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Fermion and Sfermion Effects in e^+e^- charged Higgs pair production [†]

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Abstract

Top and bottom one loop corrections to $e^+e^- \to H^+H^-$ are considered at NLC energies and found to be significantly large and negative over the full range of $tan\,\beta$. Moreover, loop effects of moderately heavy squarks and sleptons in the minimal supersymmetric extension of the standard model, tend to cancel partly or fully these large corrections, thus providing a useful indirect information about the supersymmetric sector. The overall effect can still range between -25% and $+25\%$, however lies typically around -10% in a wide range of the model parameters.

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Introduction.

The scalar sector of the Standard Model (SM) still lacks direct experimental evidence. Yet the recent LEP precision measurements, together with the dogma of grand unification seem to favour not just an extended Higgs sector, but rather a full-fledged supersymmetric extension of the SM [1].

Furthermore, it appears,[2] , that the minimal supersymmetric extension of the SM, MSSM [3], can be at least as successful as the SM itself, when confronted with the LEP data, even for not too heavy susy particles. All this, together with the general strong theoretical motivation for the MSSM when viewed as the low energy effective theory of some grand unified supergravity theory [4], makes it rather compelling to expect that the next generation of e^+e^- machines (LEPII, NLC) and/or hadron colliders (LHC and possibly an upgraded Tevatron) could discover neutral and/or charged Higgs scalars [5,6], and perhaps establish first evidence for the existence of SUSY particles.

However even then, one might still have to face the situation where a substantial part of the SUSY spectrum remains unidentified. Typically, squarks and gluinos can in principle be discovered at future hadron colliders up to masses in the $1-2$Tev range while sleptons would become invisible to LHC if heavier than ~ 250GeV or so [7], due to their weak coupling and a prominent background. It is then natural to ask to what extent can the phenomenological study of the Higgs sector give complementary, though indirect, information about susy spectrum. Furthermore it is clearly important to investigate the extent to which loop corrections can modify the tree-level based assessment of expected Higgs production rates at future machines.

Hereafter we study this question in the context of charged Higgs production at the projected e^+e^- machine with c.m. energy $\sqrt{s} = 500GeV$.

The present experimental lower bounds on $m_{H^\pm}$ lie around $35 - 40$ GeV [8], (to be contrasted with those of the Standard Higgs, 60 GeV [9], and the neutral MSSM Higgses, 44 (resp. 20) GeV for h (resp. A))[10]. Recently there has also been some renewed interest in constraints on $m_{H^\pm}$ which can be inferred from the present upper bound on the branching ratio $Br(b \to s\gamma)$ [11], where $H^\pm$ as well as charginos and stops enter at the one-loop level, [12]. However a charged Higgs lighter than 250 GeV seems to be still allowed in the context of the MSSM, [13], even though the region under 200GeV is rather not favoured in a general analysis of susy unification constraints * [14]. If so, one would still expect typically a few 100 events from the pair production at NLC energies, assuming a nominal luminosity of 10 fb^{-1} and $M_{H^\pm} \sim 200 - 230$ GeV, [15]. It is worth recalling here that the charged Higgs production via the Z^o or the photon at the tree level has the peculiar feature of being independent of the non-standard parameters (m_A, tanβ, $m_{\tilde{f}}$, $m_{\tilde{q}}$, $\cdots$) except of course for the charged Higgs mass itself. (One should keep in mind the $\tan\beta$ dependence at tree level in the charged Higgs leptonic and hadronic partial decay widths. In the present study however, we concentrate exclusively on the production process). In the process $e^+e^- \to H^+H^-$ all non-standard parameters will first occur at the one loop level and the hope is that this would increase the sensitivity to the susy spectrum

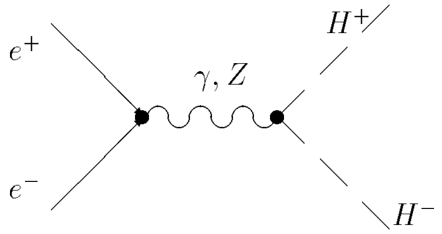
*We thank L. Roszkowski for a discussion on this point itself, once the well known SM one loop corrections are properly subtracted. This would complement the information from the neutral Higgs sector where the free parameters at the tree level render the analysis somewhat more intricate [16]. It thus appears that quantum corrections to $e^+e^- \to H^+H^-$ could provide useful indirect indications for a heavy susy spectrum. However one still has to control the potentially large top-bottom contributions which are sensitive to $tan\,\beta$, the relative strength of the Higgs vacuum expectation values in two Higgs doublets models.

