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A Bound on the Energy Loss of Partons in Nuclei*

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ABSTRACT

We derive a quantum mechanical upper bound on the amount of radiative energy loss suffered by high energy quarks and gluons in nuclear matter. The bound shows that the nuclear suppression observed in quarkonium production at high x_F cannot be explained in terms of energy loss of the initial or final parton states. We also argue that no nuclear suppression is expected in the photoproduction of light hadrons at large x_F.

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In high energy inelastic hadron – nucleus (hA) collisions, the projectile rarely retains a major fraction of its momentum after traversing the nucleus[1]. Rather, its momentum is shared by several produced particles, which form a hadron jet in the forward direction. The classical description of this phenomena is that the hadron projectile suffers multiple collisions and repeated energy loss in the nucleus. However, the quantum mechanical situation in QCD is much more interesting. It is convenient to decompose the wavefunction of the incoming hadron of lab momentum P in terms of its free quark and gluon Fock states. Each Fock state of invariant mass $\mathcal{M}$ then persists for a time $2P/(\mathcal{M}^2 - M_h^2)$ which for large P is long compared to the transit time through the nucleus.[2] Due to time dilation, constituents which are separated by a typical distance of 1 fermi in impact space have no time to communicate while in the nucleus. Thus at high energies the quark and gluon constituents of the hadron typically interact independently of each other. Each quark or gluon constituent can lose a finite fraction of its energy in its first collision in the target due to QCD bremsstrahlung. However, since there is insufficient time to regenerate its self-field, repeated collisions (of similar hardness) by the same parton in the target do not significantly increase its total energy loss.[4] For similar reasons, final state hadrons are formed only after the projectile constituents have left the nucleus. Thus the nuclear interactions of a high energy hadron can be most simply described in terms of the individual interactions of its quarks and gluons. It is only necessary to take into account coherent interactions between constituents for the rare Fock components having a small transverse size.[3]

The uncorrelated interactions of a high energy hadron's constituents in nuclear matter imply that the constituents will normally hadronize independently of each other outside the nucleus, giving rise to overlapping jets in the forward direction.

The rare case where a single hadron h' carries a large fraction x_F of the beam momentum thus most likely occurs when h' is formed from a transversely compact Fock state which can retain its coherence while traversing the nucleus. As we showed in an earlier paper[5], if the A-dependence of the inclusive cross section on nuclei $d\sigma/dx_F(hA \to h'X)$ is parametrized as $A^{\alpha}(x_F)$, then this restriction to compact states implies a monotonic decrease of α with x_F, which is consistent with the trend of the data[1].

The time dilation argument given above implies that the fractional energy loss of a high energy quark or gluon which participates in a hard collision will not depend on the size of the nuclear target. In fact, there is convincing empirical evidence that high energy quarks can penetrate even very heavy nuclei with insignificant mean energy loss. In deep inelastic muon scattering[6] on heavy nuclei, the struck quark emerges from the nucleus with close to the full energy ν transferred by the virtual photon, provided $\nu > 20$ GeV. Secondly, the production cross section of high mass muon pairs by hadrons on nuclei appears to be closely proportional to the atomic number A of the target[7]. According to the Drell – Yan mechanism, this implies that the projectile quark (or antiquark) carries its full initial energy even at the time of its annihilation with an antiquark (or quark) at the back side of the target nucleus. Both the deep inelastic lepton scattering and the large mass lepton pair production data are thus incompatible with a significant mean energy loss of quarks in nuclear matter. The same conclusion follows more generally from the factorization theorems of perturbative QCD: the structure functions of the projectile hadron and the fragmentation functions of the final state partons are unmodified at leading twist by the nuclear target.

