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RARE DECAYS*

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Recent theoretical and experimental progress on rare decays is sumarized, principally for K and B mesons, as discussed in the parallel sessions of this workshop.

1. INTRODUCTION

The primary aim of much of the present program of looking at rare decays is part of a much broader effort of seeking physics beyond the Standard Model. Such physics could appear directly in rare decays in the form of new particles, *e.g.*, a low-mass Higgs boson in radiative meson decay. More commonly, one is sensitive to new physics through the indirect effects of virtual, *heavy* particles. These, through precision measurements at "low energies," give us a window on the high-energy world which others attack directly.

In examining the possibilities for new physics, aside from direct searches for new particles, we consider:

(*a*) Processes forbidden in the Standard Model, such as would be induced by lepton flavor-changing neutral currents.

(*b*) Indications that CP-violating phenomena have an origin other than from the nontrivial phase in the quark flavor-mixing matrix of the Standard Model.

(*c*) Deviations from expected rates, especially for rare processes which are sensitive to virtual, heavy particles (from a fourth generation, supersymmetry, left-right electroweak gauge symmetry, etc.). This is particularly true of CP-violating processes, which, in some cases, are especially sensitive to high-mass particles.

A subtheme of this talk is that one such high-mass particle lies inside the Standard Model: the top quark. We now know experimentally[1-3] that m_t is comparable to M_W. This has important consequences for rare K and B decays. As we will see later in specific examples, the contributions from virtual top quarks in one-loop diagrams become the dominant ones within the three-generation Standard Model, with previously (justifiably) neglected diagrams playing an important role.

2. MASS MATRICES AND MIXING ANGLES

Two of the pieces of physics from beyond the Standard Model which we have in our hands right now are the quark masses and the weak mixing angles; these are quantities which are not determined within the Standard Model and are presently put in by hand. One time-honored way of proceeding is to "guess" at the Yukawa couplings of the Higgs to quarks in the weak eigenbasis. Diagonalization of the up (charge $2e/3$) and down (charge $-e/3$) matrices of Yukawa couplings is by definition transforming to a mass eigenbasis: the eigenvalues are proportional to the quark masses and the unitary matrix between the up and down sectors is just the Kobayashi–Maskawa matrix. A judicious choice of zeros in the Yukawa couplings, possibly justified *a posteriori* by imposing a symmetry principle, in general will

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produce relations between the masses and mixing angles. Such is the case for the original work of Fritzsch,[4] which has been followed by a large literature of alternates and extensions to this basic idea, including several talks here.[5–7] Present experimental information on the mixing angles and the masses of all the quarks except top, together with the Fritzsch form for the Yukawa coupling matrices, requires that the top quark reside not far from M_W.[8]

Ultimately, of course, we want to proceed the other way: start with a full theory that goes beyond the Standard Model and derive from it the form of the Yukawa coupling matrices, with the symmetries given *a priori* by the theory of everything. This is the domain, for example, of superstrings, and intriguing work in this direction was described here by Ross.[9]

3. THE SEARCH FOR A LIGHT HIGGS BOSON

In the past few years, experiments on decays of the K, B, and Υ mesons have reached the level of sensitivity where one is able use them to exclude certain mass regions for the Higgs boson of the Standard Model. Much better limits,[10] on $B(K_L \to \pi^0 + H)$ followed by $H \to e^+e^-$ from NA31, were announced here, as was an upper limit from BNL[11] of 1.5×10^{-7} on $B(K^+ \to \pi^+ H)$ followed by $H \to \mu^+\mu^-$. A major improvement has also been achieved recently by CLEO[12] in searching in B decays for the process which, at the quark level, is $b \to s + H$. On the theoretical side, there has been clarification of the predictions in K decay[13] and in B decay.[14] Together, the recent experimental and theoretical results make it such that[15] a Standard Model Higgs boson below 3.6 GeV is, to borrow an overused word from our experimental colleagues, "unlikely."

4. "FORBIDDEN" LEPTONIC FLAVOR-CHANGING NEUTRAL CURRENTS

The advent of gauge theories, and more specifically, of the Standard Model, has helped put into focus many old problems. The question of "who ordered the muon" was generalized to "why three generations," and this has brought to the fore the possibility of interactions which would connect quarks and leptons of different generations, producing flavor-changing neutral currents. The origin of generations might be from symmetries or from dynamics, as we were reminded here,[16] and one might imagine these interactions to be fundamental, as in a theory with so-called horizontal gauge bosons, or only effective, as in some theories where quarks and leptons are themselves composite. While such effects can be sought at high energy and momentum transfer,[17] the primary emphasis here has been on rare decays.