In this letter we concentrate on the estimate of the effects from the standard heavy fermions and their susy scalar partners which occur in self-energies and vertices on the integrated cross-section, leaving the inclusion of the full radiative corrections from the MSSM to a forthcoming paper. The leading (top-bottom) fermionic contributions will depend solely on $\tan\beta$ as a non-standard free parameter and are actually those of a (type II) two-Higgs doublets model without supersymmetry. Those of the scalar partners bring in the dependence on the soft susy breaking parameters, the left-right mixing angles, the squark and slepton masses, etc... As it will turn out, the effect from top-bottom will be at least a -8% to -10% dip in the cross section, for $m_{top} \sim 165-175 GeV$ (but actually much larger if $\tan\beta < 5$ or $\tan\beta > 10$), while that from squarks and sleptons is found to be generically positive. However the possibility of large cancellation between the above contributions is possible only in a marginal region of parameter space. A generic -10% overall effect is found over a wide range of the parameter space and for moderately heavy squarks and sleptons. Yet there will still be regions where the effect is much larger and with either signs.

Before discussing the results in more detail, we describe briefly in the next section the renormalization scheme and the parametrization we use.

Renormalization and parametrization

At tree level, the leading contribution (i.e. neglecting the electron mass) to the angular distribution corresponds to the Feynman diagram drawn below, and is given by,



$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\kappa}{64\pi^2 s}\left(\sum_{spin}|\frac{1}{2}\mathcal{M}_{Born}|^2\right), \tag{1}$$

with

$$\sum_{spin}|\frac{1}{2}\mathcal{M}_{Born}|^2 = \frac{1}{2}e^4\kappa^2\left(1 + g_H^2\frac{g_V^2+g_A^2}{(1-m_Z^2/s)^2} - \frac{2g_H g_V}{1-m_Z^2/s}\right) sin^2\theta, \tag{2}$$

$\kappa^2 = 1 - \frac{4m_{H^\pm}^2}{s}$, $m_{H^\pm}$ the mass of the charged Higgs, m_Z the mass of the neutral Z boson, s the energy in the center of mass frame , $g_V = (1 - 4s_w^2)/(4c_w s_w)$, $g_A = 1/(4c_w s_w)$, $g_H = -(c_w^2 - s_w^2)/(2c_w s_w)$, $c_w \equiv \cos\,\theta_w$, $s_w \equiv sin\,\theta_w$ and θ the scattering angle.
The total cross-section is, [15]:

$$\sigma_0 = \frac{\pi\alpha^2\kappa^3}{3s}\left(1 + g_H^2 \frac{g_V^2 + g_A^2}{(1 - m_Z^2/s)^2} - \frac{2g_H g_V}{1 - m_Z^2/s}\right). \tag{3}$$

The parameters entering the above tree level observables are all standard except for the charged Higgs mass. Furthermore, the non-standard parameters which will appear at the one–loop level can be consistently taken as bare in our computation. In particular issues like the renormalization scheme dependence of the separation between large and small tan β which occurs in the case of neutral Higgs [17] can be ignored in our case as being a higher ordre effect.
On the other hand, in the process of comparing the non-susy case, (i.e. mainly the top-bottom contributions) with the MSSM case, (i.e. fermions plus their scalar partners), one should in principle take into account the fact that in the first case all Higgs masses are independent quantities while Higgs mass sum rules exist in the second case.

Let us recall here that these tree-level relations [18], can be largerly violated due to radiative corrections, in the case of neutral Higgses [19], whereas the relation involving the charged and neutral CP-odd Higgses was shown to be much less sensitive to loop corrections, apart from a small region in parameter space [20].

Because of such Higgs mass relations one is not at liberty to choose the Higgs mass counterterms such that *all* the physical Higgs masses be identified with the tree-level corresponding parameters in the lagrangian. For instance in ref.[17] the above choice is made for the neutral CP–odd Higgs. The renormalized masses of the other Higgses are then automatically determined by the corresponding sum rules without further subtraction conditions, (provided of course the standard Z and W masses have been renormalized). In the present paper we also adopt an on-shell scheme as defined in ref.[22], however we choose to identify the physical charged Higgs mass with the corresponding parameter in lagrangian, i.e., the mass counterterm is given by

$$\delta m_{H^\pm}^2 = Re \sum_{H^+H^+} (m_{H^\pm}^2) \tag{4}$$

where $\sum_{H^+H^-}(p^2)$ is the charged Higgs bare self-energy. The above choice is the simplest in our case since we consider only the charged Higgs and do not look at constraints from the violation of the tree level relation between $m_{H^\pm}$ and m_A. Indeed the only sizeable constraint seems to come from the light charged Higgs scenario ($m_{H^\pm} < M_W$) [20], which we disregard here. On the other hand eq.(4) corresponds to the natural on-shell prescription in the non susy case and in any case implies that the one-loop correction to the $H^\pm$ self-energy cancels out when the Higgs pair is produced on shell. All the remaining divergencies are then absorbed in the renormalization of the electric charge, M_Z, M_W, m_e and the wave functions following [22]. Hence the renormalization procedure will involve essentially a set of (standard) parameters, which would facilitate a simultaneous treatment and comparison of susy and non-susy cases. The purely standard contributions to these corrections can be obtained by running their known expressions up to NLC energies. (The treatment of the hadronic contribution to the self-energies would require however the knowledge of $e^+e^- \to \gamma^* \to hadrons$ up to these energies). These purely standard effects will not be included in the present study, except for the non-universal contributions coming from the heavy top-bottom doublet. There are also the pure susy contributions to the standard parameters. In the energy range $500GeV - 1TeV$, one no more expects all the susy particles to decouple, since some or all of the charginos and neutralinos are expected to be much lighter. The loop contributions of charginos and neutralinos to the bosonic self-energies and the $(\gamma)ZH^+H^-$ vertices can be comparable to those from other sectors. (Note however that in the present case the potential enhancement of their coupling to the lepton-slepton sector for large $tan\,\beta$ is suppressed by the electron mass). Only the squark and slepton corrections enjoy further enhancement from the free parameters $\mu, A_f, tan\,\beta$, which are the ones we are interested in here. In this case the $(\gamma)ZH^+H^-$ vertex counterterms are related to the Higgs wave function renormalization (see also [23]).

