

Vacuum Tracking ¹

V. ZIEMANN
*Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309*

Abstract

A method to determine longitudinal pressure profiles in the presence of pumps and outgassing elements in conductance limited vacuum systems by a transfer matrix formalism is discussed. The algorithm is capable of dealing with multiple connected vacuum systems. An implementation of this method in the computer codes VAKTRAK and VAKLOOP is briefly described.

1 Introduction

The longitudinal pressure profile $P(z)$ in a conduction limited vacuum system obeys the following linear differential equation in the presence of pumps and outgassing elements [1]

$$c \frac{d^2 P}{dz^2} - sP = -q \quad (1)$$

where the symbols are explained in table 1. We stress that the presented method is only applicable in the molecular flow regime, where the mean free path of the molecules is much larger than the dimensions of the pipes and manifolds such that the viscosity of the gas is negligible.

¹work supported by the Department of Energy Contract DE-AC03-76SF00515.

Quantity	Units	Explanation
P	[Torr]	pressure
$Q = -cdP/dz$	[Torr l/s]	gas flow
z	[m]	longitudinal position
c	[m l/s]	specific molecular conductance
s	[l/s m]	linear pumping speed
q	[Torr l/s m]	specific outgassing rate
$\alpha = \sqrt{s/c}$	[1/m]	normalized pumping speed
C	[l/s]	conductance
S	[l/s]	integrated pump speed
ΔQ	[Torr l/s]	outgassing strength

Table 1: *Definitions of symbols used.*

Under the assumption that the specific conductance c , the specific pumping speed s , and the specific outgassing rate q are piecewise constant, equation 1 is an ordinary differential equation with (piecewise) constant coefficients. Thus it can be solved by the method of transfer matrices.² The general transfer matrix can be easily found from the sum of homogeneous and inhomogeneous solution of eq. 1, namely

$$P(z) = C_1 e^{\alpha z} + C_2 e^{-\alpha z} + \frac{q}{s} \quad (2)$$

with $\alpha = \sqrt{s/c}$. Solving for initial conditions we obtain the following transfer matrix

$$\begin{pmatrix} P(L) \\ dP/dz(L) \\ 1 \end{pmatrix} = \begin{pmatrix} \cosh(\alpha L) & \frac{\sinh(\alpha L)}{\alpha} & -\frac{q}{c} \frac{\cosh(\alpha L) - 1}{\alpha^2} \\ \alpha \sinh(\alpha L) & \cosh(\alpha L) & -\frac{q}{c} \frac{\sinh(\alpha L)}{\alpha} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ dP_0/dz \\ 1 \end{pmatrix} \quad (3)$$

which maps the initial conditions P_0 and dP_0/dz to their value $P(z)$ and $dP/dz(z)$ after a length $z = L$ in the presence of outgassing and pumping.

For practical purposes the quantities appearing in eq. 3 are not convenient and we therefore rewrite the equation in terms of pressure P and gas flow Q , instead of pressure P and pressure gradient dP/dz . Assuming an element of length L , conductance $C = c/L$, integrated pump strength $S = sL$, and total desorbing gas flow $\Delta Q = qL$ we can rewrite

²This approach was originally initiated by the author at the Universität Dortmund and led to the development of a PC based program CLOBIVAC [2] by M. Michel.

eq. 3 as

$$\begin{aligned}
\begin{pmatrix} P(L) \\ Q(L) \\ 1 \end{pmatrix} &= \begin{pmatrix} \cosh(\sqrt{S/C}) & -\frac{1}{C} \frac{\sinh(\sqrt{S/C})}{\sqrt{S/C}} & -\frac{\Delta Q}{C} \frac{\cosh(\sqrt{S/C})-1}{S/C} \\ -C\sqrt{S/C} \sinh(\sqrt{S/C}) & \cosh(\sqrt{S/C}) & \Delta Q \frac{\sinh(\sqrt{S/C})}{\sqrt{S/C}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \cosh(\sqrt{s/cL}) & -\frac{L}{c} \frac{\sinh(\sqrt{s/cL})}{\sqrt{s/cL}} & -\frac{qL^2}{c} \frac{\cosh(\sqrt{s/cL})-1}{(s/c)L^2} \\ -c\sqrt{s/c} \sinh(\sqrt{s/cL}) & \cosh(\sqrt{s/cL}) & qL \frac{\sinh(\sqrt{s/cL})}{\sqrt{s/cL}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix} \quad (4)
\end{aligned}$$