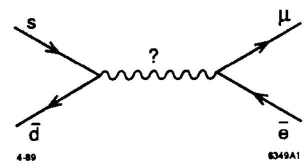

FIGURE 1
Tree-level diagram involving a flavor-changing gauge boson.

To parametrize such flavor-changing effects, it is convenient to think in terms of the exchange of a horizontal gauge boson as in figure 1, so that the amplitude behaves as

$$A \propto \frac{g_H^2}{M_H^2} \quad . \tag{1}$$

The width is then proportional to M_H^{-4}. It has become usual[18] to set $g_H = g$, the weak gauge coupling; then, an upper bound on the branching ratio for a given process can be converted into a lower bound on M_H. In doing so, one must always be careful not to apply equation (1) blindly: Not only do kinematic factors and the space-time structure of the currents (*e.g.*, possible "helicity suppression") need to be taken into account, but also there may be additional mixing angles or selection rules associated with the underlying physics which partially or totally suppress a given process.

At this meeting, the old limit[19] of 7×10^{-9} on $B(K_L \to \mu e)$ has been superseded by limits of 3×10^{-10} from BNL[20] and 4.2×10^{-10} from KEK,[21] which, translated as discussed above into a lower limit on M_H, correspond to a mass scale of about 50 TeV. A preliminary limit[20] from BNL experiment E777 on $B(K^+ \to \pi^+\mu^+e^-)$ is 2.3×10^{-10}, corresponding to a mass scale of close to 40 TeV.

Byproducts of these limits are the measurements:[21]

$$B(K_L \to e^+e^-) < 5.4 \times 10^{-10}$$
$$B(K_L \to \mu^+\mu^-) = 8.4 \pm 1.1 \times 10^{-9} \quad ,$$

with results on these reactions also expected soon from BNL experiment E791. Within a year, one can expect the limits on forbidden, flavor-changing K decays to go below 10^{-10}, and within a few years, to below the 10^{-11} to 10^{-12} level.

Plans are in progress to make "factories" to produce other light mesons—i.e., η, η', ρ, ω, ϕ—at rates of order 10^{12} per year, in some cases. Now "rare" decays may be only "ordinary" weak decays of these states with dominant strong and/or electromagnetic decays, but they are nonetheless interesting, as they give us another handle on many old questions having to do with nonleptonic decays and the interplay of strong and electroweak interactions.[22]

5. "ALLOWED" QUARK FLAVOR-CHANGING NEUTRAL CURRENTS—ONE-LOOP PROCESSES

5.1 $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ Mixing

The grandfather of all the calculations of amplitudes which are forbidden in the lowest order of the electroweak theory is that of the off-diagonal elements of the $K^0 - \bar{K}^0$ mass matrix which generate the $K_L - K_S$ mass difference and ϵ. This still provides the tightest constraint on quark flavor-changing neutral currents (provided, of course, that they contribute to this process). The one-loop contribution is shown as the box diagram in figure 2. For the imaginary part of the off-diagonal $K^0 - \bar{K}^0$ matrix element (*i.e.*, ϵ), we expect the short-distance contribution to be the dominant one. This is also expected to be the case for the real part of the $B^0 - \bar{B}^0$ matrix element (*i.e.*, ΔM_B), and, consistent with this, explicit consideration of the long-distance contributions shows a strong cancellation.[23] In the Standard Model, the box diagram involving top quarks is almost the whole story, and the experimental measurement of this quantity then provides important constraints on a combination of the top quark mass and associated weak mixing angles.

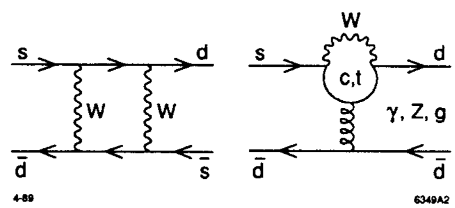

FIGURE 2
One-loop diagrams giving rise to flavor-changing processes.