In the more general case however, the vertex counterterms will be also related to the one loop renormalization of the photon and Z self energies. †. Accordingly, the full vertex counterterm is given by, (using notations of ref. [22])

$$\begin{aligned}
\delta(A_\mu H^+H^-) &= -ie[\delta Z^{H^\pm} + (\delta Z_1^\gamma - \delta Z_2^\gamma) + g_H(\delta Z_1^{\gamma z} - \delta Z_2^{\gamma z})](p+q)_\mu \\
\delta(Z_\mu H^+H^-) &= ieg_H[\delta Z^{H^\pm} + (\delta Z_1^z - \delta Z_2^z) + \frac{1}{g_H}(\delta Z_1^{\gamma z} - \delta Z_2^{\gamma z})](p+q)_\mu \qquad (5)
\end{aligned}$$

all other MSSM parameters being consistently kept at their tree level values.
Specifying to the quarks, leptons and their scalar partners, the relevant one-loop diagrams are depicted in fig.8 and have been generated and computed as part of the full one loop corrections in the 't Hooft-Feynman gauge, using FeynArts and FeynCalc packages [24] supplied with a full MSSM Feynman rules code [25]. We also used the Fortran FF-package [26], in the numerical analysis.

It is worth noting that the set of diagrams in fig.8 is gauge invariant by itself and thus can be studied separately. (Note also that the 1PI contribution to $Z(\gamma)H^+H^-$ with $\tilde{q}\tilde{q}H^+H^-$ coupling is trivially vanishing). Apart from the top quark mass, we have *generically* nine non-standard free parameters to deal with, which one can choose as,

- the charged Higgs mass $m_{H^\pm}$.
- the ratio of the two vacuum expectation values tanβ.
- the μ-parameter parametrizing the (susy invariant) Peccei-Quinn symmetry breaking term in the superpotential.

† as for the infrared, part it will be included in the full one-loop computation elsewhere, [21].

- the squark and slepton masses $m_{\tilde{q}_{1,2}}$, $m_{\tilde{l}_{1,2}}$.
- the left-right mixing angles $\theta_{u,d}$ and θ_l.

Furthermore we will neglect CKM and super-CKM flavor mixing effects. The soft supersymmetry breaking parameters A_f are then connected to the previous ones through,

$$A_{(u,d)} = \frac{\sin(\theta_{(u,d)})\ \cos(\theta_{(u,d)})}{m_{(u,d)}}(m^2_{(\tilde{u}_1,\tilde{d}_1)} - m^2_{(\tilde{u}_2,\tilde{d}_2)}) - \mu\,(cot\beta, tan\beta) \tag{6}$$

where u and d denote up and down flavors for quarks or leptons and $m_{(u,d)}$, $m_{(\tilde{u}_1,\tilde{d}_1)}$ and $m_{(\tilde{u}_2,\tilde{d}_2)}$ are the *physical* masses of the respective scalar partners .(In eq.(5) we absorbed the soft susy breaking mass parameter "m_6" in the definition of $A_{(u,d)}$, see ref.[3] for notations). On the other hand we will carry later discussions in terms of degenerate or non degenerate scalar partners (at least for the scalar tops), rather than in terms of squark (or slepton) mixing. Among others, a significant consistency check of our one–loop expressions is their independence of θ_u (resp. θ_d) when the up–squarks (resp. down–squarks) of a given family are mass degenerate.

