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Towards Quantum Cosmology without Singularities†

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Abstract

In this paper, we investigate the vanishing of cosmological singularities by quantization. Starting from a five-dimensional (5-D) Kaluza-Klein approach we quantize, as a first step, the nonspherical metric part and the dilaton field. These fields, which are classically singular, become smooth after quantization. In addition, we argue that the incorporation of nonperturbative quantum corrections form a dilaton potential. Technically, the procedure corresponds to the quantization of two-dimensional (2-D) dilaton gravity and we discuss several models. From the four-dimensional (4-D) point of view, this procedure is a semiclassical approach where only the dilaton and moduli matter fields are quantized.

We consider a cosmological string solution which has classical singularities (Big Bang). Near these singularities, the theory factorizes in a smooth spherical part and a singular 2-D part. This singular part is the well-known dilaton gravity (see, e.g., Refs. [1]–[4]) and as a first step we are going to quantize this part with the result that all singularities disappear. This procedure is also known as s-wave reduction and has been so far used for 4-D BH physics.

Classical theory. Our 4-D classical model is given by

$$S = \int d^4x \sqrt{\tilde{G}} e^{-2\phi} \left(R + 4(\partial\phi)^2 - (\frac{\partial\rho}{\rho})^2 - \frac{1}{12} H^2 \right) \ , \tag{1}$$

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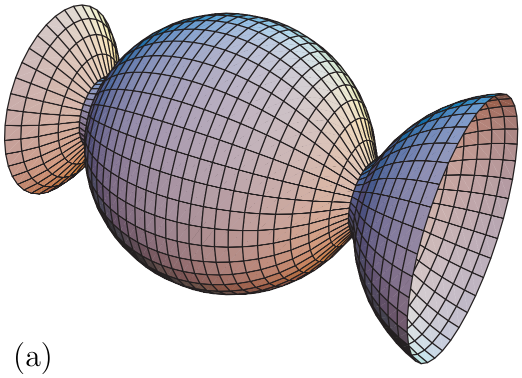
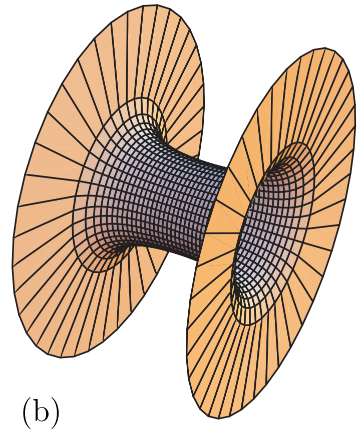


(a)



(b)

Figure 1: In (a) we have plotted the closed oscillating solution for $k = 1$; (b) is the wormhole solution for $k = -1$.

where ϕ is a dilaton field, $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$ is the torsion corresponding to the antisymmetric tensor field $B_{\mu\nu}$ and ρ is a modulus field. A cosmological solution to this model is given by [5]

$$\begin{aligned} ds^2 &= -\frac{dt^2}{\left(-k+(t_+/t)^2\right)\left(1-(t_-/t)^2\right)} + t^2 d\Omega_k^2 \quad , \quad \rho^2 = \frac{-k+(t_+/t)^2}{1-(t_-/t)^2} \\ H &= 2t_+t_- \left(\sin\sqrt{k}\chi/\sqrt{k}\right)^2 \sin\theta\, d\chi\wedge d\theta\wedge d\varphi \quad , \quad e^{-2\phi} \sim \sqrt{\left(1-(t_-/t)^2\right)\left(-k+(t_+/t)^2\right)}\ . \end{aligned} \tag{2}$$

After a time reparameterization, one obtains the standard Friedmann-Robertson-Walker (FRW) metric with $d\Omega_k^2$ as three-dimensional (3-D) volume form corresponding to the spatial curvature k $(= 0, -1, +1)$. The parameter t_- is the minimal extension and, for $k = 1$, the parameter t_+ denotes the maximal extension of the universe. This is obvious after transforming the solution to the conformal time

$$t^2 = t_-^2 + (t_+^2 - kt_-^2)\left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2 \tag{3}$$

for which the metric is

$$ds^2 = \left\{ t_-^2 + (t_+^2 - kt_-^2)\left(\frac{\sin\sqrt{k}\eta}{\sqrt{k}}\right)^2 \right\} \left[-d\eta^2 + d\Omega_k^2\right]\ . \tag{4}$$