5.2 $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$

Both of these processes receive short-distance contributions from the "electromagnetic penguin" with a charm quark in the loop. However, there are very large QCD corrections[24] (so big as to change the sign of the amplitude), and the result is very untrustworthy. Not surprisingly, for the real, CP-conserving part of the amplitude which enters both these processes, it is necessary to understand significant long-distance contributions. These may be best calculable in chiral perturbation theory.[25]

The measured branching ratio for $K^+ \to \pi^+e^+e^-$ is[19] $2.7 \pm 0.5 \times 10^{-7}$. We may expect hundreds, if not thousands, of events from ongoing experiments (BNL experiment E777), as well as some events of $K^+ \to \pi^+\mu^+\mu^-$ (BNL experiment E787 now quotes a limit[11] of 2.3×10^{-7}). The predicted branching ratio for $K_S \rightarrow \pi^0 \ell^+ \ell^-$ is in the neighborhood of several times 10^{-9}, and will be of importance both for a check on the chiral perturbation theory calculations[25] and for CP violation in the decay $K_L \rightarrow \pi^0 \ell^+ \ell^-$, to be discussed later.

5.3 $K_L \rightarrow \pi^0 e^+ e^-$

If we define K_1 and K_2 to be the even and odd CP eigenstates, respectively, of the neutral K system, then $K_L \rightarrow \pi^0 e^+ e^-$ has three contributions:

(1) Through a two-photon intermediate state:

$$K_2 \rightarrow \pi^0 \ \gamma\gamma \rightarrow \pi^0 e^+ e^- \quad .$$

This is higher order in α, but is CP conserving. With two real photons, there are two possible Lorentz invariant amplitudes for $K_L \rightarrow \pi^0 \ \gamma\gamma$. One is the coefficient of $F^{(1)}_{\mu\nu} \ F^{(2)}_{\mu\nu}$, which corresponds to the two photons being in a state with total angular momentum zero. Consequently, it picks up a factor of m_e when contracted with the QED amplitude for $\gamma\gamma \rightarrow e^+ e^-$, as the interactions are all chirality conserving, and its contribution to the $K_L \rightarrow \pi^0 e^+ e^-$ decay rate is totally negligible.[26] The other invariant amplitude is the coefficient of a tensor which contains two more powers of momentum, and one might hope for its contribution to be suppressed by angular momentum barrier factors. In chiral perturbation theory, an order-of-magnitude estimate[25] for the resulting branching ratio of $K_2 \rightarrow \pi^0 \ e^+ e^-$ is 10^{-14}. However, a vector dominance, pole model predicts[27] a much bigger result: a branching ratio of order 10^{-11}, roughly at the same level as that arising from the CP-violating amplitudes (see below). The experimental upper limit on the branching ratio for $K_L \rightarrow \pi^0 \gamma\gamma$ has very recently been considerably improved,[28] and now is only a few times larger than some of the predictions.[27,25] In the future, we might have not only a measurement of the branching ratio, but a Dalitz plot distribution which could help distinguish between models. The final answer for this contribution remains to be seen, both theoretically and experimentally.

(2) Through the small (proportional to ϵ) part of the K_L, *i.e.*, K_1, due to CP violation in the mass matrix:

$$K_L \approx K_2 + \epsilon K_1$$

$$K_1 \rightarrow \pi^0 \ \gamma_{virtual} \rightarrow \pi^0 e^+ e^- \quad .$$

We call this "indirect" CP violation and may calculate its contribution to the decay rate once we know the width for the CP-conserving process $K_1 \rightarrow \pi^0 e^+ e^-$. Eventually, there will presumably be an experimental measurement of $\Gamma(K_S \rightarrow \pi^0 e^+ e^-)$, which will take all the present theoretical model dependence away. For now, equating this width to the measured one for $K^+ \rightarrow \pi^+ e^+ e^-$ gives the estimate:

$$B(K_L \rightarrow \pi^0 e^+ e^-)_{\text{indirect}} = 0.58 \times 10^{-11} \quad . \quad (2)$$

(3) Through the large part of the K_L, *i.e.*, K_2, due to CP violation in the decay amplitude:

$$K_2 \rightarrow \pi^0 \ \gamma_{virtual} \rightarrow \pi^0 e^+ e^- \quad .$$

We call this "direct" CP violation, and the amplitude for it arises from the diagrams shown in figure 3. For values of $m_t \ll M_W$, it is the electromagnetic penguin that gives the dominant short-distance contribution to the amplitude, which is summarized in the Wilson coefficient, C_{7V}, of the appropriate operator,

$$Q_{7V} \ = \ \alpha \ \left(\bar{s}\gamma_\mu(1-\gamma_5)d\right)\left(\bar{e}\gamma^\mu e\right) \quad . \qquad (3)$$