Recently, it has been pointed out[8] that data on fast hadron leptoproduction even for (almost) real and virtual photons show no nuclear effect. For large photon energies $\nu > 100$ GeV the hadron momentum spectra observed in the E665 experiment are essentially independent of the size of the nuclear target. Even more remarkably, the hadron spectrum in the kinematic region where a strong absorption effect is seen in the low Q^2 inelastic cross section ($x_{Bj} < .005, Q^2 < 1$ GeV^2) is very similar to the spectrum observed in the non-shadowing region ($x_{Bj} > 0.03, Q^2 > 2$ GeV^2).

We can understand also these effects in terms of the time dilation and the QCD Fock state decomposition. Clearly a major fraction of the real or low Q^2 photoabsorption cross section occurs via vector meson dominance. However, aside from diffractive processes, the hadrons produced by the intermediate vector mesons mostly have small longitudinal momentum fractions, just as in hadron-nucleus scattering. By triggering on fast hadrons in the photoabsorption cross section, we essentially eliminate the VMD component and select the mechanism whereby the photon scatters only via the $q\bar{q}$ intermediate Fock state. As is well-known (see, *e.g.*, Ref. 9), the $q\bar{q}$ state scatters most strongly in an asymmetric configuration where one of the quarks carries only a small fraction $1 - x \ll 1$ of the photon energy. The fluctuation of the high energy photon to the $q\bar{q}$ state occurs at a time $\tau \simeq 2\nu/(\mathcal{M}^2 + Q^2)$, well before interactions occur in the target. The transverse size $r_\perp$ of the $q\bar{q}$ Fock state on arrival at the target nucleus can be estimated from the transverse velocity $v_\perp = p_\perp/(1-x)\nu$ of the soft quark. In the low Q^2 region, for $\mathcal{M}^2 \simeq p_\perp^2/(1-x) \gg Q^2$, we get a large size $r_\perp = v_\perp \tau \simeq 2/p_\perp$, of the order of 1 fm for typical values of $p_\perp$. Hence a soft interaction will occur on the nuclear surface and the inner parts of the nucleus are shadowed.[10] For large $Q^2 \gg \mathcal{M}^2$ on the other hand, $r_\perp p_\perp \simeq 2\mathcal{M}^2/Q^2$ is small, and there is no shadowing. Note that in *either case* the major fraction x of the photon momentum is carried by the fast quark, which has no time to build up a self-field and thus loses little energy in the nucleus. The fast hadrons, which are formed by the fragmentation of this fast quark outside the nucleus, will thus have a momentum spectrum which is independent of the nuclear size both in the (shadowing) region of small Q^2 and in the (transparent) region of large Q^2.

The data[11,12,13] for the hadroproduction of heavy quarkonium states on nuclei show a strong nuclear suppression at large x_F, which is in striking contrast to the minimal effects seen in continuum lepton pair production. Since the nuclear dependence does not factorize[14] as a function of the nuclear parton fraction x_2, the effect cannot be due to gluon shadowing. The breakdown of factorization also implies that the nuclear dependence must be associated with a higher twist mechanism. Furthermore, the nuclear suppression seen in the E772 experiment[13] is essentially identical for J/ψ, ψ', and ψ'' production even though these states have drastically different sizes; thus the nuclear effect cannot be attributed to final state hadronic absorption. In fact, at high x_F the $c\bar{c}$ pair does not form the quarkonium state until it is well beyond the nuclear volume.[15] Thus final state absorption of heavy quarkonium is predicted to decrease with growing x_F, contrary to the observed nuclear attenuation.[16]

Recently, some authors have claimed[17,18,19] that the anomalous suppression of large x_F J/ψ production on nuclear targets[11,12,13] can be explained by postulating a significant energy loss for fast gluons and quarks as they propagate through the nucleus. The nuclear effect is assumed to be higher twist so that it would not conflict with the PQCD factorization theorems. Any parton energy loss implies that the structure function of the projectile is sampled at a larger value of x_1 than would otherwise be inferred from the x_F of the J/ψ.

