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THE FORM FACTORS OF THE NUCLEONS

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Abstract

We demonstrate that a relativistic constituent quark model can give nucleon form factors that agree well with recent, accurate measurements. The relativistic features of the model and the specific form of the wave function are essential for the result.

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The study of the electromagnetic form factors of the nucleons is of fundamental importance in understanding the nucleon structure. The form factors contain all the information about the deviation from pointlike structure of the charge and magnetic current distributions of the nucleons. Recent measurements [1] from Rosenbluth separations in elastic electron-proton and quasielastic electron-deuteron scattering have doubled the Q^2 range of previous data on the nucleon form factors and reduced the error bars in the region of overlap. None of the existing models is in good agreement with all form factor results at all values of Q^2, although for several models, the fit could be improved by adjusting free parameters. In addition, in the near future new accelerators like CEBAF will get precise data on all form factors up to 6 GeV^2. Thus it seems that there is a need for reliable theoretical predictions of the nucleon form factors for moderately large values of Q^2. In the present work we investigate a relativistic constituent quark model in the light of the recent data [1].

We formulate the constituent quark model on the light-cone [2]. The wave function is constructed as the product of a momentum wave function, which is spherically symmetric and invariant under permutations, and a spin-isospin wave function, which is uniquely determined by SU(6)-symmetry requirements. A Wigner (Melosh) rotation [3] is applied to the spinors, so that the wave function of the proton is an eigenfunction of J^2 and J_z in its rest frame [4]. For the momentum wave function we choose a simple function of the invariant mass $\mathcal{M}$ of the quarks:

$$\psi(\mathcal{M}^2) = N(1 + \mathcal{M}^2/\beta^2)^{-p} \tag{1}$$

where β sets the scale of the nucleon size and $p = 3.5$. The invariant mass $\mathcal{M}$ can be written as

$$\mathcal{M}^2 = \sum_{i=1}^{3} \frac{\vec{k}_{\perp i}^2 + m^2}{x_i} \tag{2}$$

where we used the longitudinal light-cone momentum fractions $x_i = p_i^+/P^+$ (P and p_i are the nucleon and quark momenta, respectively, with $P^+ = P_0 + P_z$). The internal momentum variables $\vec{k}_{\perp i}$ are given by $\vec{k}_{\perp i} = \vec{p}_{\perp i} - x_i \vec{P}_\perp$ with the constraints $\sum \vec{k}_{\perp i} = 0$ and $\sum x_i = 1$. It has been shown [5] that observables for $Q^2 = 0$ are independent of the wave function ψ.

The Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ of the nucleons are given by the spin-conserving and the spin-flip vector current J_V^+ matrix elements ($Q^2 = -q^2$)

$$F_1(Q^2) = \langle p+q, \uparrow | J_V^+ | p, \uparrow \rangle, \tag{3}$$

$$(Q_1 - iQ_2) F_2(Q^2) = -2M \langle p+q, \uparrow | J_V^+ | p, \downarrow \rangle. \tag{4}$$

The Sachs form factors shown in the figures 1 to 5 are given by

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \tag{5}$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2). \tag{6}$$

The measurements can be roughly described by the dipole fit $G_D(Q^2) = (1 + Q^2/0.71)^{-2}$: $G_{Ep}(Q^2) \sim G_{Mp}(Q^2)/\mu_p \sim G_{Mn}(Q^2)/\mu_n \sim G_D(Q^2); G_{En}(Q^2) \sim 0$.

The only parameters used in Ref. [2] are the constituent quark mass m and the scale parameter β. For $m = 263$ MeV and $\beta = 607$ MeV we get the dashed line in figures 1 to 5. The form factor G_{Ep} contradicts the new data (while it was still fine with the old data). By including anomalous magnetic moments for the quarks [6] we can fix this problem. The continuous lines in figures 1 to 5 give the form factors for the values $m = 240$ MeV, $\beta = 670$ MeV, $F_{2u} = -0.035$, and $F_{2d} = 0.015$. These are the optimal values, and the precise measurements show some deviations for the G_{Mn} which cannot be resolved by changing parameters. Additional physics like higher Fock states (pion cloud, gluons, strange quarks) contribute to the form factors.

ACKNOWLEDGMENTS

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FIGURES

FIG. 1. Proton form factor G_E as calculated in the relativistic constituent quark model compared with data from Ref. [1]. Solid line, calculation with nonzero F_{2q}; dashed line, calculation with $F_{2q} = 0$.

FIG. 2. Proton form factor G_M as calculated in the relativistic constituent quark model compared with data from Ref. [1]. The same line code as in Fig. 1 is used.