The proliferation of free parameters is of course due to the fact that we do not assume any specific underlying theory at higher energies. However in the numerical study we will generally remain close to the typical range of values suggested by the underlying assumptions of supersymmetric grand unification and radiative electroweak breaking[28]. For completeness let us simply recall how the free parameters enter the charged Higgs couplings to quarks, leptons and their scalar partners, in the left-right basis, (see ref.[3,18] for more details) :

$$H^{\pm}ud: \ \frac{ig}{2\sqrt{2}m_W}(m_d tan\beta + m_u cot\beta \pm (m_d tan\beta - m_u cot\beta)\gamma_5) \tag{7}$$

$$H^{+}\tilde{u}_{(R,L)}\tilde{d}_{(L,R)}: \ H_{(RL,LR)} \equiv \frac{-ig}{\sqrt{2}}\frac{m_{(u,d)}}{m_W}(\mu - (A_u cot\beta, A_d tan\beta)) \tag{8}$$

$$H^{+}\tilde{u}_L\tilde{d}_L: \ H_{LL} \equiv \frac{-ig}{\sqrt{2}}m_W(sin(2\beta) - \frac{m_d^2 tan\beta + m_u^2 cot\beta}{m_W^2}) \tag{9}$$

$$H^{+}\tilde{u}_R\tilde{d}_R: \ H_{RR} \equiv \frac{-ig}{\sqrt{2}}\frac{m_u m_d}{m_W}(tan\beta + cot\beta) \tag{10}$$

Furthermore the left-right mixing angles, θ_q, θ_l will enter the game whenever $\tilde{q}_L(\tilde{l}_L)$ and $\tilde{q}_R(\tilde{l}_R)$ are not mass eigenstates.

From the above dependence on the various free parameters one can understand the qualitative features of the results discussed in the next section. It is however already clear for instance that in general the bottom quark mass should not be neglected in the $H^{\pm}tb$ vertex, since its contribution becomes comparable to that of the top ($m_t \simeq 160$GeV) as soon as tan$\beta \geq 6$. On the other hand the chirality non-conserving $H\tilde{u}\tilde{d}$ couplings become insensitive to the scalar partner masses in the degenerate case, but remain very sensitive to the μ parameter. In contrast the chirality conserving ones are controlled by the fermion masses and $tan(\beta)$ only.

Results and discussion

Since we are interested here mainly in the trend of the corrections as a function of the free parameters, we will consider only the leading effect from the interference between the one loop and tree level contributions. We do not include any improved resummation and the standard model parameters will be kept at their tree level values rather than being run to the energy scale under consideration. This approximation is safe inasmuch as it somewhat underestimates the effect from the running of α_{QED}.

Perhaps more importantly, and since we are dealing with very large effects in some cases, it is important to have some criteria which allow to control the validity of the perturbative result. With this respect a more elaborate treatment using renormalization group techniques to include higher order leading logs would be appropriate. This might extend the region in parameter space, overwhich the perturbative results remain reliable. In the present study, however, we will adopt the following simple (and preliminary) criterion. We require that the contributions to $\tilde{\delta}_t$, (see eq.(11) below), which are $O(\alpha^2)$ and obtained from squaring the one-loop amplitude, do not exceed 50% of the $O(\alpha)$ ones. Although incomplete, this criterion does give some constraints as we will see, on the otherwise increasingly big effects in some regions of the parameter space. It will constrain as well the range of variation of the free parameters, over which we consider the (unimproved) one-loop results to remain meaningful. For instance allowing $tan\,\beta$ to be in the range $1 < tan\,\beta < m_t/m_b$ [27], our criterion will most of the time constrain further the valid domain as we will see. The same holds for μ which we will take a priori between 0 and 500GeV.

On the other hand the effects we are interested in appear in the integrated cross-section. There are no shape effects in the angular distribution since the self-energy and vertex corrections clearly contribute only to the leading tree-level s-wave component with the typical $\sin^2\theta$ dependence ‡. The one-loop correction $\tilde{\delta}_t$ is defined through

$$\sigma = \sigma_0\,(1 + \tilde{\delta}_t) \tag{11}$$

where σ_0 is the tree-level cross-section given by eq.(3) and

$$\tilde{\delta}_t \equiv \tilde{\delta}_t(m_t, tan\,\beta, m_{H^\pm}, \mu, m_{\tilde{q}_{1,2}}, m_{\tilde{l}_{1,2}}, \theta_{\tilde{q}}, \theta_{\tilde{l}})$$

As we will see, qualitatively significant information can be obtained in spite of the large number of free parameters especially as regards the potentially large effects that may arise for large $tan\,\beta$ and/or soft SUSY breaking scale. In fig.1 we show the expected tree level integrated cross section σ_0 as a function of $m_{H^\pm}$. Most of the subsequent discussion of the radiative corrections will be carried with $m_{H^\pm} = 220$ GeV, a value which lies in the typical range compatible with the indirect constraints from $BR(b \to s\gamma)$ and in the same time leading to an interesting production rate at a 500GeV ($10 fb^{-1}$) machine.

‡ angular effects arise only from box contributions which will be considered in [21].

