# Quantum Mechanics of the Googolplexus ${ }^{\star \dagger}$ 

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## ABSTRACT

"The Euclideau formulation of gravity is not a subject with firm foundations and clear rules of procedure; indeed it is more like a trackless swamp. I think I have threaded my way through it safely, but it is always possible that unknown to myself I am up to my neck in quicksand and sinking fast."

Sidney Coleman

## 1. Introduction

According to some very recent speculations, the laws of nature as seen by an ordinary observer inside a macroscopic universe are at least partially determined by a background of "baby universe" or wormhole operators which act on a Fock space of universes [1-5]. Thus far the most widely known formulation of this idea is due to Coleman [4] and is based on a nonexistent euclidean path integral quantization. Coleman's theory seems to lead to the remarkable conclusion that the baby universes will always adjust themselves to exactly cancel the cosmological constant.

Unfortunately, the unboundedness of the action

$$
\begin{equation*}
I_{E}=\int d^{4} x \sqrt{g}\left(\Lambda-\frac{1}{16 \pi G} R\right) \tag{1}
\end{equation*}
$$

makes it difficult to interpret the euclidean path integral of quantum gravity. In particular, Coleman's mechanism for the vanishing of the cosmological constant relies heavily on the apparent instability with respect to nucleating arbitrarily large numbers of the Euclidean four-spheres each contributing $-2 / 3 \lambda$ to the action $\left(\lambda=16 G^{2} \Lambda / 9\right)$. Unfortunately, the same instability seems to lead to a catastrophic number of macroscopically or even cosmically large wormwholes in spacetime [6]. (Resolutions of this problem have been proposed in ref. [7], but it is shown in ref. [8] that these idcas do not work.) On the other hand, there are mathematical prescription which eliminate the instabilities. They involve rotating the contours of path integration for all or part of the conformal degree of freedom of the metric. The simplest and mathematically clearest prescription due to Gibbons, Hawking and Perry [9] rotates the entire conformal factor to an imaginary value, thus eliminating any negative terms in the action (we assume that the cosmological constant is positive). Such a procedure may define a consistent quantum theory of gravity, but it surely eliminates the divergences as $\lambda \rightarrow 0$ which drive Coleman's mechanism.

Other prescriptions rotate the contour about the Euclidean saddle point or approximate saddle points associated with wormhole-connected four-spheres. In this case a careful analysis of the modes of fluctuation [10] reveals a prefactor $i^{D-2}$ in front of the Baum-Coleman-Hawking amplitude $\exp (2 / 3 \lambda)[11,12,4]$ associated with each four-sphere. The result is, once again, not favorable: in 4 dimensions $(D=4)$ the marvelous $\exp (\exp (2 / 3 \lambda))$ becomes a disappointing $\exp (-\exp (2 / 3 \lambda))$. Evidently, the Euclidean path integral is so ill-defined that it can be imaginatively used to prove anything.

In this paper we will formulate the quantum mechanics of gravity with topology change by returning to the high dry ground of Hamiltonian Hilbert space quantum mechanics ultimately to be derived from the Minkowski space path integral. We construct a simplified quantum mechanics of a system -the googolplexus-which is rich enough to describe any number of spatially spherical universes as well as worm holes which mediate transitions between states. The procedure involves a "third quantization" which replaces the Wheeler DeWitt wave function by an operator which acts on a Fock space of universes. Worm holes induce nonlinear couplings and, as in refs. $[4,5]$, make the cosmological constant into a variable with a probability distribution. We find the following results:
(1) The mean number of universes which are created by the dynamics is $\sim$ $\exp (2 / 3 \lambda)$ where $\lambda$ is the cosmological constant. In view of the known bound on $\lambda\left(\lambda \lesssim 10^{-120}\right)$ the mean number of universes is $\gtrsim 10^{10^{120}}$. This is the origin of the name googolplexus. ${ }^{\star}$
(2) The probability for a given $\lambda$ is flat at $\lambda=0$, with no enhancement of either the Baum-Hawking or Coleman type.
(3) The Coleman double exponential does occur but in the form $\exp \left(-e^{2 / 3 \lambda}\right)$. Furthermore, it does not have the interpretation of a probability for $\lambda$ but rather a transition amplitude from an initial state to an out-vacuum, i. e., it