To see the effect of such a parton energy loss explicitly, let us assume that the structure function has a behavior $F(x_1) \propto (1-x_1)^n$. The suppression corresponding to a fractional energy loss Δx_1 is

$$\frac{F(x_1+\Delta x_1)}{F(x_1)} \simeq 1 - \frac{n}{1-x_1}\Delta x_1 \simeq A^{\delta\alpha}. \tag{1}$$

Hence the effective shift $\delta\alpha$ in the nuclear power dependence is approximately given by

$$\delta\alpha = -\frac{n}{1-x_1}\frac{\Delta x_1}{\log A}. \tag{2}$$

An energy loss due to multiple scattering in the nucleus would be proportional to the nuclear diameter, $\Delta x_1 \propto A^{1/3}$. Then the dependence of $\delta\alpha$ on A in Eq. (2) is indeed quite weak; $A^{1/3}/\log A = 1$ within 10% over the range $5 \leq A \leq 200$, implying that the energy loss effect can be indeed parametrized as a power of A.

The authors of Refs. 17 and 18 assume that the average fractional momentum loss of an incident parton in a high Q^2 reaction, such as charm production, is given by

$$\frac{\Delta E_{lab}}{E_{lab}} = \Delta x_1 = C\frac{x_1}{Q^2}A^{1/3} \tag{3}$$

where C is a color-dependent constant and $A^{1/3}$ reflects the number of nuclear collisions. They propose that the coefficient of $A^{1/3}$ decreases as $1/Q^2$ because energy loss should be a higher twist effect; it is also evidently consistent with the reduced nuclear suppression observed for the Υ data. Finally, they argue that the fractional loss should be proportional to x_1 in analogy with the QCD bremsstrahlung processes.

Here we would like to show that the form (3) of the energy loss violates general quantum mechanical arguments based on the uncertainty principle. We shall show that any A-dependent energy loss ΔE_{lab} must be independent of E_{lab}. Thus the energy loss per unit length of the target is fixed[20] in the target rest frame, rather than being $\propto E_{lab}$ as in Eq. (3). In fact, the energy loss due to multiple soft scattering cannot depend on the Q^2 of the hard collision: it occurs long before the hard vertex and is thus causally independent of Q^2. Furthermore, since $Q^2 = sx_1x_2$, Eq. (3) would imply that the multiple scattering energy loss depends on the x_2 of the target parton involved in the hard collision.

The requirement of a fixed, energy-independent energy loss in the target rest frame is a direct consequence of the uncertainty principle relation $\Delta p_z \Delta L > 1$. The uncertainty principle sets a minimum longitudinal momentum transfer Δp_z from the target for any inelastic process which can be resolved as occurring between two scattering centers of separation L. The longitudinal momentum transfer to the scattered parton due to gluon radiation is $\Delta p_z \simeq \Delta \mathcal{M}^2/2E_{lab}$, where $\Delta \mathcal{M}^2 \sim k_{\perp g}^2/x_g$ is the difference between the incident parton mass squared and the mass squared of the parton–gluon system after radiation, and $k_{\perp g}$, x_g are, respectively, the transverse momentum and momentum fraction of the radiated gluon. Repeated radiation at distances less than that allowed by the uncertainty relation is cancelled because of destructive interference between the radiation emitted by the parton at the two scattering centers[21]. The minimum distance L required between repeated emissions may be interpreted as that needed for the buildup of the gluon field around the bare parton.