The contributions directly sensitive to $tan\,\beta$ and to the soft susy breaking parameters in $\tilde{\delta}_t$ come from matter fields and their susy partners in the γH^+H^- and ZH^+H^- vertices. We also included the contributions to the γ and Z self-energies from top, bottom and squarks and sleptons loops, but these remain actually numerically small. (As we mentioned before, we do not include in the present study all the remaining one loop corrections to $e^+e^- \to H^+H^-$ as they are either purely standard and in principle under control, or correspond to those susy contributions which cannot be enhanced by large free parameters. The same turned out to be true for the potentially large contributions from the various neutral and charged Higgses. These do not exceed a few percent as long as the Higgses masses remain far below the TeV scale).
The leading $O(\alpha)$ correction can be written in the form

$$\tilde{\delta}_t \equiv \frac{\alpha\,\kappa^3}{12\sigma_0} Re\{(-1+\frac{g_H g_V}{1-m_Z^2/s})(\frac{1}{s}X_\gamma + \frac{(-1+4s_w^2)}{s-m_Z^2}X_Z) - \frac{g_H g_A}{1-m_Z^2/s}\frac{X_Z}{s-m_Z^2}\} \qquad (12)$$

where X_γ (resp. X_Z) collect the one-loop corrections to the γH^+H^- (resp. ZH^+H^-) renormalized vertices as well as the γ and Z self-energies and we drop the Z width in the propagator. (We also absorb in X_γ and X_Z the corresponding couplings to the initial state). The full analytic expressions of the leading contributions, namely the renormalized vertex corrections due to top-bottom and stop-sbottom loops are given in the appendix. The leading effect from the top and bottom quarks is shown in fig.2a and 2b. as a function of m_t or $tan\,\beta$ (with $m_b \simeq 4.5GeV$). The contribution turns out to be always negative and significant, in the full $tan\,\beta$ range. It reaches at least -8% to -10% if $m_t \geq 165$ GeV for $tan\,\beta$ between 5 and 10 and becomes much larger away from this range. Also the fact that $\tilde{\delta}_t$ is bounded from above (for a given value of m_t) is a direct consequence of the form of the $H^\pm tb$ coupling in eq.(6), and corresponds to the value of $tan\,\beta$ where the top and bottom effects in $H^\pm tb$ vertex become comparable. In any case the main feature here is that the top-bottom effects are generally significantly large and *negative* even for not too large $tan\,\beta$ and that in a non-susy two Higgs doublet model, they can easily overwhelm the remaining one loop corrections and lead to a significant dip in the expected charged Higgs production rate. Of course the effect can become eventually too large ($\sim -50\%$) for very large $tan\,\beta$ so that one should start worrying about higher order corrections. Actually, according to our perturbation criterion, $O(\alpha^2)$ effects can become sizeable if $tan\,\beta > 20$ or $tan\,\beta < 1.5$ in which case $\tilde{\delta}_t$ reachs respectively -25% and and -20%. This suggests that (conservatively) the magnitude of the real effect would not exceed the values quoted above, leading anyway to a dip in the total cross-section. For a further assessment of the effect we show in fig.3 the sensitivity to the charged Higgs mass.

The situation becomes drastically different in the susy case. As illustrated in fig.4a-b, the contributions of squarks and sleptons are generically positive and become increasingly significant for large μ and $tan\,\beta$ as far as squark and slepton masses are not in the TeV range. Note that we assumed all squarks (resp. sleptons) degenerate in mass, which is a typical feature when running these masses down from a common value at a higher unification scale [28], except for the susy partners of the top quark which are non-degenerate. We will consider this more realistic case later on. For now it is important to stress that

the squark and slepton contributions can largely reduce the top-bottom effect. This is illustrated in fig. 5 for various values of μ and $tan\,\beta$. (Note that since squarks (resp. sleptons) are taken degenerate in mass, the parameter A_f in eq.(5) is no longer free, also the physical observables do not depend in this case on the actual value of the (arbitrary) left-right mixing angles θ_q and θ_l).
As one can see from fig.5 , the inclusion of squarks and sleptons, (actually mainly the susy partners of (t,b) and (ν,τ)) levels the overall negative effect to about -10%, for a wide range of $tan\,\beta$, but a complete cancellation is also possible and even a significant overall positive effect. The details depend of course on the chosen values of $m_{\tilde{l}}, m_{\tilde{q}}, m_t$ and $m_{H^\pm}$, however varying μ and $tan\,\beta$ simultaneously and assuming $m_{\tilde{l}}$ and $m_{\tilde{q}} > 250$GeV one finds a large and negative effect over a fairly large region of parameter space. This is illustrated by the contour plots of fig.9 where we took for illustration $m_{\tilde{l}} = 300$ GeV and $m_{\tilde{q}} = 400$GeV. In this case a negative effect reaching down to -25% is expected in a large domain comprising virtually any value of tan β and μ, although the larger tan β the smaller μ should be. In contrast, positive overall effects, reaching up to $+25\%$ are squeezed in a much smaller region corresponding to tan β and μ simultaneously large. Note also that a cancellation between the top–bottom and squark-slepton contributions occurs only in a pencil-like region of large parameters. (The maximum values $\pm 25\%$ correspond to the maximum effect allowed within our perturbativity criterion which requires in this case tan $\beta < 25$ (resp. < 30) iﬁ $\mu < 50$GeV (resp. > 400GeV). Furthermore the region between say, -8% and -12% tends to be much less sensitive to the values of μ and tan β than the rest of the negative region, while for tan $\beta < 5$ the leading effect is from top–bottom loops.
We turn now briefly to two other possibilities, namely, a) squarks are very heavy (~ 1 TeV) and sleptons much lighter but heavier than $\sim 250 GeV$; b) the susy partners of the top are non-degenerate ($m_{\tilde{t_1}} - m_{\tilde{t_2}} \sim 100 - 300 GeV$); both cases a) and b) would modify the previous discussion.
As we mentioned previously, case a) corresponds to a situation where the squarks can still be detected at an LHC machine but not the sleptons. In addition in this case the squarks contibution to $\tilde{\delta}_t$ becomes negligible (see fig. 4a), a direct consequence of the decoupling of the heavy susy sector from "low energy" physics [29], as far as the parameter μ present in the Higgs-squark-squark vertex is not very large ($\mu < 500$ GeV). In such a scenario a slepton in the $300 - 400$GeV mass range could still give a significant effect. For instance as it can be seen from fig.6 and fig.2b, a 300 GeV scalar-tau can reduce the (negative) top-bottom effect down to -15% if $\mu \simeq 450$GeV and $tan\,\beta \simeq 30$. However taking into account our criterion for perturbativity the acceptable effect would be around only 4%. Although this might be a marginal possibility as far as the parameter space is concerned, it remains worth considering as an indirect signature of sleptons heavier than $250 GeV$, given the rather limited discovery potential at LHC, as well as at a 500 GeV e^+e^- NLC machine §.

