 5.5.

The fitted values of $\alpha_{s}$ and their errors are summarized on table 3. In all cases the theoretical error, which consists mainly of the scale ambiguity, dominates. This error varies from $4 \%$ for the AEEC to $20 \%$ for $B_{T}$. The $\alpha_{s}$ values from the fifteen observables are consistent within these theoretical errors. Since the same data are used for all observables we combine these results using an unweighted average to obtain

$$
\alpha_{s}\left(M_{Z^{0}}^{2}\right)=0.121 \pm 0.003 \text { (tot. exp.) } \pm 0.011 \text { (tot. theor.) }
$$

where the experimental error is the quadratic sum of the average statistical $( \pm 0.001)$ and average experimental systematic ( $\pm 0.002$ ) errors, corresponding to the assumption that all are completely correlated. The theoretical error is the quadratic sum of the average hadronization ( $\pm 0.002$ ) and average scale ( $\pm 0.011$ ) uncertainties.

### 5.3 Measurement of $\alpha_{s}$ using $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations

We next measured $\alpha_{s}$ by comparing the $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations with the corrected data at the parton level for those observables for which the $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations exist, i.e. thrust, heavy jet mass, total and wide jet broadening measures, differential 2 -jet rate ( D -algorithm), and energy-energy correlations. We considered all four matching schemes, $\ln R$-, modified $\ln R-, R$-, and modified $R$-matching, where possible. Modified $R$-matching is not applicable to $D_{2}$ because the subleading term $G_{21}$ is not calculated for $D_{2}$. For the EEC $\ln R$-matching and modified $\ln R$-matching schemes cannot be applied reliably [40].

The fit ranges were initially chosen to be the same ranges as for the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ fits except for $E E C$. We performed the fits to the $E E C$ within the angular range $90^{\circ}-154.8^{\circ}$, where the lower limit is the kinematic limit for the $\operatorname{NLL}+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculation. Figures 2.1(a)-2.4(a), 2.11(a), and 2.13(a) show the results of the $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right) \mathrm{QCD}$ fits using the modified $\ln R$-matching scheme with the renormalization scale factor $f=1$. For the fit to $D_{2}$ (D-algorithm) we adopted a procedure [5], using the matched calculation
for $0.01 \leq y_{\text {cut }} \leq 0.04$ and the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculation for $0.05 \leq y_{\text {cut }} \leq 0.33$. Figures 4.1-4.4 show (a) $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ and (b) the corresponding $\chi_{\text {dof }}^{2}$, derived from the fits for different values of $f$ for the various matching schemes. We found the fit qualities for the total and wide jet broadening measures using $R$-matching scheme were poor for all $f$.

We applied the same analysis as for the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations to each combination of matching scheme and observable. Table 4 summarizes the $f$ ranges, central values of $\alpha_{s}$ and scale uncertainties. The experimental and hadronization systematics were estimated by the method described above and found to be the same as in the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ analysis. The $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations are able to fit the data in much reduced ranges of $f$ and show reduced scale dependence. However, the measured $\alpha_{s}$ is sensitive to the choice of matching scheme. We averaged over matching schemes to obtain a value of $\alpha_{s}$ for each observable, and considered the matching ambiguity, defined as the maximum deviation from the average, as an additional theoretical uncertainty. This uncertainty also reflects a lack of higher order terms in the calculations. The theoretical errors are reduced by typically a factor of two relative to the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ analysis, but still dominate the measurement of $\alpha_{s}$. The values of $\alpha_{s}$ are lower by about $3-9 \%$ for the event shapes and higher by 5 and $6 \%$ for $D_{2}(D)$ and $E E C$ respectively than those from the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ analysis.

The fitted $\alpha_{s}$ values and their errors of $\alpha_{s}$ from six observables are summarized in Table 5. By averaging over six $\alpha_{s}$ values we obtain

$$
\alpha_{s}\left(M_{Z^{0}}^{2}\right)=0.119 \pm 0.003 \text { (tot. exp.) } \pm 0.007 \text { (tot. theor.) }
$$

where the total experimental error is the sum in quadrature of the statistical $( \pm 0.001)$ and experimental systematic errors ( $\pm 0.002$ ), and the total theoretical error is the sum $=\quad$ in quadrature of the hadronization $( \pm 0.002)$ and scale and matching uncertainties $( \pm 0.007)$.

From Figs. 2.1(a)-2.4(a), 2.11(a) and 2.13(a), it is clear that as expected the $N L L+$ $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations are more successful than the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations in describing the back-to-back region (Sudakov region) of each distribution. This would imply that multiple emissions of soft gluons which are taken into account in the $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations significantly contribute to this region. We therefore extended the fit regions toward the back-to-back region using criteria similar to those used in $\mathcal{O}\left(\alpha_{s}^{2}\right)$ analysis. For $D_{2}$, we required the 5 -jet production rate, $R_{5}$, to be less than $1 \%$. For the $E E C$ the upper limit of fit range was extended to $162^{\circ}$ by applying the empirical criterion $\chi_{d o f}^{2}<5$. The requirements of $\left|1-C_{H, D}\right|<40 \%$ and $\delta\left|C_{H, D}\right|<30 \%$ were kept.

Table 6 summarizes the fit ranges, as well as the values of $\alpha_{s}$, the scale uncertainty, and the $f$ ranges defined as above. We could not derive good fits using the $R$-matching scheme for the $\tau, \rho, B_{T}, B_{W}$, nor $D_{2}$ (D-algorithm) even if we chose the fit range which gave the best fit. The extension of the fit range has little effect on the $\alpha_{s}$ values, reducing them by $0-0.004$ except for $B_{W}$ with modified ln $R$-matching which decreases by 0.011 .