[^1]is the amplitude to create no universes. The $\exp (2 / 3 \lambda)$ universes are rather like the soft photons emitted in electrodynamics and the factor $\exp \left(-e^{2 / 3 \lambda}\right)$ is the analogue of the soft photon factor which suppresses individual exclusive states with a finite number of soft photons.
(4) The $e^{2 / 3 \lambda}$ of emitted "soft" universes arc cold, empty and uninteresting. We do not live in one of them. The probability to find an interesting warm universe is like an inclusive probability which sums over the unobserved soft universes. For these inclusive probabilities no Coleman or Baum-Hawking factor appears. Thus we find no reason to think that wormholes drive $\lambda \rightarrow 0$. In fact with our present understanding it seems that worm holes make $\lambda$ a random variable.

## 2. Second Quantization of Minisuperspace

Consider the minisuperspace model of quantum gravity. This model includes spatially spherical geometries with metric

$$
\begin{equation*}
d s^{2}=\frac{2 G}{3 \pi}\left(-d \tau^{2}+a^{2}(\tau) d \Omega_{3}^{2}\right) \tag{2}
\end{equation*}
$$

where $d \Omega_{3}^{2}$ is the metric of a unit 3 -sphere. We also include a number of spatially constant scalar fields $\phi_{i}(\tau)$. The Einstein-Hilbert action with mattcr coupling becomes

$$
\begin{equation*}
I=\frac{1}{2} \int d \tau\left[-a \dot{a}^{2}+a-a^{3}\left(\lambda+V\left(\phi_{i}\right)\right)+a^{3}\left(\dot{\phi}_{i}\right)^{2}\right] \tag{3}
\end{equation*}
$$

This action defines a hamiltonian

$$
\begin{equation*}
H=\frac{1}{2 a}\left(-\Pi^{2}-a^{2}+\lambda a^{4}+\frac{1}{a^{2}} \Pi_{\phi}^{2}+a^{4} V\left(\phi_{i}\right)\right) \tag{4}
\end{equation*}
$$

which is the generator of translations in the parameter time $\tau$. Since the theory is general coordinate invariant, this generator must annihilate the physical states.

This defines the Wheeler-DeWitt equation which the "wave function of the universe" $\Phi\left(a, \phi_{i}\right)$ must satisfy:

$$
\begin{equation*}
\left[\frac{\partial^{2}}{\partial a^{2}}-\frac{1}{a^{2}} \frac{\partial}{\partial \phi_{i}} \frac{\partial}{\partial \phi_{i}}-a^{2}+a^{4}\left(\lambda+V\left(\phi_{i}\right)\right)\right] \Phi\left(a, \phi_{i}\right)=0 \quad ; \tag{5}
\end{equation*}
$$

There is some ambiguity in eq. (5) due to operator ordering, but the results in Sections II and III are completely independent of assumptions about operator ordering.

Sometimes eq. (5) is regarded as gravity's Schrödinger equation, but it is obviously more like gravity's Klein-Gordon equation. Indeed, many authors, beginning with DeWitt[13] have noted the similarity between the scale factor $a$ and an embedding time. In $1+1$ dimensional gravity this correspondence is precise [14]. As in the case of the Klein-Gordon equation, the lack of a positive definite probability density makes it more appropriate to think of $\Phi$ as a quantum field rather than a state amplitude [5, 15-20]. Of course, there is a second, independent, motivation: in order to have a Hilbert space theory with spaces of arbitrary topology, we need an operator which connects sectors with different numbers of connected components.