Now all appearing quantities are easily accessible.

From eq. 4 we can now deduce special cases for elements which do only outgassing or pumping. We will start with a piece of beam line which neither pumps or outgasses. It is given by eq. 4 in the limit $\Delta Q \rightarrow 0$ and $S \rightarrow 0$. We obtain

$$\begin{pmatrix} P(L) \\ Q(L) \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -1/C & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix}. \quad (5)$$

A long pump is given by $\Delta Q \rightarrow 0$

$$\begin{aligned}
\begin{pmatrix} P(L) \\ Q(L) \\ 1 \end{pmatrix} &= \begin{pmatrix} \cosh(\sqrt{S/C}) & -\frac{1}{C} \frac{\sinh(\sqrt{S/C})}{\sqrt{S/C}} & 0 \\ -C\sqrt{S/C} \sinh(\sqrt{S/C}) & \cosh(\sqrt{S/C}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix} \quad (6) \\
&= \begin{pmatrix} \cosh(\sqrt{s/cL}) & -\frac{L}{c} \frac{\sinh(\sqrt{s/cL})}{\sqrt{s/cL}} & 0 \\ -c\sqrt{s/c} \sinh(\sqrt{s/cL}) & \cosh(\sqrt{s/cL}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix}.
\end{aligned}$$

In the limit of a very short pump with $L \rightarrow 0$ but constant integrated pump strength S we obtain

$$\begin{pmatrix} P(L) \\ Q(L) \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -S & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix}. \quad (7)$$

The transfer matrix of a long outgassing element is given by eq. 4 in the limit of $S \rightarrow 0$. We get

$$\begin{aligned} \begin{pmatrix} P(L) \\ Q(L) \\ 1 \end{pmatrix} &= \begin{pmatrix} 1 & -1/C & -\frac{\Delta Q}{2C} \\ 0 & 1 & \Delta Q \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -L/c & -\frac{qL^2}{2c} \\ 0 & 1 & qL \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix} \end{aligned} \quad (8)$$

which, for short gas sources reduces in the limit $L \rightarrow 0$ which implies infinite conductance to

$$\begin{pmatrix} P(L) \\ Q(L) \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \Delta Q \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ Q_0 \\ 1 \end{pmatrix}. \quad (9)$$

Given these transfer matrices it is possible to *track* a given pressure profile through a *vacuum lattice* if the initial values P_0 and Q_0 are known. Another option is to find the initial values under the assumption of periodicity, i.e., if one considers a sequence of equal *vacuum cells*. Given a cumulative transfer matrix \tilde{M} through a vacuum structure

$$\tilde{M} = \begin{pmatrix} M & \vec{v} \\ 0 & 1 \end{pmatrix}, \quad (10)$$

where $M_{ij} = \tilde{M}_{ij}$ for $i, j = 1, 2$ and $v_i = \tilde{M}_{i3}$ for $i = 1, 2$, the periodic solution \vec{P}_0 is given by

$$\vec{P}_0 = (1 - M)^{-1} \vec{v}, \quad (11)$$

where \vec{P}_0 stands for the vector (P_0, Q_0) .

Observe that a transfer matrix represents two equations that relate two components of the input vector \vec{P}_0 to two components of the vector at the end of the vacuum beam line. By specifying two of the four values the system can be solved for the other two. In this way more general boundary conditions can be taken into account.