## 6 Conclusions

We have measured the value of the strong coupling, $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$, by analyses of fifteen different observables describing the hadronic final states of $60,000 Z^{0}$ decays recorded by the SLD experiment. The observables comprise six event shapes $\left(\tau, \rho, B_{T}, B_{W}\right.$, $O$, and $C$ ), differential 2-jet rates defined by six different jet resolution/recombination schemes ( $\mathrm{E}, \mathrm{E} 0, \mathrm{P}, \mathrm{P} 0, \mathrm{D}$, and G), energy-energy correlations and their asymmetry, and the jet cone energy fraction ( $J C E F$ ). The new quantity, $J C E F$, has been measured for the first time. Our measured distributions of these observables are reproduced by the JETSET and HERWIG Monte Carlo simulations and are consistent with previous measurements at the $Z^{0}$. The coupling is determined by fitting analytic QCD calculations to the data distributions corrected to the parton level. Perturbative QCD calculations complete to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ were used for all 15 observables and $N L L$ calculations were matched to the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations using four matching schemes and applied to the six observables for which $N L L$ calculations are available.

We find that the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations are able to describe the data in the hard 3 -jet region of all 15 distributions for a wide range of the QCD renormalization scale $f$. The fitted $\alpha_{s}$ depends strongly on the choice of $f$, which limits the precision of each $\alpha_{s}$ measurement. The $A E E C$ shows the smallest renormalization scale uncertainty of $3 \%$, which is just larger than experimental errors. The $\alpha_{s}$ from each observable is consistent with previous measurements within experimental errors. The $\alpha_{s}$ values from the various observables are consistent with each other within the scale uncertainties.

The $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations show a reduced $f$-dependence and are able to fit a wider region of each distribution and give similar fitted values of $\alpha_{s}$. However the different matching schemes give different $\alpha_{s}$ values reflecting a residual uncertainty in the inclusion of terms in the $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations. This theoretical uncertainty is smaller than in the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ case, but still dominates the uncertainty on the extraction of $\alpha_{s}$. Again, the individual $\alpha_{s}$ values are consistent with previous measurements within experimental errors, and the values from the six observables are consistent within theoretical errors.

Figure 5 summarizes the measured $\alpha_{s}$ values from fifteen observables using $\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations and the six observables using $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations. Since the same data are used for all observables, we combine the results by taking an unweighted average of the $\alpha_{s}$ value and experimental and theoretical errors, obtaining

$$
\alpha_{s}\left(M_{Z^{0}}^{2}\right)=0.121 \pm 0.003(\text { exp. }) \pm 0.011 \text { (theor.) } \quad \mathcal{O}\left(\alpha_{s}^{2}\right)
$$

and

$$
\alpha_{s}\left(M_{Z^{0}}^{2}\right)=0.119 \pm 0.003(\text { exp. }) \pm 0.007(\text { theor. }) \quad N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)
$$

Averaging these two results, we obtain

$$
\alpha_{s}\left(M_{Z^{0}}^{2}\right)=0.120 \pm 0.003(\text { exp. }) \pm 0.009 \text { (theor.) }
$$

where the theoretical error is dominated by the uncertainty in the missing higher order terms in the various calculations. This result is consistent with our previous results and with results from other experiments [ $33,34,40,41,42$ ]

## Acknowledgements

We thank the personnel of the SLAC accelerator department and the technical staffs of our collaborating institutions for their efforts which resulted in the successful operation of the SLC and the SLD. We also thank S.J. Brodsky, S.D. Ellis, K. Kato, P. Nason and D. Ward for helpful comments and suggestions relating to this analysis.

## References

[1] SLD Collab., K. Abe et al., SLAC-PUB-6456 (1994); to appear in Phys. Rev. Lett. ALEPH Collab., D. Buskulic et al., Z. Phys. C60 (1993) 71.
DELPHI Collab., P. Aarnio et al., Nucl. Phys. B367 (1991) 511.
L3 Collab., O. Adriani et al., Phys. Rept. 236 (1993) 1.
OPAL Collab., R. Akers et al., Z. Phys. C61 (1994) 19.
[2] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B47 (1973) 365.
D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343.
H. D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.
[3] W.A. Bardeen et al., Phys. Rev. D18 (1978) 3998.
[4] S.J. Brodsky and H.J. Lu, SLAC-PUB-6481 (1994); to appear in Phys. Rev.
[5] SLD Collab., K. Abe et al., Phys. Rev. Lett. 70 (1993) 2528.
[6] SLD Collab., K. Abe et al., Preprint SLAC-PUB-6451 (1994).
[7] Z. Kunszt, P. Nason, G. Marchesini and B.R. Webber, Proceedings of the Workshop on Z Physics at LEP I, eds. G. Altarelli, R. Kleiss and C. Verzegnassi, CERN Report 89-08 (1989).
[8] S. Bethke et al., Nucl. Phys. B370 (1992) 310.
[9] C. L. Basham et al., Phys. Rev. Lett. 41 (1978) 1585; Phys Rev. D17 (1978) 2298; Phys. Rev. D19 (1979) 2018.
[10] S. Catani, G. Turnock, B.R. Webber and L. Trentadue, Phys. Lett. B263 (1991) 491.
[11] S. Catani, G. Turnock and B.R. Webber, Phys. Lett. B272 (1991) 368.