We will therefore consider a quantum field theory of the Wheeler-De Witt equation, guided by the analogy between eq. (5) and the Klein-Gordon equation. To emphasize this we rename

$$
\begin{gather*}
a \rightarrow t \\
\phi_{i} \rightarrow x_{i} \tag{6}
\end{gather*}
$$

so that eq. (5) becomes

$$
\begin{equation*}
\left[\frac{\partial^{2}}{\partial t^{2}}-\frac{1}{t^{2}} \frac{\partial}{\partial x_{i}} \frac{\partial}{\partial x_{i}}-t^{2}+t^{4}\left(\lambda+V\left(x_{i}\right)\right)\right] \Phi\left(t, x_{i}\right)=0 \tag{7}
\end{equation*}
$$

In order not to confuse the reader we emphasize that the $x_{i}$ are not ordinary spatial positions and $\Phi\left(t, x_{i}\right)$ is not an ordinary field. $\Phi\left(t, x_{i}\right)$ creates or annihilates
whole universes with scale factor $t$ and field values $x_{i} . t$ and $x_{i}$ are the coordinates in a new space - the googolplexus - where particles are universes.

Eq. (7) is a Klein-Gordon equation with position-dependent mass-squared,

$$
\begin{equation*}
m^{2}(t, \vec{x})=-t^{2}+t^{4} \lambda+t^{4} V(\vec{x}) \tag{8}
\end{equation*}
$$

and time dependent metric $g_{t t}=-1, g_{x x}=t^{2}$. The Hilbert space consists of all functionals of $\Phi$ at a fixed time $t_{0},|\Psi\rangle \sim \Psi\left(\Phi\left(t_{0}, \vec{x}\right)\right)$. The field $\Phi\left(t_{0}, \vec{x}\right)$ is a complete set of commuting operators, the time derivative $\partial_{t} \Phi\left(t_{0}, \vec{x}\right)$ is the conjugate operator $\Pi\left(t_{0}, \vec{x}\right)$, and the Wheeler-DeWitt equation (7), interpreted as a Heisenberg field equation, determines the field at all other times in terms of these.

We now have two issues to consider: what determines $|U\rangle$, the wave function of the googolplexus, and, given $|U\rangle$, what are the observables? A meta-observer, able to couple to the googolplexus via arbitrary sources, could determine a general expectation value $\langle U| \Phi\left(t_{1}, \vec{x}_{1}\right) \cdots \Phi\left(t_{k}, \vec{x}_{k}\right)|U\rangle$. We, on the other hand, are interested in single-universe observables: what is the probability that a universe will have given properties -- for example, that the matter fields and energy density will have given values when the radius of the universe takes some value $a_{0}$. We are using "universe" to mean a single connected spatial component, corresponding to a "particle" in the Klein-Gordon analogy. So, we have the problem of identifying these one-universe observables in the second quantized Hilbert space, analogous to the problem of identifying the position of one particle in quantum field theory. Now there is a subtlety: it is not in general possible to identify particles in a field theory, even in the absence of interactions, if the Hamiltonian is time-dependent. It will be possible only in regions where $m^{2}$ is positive and if the variations of $m^{2}$ in eq. (8) and the metric are adiabatic. In the present case one verifies that this is the case.

$$
\begin{equation*}
m^{2}>0, \quad \frac{\dot{m}}{m^{2}} \ll 1, \frac{\dot{g}_{x x}}{m g_{x x}} \ll 1 \tag{9}
\end{equation*}
$$

at sufficiently large $t$, provided that $\lambda+V(\vec{x})$ is positive. For convenience, we will assume the latter to be true in this paper. Then at large $t$ we can identify
universes, which we will refer to as "out-universes". In particular, the late-time adiabatic vacuum state $\mid$ out $\rangle$ is the state with no universes as $t \rightarrow \infty$,