Unfortunately, in Ref. [5] no analytic results for the world radius $a(\tau)$ in the standard parameterization of the FRW metric $ds^2 = -d\tau^2 + a(\tau)^2 d\Omega_k^2$ could be found. We have plotted numerical results in Fig. 1. Figure 1(a) shows the oscillating solution for $k = 1$ and Fig. 1(b) shows the wormhole solution for $k = -1$. For $k = 0$, the solution has again the geometry of Fig. 1(b) but with the difference that there are no asymptotic flat regions as for $k = -1$. Remarkably, the scalar fields ρ and ϕ have divergencies, although the metric behaves completely smooth for all times. To understand this phenomena, one has to go back to the 5-D origin. In the sense of a Kaluza–Klein approach solution Eq. (2) can be obtained by dimensional reduction of a 5-D theory. Then, the modulus field ρ corresponds to a time-dependent compactification radius of the fifth coordinate. The corresponding action is given by the effective string action

$$S^{(5)} = \int d^5x\sqrt{G}e^{-2\psi}\left(R + 4(\partial\psi)^2 - \frac{1}{12}H^2\right) \tag{5}$$

and the 5-D solution can be written as

$$\begin{aligned} ds^2 &= \left(\sqrt{k}/\tan\sqrt{k}\eta\right)^2 dw^2 + \left\{t_-^2 + (t_+^2 - kt_-^2)\left(\sin\sqrt{k}\eta/\sqrt{k}\right)^2\right\}\left[-d\eta^2 + d\Omega_k^2\right] \\ e^{2(\psi-\psi_0)} &= 1 + \frac{t_-^2}{(t_+^2 - kt_-^2)\left(\sin\sqrt{k}\eta/\sqrt{k}\right)^2}\ . \end{aligned} \tag{6}$$

The 5-D dilaton is related to ϕ by $2\psi = 2\phi + \log\chi$ and $\rho = G_{55}$. For $k = 1$ and after switching the signature of the metric ($dw^2 \to -dw^2$ and $d\eta^2 \to -d\eta^2$), this 5-D solution is just the 5-D BH solution (in the conformal time η) discussed by Horowitz and Strominger in Ref. [6]. Here, $t_\pm$ define the two horizons of the theory and our cosmological solution lives between these horizons.

S-wave reduction. We are particularly interested in the fate of the singularities if one starts to quantize the theory. Therefore, it is reasonable to restrict ourselves to the region near the singularities ($\sin\sqrt{k}\eta \simeq 0$). In this region, one can assume that quantum corrections become important. Furthermore, as one can see from (Eq. 6) in this limit the 5-D solution decouples in a 3-D spherical part ($\sim d\Omega_k^2$) and a 2-D (w, η) part which is the known dual 2-D BH [7]. In the figure, it is just the region of minimal extension; e.g., inside the wormhole of Fig. 1(b).

Before we can start to quantize the 2-D part, we have to reduce the 5-D action [in Ref. 5] down to a 2-D theory.This procedure is motivated by the assumption that the quantum corrections respect the spherical symmetry. Generally, this is not the case—but it is sufficient for a first approximation. In BH physics, this procedure is also known as s-wave reduction. For the 5-D metric we make the ansatz

$$ds^2 = g_{ab}^{(2)}dz^a dz^b + e^{2\chi}d\Omega_k^2\ , \tag{7}$$

where $g_{ab}^{(2)}$ is the 2-D metric part. In what follows, we quantize only $g_{ab}^{(2)}$ and the dilaton ψ, (respectively ϕ, see below). The remaining fields are assumed to be classical backgrounds given by Eqs. (2) or (6). After integrating out the angular degrees of freedom and using the H field from (Eq. 2) we obtain for Eq. (5)

$$S^{(2)} = \int d^2z\sqrt{g}e^{-2\phi}\left(R^{(2)} + 4(\partial\phi)^2 - 3(\partial\chi)^2 + V(\chi)\right) \tag{8}$$