- [12] S. Catani, Yu.L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, Phys. Lett. B269 (1991) 432.
[13] S. Catani, G. Turnock and B.R. Webber, CERN-TH-6570/92 (1992).
[14] G. Turnock, Preprint Cavendish-HEP-92/3 (1992).
[15] S. Catani, L. Trentadue, G. Turnock and B.R. Webber, Nucl. Phys. B407 (1993) 3.
[16] SLD Design Report, SLAC Report 273 (1984).
[17] C. J. S. Damerell et al., Nucl. Inst. Meth. A288 (1990) 288.
[18] D. Axen et al., Nucl. Inst. Meth. A328 (1993) 472.
[19] A. C. Benvenuti et al., Nucl. Inst. Meth. A290 (1990) 353.
[20] S. Brandt et al., Phys. Lett. 12 (1964) 57. E. Farhi, Phys. Rev. Lett. 39 (1977) 1587.
[21] T. Sjöstrand and M. Bengtsson, Comp. Phys. Comm. 43 (1987) 367.
[22] G. Marchesini et al., Comp. Phys. Commun. 67 (1992) 465.
[23] MARKJ Collab., D. P. Barber et al., Phys. Rev. Lett. 43 (1979) 830; Phys. Lett. B89 (1979) 139.
[24] G. Parisi, Phys. Lett. B 74 (1978) 65.
J. F. Donoghue, F. E. Low and S. Y. Pi, Phys. Rev. D20 (1979) 2759.
[25] L. Clavelli, Phys. Lett. B85 (1979) 111.
[26] S. Catani, G. Turnock and B. R. Webber, Phys, Lett. B295 (1992) 269.
[27] MARK II Collab., S. Komamiya et al., Phys. Rev. Lett. 64 (1990) 987.
[28] JADE Collab., W. Bartel et al., Z. Phys. C33 (1986) 23.
[29] Y. Ohnishi and H. Masuda, SLAC-PUB-6560 (1994).
[30] ALEPH Collab., D. Decamp et al., Phys. Lett. B257 (1991) 479.
[31] L3 Collab., B. Adeva et al., Phys. Lett. B257 (1991) 469.
[32] OPAL Collab., P. D. Acton et al., Phys. Lette. B276 (1992) 547.
[33] DELPHI Collab., P. Abreu et al., Z. Phys. C59 (1993) 21.
[34] ALEPH Collab., D. Decamp et al., Phys Lett. B284 (1992) 163.
- [35] B.R. Webber, Preceedings of the Workshop "QCD 20 years later", Aachen, June 9-13, 1992.
[36] T. Sjöstrand, CERN-TH-6488-92 (1992).
[37] P. N. Burrows, Z. Phys. C41 (1988) 375, OPAL Collab., M. Z. Akrawy et al., Z. Phys. C47 (1990) 505.
[38] CLEO Collab., R. Giles et al., Phys. Rev. D30 (1984) 2279. ARGUS Collab., Albrecht et al., Z. Phys. C54 (1992) 13, Z. Phys. C58 (1993) 191.
[39] P.N. Burrows and H. Masuda, SLAC-PUB-6394 (1993); to appear in Z. Phys. C.
[40] OPAL Collab., P. D. Acton et al., Z. Phys. C59 (1993) 1.
[41] L3 Collab., O. Adriani et al., Phys. Lett. B284 (1992) 471.
[42] S. Bethke, in Proceedings of the XXVI ${ }^{\text {th }}$ ICHEP, Dallas, (1992) 81.

Table 1: Distributions of the variables defined in the text. The data are corrected for acceptance and resolution of the detector and for initial state photon radiation. The first error represents the statistical uncertainty and the second error represents the experimental systematic uncertainty.

| $\tau$ | $\frac{1}{\sigma} \frac{d \sigma}{d \tau} \pm($ stat. $) \pm($ exp. $)$ | $\rho$ | $\frac{1}{\sigma} \frac{d \sigma}{d \rho} \pm($ stat. $) \pm($ exp. $)$ | $B_{T}$ | $\frac{1}{\sigma} \frac{d \sigma}{d B_{T}} \pm($ stat. $) \pm($ exp. $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.01 | $7.01 \pm 0.10 \pm 0.50$ | 0.01 | $10.53 \pm 0.12 \pm 0.41$ | 0.01 | $0.018 \pm 0.005 \pm 0.007$ |
| 0.03 | $16.10 \pm 0.15 \pm 0.15$ | 0.03 | $17.38 \pm 0.15 \pm 0.14$ | 0.03 | $1.36 \pm 0.04 \pm 0.18$ |
| 0.05 | $8.67 \pm 0.11 \pm 0.05$ | 0.06 | $6.21 \pm 0.07 \pm 0.16$ | 0.05 | $8.81 \pm 0.11 \pm 0.32$ |
| 0.07 | $5.08 \pm 0.08 \pm 0.16$ | 0.10 | $2.39 \pm 0.04 \pm 0.09$ | 0.07 | $10.64 \pm 0.12 \pm 0.16$ |
| 0.10 | $2.91 \pm 0.04 \pm 0.06$ | 0.15 | $1.08 \pm 0.02 \pm 0.04$ | 0.10 | $6.52 \pm 0.07 \pm 0.10$ |
| 0.14 | $1.57 \pm 0.03 \pm 0.05$ | 0.21 | $0.404 \pm 0.014 \pm 0.021$ | 0.14 | $3.65 \pm 0.05 \pm 0.04$ |
| 0.18 | $0.917 \pm 0.025 \pm 0.028$ | 0.28 | $0.102 \pm 0.006 \pm 0.010$ | 0.18 | $2.10 \pm 0.04 \pm 0.06$ |
| 0.23 | $0.495 \pm 0.015 \pm 0.025$ | 0.36 | $0.0047 \pm 0.0013 \pm 0.0008$ | 0.23 | $1.12 \pm 0.02 \pm 0.03$ |
| 0.29 | $0.227 \pm 0.010 \pm 0.016$ |  |  | 0.29 | $0.384 \pm 0.013 \pm 0.023$ |
| 0.35 | $0.061 \pm 0.005 \pm 0.006$ |  |  | 0.35 | $0.050 \pm 0.005 \pm 0.011$ |
| 0.41 | $0.003 \pm 0.001 \pm 0.003$ |  |  |  |  |