$$
\begin{equation*}
\langle\Phi, t \mid o u t\rangle \sim \exp \left\{-\frac{1}{2} \int d \vec{x} \Phi(t, \vec{x})\left(m^{2}(t, \vec{x})-t^{-2} \vec{\nabla} \cdot \vec{\nabla}\right)^{1 / 2} \Phi(t, \vec{x})\right\} \tag{10}
\end{equation*}
$$

for $\ell$ large. The field $\Phi(t, \vec{x})$ is expanded in a complctc sct of solutions of the Wheeler-DeWitt equation (6),

$$
\begin{equation*}
\Phi(t, \vec{x})=\sum_{I} A_{I} f_{I}(t, \vec{x})+A_{I}^{\dagger} f_{I}^{*}(t, \vec{x}) \tag{11}
\end{equation*}
$$

The division into raising operators $A_{I}^{\dagger}$ and lowering operators $A_{I}$ is made using the out-vacuum (10),

$$
\begin{equation*}
A_{I}|o u t\rangle=0 \tag{12}
\end{equation*}
$$

The $f_{I}$ are orthonormal,

$$
\begin{align*}
& W\left(f_{I}^{*}, f_{J}\right)=-i \delta_{I J}  \tag{13}\\
& W\left(f_{I}, f_{J}\right)=W\left(f_{I}^{*}, f_{J}^{*}\right)=0
\end{align*}
$$

where the Wronskian

$$
\begin{equation*}
W(f, g)=\int d \vec{x} f(t, \vec{x}) \dot{g}(t, \vec{x})-\dot{f}(t, x) g(t, \vec{x}) \tag{14}
\end{equation*}
$$

is time independent. Then

$$
\begin{equation*}
\left[A_{I}, A_{J}^{\dagger}\right]=\delta_{I J} \tag{15}
\end{equation*}
$$

Let $\mathcal{O}$ be an observable defined in a single universe, and let $\mathcal{O}_{I}$ be its expectation value in the universe described by solution $f_{I}$. Then its expectation value
is

$$
\begin{equation*}
\langle\mathcal{O}\rangle=\frac{\sum_{I} \mathcal{O}_{I} n_{I}}{\sum_{I} n_{I}}, \tag{16}
\end{equation*}
$$

where $n_{I}=\langle U| A_{I}^{\dagger} A_{I}|U\rangle$ is the number of out-universes of type $I$. More generally, if $\mathcal{O}$ is not diagonal in the basis $I$, this would become

$$
\begin{equation*}
\langle\mathcal{O}\rangle=\frac{\operatorname{tr} \mathcal{O} \rho}{\operatorname{tr} \rho} \tag{17}
\end{equation*}
$$

where $\rho_{I J}=\langle U| A_{I}^{\dagger} A_{J}|U\rangle$ is the effective density matrix for single universes. Finally, we will usually be interested in conditional expectations values - conditional, in particular, on the presence of cosmological conditions suitable for our existence. 'I'hus, $\langle\mathcal{O}\rangle \sim \operatorname{tr} \mathcal{O} W \rho / \operatorname{tr} W \rho$, where $W$ is some appropriate weight operator.

Next, what determines $|U\rangle$ ? The Minkowski nature of the googolplexus field equations (7) suggest that $|U\rangle$ is determined at $t=0$. It is possible that $|U\rangle$ is determined at $t=0$ by a smooth match on to short distance physics (this is similar in spirit to the Hartle-Hawking idea [22] extended to the many-universe context). The idea is that the universes at $t=0$ are the "baby universes" whose state determines the local lagrangian of our world. We shall consider a number of hypotheses for $|U\rangle$. However we will see that the main features of the physics we are interested in is largely insensitive to the details of $|U\rangle$ so long as $|U\rangle$ does not have a fine tuned singular dependence on $\lambda$. The interesting features arise from evolution between $t=0$ and large $t$. We will discuss $|U\rangle$ in more detail in Sec. IV.