with $\phi = \psi + 3/2\chi$ and $V(\chi) = 6ke^{-2\chi} - 2t_+^2 t_-^2 e^{-6\chi}$. Near the singularity ($\eta \simeq 0$), the background field χ is smooth, $\partial/\partial\eta\ \chi|_0 = 0$ [see Eqs. (6) and (7)] and up to the second order in η we can approximate the χ terms by a constant

$$S^{(2)} = \int d^2 z \sqrt{g} e^{-2\phi} \left(R^{(2)} + 4(\partial\phi)^2 + \lambda \right) \tag{9}$$

with $\lambda = 2/t_-^2 \, (3k - (t_+/t_-)^2)$. A classical solution in conformal coordinates is given by Ref. [7]

$$ds^2 = e^{2\sigma} dz^+ dz^- \qquad , \qquad e^{-2\phi} \sim e^{-2\sigma} = u - \lambda z^+ z^- \tag{10}$$

where u is constant. This solution can be transformed to the 2-D (w, η) part of Eq. (6) where $\eta \simeq 0$ corresponds to $u \simeq \lambda z^+ z^-$.

Quantization. The quantization of Eq. (9) has been studied in various papers (see Refs. [1]–[4], [8], [9]). Each of these models will be discussed but, before we do so in detail, let us come back to the classical solution once more. The fact that Eq. (6) and Eq. (2) are independent of the fifth coordinate leads to the dual solution

$$\begin{aligned} ds^2 &= \left(\tan \sqrt{k}\eta/\sqrt{k}\right)^2 dw^2 + \left\{ t_-^2 + (t_+^2 - k t_-^2) \left(\sin \sqrt{k}\eta/\sqrt{k}\right)^2 \right\} \left[-d\eta^2 + d\Omega_k^2\right] \\ e^{2(\psi-\psi_0)} &= \left(\tan \sqrt{k}\eta/\sqrt{k}\right)^2 + t_-^2 / \left[(t_+^2 - k t_-^2) \cos^2 \sqrt{k}\eta \right] \ . \end{aligned} \tag{11}$$

Both solutions have a significant difference. Singular points of Eq. (6) are regular in Eq. (11) and vice-versa. Furthermore, in the region that we are interested in ($\sin \sqrt{k}\eta \simeq 0$) the solution Eq. (6) is in the strong coupling region ($e^{2\psi} \to \infty$) whereas the dual solution Eq. (11) is in the weak coupling region ($e^{2\psi} \ll 1$; if we assume that $t_+ \gg t_-$ which is reasonable, since $t_\pm$ corresponds to the maximal/minimal extension of the universe) Again, the 2-D part decouples and can be transformed into Eq. (10) where $\eta \simeq 0$ corresponds to $z^+ z^- \simeq 0$.

When quantizating this theory, we are especially interested in what happens in the strong coupling region, i.e. the fate of the singularity in Eq. (6). As a consistency condition of this procedure, we have to ensure that in the classical limit (weak coupling region) we get back our classical result Eq. (11) which is nonsingular for $\eta \simeq 0$. We are following the procedure of de Alwis [8] and later on we discuss the modification concerning the other models. After choosing the conformal gauge

$$g_{ab} = e^{2\sigma} \hat{g}_{ab} \tag{12}$$

we can rewrite Eq. (9) as a general 2-D σ model

$$S = - \int d^2 z \sqrt{\hat{g}} \left[\hat{g}^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \hat{R} \, \Phi(X) + T(X) \right] \tag{13}$$

with $X^\mu = \{\phi, \sigma\}$. Thus, the quantization of the dilaton gravity is reduced to the quantization of a 2-D σ model with the target space spanned by ϕ and σ. This model, however, is well defined only if the background fields $G_{\mu\nu}, \Phi$ and T define a 2-D conformal field theory. This symmetry is a consequence of the fact that the original theory depends only on g and not on $\hat{g}$, and thus has to respect the symmetry $\hat{g} \to e^{2\rho}\hat{g}$ and $\sigma \to \sigma - \rho$ [see Eq. (12)]. We transform the theory to an exact model and define the quantum theory by this (exact) conformal field theory (see, e.g., [8], [9]). Following this approach, we first note that the target space metric $G_{\mu\nu}$ has the general structure