Finally let us comment briefly case b). Note that in this case the soft susy breaking parameter A_t (or equivalently the left-right mixing angle θ_t between the two scalar partners

§ actually even a laser back scattered photon could scan only selectrons somewhat above the beam energy, through $e\gamma \to \tilde{e}\tilde{\gamma}$, but not $s-taus$ which give the leading effect in our case.

of the top quark, see eq.(5)) becomes a free parameter.The effects are found to remain comparable to those in the degenerate case as long as θ_t is small (< 0.2rad.), but they are generally very sensitive to this angle. In fig.7a–b we illustrate the squarks contribution for two values of the stop mass splitting with $\theta_t = 0.4$ rad. and where we assumed for definiteness the lighter stop mass to be above 150 GeV and $\mu = 250$ GeV. (Note also that the sensitivity to the sign of μ turns out to be weak for the chosen values in the figures). It should be stressed that unlike the case of the one–loop corrected mass relation between $m_{H^\pm}$ and m_A [20], the effect under study does not exhibit a new enhancing m_t dependence in the presence of left–right mixing. (see expressions in appendix b)). In any case a full 3–parameter study becomes mandatory and is out of the scope of this letter. However in the special case of figure 7a for instance, one finds that the perturbative result remains reliable even for $m_{\tilde{t}_1}$ as low as 300GeV and $\tan\beta$ as large as 20, in which case the total effect (including top-bottom contribution) would be around $+30\%$. Finally it is worth noting that one can get even larger total positive effects, up to 50% which remain rather safe from the perturbative point of view (the $O(\alpha^2)$ contributions being only a few percent of the $O(\alpha)$ ones) typically if the left–right mixing angle is $\sim \pi/2$ and $\tan\beta \sim 30$.

To conclude, we showed in this letter that potentially large and model– dependent one–loop corrections to the (model–independent) tree–level charged Higgs production cross section can occur, for generic values of the model parameters, at $\sqrt{s} = 500$ GeV e^+e^- collisions. These should be taken into account when discussing production rates at NLC. A further investigation at higher NLC energies and for heavier charged Higgses is however needed, as well as the inclusion of loop effects from the rest of the particle spectrum and eventually an improved resummation in the region where the effects are very large.

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Appendix

a) top-bottom contributions to the γH^+H^- and ZH^+H^- vertices :
Using the following intermediate functions,

$$f(x,y,z) = s + 2(x+y-z)$$

$$g(x,y) = x^2 - ys - (m_H^2 - y)^2$$

$$I(x) = 3(2m_H^2 - s)x - 6\cot^2\beta(\tan^2\beta m_b^2 + m_t^2)^2$$

$$K_0(m_b^2, m_t^2) = (\tan^2\beta m_b^2 + \cot^2\beta m_t^2) f(m_t^2, m_b^2, m_H^2) + 8m_b^2 m_t^2$$

$$K_1(x,y,m_1^2,m_2^2) = x(\tan^2\beta m_b^2 + \cot^2\beta m_t^2) g(m_1^2, m_2^2) + y m_b^2 m_t^2 f(m_2^2, -m_1^2, m_H^2)$$

$$K_2^Z(t^2, m_1^2, m_2^2) = 3m_1^2(m_2^2 + m_H^2)(\frac{m_1^2}{t^2} - t^2 m_2^2) - 3t^2 m_2^2(g(m_1^2, m_2^2) - m_1^4) - 3\frac{m_1^6}{t^2}$$