We now apply these ideas. We specialize to pure gravity (with $\lambda>0$ ) for which second quantized minisuperspace reduces to a harmonic oscillator with $t$ dependent frequency. The striking feature of the dynamics is this: the generic in-state $|U\rangle$, with few or no universes at $t=0$, may contain a large number of universes as $t \rightarrow \infty$. The point is that the frequency-squared (8) is negative at small $t$, changing sign at $t=\lambda^{-1 / 2}$. Before $t=\lambda^{-1 / 2}$, the potential is upside-down and the wave packet spreads. As a result, the wavefunction $\langle\Phi, t \mid U\rangle$ is a highly
excited state of the Hamiltonian in the late time, adiabatic, region. This state contains a large number of quanta, which are the out-universes.

We see this quantatively as follows. With no matter fields, the index $I$ takes only one value; the solution $f(t)$, normalized as in (13), is found in the WKB approximation to be

$$
\begin{equation*}
f(t) \sim\left(4 \lambda t^{4}\right)^{-1 / 4} e^{-i\left(t^{3} \lambda^{1 / 2} / 3+\delta\right)} ; \quad t \gg \lambda^{-1 / 2} \tag{18}
\end{equation*}
$$

There is a single lowering operator,

$$
\begin{equation*}
A=i W\left(f^{*}, \Phi\right) \tag{19}
\end{equation*}
$$

The number of universes in the adiabatic region is

$$
\begin{equation*}
n=\langle U| A^{\dagger} A|U\rangle \tag{20}
\end{equation*}
$$

'This number is readily estimated by using the time-independence of the Wronskian to express $A$ in terms of the $t=0$ Heisenberg field,

$$
\begin{equation*}
A=i f^{*}(0) \Pi(0)-i \dot{f}^{*}(0) \Phi(0) \tag{21}
\end{equation*}
$$

For small $\lambda$, the behavior of $\int(t)$ is determined near $t=0$ from the WKB approximation. For convenience, we choose the phase $\delta$ in eq. (18) so that $\operatorname{Re} \dot{f}(0)=0$. The solution $\operatorname{Re} f(t)$, being fixed by a $t=0$ boundary condition (but normalized at large $t$ by eq. (18)) is $O(1) \cdot \exp \left[\left(1-\lambda t^{2}\right)^{3 / 2} / 3 \lambda\right]$ in the region $1 \lesssim t \lesssim \lambda^{-1 / 2}$, while $\operatorname{Im} f(t)$, being fixed at large $t$ (by the condition that it is $\pi / 2$ out of phase with $\operatorname{Re} f(t))$ is $O(1) \cdot \exp \left[2 / 3 \lambda-\left(1-\lambda t^{2}\right)^{3 / 2} / 3 \lambda\right]$. In all,

$$
\begin{align*}
& f(0)=-c_{2}^{-1} e^{-1 / 3 \lambda}+i c_{1} e^{1 / 3 \lambda} \\
& \dot{f}(0)=+i c_{2} e^{1 / 3 \lambda} \tag{22}
\end{align*}
$$

where $c_{1}$ and $c_{2}$ are of order 1 and we have used the conservation of the Wronskian.

Then from eqs. (20), (21), and (22),

$$
\begin{equation*}
n \simeq e^{2 / 3 \lambda}\langle U|\left\{c_{2} \Phi(0)+c_{1} \Pi(0)\right\}^{2}|U\rangle \sim e^{2 / 3 \lambda} \tag{23}
\end{equation*}
$$

By our assumption about the nature of $|U\rangle$, the matrix element is non-singular in $\lambda$, and so the number of universes diverges as $e^{2 / 3 \lambda}$ for $\lambda \rightarrow 0^{+}$. This is the large exponential factor of Baum [11] and Hawking [12]. While refs. [11] and [12] were in the context of a single universe (single connected spatial component), in our approach this large factor is interpreted as the average number of universes that is, as the average number of connected components of space. In the following sections we will discuss the relevance of this factor to the determination of the cosmological constant.

