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CP Violation in $B^\pm \to \gamma\pi^\pm\pi^+\pi^-$ *

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Abstract

We consider CP violating effects in decays of the type $B^\pm \to \gamma a^\pm_{1,2}$ where $a_{1,2}$ are the $J^P = 1^+$ and 2^+ resonances each decaying to the common final state via $a^\pm_{1,2} \to \pi^\pm \rho^0$. The resonances enhance the CP asymmetries and also knowledge of their masses and widths facilitates calculations of the effects. Several types of CP asymmetries are sizable ($\sim$ 10–30%) requiring about (3–10) $\times 10^8$ $B^\pm$ mesons for detection at the 3σ level thereby providing a method for measuring the angle α in the unitarity triangle.

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Unlike the neutral B system,[1] wherein for several decay modes predictions for CP asymmetries can be made with considerable confidence, in charged B decays reliable quantitative predictions for CP violation are very difficult to make due to the traditional problems in calculating hadronic matrix elements. To alleviate this outstanding problem we consider final states that are dominated by at least two neighboring resonances.[2] This has the advantage that, to the extent that the resonances dominate the channels, the known widths and masses of the resonances give a crucial handle on reliably calculating the CP violating asymmetries. Furthermore, dominance of the channels by the resonances and coherent superposition of the contributing amplitudes from the resonances can lead to significant enhancements in the asymmetries.[2] Let us also briefly recall, in passing, that the charged B meson system has the advantage that 1) all CP violation is unambiguously of the "direct" type, 2) no tagging of "the other" B is necessary and 3) experiments can be performed at the conventional machines (e.g. CESR) as well as at the asymmetric B-factories that are under construction at SLAC and at the KEK.

We are thus led to investigate the prospects for CP violation in radiative decays of $B^\pm$ mesons to pionic final states, i.e. $B^\pm \to \gamma\pi^\pm\pi^+\pi^-$. The key feature of this reaction that we wish to exploit is in the region where it is dominated by two overlapping resonances, namely, the $J^P = 1^+$, a_1 ($M_{a_1} = 1260$ MeV, $\Gamma_{a_1} \sim 400$ MeV) and $J^P = 2^+$, a_2 ($M_{a_2} = 1318$ MeV, $\Gamma_{a_2} = 110$ MeV). So the reactions of interest are:

$$B^\pm \to \gamma a_1^\pm \quad , \quad a_1^\pm \to \rho^0\pi^\pm \quad , \quad \rho^0 \to \pi^+\pi^- \tag{1}$$
$$B^\pm \to \gamma a_2^\pm \quad , \quad a_2^\pm \to \rho^0\pi^\pm \quad , \quad \rho^0 \to \pi^+\pi^- \tag{2}$$

The formalism for assessing CP violation effects in presence of interfering resonances was given in Ref. 2 where, as an illustration, it was used for radiative decays of B-mesons to final states that are dominated by kaonic resonances, i.e. $B \to \gamma K^*(892)$, $\gamma K_1(1270)$, $\gamma K_1(1400)$, $\gamma K^*(1410)$ and $\gamma K_2(1430)$. This class of reactions are, of course, driven largely by the $b \to s$ penguin transition whereas what we will report in the present study are purely pionic final states which therefore result from $b \to d$ quark transitions. Since in the Standard Model (SM) all CP violation has to proceed via a single, unique, invariant quantity[3] and since $b \to d$ transitions are relatively suppressed compared to $b \to s$, it is therefore clear that CP violating asymmetries should be larger in reactions of the type (1–2) compared to our previous study involving $B \to \gamma K^*$-like resonances.

These reactions receive contributions from the penguin and the annihilation graph as well. However, since due to the Cabibbo angle the annihilation graph for $b \to d$ reactions is larger than it is for the reactions $b \to s$, the two contributing graphs (namely the penguin and the annihilation) tend to become of comparable strength and that too enhances the prospects for larger

CP asymmetries for reactions (1–2). Indeed, asymmetries are typically several tens of percents so that effects at the 3σ level should be observable with about 5×10^8 $B^{\pm}$ mesons. Furthermore, such a final state is expected to reveal CP-conserving asymmetries as well which depend on the CP conserving "interaction" phase(s) originating from strong interactions thus giving a better handle on deducing the underlying CP-violating CKM phase.