$$K_2^\gamma(t^2, m_1^2, m_2^2) \equiv 0$$

$$dB(m^2) = B_0(m_H^2, m_b^2, m_t^2) - B_0(s, m^2, m^2)$$

$$C(m_t^2, m_b^2) = C_0(m_H^2, m_H^2, s, m_t^2, m_b^2, m_t^2)$$

$$B_0'(m_H^2, ...) \equiv \frac{\partial}{\partial s} B_0(s, ...)|_{s=m_H^2}$$

the (renormalized) vertex correction from the top-bottom loop can be written in the form, $(V = \gamma, Z)$

$$\mathcal{V}_V^{(top-bottom)} =$$
$$\frac{N_c \delta^V \alpha^2}{c_w^2 s_w^2 M_Z^2 (s - 4m_H^2)} [(\delta_1^V K_0(m_b^2, m_t^2) + I(\cot^2\beta m_t^2)) dB(m_b^2) +$$
$$(2\delta_1^V K_0(m_b^2, m_t^2) + I(\tan^2\beta m_b^2)) dB(m_t^2) +$$
$$2(K_1(x^V, y^V, m_b^2, m_t^2) + K_2^V(\cot^2\beta, m_b^2, m_t^2)) C(m_t^2, m_b^2) +$$
$$4(K_1(x^V, y^V, m_t^2, m_b^2) + \frac{1}{2}K_2^V(\tan^2\beta, m_t^2, m_b^2) - \frac{3}{2} m_b^2 m_t^2 f(m_b^2, -m_t^2, m_H^2)) C(m_b^2, m_t^2) +$$
$$3\delta_2^V(s - 4m_H^2)((\tan^2\beta m_b^2 + \cot^2\beta m_t^2)(m_H^2 - m_b^2 - m_t^2) - 4m_b^2 m_t^2) B_0'(m_H^2, m_b^2, m_t^2)] \quad (13)$$

where $\delta^\gamma = \delta_1^\gamma = \delta_2^\gamma = 1$, $\delta^Z = \frac{1}{8c_w^2 s_w^2}$, $\delta_1^Z = x^Z = 2s_w^2$, $\delta_2^Z = (-1 + 2s_w^2)$, $x^\gamma = 1$, $y^\gamma = -2$ and $y^Z = 3 - 4s_w^2$; $N_c(= 3)$ is the color factor, m_H, m_t and m_b denote respectively the charged higgs, the top quark and the bottom quark masses. (The above expressions are valid for any up-down quark doublet).

b) Squark contributions to the γH^+H^- and ZH^+H^- vertices :
In the following we assume the bottom squarks to be mass degenerate. Using the following intermediate function,

$$\begin{aligned}\mathcal{F}^V_{\tilde{Q}_i\tilde{q}_j} &= 2B_0(s, m^2_{\tilde{Q}_i}, m^2_{\tilde{Q}_i}) - 2B_0(s, m^2_{\tilde{q}_j}, m^2_{\tilde{Q}_i}) \\ &+ f(m^2_{\tilde{q}_j}, -m^2_{\tilde{Q}_i}, m^2_H)C_0(m^2_H, m^2_H, s, m^2_{\tilde{Q}_i}, m^2_{\tilde{q}_j}, m^2_{\tilde{Q}_i})\end{aligned} \tag{14}$$

where $V = \gamma, Z$ we can write the (renormalized) vertex corrections from squark loops associated to one quark generation (denoted generically by "t" and "b") in the following condensed form :

$$\mathcal{V}_V^{(squarks)} = \frac{N_c\alpha^2 c_V}{s-4m^2_H}(G^V_1\mathcal{F}^V_{\tilde{t}_1\tilde{b}} + G^V_2\mathcal{F}^V_{\tilde{t}_2\tilde{b}} - G^V_3\mathcal{F}^V_{\tilde{b}\tilde{t}_1} - G^V_4\mathcal{F}^V_{\tilde{b}\tilde{t}_2} + G^V_5 X + d_V(s-4m^2_H)\delta B) \tag{15}$$

where $c_\gamma = 2$, $c_Z = 1/(2c_w s_w)$, $d_\gamma = 1$, $d_Z = (2s^2_w - 1)/(2c_w s_w)$ and

$$\begin{aligned}X = \;& 4B_0(s, m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}) - 2B_0(s, m^2_{\tilde{b}}, m^2_{\tilde{t}_1}) - 2B_0(s, m^2_{\tilde{b}}, m^2_{\tilde{t}_2}) \\ &+ f(m^2_{\tilde{b}}, -\tfrac{m^2_{\tilde{t}_1}+m^2_{\tilde{t}_2}}{2}, m^2_H)\, C_0(m^2_H, m^2_H, s, m^2_{\tilde{t}_1}, m^2_{\tilde{b}}, m^2_{\tilde{t}_2})\end{aligned} \tag{16}$$