$$dS^2 = -4e^{-2\phi}[1 + h(\phi)]d\phi^2 + 4e^{-2\phi}[1 + \bar{h}(\phi)]d\sigma d\phi + \kappa d\sigma^2 \tag{14}$$

where h and $\hat{h}$ are model-dependent functions of ϕ or X^1. For $h = \bar{h} = 0$ we have the CGHS model [1]; for $2h = \bar{h} = -e^{2\phi}$, we have the model from Strominger [3]; $h = 0$ and $\bar{h} = -\kappa/4e^{2\phi}$ describe the RST model [4]. The parameter $\kappa = (24 - N)/6$ originates from the definition of the functional integration measure and N corresponds to additional conformal matter. As next step, we introduce new target space coordinates

$$\begin{aligned} x &= 2/\sqrt{\kappa} \textstyle\int d\phi\, e^{-2\phi}\sqrt{(1+\bar{h})^2 + \kappa e^{2\phi}(1+h)} \\ y &= -\sqrt{\kappa}\left(\sigma - 1/\kappa e^{-2\phi} + 2/\kappa \textstyle\int d\phi e^{-2\phi}\bar{h}\right) \end{aligned} \tag{15}$$

and obtain a flat metric

$$dS^2 = -dx^2 + dy^2 \ . \tag{16}$$

(for negative κ, we have to perform a Wick rotation in x and y). For this flat metric, it is easy to find the dilaton Φ and tachyon T that define a conformal field theory. The general solution of the corresponding β equations is

$$\begin{aligned} &\Phi(x) = ax + by &&\text{with} && a^2 - b^2 = -\kappa \quad , \\ &T(x) \sim e^{\alpha x + \beta y} &&\text{with} && \tfrac{1}{2}(\alpha^2 - \beta^2) - a\alpha + b\beta - 2 = 0 \ . \end{aligned} \tag{17}$$

The demand to get the classical model in Eq. (9) in the weak coupling limit yields a further restriction to Φ and T. Following the suggestion of de Alwis we set $a = 0$ and $\alpha = -\beta = -2/\sqrt{\kappa}$ and get the known Liouville theory (y as Liouville field) that couples to the matter field x

$$S = -\int d^2z\sqrt{\hat{g}}\left[-(\partial x)^2 + (\partial y)^2 + \sqrt{\kappa}\,\hat{R}\,y + \lambda\, e^{+\frac{2}{\sqrt{\kappa}}(x-y)}\right] \ . \tag{18}$$

This describes a well-defined 2-D gravity theory on the classical as well as on the quantum level. The strategy is to define the quantum theory in terms of *this* action and to regard Eq. (9) as the classical limit.

As second step, we have to find solutions of the equations of motion for x and y ($\hat{R}^{(2)} = 0$)

$$-\partial^2 x = \frac{\lambda}{\sqrt{\kappa}} e^{-2/\sqrt{\kappa}(x-y)} \qquad , \qquad \partial^2 y = \partial^2 x \ . \tag{19}$$

Solving these equations, we have to restrict ourselves to solutions that reproduce the BH solution, in Eq. (10) in the classical limit. Therefore we are interested in a solution depending on z^+z^- only and find

$$x = y = -\frac{1}{\sqrt{\kappa}}\left(u - \lambda z^+ z^-\right) \tag{20}$$

(u =constant). Using the transformation in Eq. (15), we can express this solution in ϕ and σ. In doing so, we have to fix the up-to-now arbitrary functions $h(\phi)$ and $\bar{h}(\phi)$. Let us start with parameterization suggested by de Alwis: $h = 0$, $\bar{h} = -1/2\kappa e^{2\phi}$. This choice is motivated by the fact that for all values of ϕ and σ the transformation in Eq. (15) is nonsingular and also that the range of x and y goes from $-\infty$ to $+\infty$ if ϕ and σ do so. For x and y, one gets

$$\begin{aligned} x &= 1/\sqrt{4\kappa}\left(-\sqrt{\kappa^2 + 4e^{-4\phi}} + \sqrt{\kappa}\,\mathrm{arcsinh}\kappa/2e^{2\phi}\right) \\ y &= -\sqrt{\kappa}\left(\sigma - 1/\kappa\ e^{-2\phi} - \phi\right) \end{aligned} \tag{21}$$