As an example consider a synchrotron radiation beam line. At one end it is linked to the storage ring which can be assumed to be held at constant pressure and at the other end it is closed off, such that the gas flow is zero. If the vacuum components in the beam line are specified the system can be solved for the incoming gas flow from the storage ring and the pressure at the closed off end.

2 A simple analytic Example

In order to illustrate the use of the vacuum transfer matrices we consider the following problem. Assume a vacuum pipe with a given specific conductance c and outgassing rate q and (short) pumps with a pump speed S . The question now is how to space the pumps in order for the pressure not to exceed a given maximum pressure P_0 .

From symmetry considerations we deduce that the maximum pressure is half way between the pumps. We therefore consider a vacuum lattice consisting of two pieces of outgassing pipe (eq. 8) and one pump sandwiched in between. The total transfer matrix for such a system can be easily calculated as

$$\begin{pmatrix} 1 + \frac{SL}{c} & -\frac{L}{c} - \frac{SL^2}{c^2} & -\frac{2q}{c}L^2 - \frac{Sq}{2c^2}L^3 \\ -S & 1 + \frac{SL}{c} & 2qL + \frac{Sq}{2c}L^2 \\ 0 & 0 & 1 \end{pmatrix}. \quad (12)$$

We can now use eq. 11 to solve for equilibrium pressure and gas flow but from symmetry considerations we can deduce that the gas flow at the starting point is zero and we only have to solve the following equation for the peak pressure at the symmetry point P_0

$$P_0 = \left(1 + \frac{SL}{c}\right) P_0 - \frac{2q}{c}L^2 - \frac{Sq}{2c^2}L^3 \quad (13)$$

which yields

$$P_0 = \frac{2q}{S}L + \frac{q}{2c}L^2. \quad (14)$$

In order to find the pump spacing $2L$ we solve the previous equation for L and obtain the final result

$$2L = \frac{4c}{S} \left[\sqrt{1 + \frac{S^2}{2qc}P_0} - 1 \right]. \quad (15)$$

The average pressure in the section under consideration, \bar{P} can be easily derived from averaging $P(x)$ over a half-cell where $P(x)$ is given by eq. 8 and initial conditions eq. 14. We then get

$$\bar{P} = \frac{1}{L} \int_0^L dx P(x) = \frac{2q}{S}L + \frac{q}{3c}L^2. \quad (16)$$

This small example exhibits the usefulness of the transfer matrix approach to vacuum calculations. Analyzing more complex systems analytically, however, is not practical. In order to overcome this problem we present two computer codes, similar to the one in ref. 2 in which the above transfer matrices are implemented.

3 The code VAKTRAK

The code VAKTRAK contains an implementation of the matrices given by eqs. 4 through 9. It is organized as a collection of Fortran subroutines that perform specific tasks such as reading an input vacuum lattice file, calculating the transfer matrix between points, calculating the periodic solution or solutions for other boundary conditions and generating a plot. Using this collection of routines one program has already been assembled that reads a vacuum lattice, prompts for the desired boundary condition, calculates the solution and plots it. In the next paragraphs we will briefly describe how to run it.

First an input file named VAKTRAK.INPUT has to be generated that contains the lattice information. The first line must be a comment line not exceeding 40 characters enclosed in quotes that describes the input file. Each subsequent line in this file represents a single vacuum element. The syntax is:

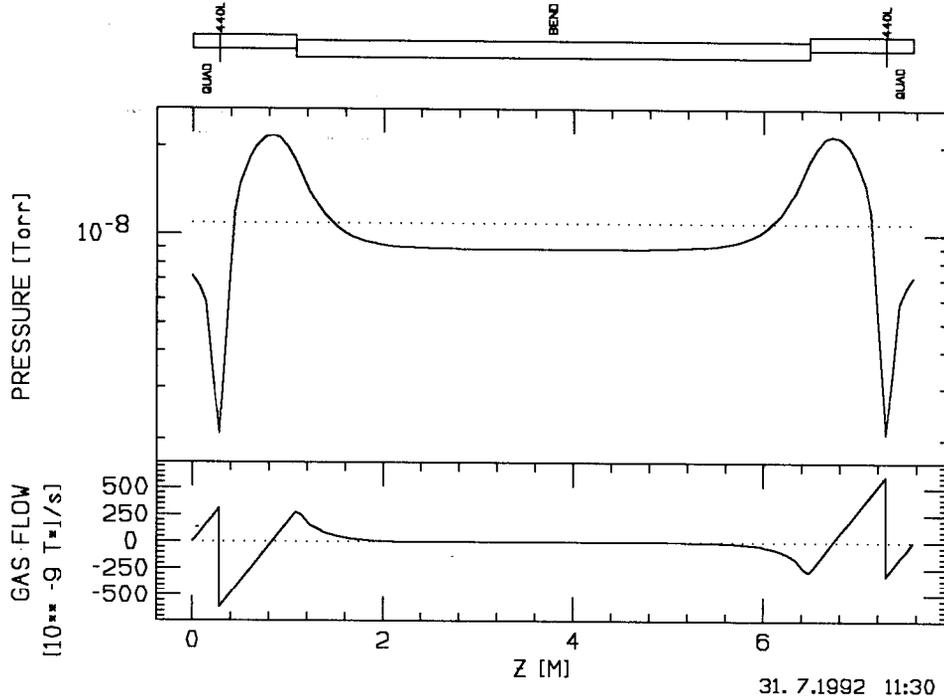
ICODE, LENGTH, CONDOC, PUMP, OUTGAS, 'NAME'

where ICODE is an integer specifying the type of element. The following four real variables are LENGTH, the length of the element, CONDOC, the conductance, PUMP, the pumping strength, and OUTGAS is the outgassing strength. NAME is an arbitrary name that can be given to the element. The following table explains the details of the input quantities.

ICODE	LENGTH	CONDOC	PUMP	OUTGAS	Comment
1	L	C	0	0	no pump, no outgas, eq. 5
21	L	c	0	0	no pump, no outgas, eq. 5
2	0	0	S	0	short pump, eq. 7
3	L	C	S	0	long pump, eq. 6
23	L	c	s	0	long pump, eq. 6
4	0	0	0	ΔQ	short gas source, eq. 9
5	L	C	0	ΔQ	long gas source, eq. 8
25	L	c	0	q	long gas source, eq. 8
6	L	C	S	ΔQ	long pump and outgas, eq. 4
26	L	c	s	q	long pump and outgas, eq. 4

The elements with code numbers $2n$ correspond to those with code n , except that they accept longitudinal densities as input quantities, i.e. $c = CL$ and $s = S/L$. Thus a typical input file for a simple vacuum lattice looks like

VAKTRAK: B-FAC - ARCS 1/2 CELL.
P/Q(START) = 0.7138E-08/ -0.5871E-28, P/Q(END) = 0.7138E-08/ -0.1916E-15
AVERAGE P/Q = 0.1090E-07/ -0.7572E-17



31. 7.1992 11:30

Figure 1: Longitudinal Pressure Profile

```
'B-FAC - ARCS 1/2 CELL.'
```

25	0.28	8.64	0	1.113E-6	'QUAD'
2	0	0	440	0	'440L'
25	0.80	8.64	0	1.113E-6	' '
26	5.4	8	119	1.063E-6	'BEND'
25	0.80	8.64	0	1.113E-6	' '
2	0	0	440	0	'440L'
25	0.28	8.64	0	1.113E-6	'QUAD'

This is a sequence of desorbing pieces of beam line and a few pumps. The elements labelled 440L are short 440l pumps. The element labelled BEND constitutes a long ion getter pump, embedded in a bending magnet. The other elements only desorb gas.