| $B_{W}$ | $\frac{1}{\sigma} \frac{d \sigma}{d B_{W}} \pm$ (stat. $) \pm$ (exp. $)$ | $\bigcirc$ | $\frac{1}{\sigma} \frac{d \sigma}{d O} \pm$ (stat. $) \pm$ (exp. $)$ | C | $\frac{1}{\sigma} \frac{d \sigma}{d C} \pm($ stat. $) \pm$ (exp. $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.01 | $0.570 \pm 0.028 \pm 0.213$ | 0.01 | $9.07 \pm 0.11 \pm 0.19$ | 0.02 | $0.166 \pm 0.011 \pm 0.015$ |
| 0.03 | $13.86 \pm 0.14 \pm 0.45$ | 0.03 | $11.28 \pm 0.12 \pm 0.20$ | 0.06 | $1.76 \pm 0.03 \pm 0.04$ |
| 0.05 | $11.71 \pm 0.13 \pm 0.20$ | 0.06 | $5.98 \pm 0.06 \pm 0.07$ | 0.10 | $4.01 \pm 0.05 \pm 0.09$ |
| 0.07 | $7.38 \pm 0.10 \pm 0.11$ | 0.10 | $3.16 \pm 0.05 \pm 0.06$ | 0.15 | $3.57 \pm 0.04 \pm 0.10$ |
| 0.10 | $4.29 \pm 0.05 \pm 0.08$ | 0.15 | $1.77 \pm 0.03 \pm 0.03$ | 0.21 | $2.30 \pm 0.03 \pm 0.02$ |
| 0.14 | $2.185 \pm 0.038 \pm 0.128$ | 0.21 | $0.935 \pm 0.021 \pm 0.028$ | 0.28 | $1.54 \pm 0.02 \pm 0.016$ |
| 0.18 | $1.12 \pm 0.028 \pm 0.061$ | 0.28 | $0.523 \pm 0.013 \pm 0.013$ | 0.36 | $1.07 \pm 0.02 \pm 0.03$ |
| 0.23 | $0.403 \pm 0.014 \pm 0.025$ | 0.36 | $0.223 \pm 0.009 \pm 0.010$ | 0.46 | $0.718 \pm 0.013 \pm 0.024$ |
| 0.29 | $0.030 \pm 0.004 \pm 0.005$ | 0.45 | $0.052 \pm 0.004 \pm 0.003$ | 0.58 | $0.491 \pm 0.011 \pm 0.013$ |
|  |  |  |  | 0.70 | $0.311 \pm 0.008 \pm 0.022$ |
|  |  |  |  | 0.82 | $0.146 \pm 0.006 \pm 0.012$ |
|  |  |  |  | 0.94 | $0.012 \pm 0.002 \pm 0.001$ |


| $y_{\text {cut }}$ | $D_{2}(\mathrm{E}) \pm($ stat. $) \pm($ exp. $)$ | $D_{2}(\mathrm{E} 0) \pm($ stat. $) \pm($ exp. $)$ | $D_{2}(\mathrm{P}) \pm($ stat. $) \pm($ exp. $)$ |
| :---: | :---: | :---: | :---: |
| 0.005 | $0.669 \pm 0.060 \pm 0.080$ | $28.95 \pm 0.39 \pm 1.44$ | $41.80 \pm 0.47 \pm 2.43$ |
| 0.010 | $2.60 \pm 0.12 \pm 0.12$ | $25.25 \pm 0.37 \pm 0.50$ | $31.06 \pm 0.41 \pm 0.63$ |
| 0.015 | $7.07 \pm 0.20 \pm 0.27$ | $19.93 \pm 0.33 \pm 0.53$ | $21.24 \pm 0.34 \pm 0.28$ |
| 0.02 | $10.48 \pm 0.24 \pm 0.66$ | $15.85 \pm 0.29 \pm 1.04$ | $14.96 \pm 0.28 \pm 0.54$ |
| 0.03 | $12.28 \pm 0.18 \pm 0.39$ | $11.66 \pm 0.18 \pm 0.15$ | $10.82 \pm 0.17 \pm 0.37$ |
| 0.05 | $10.89 \pm 0.12 \pm 0.34$ | $7.01 \pm 0.10 \pm 0.19$ | $6.35 \pm 0.09 \pm 0.23$ |
| 0.08 | $7.22 \pm 0.08 \pm 0.22$ | $3.85 \pm 0.06 \pm 0.05$ | $3.16 \pm 0.05 \pm 0.09$ |
| 0.12 | $3.81 \pm 0.05 \pm 0.11$ | $2.02 \pm 0.04 \pm 0.07$ | $1.61 \pm 0.03 \pm 0.08$ |
| 0.17 | $1.97 \pm 0.03 \pm 0.05$ | $1.08 \pm 0.02 \pm 0.04$ | $0.791 \pm 0.021 \pm 0.037$ |
| 0.22 | $0.987 \pm 0.023 \pm 0.034$ | $0.537 \pm 0.017 \pm 0.026$ | $0.317 \pm 0.013 \pm 0.024$ |
| 0.28 | $0.467 \pm 0.015 \pm 0.017$ | $0.204 \pm 0.010 \pm 0.015$ | $0.069 \pm 0.006 \pm 0.005$ |
| 0.33 | $0.178 \pm 0.009 \pm 0.024$ | $0.068 \pm 0.006 \pm 0.021$ | $0.008 \pm 0.002 \pm 0.007$ |


| $y_{\text {cut }}$ | $D_{2}(\mathrm{P} 0) \pm($ stat. $) \pm($ exp. $)$ | $D_{2}(\mathrm{D}) \pm($ stat. $) \pm($ exp. $)$ | $D_{2}(\mathrm{G}) \pm($ stat. $) \pm($ exp. $)$ |
| :---: | :---: | :---: | :---: |
| 0.005 | $39.78 \pm 0.46 \pm 2.41$ | $101.06 \pm 0.74 \pm 2.29$ | $7.67 \pm 0.20 \pm 1.01$ |
| 0.010 | $29.85 \pm 0.40 \pm 0.78$ | $26.85 \pm 0.38 \pm 0.34$ | $33.63 \pm 0.43 \pm 0.84$ |
| 0.015 | $20.49 \pm 0.33 \pm 0.36$ | $14.13 \pm 0.28 \pm 0.40$ | $31.71 \pm 0.41 \pm 1.01$ |
| 0.02 | $14.52 \pm 0.28 \pm 0.23$ | $9.00 \pm 0.22 \pm 0.44$ | $20.46 \pm 0.33 \pm 0.55$ |
| 0.03 | $10.65 \pm 0.17 \pm 0.37$ | $6.02 \pm 0.13 \pm 0.17$ | $11.71 \pm 0.18 \pm 0.20$ |
| 0.05 | $6.36 \pm 0.09 \pm 0.19$ | $3.30 \pm 0.07 \pm 0.11$ | $5.55 \pm 0.09 \pm 0.12$ |
| 0.08 | $3.21 \pm 0.05 \pm 0.12$ | $1.66 \pm 0.04 \pm 0.07$ | $3.20 \pm 0.05 \pm 0.06$ |
| 0.12 | $1.64 \pm 0.03 \pm 0.07$ | $0.831 \pm 0.024 \pm 0.038$ | $1.92 \pm 0.04 \pm 0.05$ |
| 0.17 | $0.944 \pm 0.023 \pm 0.057$ | $0.406 \pm 0.015 \pm 0.033$ | $1.25 \pm 0.03 \pm 0.03$ |
| 0.22 | $0.433 \pm 0.015 \pm 0.038$ | $0.173 \pm 0.010 \pm 0.011$ | $0.768 \pm 0.020 \pm 0.027$ |
| 0.28 | $0.169 \pm 0.009 \pm 0.015$ | $0.084 \pm 0.006 \pm 0.013$ | $0.409 \pm 0.014 \pm 0.019$ |
| 0.33 | $0.034 \pm 0.004 \pm 0.008$ | $0.027 \pm 0.004 \pm 0.048$ | $0.111 \pm 0.007 \pm 0.018$ |