Values of $m_t \sim M_W$ allow the "Z penguin" and "W box" contributions to become comparable to that of the electromagnetic penguin, and bring in another operator,

$$Q_{7A} \ = \ \alpha \ \left(\bar{s}\gamma_\mu(1-\gamma_5)d\right)\left(\bar{e}\gamma^\mu\gamma_5 e\right) \quad . \qquad (4)$$

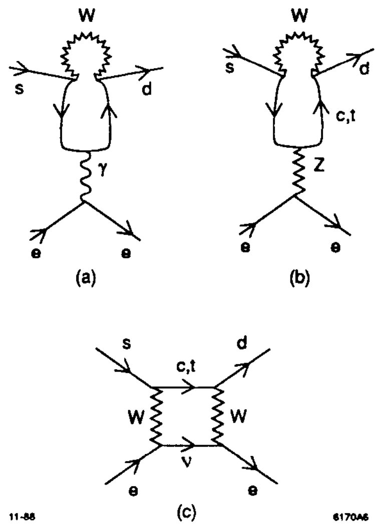

FIGURE 3
Three diagrams giving a short distance contribution to the process $K \to \pi \ell^+ \ell^-$: (a) the electromagnetic penguin; (b) the Z penguin; (c) the W box.

The QCD corrections are substantial for the electromagnetic penguin contribution and have been redone for the case[29,30] when $m_t \sim M_W$. In contrast, the top quark contributions from the Z penguin and W box live up at the weak scale and get only small QCD corrections. Still, the coefficient C_{7V} comes largely from the electromagnetic penguin, even after its reduction from QCD corrections. On the other hand, the electromagnetic penguin cannot contribute to C_{7A}, and here it is the Z penguin which gives the dominant contribution. The overall decay rate due to the direct CP-violating amplitude can be obtained by relating the hadronic matrix elements of the operators Q_{7V} and Q_{7A} to that which occurs in K_{e3} decay. Then, we find that

$$B(K_L \to \pi^0 e^+ e^-)_{direct} \approx 1 \times 10^{-5} \, (s_2 s_3 s_6)^2 \, [|\widetilde{C}_{7V}|^2 + |\widetilde{C}_{7A}|^2] \quad . \tag{5}$$

The last factor, shown in figure 4, ranges[29] between about 0.1 and 1.0. As $s_2 s_3 s_6$ is typically of order 10^{-3}, the corresponding branching ratio induced by this amplitude alone for $K_L \to \pi^0 e^+ e^-$ is around 10^{-11}. Note that when $m_t \gtrsim 150$ GeV, the contribution from C_{7A}

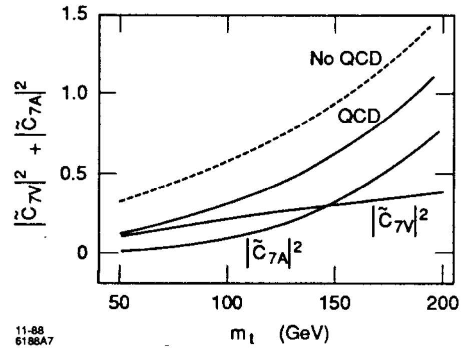

FIGURE 4
The quantities $(\widetilde{C}_{7V})^2$ and $(\widetilde{C}_{7A})^2$ as a function of m_t, and their sum, $(\widetilde{C}_{7V})^2 + (\widetilde{C}_{7A})^2$, with (solid curve, $\Lambda_{QCD} = 150$ MeV) and without (dashed curve) QCD corrections, which enters the branching ratio induced for $K_L \to \pi^0 \ell^+ \ell^-$ by CP violation in the decay amplitude. From Ref. 29.

overtakes that from C_{7V}, and it is the Z penguin and W box, coming from the top quark with small QCD corrections, which dominate the decay rate.

Thus, it appears at this point that the three contributions from (1) CP-conserving, (2) indirect CP-violating, and (3) direct CP-violating amplitudes could all be comparable. The weighting of the different pieces in $K_L \to \pi^0 e^+ e^-$ is entirely different from that in $K \to \pi\pi$. The present experimental upper limit[31,32] is 4×10^{-8}, with prospects of getting to the Standard Model level of around 10^{-11} in the next several years.[33] Hopefully, the CP-conserving and indirect CP-violating amplitudes will be pinned down much better by then, permitting an experimental measurement of this decay to be interpreted in terms of the magnitude of the direct CP-violating amplitude.