FIG. 3. The ratio of the proton form factors F_2 and F_1 as calculated in the relativistic constituent quark model compared with data from Ref. [1]. The same line code as in Fig. 1 is used.

FIG. 4. Neutron form factor G_E as calculated in the relativistic constituent quark model compared with data from Ref. [1]. The same line code as in Fig. 1 is used.

FIG. 5. Neutron form factor G_M as calculated in the relativistic constituent quark model compared with data from Ref. [1]. The same line code as in Fig. 1 is used.

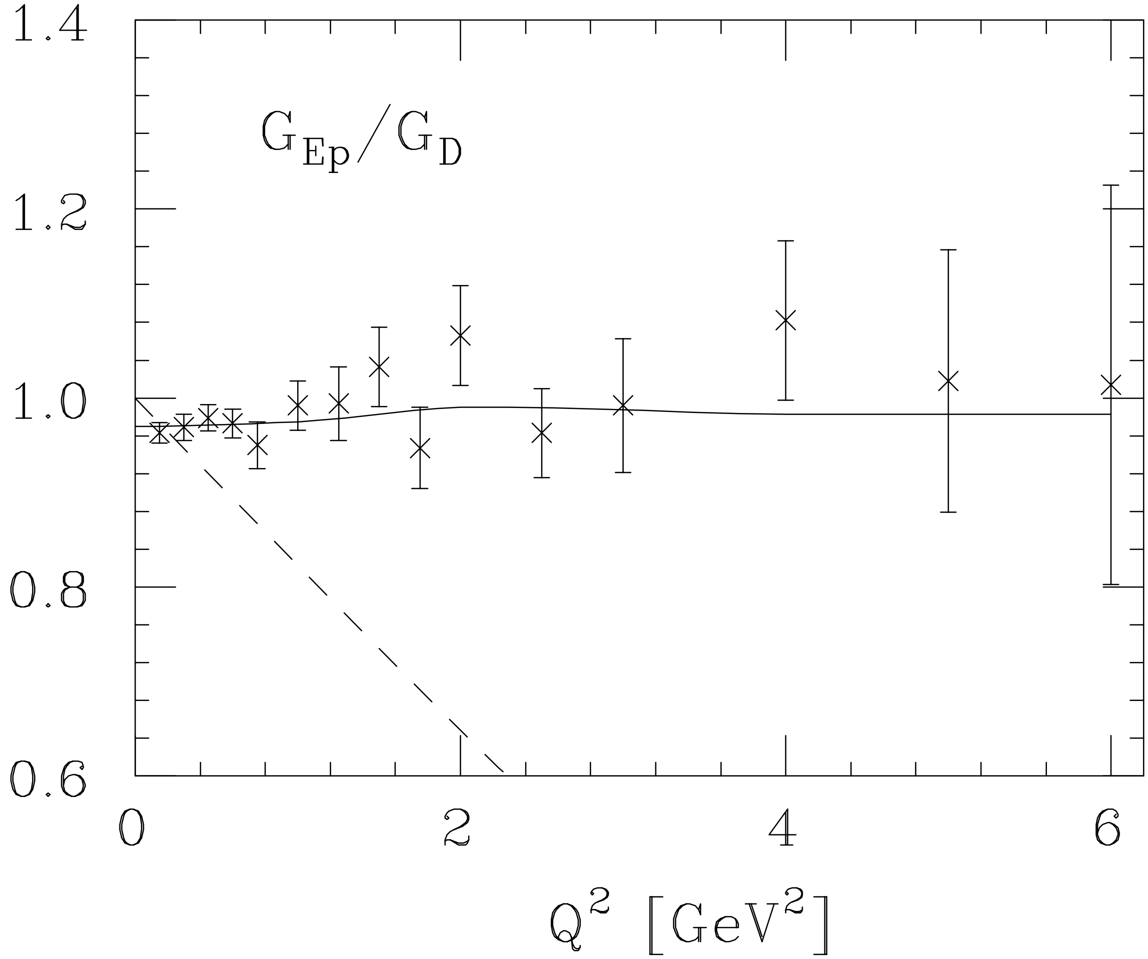

Figure 1