| $\chi$ (deg.) | $E E C\left(r a d^{-1}\right) \pm($ stat. $) \pm($ exp. $)$ | $A E E C\left(\mathrm{rad}^{-1}\right) \pm(\mathrm{stat}) \pm.($ exp. $)$ | $J C E F\left(\mathrm{rad}^{-1}\right) \pm($ stat. $) \pm($ exp. $)$ |
| :---: | :---: | :---: | :---: |
| 1.8 | $2.265 \pm 0.006 \pm 0.055$ | $-1.506 \pm 0.006 \pm 0.060$ |  |
| 5.4 | $1.316 \pm 0.006 \pm 0.032$ | $-0.0403 \pm 0.0053 \pm 0.0001$ |  |
| 9.0 | $0.874 \pm 0.004 \pm 0.020$ | $0.224 \pm 0.010 \pm 0.002$ |  |
| 12.6 | $0.598 \pm 0.003 \pm 0.019$ | $0.249 \pm 0.009 \pm 0.005$ |  |
| 16.2 | $0.425 \pm 0.002 \pm 0.011$ | $0.211 \pm 0.006 \pm 0.005$ |  |
| 19.8 | $0.310 \pm 0.002 \pm 0.014$ | $0.181 \pm 0.004 \pm 0.005$ |  |
| 23.4 | $0.241 \pm 0.001 \pm 0.005$ | $0.148 \pm 0.004 \pm 0.006$ |  |
| 27.0 | $0.199 \pm 0.001 \pm 0.005$ | $0.121 \pm 0.003 \pm 0.004$ |  |
| 30.6 | $0.168 \pm 0.001 \pm 0.006$ | $0.0972 \pm 0.0024 \pm 0.0029$ |  |
| 34.2 | $0.146 \pm 0.001 \pm 0.005$ | $0.0785 \pm 0.0022 \pm 0.0062$ |  |
| 37.8 | $0.128 \pm 0.001 \pm 0.004$ | $0.0645 \pm 0.0017 \pm 0.0024$ |  |
| 41.4 | $0.118 \pm 0.001 \pm 0.003$ | $0.0513 \pm 0.0020 \pm 0.0026$ |  |
| 45.0 | $0.1099 \pm 0.0008 \pm 0.0026$ | $0.0413 \pm 0.0015 \pm 0.0027$ |  |
| 48.6 | $0.1014 \pm 0.0009 \pm 0.0031$ | $0.0346 \pm 0.0016 \pm 0.0021$ |  |
| 52.2 | $0.0935 \pm 0.0008 \pm 0.0027$ | $0.0275 \pm 0.0013 \pm 0.0060$ |  |
| 55.8 | $0.0901 \pm 0.0009 \pm 0.0021$ | $0.0213 \pm 0.0010 \pm 0.0024$ |  |
| 59.4 | $0.0867 \pm 0.0008 \pm 0.0023$ | $0.0163 \pm 0.0008 \pm 0.0073$ |  |
| 63.0 | $0.0827 \pm 0.0009 \pm 0.0023$ | $0.0141 \pm 0.0007 \pm 0.0026$ |  |
| 66.6 | $0.0802 \pm 0.0010 \pm 0.0018$ | $0.0129 \pm 0.0010 \pm 0.0008$ |  |
| 70.2 | $0.0764 \pm 0.0009 \pm 0.0031$ | $0.0110 \pm 0.0007 \pm 0.0025$ |  |
| 73.8 | $0.0770 \pm 0.0010 \pm 0.0010$ | $0.0064 \pm 0.0005 \pm 0.0017$ |  |
| 77.4 | $0.0752 \pm 0.0008 \pm 0.0031$ | $0.0058 \pm 0.0006 \pm 0.0029$ |  |
| 81.0 | $0.0736 \pm 0.0008 \pm 0.0013$ | $0.0041 \pm 0.0004 \pm 0.0020$ |  |
| 84.6 | $0.0751 \pm 0.0010 \pm 0.0015$ | $0.0012 \pm 0.0002 \pm 0.0038$ |  |
| 88.2 | $0.0744 \pm 0.0010 \pm 0.0014$ | $0.0017 \pm 0.0008 \pm 0.0016$ |  |
| 91.8 | $0.0761 \pm 0.0009 \pm 0.0013$ |  | $0.0274 \pm 0.0016 \pm 0.0010$ |
| 95.4 | $0.0764 \pm 0.0009 \pm 0.0025$ |  | $0.0403 \pm 0.0020 \pm 0.0012$ |
| 99.0 | $0.0777 \pm 0.0009 \pm 0.0023$ |  | $0.0442 \pm 0.0026 \pm 0.0010$ |
| 102.6 | $0.0809 \pm 0.0012 \pm 0.0016$ |  | $0.0523 \pm 0.0029 \pm 0.0023$ |
| 106.2 | $0.0834 \pm 0.0010 \pm 0.0024$ |  | $0.0566 \pm 0.0029 \pm 0.0024$ |
| 109.8 | $0.0874 \pm 0.0010 \pm 0.0022$ |  | $0.0613 \pm 0.0034 \pm 0.0026$ |
| 113.4 | $0.0931 \pm 0.0013 \pm 0.0015$ |  | $0.0725 \pm 0.0039 \pm 0.0017$ |
| 117.0 | $0.0968 \pm 0.0012 \pm 0.0038$ |  | $0.0832 \pm 0.0055 \pm 0.0046$ |
| 120.6 | $0.1030 \pm 0.0012 \pm 0.0070$ |  | $0.0858 \pm 0.0051 \pm 0.0016$ |
| 124.2 | $0.111 \pm 0.001 \pm 0.002$ |  | $0.0944 \pm 0.0043 \pm 0.0024$ |
| 127.8 | $0.121 \pm 0.001 \pm 0.007$ |  | $0.1051 \pm 0.0061 \pm 0.0055$ |
| 131.4 | $0.136 \pm 0.002 \pm 0.003$ |  | $0.114 \pm 0.005 \pm 0.002$ |
| 135.0 | $0.151 \pm 0.002 \pm 0.004$ |  | $0.131 \pm 0.005 \pm 0.005$ |
| 138.6 | $0.170 \pm 0.002 \pm 0.005$ |  | $0.148 \pm 0.005 \pm 0.006$ |
| 142.2 | $0.193 \pm 0.002 \pm 0.006$ |  | $0.169 \pm 0.007 \pm 0.004$ |
| 145.8 | $0.225 \pm 0.002 \pm 0.008$ |  | $0.188 \pm 0.007 \pm 0.005$ |
| 149.4 | $0.265 \pm 0.002 \pm 0.007$ |  | $0.228 \pm 0.008 \pm 0.009$ |
| 153.0 | $0.320 \pm 0.003 \pm 0.008$ |  | $0.275 \pm 0.009 \pm 0.010$ |
| 156.6 | $0.390 \pm 0.003 \pm 0.013$ |  | $0.329 \pm 0.011 \pm 0.013$ |
| 160.2 | $0.491 \pm 0.003 \pm 0.017$ |  | $0.414 \pm 0.011 \pm 0.019$ |
| 163.8 | $0.636 \pm 0.004 \pm 0.012$ |  | $0.551 \pm 0.012 \pm 0.013$ |
| 167.4 | $0.847 \pm 0.006 \pm 0.007$ |  | $0.751 \pm 0.021 \pm 0.021$ |
| 171.0 | $1.098 \pm 0.005 \pm 0.009$ |  | $1.095 \pm 0.024 \pm 0.019$ |
| 174.6 | $1.276 \pm 0.007 \pm 0.044$ |  | $1.639 \pm 0.032 \pm 0.034$ |
| 178.2 | $0.764 \pm 0.007 \pm 0.050$ |  | $1.530 \pm 0.039 \pm 0.049$ |