In terms of Eq. (20), one finds in the weak coupling limit ($e^{2\phi} \ll 1$) the desired classical solution Eq. (10)

$$e^{-2\phi} = u - \lambda z^+ z^- \qquad , \qquad \sigma = \phi\ . \tag{22}$$

Since we are in the weak coupling region, this solution corresponds to our dual solution in Eq. (11) which is nonsingular for $\eta \simeq 0$. In the strong coupling limit ($e^{2\phi} \gg 1$), we obtain

$$\phi = -\frac{1}{\sqrt{\kappa}}(u - \lambda z^+ z^-) \qquad , \qquad \sigma = \frac{1}{\kappa}e^{-2\phi}\ . \tag{23}$$

Therefore, after incorporation of quantum corrections [$\sim \mathcal{O}(e^{2\phi})$] the black hole solution gets smooth also in the strong coupling region. Note that in dilaton gravity, a singularity in the metric has to be accompanied by a singularity in the dilaton; i.e., singularities can only appear in the strong or weak coupling region. For the other models, the picture is qualitatively the same. In the CGHS model ($h = \bar{h} = 0$) one obtains for Eq. (15)

$$\begin{aligned} x &= -1/\sqrt{\kappa}e^{-2\phi}\sqrt{1+\kappa e^{2\phi}} - \sqrt{\kappa}/2\log\left[\kappa + 2e^{-2\phi}(1+\sqrt{1+\kappa e^{2\phi}})\right] \\ y &= -\sqrt{\kappa}\left(\sigma - 1/\kappa\ e^{-2\phi}\right)\ . \end{aligned} \tag{24}$$

and in strong coupling region this model gives

$$e^{-\phi} \sim u - \lambda z^+ z^- \qquad , \qquad \sigma = \frac{1}{\kappa}(u - \lambda z^+ z^-) \tag{25}$$

For the Strominger model ($2h = \bar{h} = -e^{2\phi}$) we find $[F(\phi) = \sqrt{e^{-4\phi} - (2-\kappa)e^{-2\phi} + (2-\kappa)/2}\,]$

$$\begin{aligned} x &= -1/\sqrt{\kappa}\,\Big[F(\phi) + (\kappa-2)/2\log[F(\phi) + e^{-2\phi} + (\kappa-2)/2] - \\ &\quad - \sqrt{(2-\kappa)/2}\log\Big(\sqrt{2(2-\kappa)}F(\phi) + (2-\kappa)e^{2\phi} - (2-\kappa)\Big)\Big] \\ y &= -\sqrt{\kappa}\left(\sigma - 1/\kappa\; e^{-2\phi} - 2/\kappa\;\phi\right)\ . \end{aligned} \tag{26}$$

which gives in the strong coupling region

$$\phi = -\frac{1}{\sqrt{2(2-\kappa)}}(u - \lambda z^+ z^-) \qquad , \qquad \sigma = \frac{1}{\kappa}\left(\sqrt{2(2-\kappa)} - 2\right)\phi\ . \tag{27}$$

And finally, in the RST model ($h = 0$, $\bar{h} = -\kappa/4\; e^{2\phi}$), the general solution is given by

$$\begin{aligned} x &= -1/\kappa\; e^{-2\phi} + \kappa/2\;\phi \\ y &= -\sqrt{\kappa}\left(\sigma - 1/\kappa\; e^{-2\phi} - 1/2\;\phi\right)\ . \end{aligned} \tag{28}$$

In the strong coupling region, this model behaves like

$$\phi = -2\kappa^{-\frac{3}{2}}(u - \lambda z^+ z^-) \qquad , \qquad \sigma = \frac{1-\sqrt{\kappa}}{2}\phi\ . \tag{29}$$

Therefore we find that all models have no singularities in the strong coupling region and yield the classical result in the weak coupling region.

