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What Do We Know (and How) about the CKM Matrix*

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ABSTRACT

The way from an experimental measurement to the numerical value for a CKM matrix element is described. How do we choose the appropriate model? What are the uncertainties involved? Where should we direct our future efforts? How do loop processes help us? Finally we describe the state of the art of our knowledge of the CKM matrix.

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1. INTRODUCTION

The Standard Model (SM) with three fermion generations and with one Higgs doublet is parametrized by eighteen free parameters. Of these, ten are related to the quark sector: six quark masses and four mixing parameters. The four mixing parameters parametrize the Cabibbo - Kobayashi - Maskawa (CKM) mixing matrix which describes the complex rotation between the weak interaction eigenstates and the mass eigenstates. With three quark generations, the CKM matrix is a unitary 3×3 matrix. In general, such a matrix has nine free parameters. However, five phases can be "rotated away" by redefinition of quark phases. Thus, we are left with four parameters: three real mixing angles and one phase.

If we have several independent measurements for a given CKM matrix element, or if we find the values of the nine entries, we will have the four mixing parameters *overdetermined.* Therefore, an exact determination of the CKM matrix elements provides us with a stringent test of the SM, and with possible clues to physics beyond it.

In this work, the emphasis is put on the calculation of the three above-diagonal elements: $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. The values of the three elements are best determined from semi-leptonic meson decays. It is one simple diagram which stands at the basis of all such processes: Within the spectator quark model, the light quark in the meson does not play a role in the decay process. The heavier quark in the meson transforms into a lighter one by emitting a W-boson, and the W-boson decays leptonically. In spite of the basic simplicity of these processes, each of the three calculations has its own characteristics and difficulties.

2. THE V_{us} ELEMENT

The value of $|V_{us}|$ is best determined from semi-leptonic K decays: $K^+ \to \pi^0 e^+ \nu_e$ and $K_L^0 \to \pi^- e^+ \nu_e$. At the quark level the process is $s \to u e \nu_e$. We would like to carry out a calculation from first principles at the quark level. However, the following difficulties arise:

a. The K semi-leptonic decay is dominated by *one* final state of a single pion (the two-pion final state has a branching ratio smaller by four orders of magnitude). Quark - meson duality is expected to hold when there is a dense set of final states, which is certainly not the case here.

b. There are QCD corrections to the weak interaction diagram. These corrections depend on α_s at a scale around m_s. This scale is very close to Λ_{QCD}, the scale at which, by definition, $\alpha_s \sim 1$. Thus, a perturbative QCD expansion is meaningless.

c. The phase-space for the decay depends on the quark masses. As quark masses are running, there is a question of the energy scales at which they should be taken. Even if we were able to determine the relevant energy scales there is still an additional uncertainty: the strange quark mass, m_s, is known with a 30% uncertainty. As the decay rate depends on m_s^5, such a large uncertainty makes it impossible to calculate $|V_{us}|$ with a reasonable accuracy.

We conclude that the spectator quark model is not expected to hold for semi-leptonic K decays, and even if it does - there are practical difficulties in the calculation. Instead, $|V_{us}|$ is calculated within phenomenological models, namely at the meson level:

$$\frac{BR(K \to \pi e \nu)}{\tau_K} = \left[\frac{G_F^2 M_K^5 C^2}{192\pi^3}\right] F_{ps}\ (1+r)|f_+(0)|^2 |V_{us}|^2. \qquad (1)$$

The quantities on the left hand side of the equation are given by experiments with an overall accuracy of 1-1.5%. The quantities in square brackets (the Fermi constant G_F, the K-meson mass M_K and a Clebsh-Gordan coefficient C) are known.

The phase-space factor F_{ps} depends on an experimentally-fitted parameter, which introduces a 0.5% uncertainty. The radiative corrections (r) can be calculated to an accuracy of about 0.3%, but an ambiguity in the way these corrections were incorporated into different data analysis adds up to a 1% uncertainty in $(1+r)$.

The main theoretical difficulty is in the normalization of the form factor $|f_+(0)|$. However, in the $SU(3)$ limit ($m_u = m_d = m_s$) we have $|f_+(0)| = 1$, and deviations from this value are only second order in the symmetry breaking parameter. The approximate symmetry allows a determination of $|f_+(0)|$ with an uncertainty of only 0.8%.