For a simple and explicit example of how the uncertainty principle is upheld in perturbation theory, consider the photon radiation induced by the double scattering of a scalar electron (Fig. 1). We shall keep the times t_1, t_2 of the instantaneous Coulomb exchanges fixed — they represent two interactions in the same nucleus. The photon can be radiated at a time t before, in between, or after the Coulomb scatterings, corresponding to the three diagrams illustrated in Fig. 1. Up to a common factor (which includes the Coulomb propagators), the amplitudes of the three time orderings are

$$
\begin{aligned}
M_a(t < t_1, t_2) =& - i \exp[-iE_{a2}(t_2 - t_1)] \int_{-\infty}^{t_1} dt \, \exp[-i(E_{a1} - E)(t_1 - t)] \\
=& \frac{\vec{\varepsilon} \cdot \vec{p}}{E - E_{a1}} \exp(-iE_{a2}\Delta t) \\
M_b(t_1 < t < t_2) =& \frac{\vec{\varepsilon} \cdot (\vec{p} + \vec{\ell}_1)}{E_{b1} - E_{b2}} [\exp(-iE_{b1}\Delta t) - \exp(-iE_{b2}\Delta t)] \\
M_c(t_1, t_2 < t) =& \frac{\vec{\varepsilon} \cdot (\vec{p} + \vec{\ell}_1 + \vec{\ell}_2)}{E - E_{c2}} \exp(-iE_{c1}\Delta t)
\end{aligned}
\tag{4}
$$

Here $\Delta t = t_2 - t_1$ and $E_{a1}, \ldots, E_{c2}$ are the energies of the scattering system at the intermediate times indicated in Fig. 1.

As the initial (and final) scattering energy grows, ($E \to \infty$ at fixed fractional momentum $x_\gamma = k_\parallel / E$ of the photon), all of the intermediate energies approach E. For example,

$$
E - E_{a1} \simeq -\frac{1}{2E(1 - x_\gamma)} (x_\gamma m^2 + \frac{k_\perp^2}{x_\gamma}). \tag{5}
$$

Thus at fixed* Δt all phase factors in Eq. (4) approach $\exp(-iE\Delta t)$. The amplitudes M_a and M_c then each have the same form as the amplitude for a single Coulomb scattering with momentum exchange $\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2$. The amplitude M_b,

* Or for any $\Delta t \lesssim L_A$, as would be the case for scattering in a nucleus of diameter L_A.

which describes photon emission between the two Coulomb exchanges, is of $\mathcal{O}(1/E)$ compared to $M_a + M_b$, due to the cancellation of the phase factors in brackets. Hence the double scattering is not resolved, and the strength of the single scattering is renormalized. This is precisely the content of the uncertainty relation, stating that multiple scattering in a target of fixed length cannot induce fractional energy loss in the high energy limit.

On the other hand, we can also see from Eq. (5) that M_b is of the same order as $M_{a,b}$ if the photon momentum fraction $x_\gamma = \mathcal{O}(1/E)$. Hence multiple scattering can induce a *fixed* energy loss in the laboratory frame. In general, the fractional energy loss x_γ that can be induced by multiple scattering in a target of length L_A is limited by

$$x_\gamma < \frac{k_\perp^2 L_A}{2E} \tag{6}$$

where $k_\perp$ is the transverse momentum of the photon.

As discussed above, the same bound (6) can be obtained directly from the uncertainty relation, and thus applies equally to gluon radiation by incoming or outgoing partons in hadron scattering. Hence the bound on the fractional energy loss Δx_1 of the projectile parton appearing in Eqs. (1),(2) is given by

$$\Delta x_1 \lesssim \frac{\kappa}{x_1 s} A^{1/3}. \tag{7}$$

where we used $E = x_1 s/2M_P$ and took the nuclear radius $R_A \sim 1.2 \text{ fm} A^{1/3}$ to characterize the largest effective distance between scattering centers in the nucleus. Hence

$$\kappa \sim (1.2 \text{ fm}) M_p < k_\perp^2 >\sim 0.5 \; GeV^2. \tag{8}$$

since gluons radiated by the incident or final state partons in cold nuclear matter have a characteristic transverse momentum $< k_\perp^2 > \sim 0.1$ GeV^2.