Since resonances a_1 and a_2 have different quantum numbers the amplitudes for reactions (1) and (2) can be simply written as:

$$M_j = A_j \Pi_j b_j \tag{3}$$

with $j = 1, 2$. Here A_j describes the weak decay $B \to \gamma a_j$ and therefore contains the CP-violating CKM phase. Π_j is the Breit-Wigner propagator:

$$\Pi_j^{-1} = s - m_j^2 + i\Gamma_j m_j \tag{4}$$

and thus is one source for the CP-conserving "interaction phase". In eqn. (3) b_j describes the strong decay of the resonance a_j to the final state $\rho_0 \pi^{\pm}$. Due to its width the decay of the ρ_0 via $\rho^0 \to \pi\pi$ introduces an additional source of an interaction phase that has to be included.

As in Ref. 2 we use a bound state model[4,2] to describe the conversion from the quark level weak amplitudes to the formation of resonances in the exclusive channels via $B \to \gamma a_{(1,2)}$. We thus find that the formation of a_2 via the annihilation graph is extremely small and we consequently approximate it to zero. In addition, using[6,7] $BR(b \to s\gamma) = 2.5 \times 10^{-4}$ (corresponding to $m_t \sim 170$ GeV), and the constraints from experiment and theory on $b \to u$ and $b \to c$ transitions, K-$\bar{K}$ and B-$\bar{B}$ mixing[8,9] we find:

$$\begin{array}{lcll} BR(B \to \gamma a_1)_{\rm pen} & \equiv B_1^{\rm pen} & \simeq (1.3\text{–}2.0) \times 10^{-7} & (a) \\ BR(B \to \gamma a_1)_{\rm ann} & \equiv B_1^{\rm ann} & \simeq (1.5\text{–}4.6) \times 10^{-7} & (b) \\ BR(B \to \gamma a_2)_{\rm pen} & \equiv B_2^{\rm pen} & \simeq (1.0\text{–}1.7) \times 10^{-7} & (c) \end{array} \tag{5}$$

The CP-violating phase δ_{cp} is then given by:

$$\delta_{cp} = \mathrm{Arg}\ [A_2^{\rm pen}(A_1^{\rm ann *} + A_1^{\rm pen *})] \tag{6}$$

Using the standard Wolfenstein parameterization[11,12,8] of the CKM matrix one gets:

$$\mathrm{Arg}\ (A_2^{\rm pen} A_1^{\rm ann *}) = \mathrm{Arg}\ [(\rho + i\eta)(1 - \rho + i\eta)] \tag{7}$$

$$= \gamma + \beta = \pi - \alpha \tag{8}$$

where ρ, η are the usual parameters of that matrix and α, β and γ are the angles in the unitarity triangle.[12,8] Thus

$$\delta_{cp} = \mathrm{Arg}\ \left[\sqrt{B_1^{\mathrm{pen}}} - \sqrt{B_1^{\mathrm{ann}}}e^{-i\alpha}\right] \tag{9}$$

and therefore it follows that the charged B-mesons via modes under discussion, namely (1,2) should allow a determination of one of the angles (namely α) in the unitarity triangle. Note also that as these *branching ratios* get experimentally measured (which should happen well before the CP asymmetries become observable), the uncertainties in equation (9) due to the model dependence of equation (5) should get significantly reduced.

For the strong decay $a_1 \to 3\pi$ the amplitude is given by

$$b_1 = c_1 m_1 a_1^\mu [(p_0 - p_1)_\mu \pi_{01} + (p_0 - p_2)_\mu \pi_{02}] \tag{10}$$

where m_1 is the mass of a_1, p_1, p_2 are the momenta of the two identical pions and p_0 that of the third pion, $\pi_{ij} = [(p_i + p_j)^2 - m_\rho^2 + i\Gamma_\rho m_\rho]^{-1}$ and $i, j = 0, 1, 2$. Similarly, for $a_2 \to 3\pi$ the strong amplitude is

$$b_2 = 2c_2 a_2^{\mu\nu} [(p_0 - p_1)_\mu p_{2\nu} \pi_{01} + (p_0 - p_2)_\mu p_{1\nu} \pi_{02}] \tag{11}$$

The constant c_1 and c_2 are determined by the measured total widths[13] to be 22.75 and 28.20 respectively.