Figure 1 shows the output of a run of VAKTRAK using the previous input file. The top line shows the input conditions at the left side for the pressure P and the gas

BETAPLOT: B-FAC - ARCS 1/2 CELL.

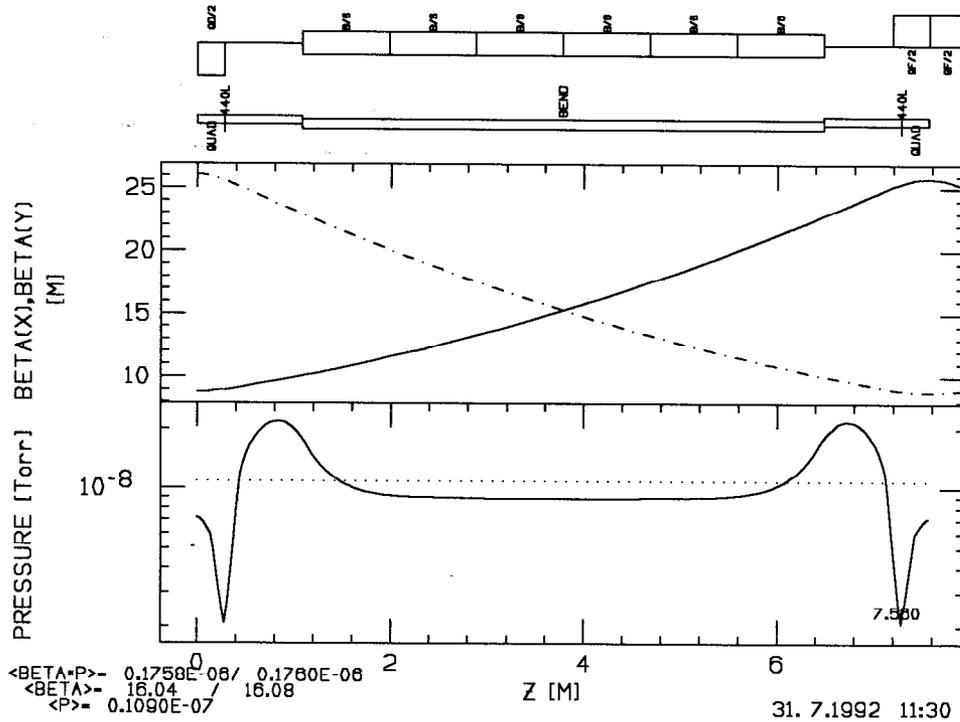


Figure 2: Beta functions and longitudinal pressure profile.

flow Q . Furthermore the average pressure and gas flow over the whole range are shown. Below the vacuum lattice is depicted. Little boxes that point upward describe desorbing elements and boxes pointing downward describe pumping elements. Elements with a code 6 that both desorb and pump point both ways. The center graph depicts the pressure profile in solid and the average pressure as a dotted line. The bottom graph shows the gas flow as a function of the longitudinal position z . Where it is positive, the flow is to the right and where it is negative the flow is toward the left. Note that the average flow is zero, which it should be in equilibrium.

Once the input file is prepared the program VAKTRAK just has to be run. VAKTRAK prompts the user for the type of boundary condition and then produces a plot like Fig. 1. One has to keep in mind that this method is numerically touchy and the calculation of the solution for very long vacuum lattices is numerically unstable, so the user must be careful in interpreting the output. The numerical instabilities are alleviated

as far as possible by using REAL*16 variables in internal calculations. However, caution is needed when interpreting the output and numerical overflows can happen. The user should check the printed transfer matrix. If the matrix elements are bigger than 10^{15} extreme caution is warranted. It should be noted that this problem mainly arises in systems with large pumps and small conductance, i.e. systems, which are physically not well designed.