The bound (7) should be contrasted with the assumption (3) of Refs. 17 and 18. Our bound is independent of Q^2, since the range of the hard interaction is short and does not affect multiple scattering elsewhere in the nucleus.* The bound (7) is also independent of the color charge of the parton, *i.e.*, this upper bound is the same for quarks, gluons and compact $c\bar{c}$ states. Most importantly, the bound (7) is inversely proportional to the laboratory energy of the radiating parton. Hence energy loss becomes insignificant at high energies, and the cross-sections obey Feynman scaling. The fact that the measured cross-sections[11,13] indeed satisfy Feynman scaling shows that the effects of finite energy loss is already negligible in the data for $E_{lab} \gtrsim 100$ GeV.

The bound (7) implies numerically insignificant effects of energy loss in the high energy data. Consider, for example, the suppression of J/ψ production on Tungsten, which for 800 GeV protons was measured by the E772 Collaboration[13] to be 60% at $x_F = 0.64$. Substituting (7) into (2) gives a suppression due to parton energy loss of $\delta\alpha = -0.008$ for $n = 5$, corresponding to only a 4% suppression of the cross section on Tungsten. This is negligible compared to the effect seen in the data.

According to (6), the average radiative energy loss per unit distance in nuclear matter is $dE/dz \lesssim \frac{1}{2} < k_\perp^2 > \simeq 0.25$ GeV/fm. A similar degradation of energy is expected from elastic scattering[22]. The total expected energy loss, $dE/dz \sim 0.5$ GeV/fm, appears to be consistent with an estimate using combined SLAC and EMC data for jet fragmentation in nuclei[22]. At high energies, such a fixed energy

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* Gluon radiation may occur in the hard process itself even at the leading twist level, as in $gg \to c\bar{c}g$. These processes are not suppressed at high Q^2 and are given by the higher order perturbative terms in the hard cross-section $\hat{\sigma}$.

loss becomes insignificant, thus explaining the lack of nuclear target dependence of jet fragmentation processes in deep inelastic lepton scattering[6] and the lack of nuclear-induced initial state energy loss of the annihilating quark in massive lepton pair production[7]. On the other hand, in a medium which is at high temperature T, such as in a quark gluon plasma, the average energy loss can be larger[22,23], since $< k_\perp > \propto T$.

The nuclear suppression of quarkonium production at high x_F is observed to be mass-dependent — the suppression measured in the E772 experiment is smaller for Υ production than for J/ψ production[13]. In view of the Q^2-independence of Eq. (7) at fixed s and x_1, this again rules out an explanation of the A-dependence in terms of nuclear-induced energy loss. It also should be emphasized that the J/ψ cross-section at large x_F is measured[11] to be *in excess* of that predicted by leading twist fusion processes for proton targets. Thus it is likely that the anomalous nuclear effects are due to higher twist effects which enhance the hard cross section on elementary targets.

Sizeable higher twist contributions at large x_F are in fact expected in QCD from intrinsic heavy quark production mechanisms[24]. In contrast to the leading twist fusion contributions such as $gg \to c\bar{c}$, the intrinsic contributions involve two or more constituents in the projectile. Although these amplitudes are relatively suppressed by powers of $1/m^2_{Q\bar{Q}}$, a greater fraction of the projectile's momentum is involved so they are less suppressed at high x_F. Since the slow spectators interact in the target, the intrinsic contributions to the large x_F cross section have nuclear dependence close to $A^{2/3}$[5]. Recently, it was shown that the above qualitative features of intrinsic production emerge in a perturbative QCD analysis[25]. However, a definitive explanation of the nuclear anomalies in heavy quark production at large

x_F in terms of higher twist contributions will require a more quantitative analysis of multiparton correlations in QCD.

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FIGURE CAPTION

Fig. 1. Photon radiation diagrams associated with the double Coulomb scattering of a (scalar) electron at the fixed times t_1 and t_2. The total initial and final energy of the scattering is E, and the intermediate energies at the times indicated in (a), (b) and (c) are denoted by E_{ai}, E_{bi} and E_{ci} ($i = 1, 2$), respectively.