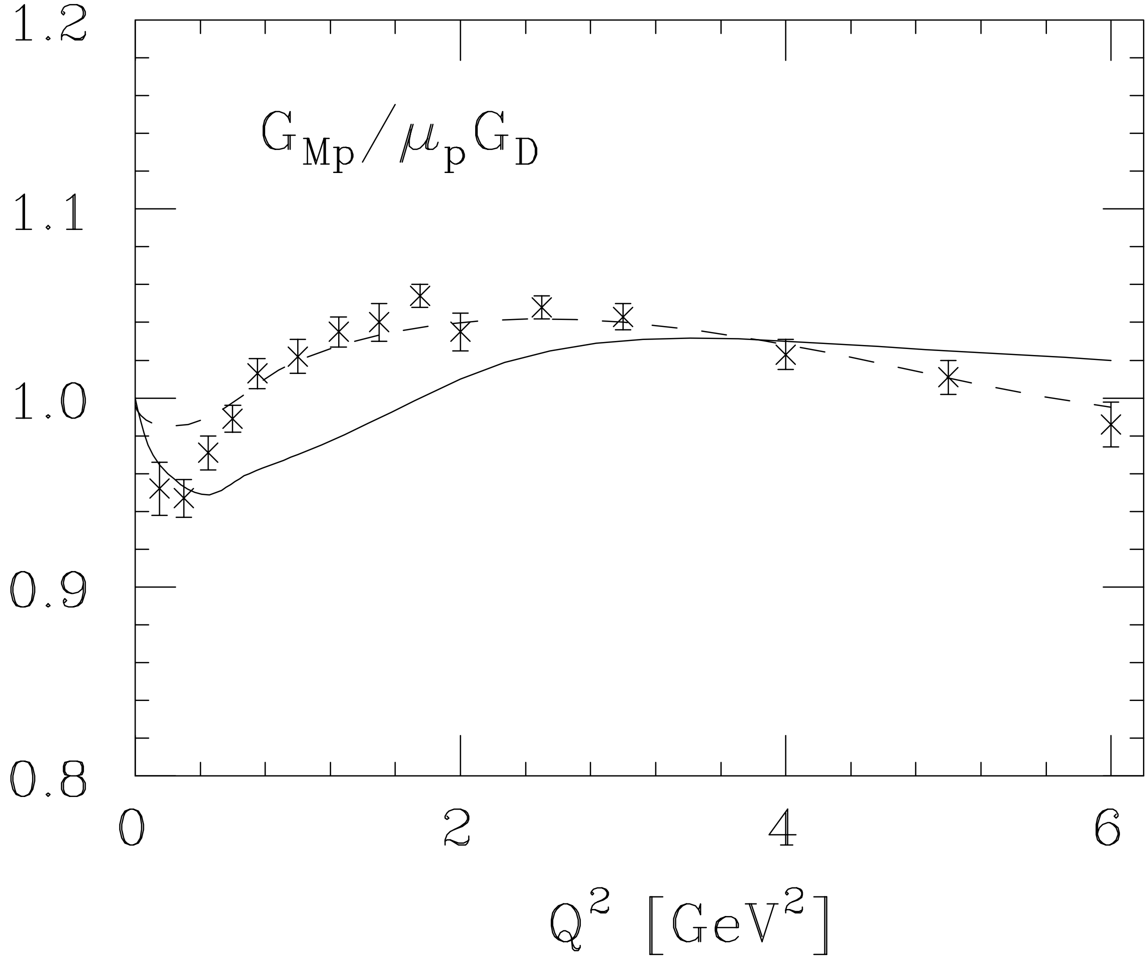

Figure 2

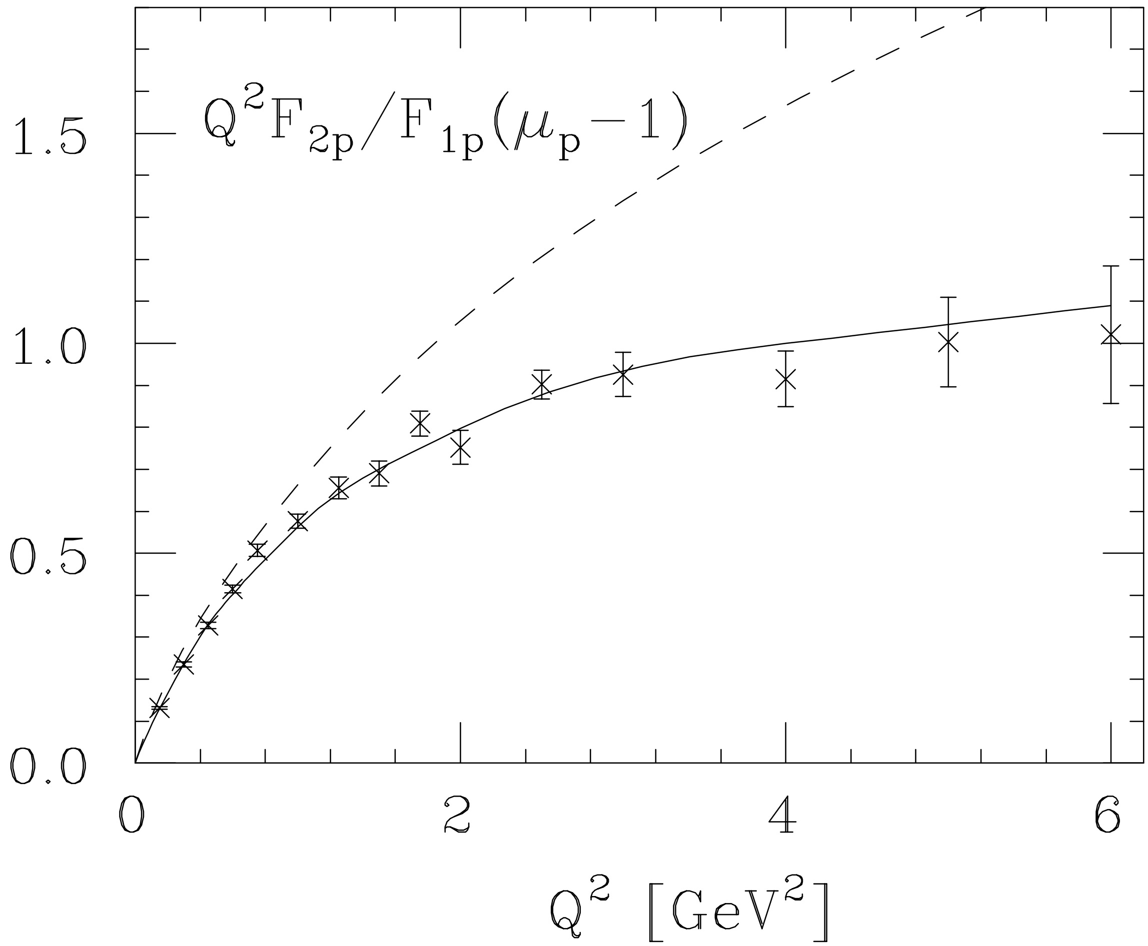

Figure 3

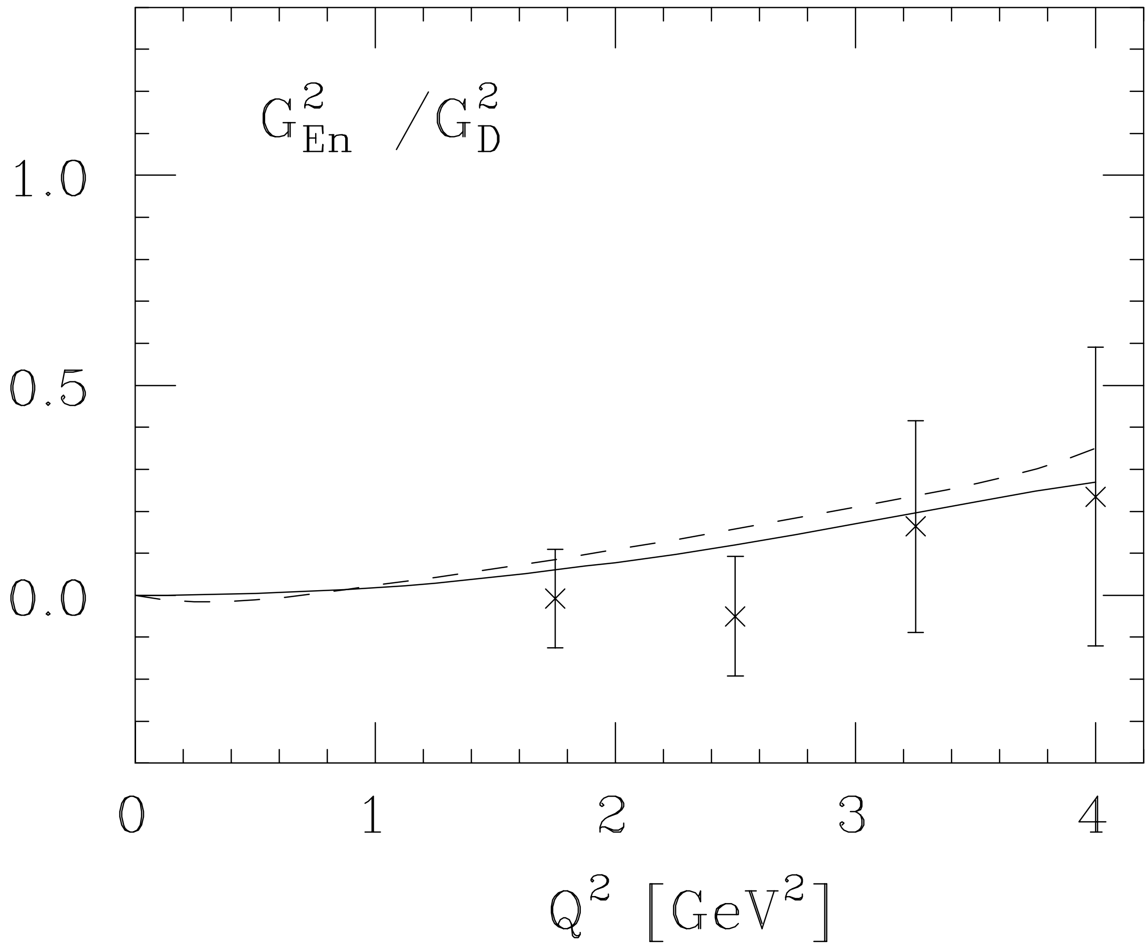

Figure 4

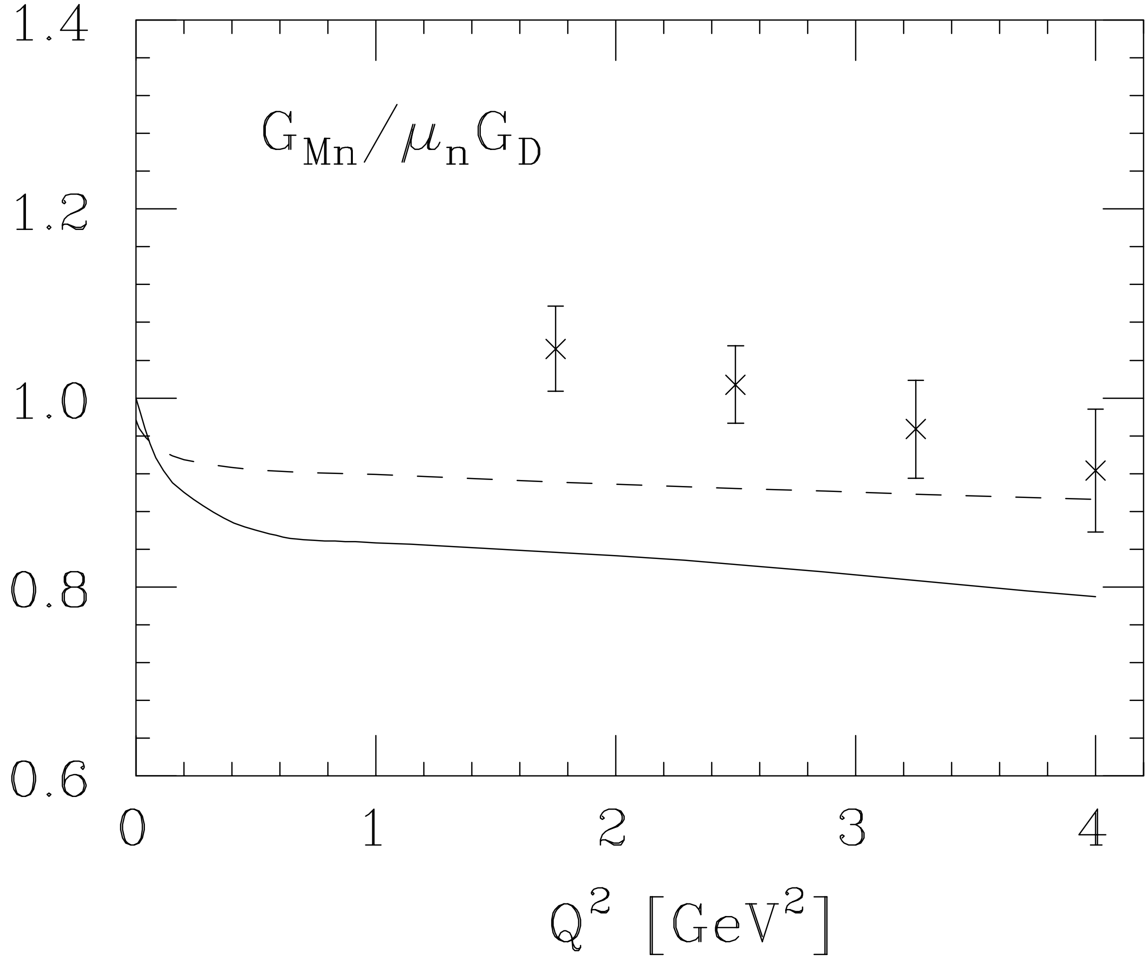

Figure 5