One can now ask what is the influence of this quantization procedure for the further evolution of the universe. For the derivation of our results, it was crucial that the solution decouples in a 2-D (dilaton gravity) part and a 3-D spherical part. This is valid only if one considers the theory; e.g.,inside the wormhole of Fig. 1(b). Extending this procedure to the region away from the wormhole seems to be difficult. But nevertheless, quantum correction inside the wormhole can form a dilaton potential which could be a source of an inflationary period in later times. A dilaton potential in our original action in Eq. (1) or Eq. (5) corresponds to an additional tachyon contribution in the 2-D action which is independent of λ [since λ was correlated to the constant χ field in the wormhole; see Eq. (8)]. The tachyon we have discussed so far is only *one* possibility. This solution has the advantage that the renormalization group β functions vanish thereby yielding a finite 2-D quantum field theory. The most general tachyon field, however, is a combination of contributions given by Eq. (17). A further additive tachyon term is given by (for $\kappa > 0$)

$$T_{np} = \mu e^{2x} \tag{30}$$

where the function x is given by Eq. (15). This term—discussed e.g., in Refs. [8] and [10]— has in the weak coupling region for all discussed models the typical non-perturbative structure

$$T_{np} \sim e^{-2/\sqrt{\kappa}e^{-2\phi}} \sim e^{-2/(\sqrt{\kappa}g_s^2)} \tag{31}$$

where $g_s = e^{\phi}$ is the string coupling constant. Therefore this term vanishes very rapidly in the weak coupling (classical) region and becomes important in the strong coupling region. Furthermore, since x is a function of the dilaton only, this tachyon term represents a candidate for a dilaton potential created by nonperturbative quantum corrections in the strong coupling region. If we insert the x values for the several models Eqs. (21), (24), (26) and (28), we obtain different potentials. But all these potentials have no local or global minima and are probably not good candidates to discuss for an inflationary period (see, e.g., Ref. [11] and references therein). It remains an open question whether another choice of the model-dependent functions h and $\bar{h}$ could yield a more appropriate potential.

Discussion. Starting with a classical solution of the low-energy string effective action, we investigated the quantization near the cosmological singularity. Via a 5-D Kaluza–Klein approach, this solution was obtained as a dimensional reduced theory. Near the singularity, the 5-D theory decouples in a 3-D nonsingular (spherical) part and a singular 2-D part. As a first step, we have quantized only this singular 2-D part (s-wave reduction). The results in Eqs. (21)–(29) show that, for all models, the singularity disappears after the quantization of the theory; i.e., the 2-D metric part and the dilaton remain finite. An interpretation of this result is that the wormhole becomes traversable via quantum corrections. In addition, we have shown that the incorporation of nonperturbative quantum corrections form a dilaton potential. The discussion of the possible structures of the potential created by this procedure remains an interesting task for further investigations.

We used the 5-D theory to get contact with the known dilaton gravity. But it is also possible to quantize the 4-D theory in Eq. (1) directly. Our approximation to quantize only the divergent 2-D part in 5 dimensions is effectively the same as to quantize the dilaton and moduli matter fields only. Note that the 2-D metric part has only one degree of freedom. In the conformal gauge in Eq. (12), this is the Liouville field σ but we can also take another gauge, e.g., $ds^2 = \rho^2 dw^2 - d\eta^2$ and then ρ is our moduli field [see Eq. (6)]. Thus, from the 4-D point of view, we replaced the dilaton and moduli contributions in the Einstein equation by its vacuum expectation value

$$R^{(E)}_{\mu\nu} - \frac{1}{2} R^{(E)} G^{(E)}_{\mu\nu} \ = < T^{(\phi,\rho)}_{\mu\nu} > + T^{(H)}_{\mu\nu} \tag{32}$$

where $G^{(E)}_{\mu\nu} = e^{-2\phi} G_{\mu\nu}$ is the metric in the Einstein frame. Classically, the 4-D string metric was smooth but the Einstein metric was singular (caused by the dilaton and moduli). However, after quantization the singularities in the scalar fields disappeared and thereby also the Einstein metric turned out to be nonsingular. This implies that, similar to the string frame, the Einstein metric describes a universe which starts and ends (for $k = 1$) **not** with a singularity but with a minimal (nonzero) extension (wormhole). Therefore, in both frames the spatial part of the universe is qualitatively given in Fig. 1. Of course, the quantization of the scalar fields near the singularity can only be a first step and future investigations have to show whether a complete quantum theory will leave this qualitative feature intact.

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