Contributions to CP-violating observables require interference between the CP-violating phase δ_{CP} with the strong rescattering phase(s). In our formulation, encapsulated in equation (3), the strong phases originate from the widths of $a_{1,2}$ as well as from the width of ρ_0. To the extent that these resonances dominate the final states, the theoretical difficulties in calculating the interaction phases are bypassed as the knowledge gained from the existing experimental information[13] of the widths and masses of the resonances suffices.

To understand the various asymmetries that arise we rewrite the propagators for $a_{1,2}$ so that the relevant rescattering phases are explicitly exhibited. Thus for the $a_{1,2}$ we write:

$$\Pi_j = \hat{\Pi}_j \exp(-i\alpha_j) \tag{12}$$

Furthermore, since there are two pions with the same charge in the final state (e.g. $B^+ \to \gamma\pi^+(p_1) + \pi^+(p_2) + \pi^-(p_0)$), therefore there are two ways in which the ρ propagator enters. For convenience, we decompose this in a symmetric (Σ) and an antisymmetric (Δ) combination:

$$\Sigma = \pi_{02} + \pi_{01} \quad ; \quad \Delta = \pi_{02} - \pi_{01}$$

Once again we factor out the phases

$$\Sigma = \hat{\Sigma} \exp(-i\rho_1) \quad ; \quad \Delta = \hat{\Delta} \exp(-i\rho_2)$$

The resulting phases that determine the asymmetries are then the differences:

$$\Delta\alpha = \alpha_1 - \alpha_2 \quad \text{and} \quad \Delta\rho = \rho_1 - \rho_2$$

Altogether there are six types of CP violating asymmetries that arise. All of the CP-odd quantities, of course, have to be proportional to $\sin\delta_{\rm CP}$. But, in addition, those observables that are odd under "naive time-reversal" (denoted by T_N and meaning time $\to$ $-$time without interchange of initial and final states) will also have to be proportional to $\cos\Delta\alpha$ or $\cos(\Delta\alpha \pm \Delta\rho)$ whereas the T_N-even ones are proportional to $\sin\Delta\alpha$ or $\sin(\Delta\alpha \pm \Delta\rho)$. Thus the square of the invariant amplitude can be expressed as:

$$|M_1 + M_2|^2 = P + \sin\delta_{\rm CP} R \tag{13}$$

where P is the CP conserving part and $R = (R_o + R_e)$ is the CP violating part. Here R_o (i.e. the C-even, P-odd, T_N-odd part) contains terms proportional to $\cos\Delta\alpha$ or $\cos(\Delta\alpha \pm \Delta\rho)$. R_e (i.e. C-odd, P-even, T_N-even part) contains terms proportional to $\sin\Delta\alpha$ or $\sin(\Delta\alpha \pm \Delta\rho)$.

Numerical results for the asymmetries are given in Table 1.[14] A simple observable that exhibits a sizable asymmetry is

$$\epsilon_{fb} = \langle Q_B \sigma(\cos\theta)\sigma(s - s_0)\rangle \tag{14}$$

where $\sigma(x) = +1$ if $x > 0$ and -1 if $x < 0$, $\cos\theta \equiv \hat{p}_0 \cdot \hat{q}$ where, $\vec{q}$ is the momentum of the photon and $\vec{p}_0$ is the momentum of the π^- (in B^+ decay) in the rest frame of $a_{1,2}$. Q_B is the charge of the $B^\pm$ meson. The quantity s is the invariant mass of the three pions and

$$s_0 = \frac{\Gamma_1 m_1 m_2^2 - \Gamma_2 m_2 m_1^2}{\Gamma_1 m_1 - \Gamma_2 m_2} \tag{15}$$

is the point at which $\sin\Delta\alpha$ switches sign. Thus ϵ_{fb} is a CP-violating forward-backward asymmetry and from Table 1 we see that it ranges from 7-11%.

In the Table we also show a simple triple product correlation asymmetry

$$\epsilon_t \equiv \langle \sigma(\sin 2\phi)\rangle \tag{16}$$

where $\sin\phi = [(\vec{p}_2 \times \vec{p}_1)\cdot\vec{q}]/|\vec{p}_1 \times \vec{p}_2|\;|\vec{q}|$; $\cos\phi = (\vec{p}_2 - \vec{p}_1)\cdot\vec{q}/|\vec{p}_2 - \vec{p}_1|\;|\vec{q}|$. For the purpose of this observable the momentum of the identical pions ($p_{1,2}$) are ordered by energy. The resulting CP violating asymmetry ranges from 7 to 10%.