$$\delta B = h(\cos\,\theta_u, \sin\,\theta_u)B_0'(m^2_H, m^2_{\tilde{b}}, m^2_{\tilde{t}_1}) + h(\sin\,\theta_u, -\cos\,\theta_u)B_0'(m^2_H, m^2_{\tilde{b}}, m^2_{\tilde{t}_2}) \tag{17}$$

and also defined

$$G^V_1 = a^V(-\cos^2\,\theta_u, e_u)h(\cos\,\theta_u, \sin\,\theta_u) \tag{18}$$

$$G^V_2 = a^V(-\sin^2\,\theta_u, e_u)h(\sin\,\theta_u, -\cos\,\theta_u) \tag{19}$$

$$G^V_3 = a^V(1, e_d)h_1(\cos\,\theta_u, \sin\,\theta_u) + a^V(0, e_d)h_2(\cos\,\theta_u, \sin\,\theta_u) \tag{20}$$

$$G^V_4 = a^V(1, e_d)h_1(\sin\,\theta_u, -\cos\,\theta_u) + a^V(0, e_d)h_2(\sin\,\theta_u, -\cos\,\theta_u) \tag{21}$$

$$\begin{aligned}G^Z_5 = \frac{1}{2c_w s_w}\cos\,\theta_u \sin\,\theta_u[&\cos\,\theta_u\sin\,\theta_u(H^2_{RL} + H^2_{RR} - H^2_{LL} - H^2_{LR}) \\ &+ (\cos^2\,\theta_u - \sin^2\,\theta_u)(H_{LL}H_{RL} + H_{RR}H_{LR})]\end{aligned} \tag{22}$$

where e_u and e_d denote the up and down quark electric charges and θ_u the up left–right mixing and

$$G^\gamma_5 = 0$$

$$a^\gamma(x, y) = y$$

$$a^Z(x, y) = \frac{x + 2s^2_w y}{2c_w s_w}$$

$$h_1(x, y) = (x\,H_{LL} + y\,H_{RL})^2$$

$$h_2(x,y) = (x\, H_{LR} + y\, H_{RR})^2$$
$$h(x,y) \equiv h_1(x,y) + h_2(x,y)$$

H_{LL}, H_{RL}, H_{RR} and H_{LR} being the couplings of the charged higgs to the various squarks (see definition in the text). Note in the above formulae the complete cancellation of the θ_d angle due to the fact that the down squarks are mass degenerate.

Figure Captions

Fig.1 : Integrated tree–level cross section as a function of the charged Higgs mass.

Fig.2 : top–bottom contribution (in percent) to the one–loop corrected integrated cross section with $m_{H^\pm} = 220$ GeV, (a) as a function of the top mass, for different values of tan $beta$, (b) as a function of tan β for three values of m_t.

Fig.3 : One–loop top–bottom contribution (in percent) to the integrated cross section as a function of the charged Higgs mass, for $m_t = 165$ GeV and three values of $tan\,\beta$. (the uppermost curve corresponds to tan $\beta = 10$.

Fig.4 : (a) One–loop squarks contribution (in percent) to the integrated cross section with $m_{H^\pm} = 220 GeV$, $\mu = 250$ GeV and $m_t = 165$ GeV, as a function of the squark mass, (all squarks are assumed degenerate in mass), and for various values of tan β. (b) same as in (a) but for sleptons.

Fig.5 : Total one–loop contribution (in percent), as a function of tan β, with $m_t = 165$ GeV $m_{\tilde{q}} = 400$ GeV and $m_{\tilde{l}} = 300$ GeV and three values of μ.

Fig.6 :One–loop sleptons contribution, as a function of μ with $m_{\tilde{l}} = 300$ GeV, $m_{H^\pm} = 220$ GeV and various values of tan β.

Fig.7 : One–loop squarks contribution assuming the scalar partners of the top quark to be non–degenerate, as a function of the mass of the heavier stop, with $\mu = 250$ GeV, $m_{H^\pm} = 220$ GeV, $m_t = 165$ GeV and a left–right mixing angle $\theta_t = 0.4$ rad., (a) $\Delta m_{\tilde{t}} \equiv m_{\tilde{t}_1} - m_{\tilde{t}_2} = 100$ GeV and $m_{\tilde{q}} = m_{\tilde{t}_1} - 50GeV$ for the other squarks; (b) $\Delta m_{\tilde{t}} = 300$ GeV and the other squarks as in (a);

Fig.8 : Feynman diagrams of the one–loop amplitude with matter fields and their scalar partners contributions. t, b (resp. $\tilde{t}, \tilde{b}$) denote generically up and down quarks or leptons (resp. squarks or sleptons). The nineth diagram is actually electron mass suppressed. The W self-energies are needed for the Z self-energie renormalization. Tadpoles are fully subtracted through renormalization;

Fig.9 : Contour plot for the total one–loop contribution to the integrated cross section, in the (tan β, μ) space and for $m_t = 165$ GeV, $m_{\tilde{q}} = 400$ GeV $m_{\tilde{l}} = 300$ GeV and $m_{H^\pm} = 220$ GeV. All squarks (resp. sleptons) are assumed degenerate in mass.