The beam-gas scattering lifetime in a storage ring is determined by the average of the pressure and the beta function over the circumference of the ring [3]. These averages are calculated if the code VAKTRAK finds a file VAKTRAK.MAGLAT, which contains the magnet lattice information as TRANSPORT style input [4]. The input for a cell of the B-Factory high energy ring is shown here

5	0.279178	-72.994109	1.00000E+00	9.00000	'QD/2'
3	0.802071	0.00000	0.00000E+00	9.00000	' '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
3	0.714883	0.00000	0.00000E+00	9.00000	' '
5	0.366366	55.80629	1.00000E+00	9.00000	'QF/2'
5	0.366366	55.80629	1.00000E+00	9.00000	'QF/2'
3	0.714883	0.00000	0.00000E+00	9.00000	' '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
4	0.9	1.819313	1.0	9.00000	'B/6 '
3	0.802071	0.00000	0.00000E+00	9.00000	' '
5	0.279178	-72.994109	1.00000E+00	9.00000	'QD/2'

Each line represents a magnet element with the syntax

ICODE, LENGTH, POLE_TIP_FIELD, APERTURE, ENERGY, 'NAME'

Currently drifts (ICODE=3), bending magnets (4), and quadrupoles (5) are supported. The parameters LENGTH and APERTURE must be given in meters, POLE_TIP_FIELD in

kilo-gauss, and ENERGY in GeV. The four character string NAME gives a description of the element. All six quantities must be present in each line.

If VAKTRAK finds the magnet lattice file it reads it and determines the periodic solution. If the length of the magnet lattice and the vacuum lattice differ, the code assumes that both lattices start at the same point and calculates the averages of the beta functions and the pressure times beta function over the length of the vacuum lattice as shown in Fig. 2. On the top the magnet lattice and the vacuum lattice are shown. The upper graph depicts the horizontal (solid) and vertical (dashed) beta function and the lower graph shows the pressure profile. The averages are shown in the lower left corner.

Using this routine it is now easy to optimize the placement of pumps in a vacuum system, and taking the detailed behavior of the beta functions into account.

4 Multiple Connected Vacuum Systems

Up to this section we are dealing with single segments of beam line. Using periodic boundary conditions is equivalent to either an infinite array of such segments or a circular segment. This approach is appropriate to calculate the pressure profile in a circular accelerator which usually consists of a large number of repeated units, called cells.

Future double ring colliders such as the SLAC-LBL-LLNL B-Factory [5] have a fundamentally different topology. They consist of two arcs that have a segment of beam line in the vicinity of the IP in common. Thus their topology is more figure-eight like and consists of 3 segments or links - two arcs and the IP region - and two nodes where the three links join. To take the topology properly into account we can not use periodic boundary conditions any more, but have to generalize the concept.

The generalization is based on Kirchhoff-like rules for vacuum systems [6,7]. They are based on the observation that in vacuum systems the pressure plays the same role as the voltage in electric circuits and the gas flow behaves analogously to the current. Using these observations we can state two rules:

1. the sum of gas flows into a node is zero,
2. the sum of pressure differences around a closed loop is zero.

These rules now take the place of periodic boundary conditions.

A further complication arises from the number of unknowns we have to solve for. For our purposes we choose the pressure at each of the n nodes and the gas flows into and

out of a link. In this way we have to deal with $n + 2l$ unknowns, where l is the number of links. Now we have to find the same number of relations among the unknowns in order to find the systems' equilibrium configuration.

The first of the above rules relates the fluxes flowing into and out of each node, thus yielding n equations among the unknowns. The second rule allows us to use the transfer matrices for each of the links j to relate the pressure at the starting node of a link P_s and the gas flow into that link Q_{sj} to the pressure of the ending node of link P_e and the gas flow out of the link Q_{ej} according to

$$\begin{pmatrix} P_e \\ Q_{ej} \end{pmatrix} = \begin{pmatrix} M_{11}^j & M_{12}^j \\ M_{21}^j & M_{22}^j \end{pmatrix} \begin{pmatrix} P_s \\ Q_{sj} \end{pmatrix} + \begin{pmatrix} v_1^j \\ v_2^j \end{pmatrix}. \quad (17)$$

In this way we obtain $2l$ more equations for a total of $n + 2l$ linear equations among the pressures at the nodes and gas flows into and out of the links. The solution of this system is simply accomplished by any routine that solves linear equations, e.g., those using a Gauss-Jordan algorithm. One should, however pay attention to numerical instabilities, because the transfer matrix elements can become very large. The cure is to use the highest available precision on the computer implementation.