From eqn. (11), following Ref. 15, the optimal observable for CP-violation is

$$\epsilon_{opt} \equiv \langle R/P\rangle \tag{17}$$

We find ϵ_{opt} to be about 20–35%. This CP violating observable can be separated into T_N-odd and T_N-even pieces. The corresponding observables, $\epsilon_o \equiv \langle R_o/P\rangle$ and $\epsilon_e = \langle R_e/P\rangle$ are about 15–20% and 20–30% respectively.

In addition to such CP violating asymmetries, the final state also exhibits rather interesting CP conserving asymmetries. As an example of this class of asymmetries we show in Table 1:

$$\zeta_{fb} \equiv \langle \sigma(\cos\theta) \rangle$$

which is about 20–25%. Measurements of such CP conserving asymmetries would be helpful in pinning down the CP-conserving interaction phase(s).

In Figure 1 we show the differential asymmetries as a function of s for the three cases mentioned above. We have assumed typical values for the CKM parameters.

In calculating the numbers given in Table 1 and in Fig. 1 we used the bound state model of Isgur *et al*[4] with modifications given in Ref. 2. The ranges in Table 1 are obtained by varying over the allowed 90%CL limits of the CKM parameters.[9] We note, in passing, that the asymmetries, being ratios of rates, tend to be less dependent on the bound state model as compared to the rates. Also, as we mentioned earlier, the model dependence should be further reduced as data on branching fractions becomes available.

As the numbers in the Table indicate these effects should be observable with about 10^8–10^9 $B^{\pm}$ mesons. This is especially notable given that we are dealing here with radiative transitions. The basic idea of interfering resonances when used in the context of purely hadronic modes should need significantly fewer B mesons. We shall discuss some of these applications in forthcoming publications.

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9. Our constraints are a little different from those in Ref. 8. The difference is primarily due to the fact that we use results from the lattice for the relevant hadronic parameters. Thus we use (in standard notation), $B_K = .8 \pm .1$, $f_B = 187 \pm 50$ MeV and $B_B = 1 \pm .2$, where the errors are our best estimates at 90% CL. As a result we find $.15 \lesssim |V_{td}/V_{ts}| \lesssim .30$, $10^\circ \lesssim |\mathrm{Arg} V_{td}| \lesssim 40^\circ$; $.035 \lesssim |V_{ub}/V_{cb}| \lesssim .130$ and $30^\circ \lesssim |\mathrm{Arg} V_{ub}| \sim < 135^\circ$. See also Ref. 10.

10. More details of this work will be published in a forthcoming article.

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14. Since the interfering resonances do not have identical quantum numbers partial rate asymmetries cannot arise. See Ref. 2.

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Table 1: Observables and their transformation properties. The ranges of the expected asymmetries are obtained by varying over the allowed region of the CKM parameters. (see Ref. 9). $N_B^{3\sigma}$ is the number of $B^\pm$ needed for detection at the 3σ level.

Observable	Transformation Property			Expected	$N_B^{3\sigma}/10^8$
	CP	P	T_N	Size	
ϵ_{fb}	$-$	$+$	$+$	7–11%	30–40
ϵ_t	$-$	$-$	$-$	7–10%	40–50
$\epsilon_{\rm opt}$	$-$	Mixed	Mixed	20–35%	3–5
ϵ_e	$-$	$+$	$+$	20–30%	5–6
ϵ_o	$-$	$-$	$-$	15–20%	8–12
ζ_{fb}	$+$	$+$	$+$	20–25%	4–10

Figure Captions:

Figure 1:

Asymmetries as a function of s for the Wolfenstein parameters $\{A = .86, \rho = .10, \eta = .45\}$. The solid line is for $|m_1^2 \frac{d\zeta_{fb}}{ds}|$; the dashed line for $|m_1^2 \frac{d\epsilon_{fb}}{ds}|$ and the dot-dashed line is for $|m_1^2 \frac{d\epsilon_t}{ds}|$.

Figure 1