Altogether, there is a nice balance between the experimental measurements and the theoretical calculations, each giving uncertainties at the 1-2% level. With two independent measurements, of the charged and neutral K decays, we are able to determine $|V_{us}|$ with a 1% uncertainty [1]:

$$|V_{us}| = 0.2196 \pm 0.0023. \tag{2}$$

Of course, we would not have confidence in this result if not for additional independent measurements. The determination from hyperon decays is less accurate, but the results are completely consistent [2].

3. THE V_{cb} ELEMENT

The value of $|V_{cb}|$ is best determined from semi-leptonic B decays: $B \to X_c e \nu_e$. At the quark level the process is $b \to c e \nu_e$. In this case:

a. The dominant semi-leptonic modes are those with $X_c = D,\ D^*$. Duality should hold for the decay rate within about 10%.

b. The relevant scale for QCD corrections is of order m_b. As $\alpha_s(m_b) \sim 0.2$, a first-order calculation should be fine to within 4% or so.

c. The mass of the b quark at a certain energy scale is known at the 2% accuracy level. Consequently, the crucial question is that of the relevant energy scales. We will argue that there is no ambiguity of energy scales for m_c or, more accurately, in the ratio m_c/m_b. However, the question of energy scale for m_b in the m_b^5 factor is still open and remains the main source of uncertainty in the calculation.

We conclude that the spectator quark model is expected to give a reasonable description of the inclusive semi-leptonic B decay rate. The major source of uncertainty in the calculation is the b mass. On the other hand, QCD corrections are under control and the mass ratio m_c/m_b to be used is unambiguous. Within the spectator quark model:

$$\frac{BR(b \to ce\nu)}{\tau_b} = \left[\frac{G_F^2}{192\pi^3}\right] m_b^5 F_{ps}(\rho_c) F_{QCD}(\rho_c) |V_{cb}|^2. \tag{3}$$

The experimantal quantities on the left hand side are known with an about 15% error, mainly from the b lifetime determination. The phase-space factor F_{ps} and the QCD correction factor F_{QCD} both depend on the mass ratio $\rho_c = m_c^2/m_b^2$. As mentioned, apriori there is an ambiguity, because quark masses are running, so that ρ depends on two scales:

$$\rho_c = \frac{[m_c(\mu_c)]^2}{[m_b(\mu_b)]^2}. \tag{4}$$

The question is what are the relevant scales μ_c and μ_b. The answer is [3] that to every choice of two scales, there corresponds a specific QCD correction factor. The modification of F_{QCD} is such that the product $F_{ps}(\rho) \cdot F_{QCD}(\rho)$ is independent of the choice of scales:

$$F_{ps}(\rho_c) F_{QCD}(\rho_c) = 0.46 \pm 0.04. \tag{5}$$

Various arguments suggest that the value of m_b should be taken as

$$m_b = 4.9 \pm 0.3\ GeV. \tag{6}$$

As the decay width depends on m_b^5, this gives a 30% uncertainty in the calculation, which makes a theoretical study of this ingredient most urgent. With the above values we get:

$$|V_{cb}| = 0.046 \pm 0.008. \tag{7}$$

Various phenomenological models are, at present, in the stage of being tested against the experimental data. However, they all tend to give $|V_{cb}|$ values which are somewhat higher than the spectator quark model value. To account for the model dependence of the calculation we take:

$$|V_{cb}| = 0.048 \pm 0.009. \tag{8}$$

4. THE V_{ub} ELEMENT

The value of $|V_{ub}|$ can be determined from semi-leptonic charmless B decays: $B \to X_u e\nu_e$. At the quark level the process is $b \to ue\nu_e$. The calculation is subject to uncertainties similar to those of V_{cb}. It is advantageous to consider the ratio $q \equiv |V_{ub}/V_{cb}|$ rather than $|V_{ub}|$ itself:

$$\frac{BR(b \to ue\nu)}{BR(b \to ce\nu)} = \frac{F_{ps}(\rho_u)}{F_{ps}(\rho_c)} \frac{F_{QCD}(\rho_u)}{F_{QCD}(\rho_c)} q^2. \tag{9}$$