Table 2: Fit ranges, scale, hadronization, and experimental systematic uncertainties used for $O\left(\alpha_{s}^{2}\right)$ QCD fits. The statistical errors are the level of 0.001 or less.

| observable | fit range | $f$-range | $\alpha_{s}$ | scale | hadro. | exp. sys. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tau$ | $0.06-0.32$ | $1.5 \times 10^{-4}-4$ | 0.124 | $\pm 0.021$ | $\pm 0.003$ | $\pm 0.002$ |
| $\rho$ | $0.04-0.32$ | $9 \times 10^{-4}-4$ | 0.123 | $\pm 0.010$ | $\pm 0.001$ | $\pm 0.002$ |
| $B_{T}$ | $0.12-0.32$ | $4 \times 10^{-3}-4$ | 0.128 | $\pm 0.022$ | $\pm 0.003$ | $\pm 0.002$ |
| $B_{W}$ | $0.06-0.26$ | $1.2 \times 10^{-3}-4$ | 0.118 | $\pm 0.008$ | $\pm 0.002$ | $\pm 0.003$ |
| $O$ | $0.08-0.32$ | $1.2 \times 10^{-1}-4$ | 0.132 | $\pm 0.009$ | $\pm 0.008$ | $\pm 0.002$ |
| $C$ | $0.24-0.76$ | $3 \times 10^{-4}-4$ | 0.122 | $\pm 0.018$ | $\pm 0.003$ | $\pm 0.002$ |
| $D_{2}(\mathrm{E})$ | $0.08-0.28$ | $5 \times 10^{-5}-4$ | 0.126 | $\pm 0.022$ | $\pm 0.002$ | $\pm 0.002$ |
| $D_{2}(\mathrm{E} 0)$ | $0.05-0.28$ | $1.2 \times 10^{-2}-4$ | 0.118 | $\pm 0.008$ | $\pm 0.001$ | $\pm 0.003$ |
| $D_{2}(\mathrm{P})$ | $0.05-0.22$ | $5.5 \times 10^{-3}-4$ | 0.122 | $\pm 0.005$ | $\pm 0.003$ | $\pm 0.003$ |
| $D_{2}(\mathrm{P} 0)$ | $0.05-0.28$ | $1.2 \times 10^{-2}-4$ | 0.120 | $\pm 0.006$ | $\pm 0.002$ | $\pm 0.003$ |
| $D_{2}(\mathrm{D})$ | $0.03-0.22$ | $1.7 \times 10^{-3}-4$ | 0.124 | $\pm 0.008$ | $\pm 0.001$ | $\pm 0.003$ |
| $D_{2}(\mathrm{G})$ | $0.12-0.28$ | $4 \times 10^{-3}-4$ | 0.118 | $\pm 0.004$ | $\pm 0.003$ | $\pm 0.001$ |
| $E E C$ | $36.0^{\circ}-154.8^{\circ}$ | $3.5 \times 10^{-3}-4$ | 0.125 | $\pm 0.012$ | $\pm 0.002$ | $\pm 0.003$ |
| $A E E C$ | $18.0^{\circ}-68.4^{\circ}$ | $9 \times 10^{-2}-4$ | 0.113 | $\pm 0.003$ | $\pm 0.002$ | $\pm 0.003$ |
| $J C E F$ | $100.8^{\circ}-158.4^{\circ}$ | $3 \times 10^{-3}-4$ | 0.115 | $\pm 0.005$ | $\pm 0.001$ | $\pm 0.003$ |

Table 3: The results of $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ values derived from various variables using $O\left(\alpha_{s}^{2}\right)$ QCD fits.