The described algorithm is implemented in the code VAKLOOP, which reads an input file VAKLOOP.INPUT that contains the segments (links) element by element just as VAKTRAK does with the exception that the segments are separated by a card of the following type

```
0      0      0      NODE_s  NODE_e  'NAME'
```

where NODE_s and NODE_e are the starting and end nodes of the segment that follows. The code VAKLOOP reads the input file, calculates the transfer matrices for each segment and sets up the linear equation as described in the previous section and solves it. Furthermore the code generates plots like those shown in Fig. 1 for each of the segments. After the calculation is done a few consistency checks are performed (also needed for accuracy) to test whether the solution actually solves the linear system. Their results are shown in the lower left corner of the output.

Note that the first Kirchhoff rule automatically takes care of open links, i.e. links leading to nodes that are only attached to a single link. On such "lonely nodes" the code automatically assumes $Q = 0$ which is equivalent to a locked end or a symmetry point in the vacuum system.³

³This was pointed out by G. Bowden.

5 Conclusions and Outlook

We presented a method to calculate longitudinal pressure profiles in very general conduction limited vacuum networks. The method is implemented in computer codes which allow fast and simple evaluation of the pressure in such systems.

This method is closely related to a matrix formalism treating charged particle beams in magnetic systems. The pressure takes the role of the transverse position x and the gas flow that of the angle x' . Some of the transfer matrices discussed in the second section of this note can be directly identified with magnetic elements (eq. 6: quadrupole; eq. 5: drift; eq. 9: corrector). Pressing this analogy a little further we can identify vacuum gauges with beam position monitors. It needs to be investigated in the future whether this can be used to determine the position of leaks in vacuum systems.

In this note the pumps are assumed to be linear ($S = Q/P$). However, this restriction can easily be overcome. The equations in this case will become weakly nonlinear, but can still be solved iteratively [2].

Furthermore, the codes currently deal with one gas at a time and the pressures are partial pressures for that gas. It is easy, at the expense of a lot of bookkeeping, to improve the codes to allow to deal with different pressures at the same time.

Acknowledgements

Discussions with M. Michel, Univ. Dortmund are gratefully acknowledged. I have profited greatly from discussions with and suggestions for improvement in both the codes and this note from the first users of VAKTRAK and VAKLOOP, B. Scott, SLAC/SSRL, G. Bowden, SLAC and J. Rifkin, SLAC. I thank M. Donald, SLAC for providing the B-Factory magnet lattice data.

References

1. O. Gröbner, *Vacuum Systems*, in CERN 85-19. p. 489.
2. M. Michel, *Überlegungen zum Ultrahoch-Vakuumsystem für DELTA*, Diploma Thesis, Universität Dortmund, 1988.
3. J. LeDuff, *Current and Density Limitations in Existing Electron Storage Rings*, Nucl. Instr. Meth. **A239**, 83, 1985.

4. K. Brown, et. al., *TRANSPORT Manual*, SLAC-91, 1977.
5. *An Asymmetric B Factory, Based on PEP*, Conceptual Design Report, SLAC-372, 1991.
6. G. Horikoshi, Y. Saito, K. Kakihara, *An analysis of complex network of vacuum components and its application*, KEK-Preprint-89-138, 1989.
7. H. Hirano, Y. Kondo, N. Yoshimura, *Matrix calculation of pressures in high-vacuum systems*, J. Vac. Sci. Technology **A6**, 2865, 1988.