The ratio is free of the uncertainties in m_b^5 and τ_b. Moreover, the ratio between the QCD correction factors does not depend (to $O(\alpha_s)$) on the choice of scale for α_s and, due to the lightness of the u quark, $F_{ps}(\rho_u) = 1$ with no uncertainty. We get:

$$F_{ps}(\rho_u) F_{QCD}(\rho_u) = 0.85. \tag{10}$$

The only theoretical uncertainty is then in $F_{ps}(\rho_c)$. We get:

$$q = (0.74 \pm 0.03) \left[\frac{BR(b \to ue\nu)}{BR(b \to ce\nu)}\right]^{1/2}. \tag{11}$$

However, experiment cannot provide us, at present, with $BR(b \to ue\nu)$ as there is no direct observation of charmless B decays. If one would try to subtract from the *measured* semi-leptonic rate the theoretically *calculated* charmed semi-leptonic decay rate, he would be left with zero and the $b \to u$ contribution "buried" within the large error bars.

Instead, $|V_{ub}|$ is determined from the electron energy spectrum. The spectator quark model is not appropriate for this analysis [4], while various phenomenological models give very different results. The strongest experimental results with the weakest theoretical constraints give [5]:

$$q \leq 0.16. \tag{12}$$

5. RESULTS

The above diagonal elements in the CKM matrix are best determined from semi-leptonic meson decays. For light mesons, or correspondingly light quarks, quark-meson duality does not hold because the spectrum is dominated by one final state. Moreover, even if the spectator quark model held, we would have practical difficulties in the calculation due to large QCD corrections and large uncertainties in the light quark masses. On the other hand, we are able to calculate rather accurately within phenomenological models, due to the approximate flavor symmetry. For heavier mesons, or correspondingly heavier quarks, the spectator quark model should give a reasonable description of the inclusive decay rate. QCD corrections are small and heavy quark masses are known rather well, though they remain the major source of uncertainty. In the case of heavy quarks, phenomenological models have no approximate symmetry to help control the hadronic matrix elements, and at this stage they should be tested against the experimental results rather than used to estimate the CKM matrix elements.

Direct measurements give:

$$|V_{us}| = 0.220 \pm 0.002, \quad |V_{cb}| = 0.048 \pm 0.009, \quad q \leq 0.16. \tag{13}$$

We have also direct measurements of $|V_{ud}|$, $|V_{cd}|$ and $|V_{cs}|$. These elements and the $|V_{us}|$ value overdetermine the Cabibbo angle, and are all consistent with $\sin\theta_C \sim .220$. They do not really test the other three parameters in the CKM matrix.

Upper bounds on the values of the CKM matrix elements in the third row can be derived from unitarity: for *any* number of generations: $\sum_{i=1}^{3}|V_{ij}|^2 \leq 1$. The information from direct measurements on the CKM matrix is then the following:

$$V = \begin{pmatrix} .9747 \pm .0011 & .220 \pm .002 & \leq .009 \\ .21 \pm .03 & \geq .60 & .048 \pm .009 \\ \leq .14 & \leq .77 & \leq .9992 \end{pmatrix} \tag{14}$$

Additional information on the matrix elements is derived from indirect measurements, namely loop processes. The most useful measurements are those of the CP violating parameter ϵ and the $B - \bar{B}$ mixing parameter x_d. To put constraints on the parameters from these measurements, one assumes that there are only three quark generations, and that there are no significant contributions from any new physics to these processes.

As quark loops are involved, the GIM mechanism is in operation and the results strongly depend on the top mass. Thus, we get constraints [6] in the three-parameter space (m_t, q, δ). The lighter the t quark, the larger portion of the range allowed by direct meaurements becomes excluded. For $m_t \leq 47\ GeV$ there is no allowed range, thus excluding this range for the top mass. For $m_t \sim 200\ GeV$ almost all of the original range is allowed. Indirect measurements give:

$$q \geq 0.015, \quad 12^o \leq \delta \leq 178^o. \tag{15}$$

Within the three generation SM, and using the unitarity conditions and all measurements (direct and indirect) we have:

$$V = \begin{pmatrix} .9750 - .9758 & .218 - .222 & .0008 - .009 \\ .217 - .223 & .9732 - .9754 & .039 - .056 \\ .006 - .020 & .038 - .055 & .9984 - .9992 \end{pmatrix} \tag{16}$$

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