| observable | $\alpha_{s}\left(M_{2^{\circ}}^{2}\right)$ | total exp. err. | total theor. err. |
| :---: | :---: | :---: | :---: |
| $\tau$ | 0.121 | $\pm 0.002$ | $\pm 0.021$ |
| $\rho$ | 0.124 | $\pm 0.002$ | $\pm 0.010$ |
| $B_{T}$ | 0.125 | $\pm 0.002$ | $\pm 0.023$ |
| $B_{W}$ | 0.116 | $\pm 0.003$ | $\pm 0.008$ |
| $O$ | 0.129 | $\pm 0.002$ | $\pm 0.012$ |
| $C$ | 0.119 | $\pm 0.002$ | $\pm 0.018$ |
| $D_{2}(\mathrm{E})$ | 0.127 | $\pm 0.002$ | $\pm 0.022$ |
| $D_{2}(\mathrm{E} 0)$ | 0.118 | $\pm 0.003$ | $\pm 0.008$ |
| $D_{2}(\mathrm{P})$ | 0.121 | $\pm 0.003$ | $\pm 0.006$ |
| $D_{2}(\mathrm{P} 0)$ | 0.119 | $\pm 0.003$ | $\pm 0.006$ |
| $D_{2}(\mathrm{D})$ | 0.125 | $\pm 0.003$ | $\pm 0.008$ |
| $D_{2}(\mathrm{G})$ | 0.119 | $\pm 0.002$ | $\pm 0.005$ |
| $E E C$ | 0.122 | $\pm 0.003$ | $\pm 0.012$ |
| $A E E C$ | 0.112 | $\pm 0.003$ | $\pm 0.004$ |
| $J C E F$ | 0.115 | $\pm 0.003$ | $\pm 0.005$ |

Table 4: Fit range, $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ and scale uncertainties $\left(\Delta \alpha_{s}\right)$ corresponding to the $f$ ranges, used for $N L L+O\left(\alpha_{s}^{2}\right)$ QCD fits.

| observable | fit range | $\ln R$ scheme | mod. $\ln R$ scheme | $R$ scheme | mod. $R$ scheme |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\alpha_{s} \pm \Delta \alpha_{s}$ | $\alpha_{s} \pm \Delta \alpha_{s}$ | $\alpha_{s} \pm \Delta \alpha_{s}$ | $\alpha_{s} \pm \Delta \alpha_{s}$ |
|  |  | $f$-range | $f$-range | $f$-range | $f$-range |
| $\tau$ | $0.06-0.32$ | $0.120 \pm 0.009$ | $0.120 \pm 0.009$ | $0.123 \pm 0.011$ | $0.119 \pm 0.009$ |
|  |  | $2.7 \times 10^{-3}-4$ | $2.7 \times 10^{-3}-4$ | $1.9 \times 10^{-3}-4$ | $2.3 \times 10^{-3}-4$ |
| $\rho$ | $0.04-0.32$ | $0.115 \pm 0.004$ | $0.116 \pm 0.005$ | $0.118 \pm 0.006$ | $0.115 \pm 0.004$ |
|  |  | $1.1 \times 10^{-2}-4$ | $1.1 \times 10^{-2}-4$ | $4.9 \times 10^{-3}-4$ | $1.0 \times 10^{-2}-4$ |
| $B_{T}$ | $0.12-0.32$ | $0.118 \pm 0.003$ | $0.121 \pm 0.002$ | - | $0.118 \pm 0.002$ |
|  |  | $6.7 \times 10^{-2}-4$ | $3.0 \times 10^{-1}-4$ |  | $3.6 \times 10^{-2}-4$ |
| $B_{W}$ | $0.06-0.26$ | $0.108 \pm 0.002$ | $0.120 \pm 0.001$ | - | $0.111 \pm 0.003$ |
|  |  | $8.2 \times 10^{-2}-4$ | $1.9 \times 10^{-1}-4$ |  | $4.9 \times 10^{-2}-4$ |
| $D_{2}(\mathrm{D})$ | $0.03-0.22$ | $0.131 \pm 0.006$ | $0.131 \pm 0.006$ | $0.125 \pm 0.005$ | $\mathrm{~N} / \mathrm{A}$ |
|  |  | $1.5 \times 10^{-1}-4$ | $1.6 \times 10^{-1}-4$ | $7.0 \times 10^{-2}-4$ |  |
| $E E C$ | $90.0^{\circ}-154.8^{\circ}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $0.124 \pm 0.005$ | $0.134 \pm 0.003$ |
|  |  |  |  | $6.1 \times 10^{-2}-4$ | $2.7 \times 10^{-1}-4$ |

Table 5: The results of $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ values derived from various variables using $N L L+$ $O\left(\alpha_{s}^{2}\right)$ QCD fits.

| observable | $\alpha_{s}\left(M_{Z^{\circ}}^{2}\right)$ | total exp. err. | total theor. err. |
| :---: | :---: | :---: | :---: |
| $\tau$ | 0.118 | $\pm 0.002$ | $\pm 0.012$ |
| $\rho$ | 0.116 | $\pm 0.002$ | $\pm 0.006$ |
| $B_{T}$ | 0.116 | $\pm 0.002$ | $\pm 0.005$ |
| $B_{W}$ | 0.107 | $\pm 0.003$ | $\pm 0.004$ |
| $D_{2}(\mathrm{D})$ | 0.130 | $\pm 0.003$ | $\pm 0.007$ |
| $E E C$ | 0.128 | $\pm 0.003$ | $\pm 0.007$ |

Table 6: Fit range, $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ and scale uncertainties ( $\Delta \alpha_{s}$ ) corresponding to the $f$ ranges, used for $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ QCD fits. The fit ranges are extended more toward the 2 -jet region than those of the $O\left(\alpha_{s}^{2}\right)$ fits.