5.4 $K^+ \to \pi^+ \nu \bar{\nu}$

Here, the short-distance contribution from charm and, especially, top quarks in Z penguin and W box graphs, provides the dominant contribution to the amplitude: all the estimates of long-distance effects show them to be negligible.[34] The QCD corrections are moderate in magnitude. They particularly need to be applied to the contribution of the charm quark. The original QCD corrections,[35] have been recently updated to the case in which the top mass is comparable to M_W.[36]

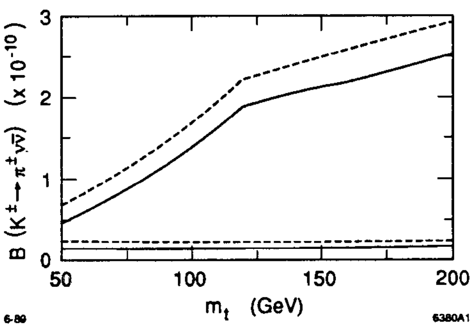

FIGURE 5
The maximum and minimum of the branching ratio (per neutrino flavor) for $K^{\pm} \to \pi^{\pm}\nu\bar{\nu}$ without (dashed curve) and with (solid curve) QCD corrections ($\Lambda_{QCD} = 150$ MeV). From Ref. 36.

The resulting branching ratio for $K^+ \to \pi^+\nu_e\bar{\nu}_e$ is shown in figure 5, with the dashed lines representing upper and lower bounds (given our present freedom in choosing Kobayashi–Maskawa parameters, particularly V_{td}) without QCD corrections and the solid lines giving the corresponding bounds with those corrections.[36] The branching ratio ranges between about 0.2 and 2×10^{-10} per neutrino flavor.

The upper limit on this process has recently been considerably improved (to 3×10^{-8}) by a dedicated Brookhaven experiment.[37] Other byproducts of this search are the upper limits

$$B(K^+ \to \pi^+ a) < 6 \times 10^{-9} \quad ,$$
$$B(\pi^0 \to \nu\bar{\nu}) < 8 \times 10^{-7} \quad ,$$

where a stands for a very low mass axion or axion-like object. There are prospects of getting to the 10^{-9} level for $B(K^+ \to \pi^+\nu\bar{\nu})$ in the next year, and eventually reaching a sensitivity where there should be a few events if the Standard Model gives the correct rate. In the meantime, there is a large window still left open for new physics between where we are now and the Standard Model prediction.

5.5 $K_L \to \pi^0\nu\bar{\nu}$

For the decay $K_L^0 \to \pi^0\nu_\ell\bar{\nu}_\ell$, there is, of course, neither an electromagnetic penguin nor a two-photon, CP-conserving contribution to the amplitude. Furthermore, the indirect CP violation arising from the neutral K mass matrix gives a negligible contribution to the decay rate. That leaves us with just the Z penguin and W box, and the V–A character of the gauge boson couplings to neutrinos allows only the operator:

$$Q_\nu = \frac{e^2}{4\pi}(\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha)(\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell) \quad . \qquad (6)$$

Being CP violating, it is the imaginary part of C_ν that is required:

$$\text{Im } C_\nu = (s_2 s_3 s_\delta)(\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}) \quad , \qquad (7)$$

which is totally dominated by the top quark contribution. The branching ratio (per neutrino flavor) is

$$B(K_L^0 \to \pi^0\nu_\ell\bar{\nu}_\ell) \approx 2 \times 10^{-5}(s_2 s_3 s_\delta)^2 \, |\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2 \quad , \qquad (8)$$

with the latter quantity shown in figure 6. Again, as $s_2 s_3 s_\delta$ is of order 10^{-3}, the branching ratio with three generations of neutrinos is of order 10^{-11}. The QCD corrections to the t quark contribution should be small, making this theoretically an ideal decay in which to study CP violation in the decay amplitude. Experimentally,[38] the problems are perhaps best represented by the statement that nobody has yet shown that a measurement of this decay is absolutely impossible.

We now turn to rare decays involving the b quark, discussing them both at the quark level and at the hadron level for exclusive processes. In general, we will find much bigger branching ratios for processes induced at one-loop than in K decays—a result of the high mass of the top quark and Kobayashi–Maskawa factors. This

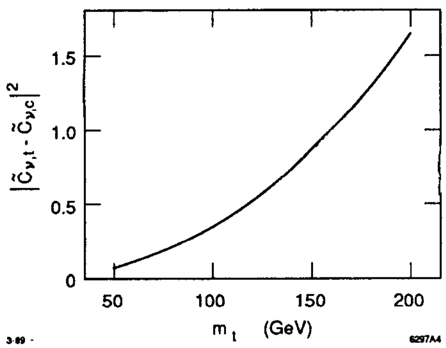

FIGURE 6
The quantity $|\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2$, which enters the branching ratio for the CP-violating decay $K_L \to \pi^0 \nu_\ell \bar{\nu}_\ell$, as a function of m_t. From Ref. 36.

also makes it so that, in the competition between short-distance and long-distance contributions, B decays are an arena in which short-distance contributions are much more likely to be dominant.[39]

5.6 $b \to s\ \ell^+\ \ell^-$

Even without restricting our attention to CP-violating amplitudes, the top quark contributions dominate the one-loop diagrams analogous to figure 3 which lead to $b \to s\ell^+\ell^-$. In the Standard Model, the decay $b \to se^+e^-$ should occur with a branching ratio[40,41] of several times 10^{-6} and is relatively weakly dependent on m_t, including modest QCD corrections. The associated exclusive modes should be roughly an order-of-magnitude smaller.[42] The benchmark process of this type at the hadron level is $B \to K\mu\bar{\mu}$. The presence of a fourth generation[43] could increase the branching ratio by perhaps an order of magnitude. Even so, the measurement of such small branching ratios still seems a way off.

5.7 $b \to s\ \gamma$

A related set of one-loop diagrams can lead to a real photon and result in the decay $b \to s + \gamma$ at the quark level, or $B \to K^* + \gamma$, $B \to K^{**} + \gamma$, etc. at the hadron level. Here, QCD corrections are absolutely critical: They change the GIM suppression in the amplitude from being in the form of a power law, $(m_t^2 - m_c^2)/M_W^2$, to being in the softer form of a logarithm, $\ln(m_t^2/m_c^2)$. This corresponds to an enhancement, depending on m_t, of one order of magnitude or more[44–46] over the rate expected from the simplest one-loop electroweak graph.[47] The inclusive process at the quark level, $b \to s\gamma$, should then occur with a branching ratio of order[44–46] 10^{-4}; exclusive modes like $B \to K^*\gamma$ and $B \to K^{**}\gamma$ are estimated at 5% to 10% of this.[39,42,44] Again, a fourth generation could enhance this rate by an order of magnitude or so.[48] The extension to a supersymmetric world is more interesting. The obvious new diagrams come from putting the supersymmetric partners of the quarks and the W in the loop of the electromagnetic penguin diagram. Much more important,[49] however, is the transition from a "penguin" to a "penguino,"—the penguin diagram involving a gluino and a squark. Because it involves strong interaction couplings rather than weak ones, it competes (and interferes) with the QCD enhanced electromagnetic penguin and produces an inclusive branching ratio that could be of order 10^{-3}.

Experiment is approaching the level of sensitivity needed to test theory for these decays. The ARGUS limit[50] on the branching ratio for $B \to K^*\gamma$ is now 2.4×10^{-4} and the limits on several other exclusive radiative B decay channels are close to this level. One can already say that these processes cannot be enhanced far beyond the standard model predictions.

5.8 $b \to s\ g \to s\ q\bar{q}$

The basic diagram under discussion here is a strong interaction or gluonic penguin, which produces an effective Hamiltonian density

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{3\pi}\ V_{tb}\ V_{ts}^*\ \ln(m_t^2/m_c^2)\ \bar{s}\gamma_\mu(1-\gamma_5)b\ \bar{q}\gamma^\mu q \quad . \tag{9}$$

The combination of the top quark mass and the Kobayashi–Maskawa angles is such that one can contemplate[39] an inclusive branching ratio of order 1%. These processes may not even be really "rare." The corresponding exclusive modes are decays like $B \to K\phi$, $B \to K\pi$, and $B \to K^*\rho$, with predicted branching ratios from a few times 10^{-5} up to roughly 10^{-4}.

Here is one place, precisely because "rare" is not so rare, in which experiment is closing in on theory. The upper limit from CLEO[51] on $B(B^0 \to K^+\pi^-)$ now stands at 9×10^{-5}, and the limits from ARGUS and CLEO on a number of other processes of this type are in the 10^{-4} range. This may well be the first place where a decay process which is unambiguously induced at one-loop by the top quark can be seen experimentally.

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