| observable | fit range | $\ln R$ scheme | mod. $\ln R$ scheme | $R$ scheme | mod. $R$ scheme |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\alpha_{s} \pm \Delta \alpha_{s}$ | $\alpha_{s} \pm \Delta \alpha_{s}$ | $\alpha_{s} \pm \Delta \alpha_{s}$ | $\alpha_{s} \pm \Delta \alpha_{s}$ |
|  |  | $f$-range | $f$-range | $f$-range | $f$-range |
| $\tau$ | $0.02-0.32$ | $0.117 \pm 0.009$ | $0.118 \pm 0.008$ | - | $0.119 \pm 0.005$ |
|  |  | $7.0 \times 10^{-2}-4$ | $1.4 \times 10^{-1}-4$ |  | $6.3 \times 10^{-1}-4$ |
| $\rho$ | $0.02-0.32$ | $0.115 \pm 0.007$ | $0.115 \pm 0.007$ | $0.114 \pm 0.005$ | $0.112 \pm 0.007$ |
|  |  | $2.6 \times 10^{-2}-4$ | $3.4 \times 10^{-2}-4$ | $2.0 \times 10^{-1}-4$ | $4.0 \times 10^{-2}-4$ |
| $B_{T}$ | $0.04-0.32$ | $0.118 \pm 0.004$ | $0.120 \pm 0.002$ | - | $0.118 \pm 0.002$ |
|  |  | $2.0 \times 10^{-1}-4$ | $6.7 \times 10^{-2}-4$ |  | $1.1 \times 10^{-1}-4$ |
| $B_{W}$ | $0.04-0.26$ | $0.108 \pm 0.002$ | $0.109 \pm 0.001$ | - | $0.111 \pm 0.003$ |
|  |  | $1.4 \times 10^{-1}-4$ | $2.8 \times 10^{-1}-4$ |  | $5.4 \times 10^{-2}-4$ |
| $D_{2}(\mathrm{D})$ | $0.01-0.22$ | $0.127 \pm 0.003$ | $0.127 \pm 0.003$ | - | $\mathrm{N} / \mathrm{A}$ |
|  |  | $1.3 \times 10^{-1}-4$ | $1.3 \times 10^{-1}-4$ |  |  |
| $E E C$ | $90.0^{\circ}-162.0^{\circ}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $0.123 \pm 0.004$ | $0.134 \pm 0.003$ |
|  | - |  |  | $6.9 \times 10^{-2}-4$ | $5.0 \times 10^{-1}-4$ |

Figures 1.1-1.15: The measured distributions of $\tau, \rho, B_{T}, B_{W}, O, C, D_{2}(\mathrm{E}, \mathrm{E} 0, \mathrm{P}, \mathrm{P} 0, \mathrm{D}$, $\mathrm{G}), E E C, A E E C$, and $J C E F$, corrected to hadron level. The curves show the predictions of the QCD parton shower models JETSET 7.3 (solid line) and HERWIG 5.5 (dashed line) as described in the text.

Figures 2.1-2.15: (a) The measured distributions of $\tau, \rho, B_{T}, B_{W}, O, C, D_{2}$ (E, E $0, \mathrm{P}, \mathrm{P} 0$, $\mathrm{D}, \mathrm{G}), E E C, A E E C$, and $J C E F$, corrected to the parton level. The curves show the predictions of the $O\left(\alpha_{s}^{2}\right)$ calculations (dashed line) and $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ calculations with modified $\ln R$-matching (solid line). The renormalization scale factor is fixed to be 1. (b) Sizes of the detector correction and (c) hadronization correction factors. The widths of the bands indicate the uncertainties of the corrections.

Figures 3.1-3.3: (a) $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ and (b) $\chi_{d o f}^{2}$ from the $\mathcal{O}\left(\alpha_{s}^{2}\right)$ fits to the various quantities as a function of renormalization scale factor $f$ (see text).

Figures 4.1-4.4: (a) $\alpha_{s}\left(M_{Z^{0}}^{2}\right)$ and (b) $\chi_{d o f}^{2}$ from the $N L L+\mathcal{O}\left(\alpha_{s}^{2}\right)$ fits to the various variables as a function of renormalization scale factor $f$ (see text).

Figure 5: Compilation of final results of $\alpha_{s}$ at $\sqrt{s}=91 \mathrm{GeV}$. The solid error bars denote experimental errors, while the dotted error bars show the total errors, including experimental and theoretical errors added in quadrature. The vertical lines represent the averaged ${ }^{-}$value of $\alpha_{s}$ and its error.


Fig. 1.1-1.6







Fig. 1.7-1.12


Fig. 1.13-1.15





Fig. 2.1-2.4


Fig. 2.5-2.8





Fig. 2.9-2.12




Fig. 2.13-2.15



Fig. 3.1 - Fig. 3.3




Fig. 4.1 - Fig. 4.3


Fig. 4.4


Fig. 5


[^0]:    * Work supported by the Department of Energy contracta: DE-FG02-91ER40676 (BU), DE-FG0392ER40701 (CIT), DE-FG03-91ER40618 (UCSB), DE-FG03-92ER40689 (UCSC), DE-FG03-93ER40788 (CSU), DE-FG02-91ER40672 (Colorado), DE-FG02-91ER40677 (Ilinois), DE-AC03-76SF00098 (LBL), DE-FG02-92ER40715 (Maenachuette), DE-AC02-76ER03069 (MIT), DE-FG06-85ER40224 (Oregon), DE-AC03-76SF00515 (SLAC), DE-FG05-91ER40627 (Tennemee), DE-AC02-76ER00881 (Wicconsin), DE-FG02-92ER40704 (Yale); National Science Foundation grante: PHY-91-13428 (UCSC), PHY-8921320 (Columbia), PHY-92-04239 (Cincinnati), PHY-88-17930 (Ratgers), PHY-88-19316 (Vanderbilt), PHY-92-03212 (Waehington); the UK Science and Enginoering Recearch Council (Brunel and RAL); the Intituto Nasionale di Fisica Nucleare of Italy (Bologan, Ferrara, Fraceati, Piea, Padova, Peragia); and the Japan-US Cooperative Reerearch Project on High Energy Physica (Nagoya, Tohoku).

[^1]:    *It has also been called $R$ - $G_{21}$-matching [33], or intermediate matching [34].
    ${ }^{\dagger}$ We took the value of $\mathrm{y}_{\text {max }}$ to be 0.5 for $\tau, 0.42$ for $\rho, 0.41$ for $B_{T}, 0.325$ for $B_{W}$, and 0.33 for $D_{2}(\mathrm{D})$. The kinematic limit depends on the parton multiplicity.
    ${ }^{\ddagger}$ To simulate B hadron decay, we have tuned the multiplicity and momentum spectra of B decay products to the $\Upsilon_{4 S}$ data [38].

[^2]:    ${ }^{\S}$ We find no differerce between JETSET 6.3 and JETSET 7.3 in parton generation or fragmentation.