After obtaining these results, we have learned that many of them have been obtained previously by other authors: Kuchar [15], Rubakov [18], Hosoya and Morikawa [19] and McGuigan [20] have observed the analogy between the WheelerDeWitt equation and the Klein-Gordon equation in a time dependent background, and have noted the resulting distinction between in-universes and out-universes. In particular, Rubakov [18] gave the interpretation of the Baum-Hawking factor as the number of out-universes. There is also some similarity with discussion of DeWitt [13], Vilenkin [23] and Strominger [24], although these were all in a one-universe context.

## 3. Topology Change in Minisuperspace

We now introduce topology change. We will do this in two different ways. The first is to introduce a baby universe field $\alpha$ into the action, following the method of refs. $[2,3,4,17,21,25]$. The free Klein-Gordon action is

$$
\begin{equation*}
S_{\mathrm{free}}=\int_{0}^{\infty} d t\left\{\frac{1}{2} \dot{\Phi}^{2}(t)+\frac{1}{2}\left(t^{2}-\lambda t^{4}\right) \Phi^{2}(t)\right\} \tag{24}
\end{equation*}
$$

Consider an interacting theory

$$
\begin{equation*}
S_{B U}=\frac{1}{2 g^{2}} \alpha^{2}+\int_{0}^{\infty} d t\left\{\frac{1}{2} \dot{\Phi}^{2}(t)+\frac{1}{2}\left[t^{2}-(\lambda+\alpha) t^{4}\right] \Phi^{2}(t)\right\} \tag{25}
\end{equation*}
$$

Here $\alpha$ is a dynamical parameter, independent of $t$. Just as in refs. [2, 3, 4, 17, 21, 25] the Feymman graphs resulting from such an action will correspond to spacetimes in which the spherical spaces described by $\Phi$ are connected by wormholes (the $\alpha$ propagator). In order to keep a Hamiltonian interpretation, we make the action local in time by means of a Lagrange multiplier $\beta(t)$ :

$$
\begin{align*}
S_{B U}^{\prime} & =\frac{1}{2 g^{2}} \alpha^{2}(0) \\
& +\int_{0}^{\infty} d t\left\{\frac{1}{2} \dot{\Phi}^{2}(t)+\frac{1}{2}\left[t^{2}-(\lambda+\alpha(t)) t^{4}\right] \Phi^{2}(t)+\beta(t) \dot{\alpha}(t)\right\} \tag{26}
\end{align*}
$$

The fields $\alpha$ and $\beta$ are a new pair of conjugate variables, so the wavefunctions can be taken to be functions of $\Phi$ and $\alpha$. The argument of $\alpha$ in the first term of eq. (26) is arbitrary, since $\dot{\alpha}=0$; this first term can be absorbed into the wavefunction $|0\rangle$. Again, we assume that the wavefunction $\Psi_{U}(\Phi, \alpha ; t)=\langle\Phi, \alpha ; t \mid U\rangle$ is some smooth function at $t=0$, detcrmincd by coordinate invariance and Planck scale physics.

Since $\dot{\alpha}=0$, the Hilbert space breaks up into sectors labcled by $\alpha$, which is independent of time. Each sector has an effective cosmological constant $\lambda_{\text {eff }}=$ $\lambda+\alpha$. What can we say about the probability distribution for the cosmological constant? As given by the ordinary rules of quantum mechanics, this is

$$
\begin{align*}
P\left(\lambda_{e f f}\right) & =\langle U| \delta\left(\lambda+\alpha-\lambda_{e f f}\right)|U\rangle \\
& =\int d \Phi\left|\Psi_{U}\left(\Phi, \lambda_{e f f}-\lambda ; 0\right)\right|^{2} \tag{27}
\end{align*}
$$

By our assumption about $|U\rangle$, this is a smooth function of $\lambda_{e f f}$ : the probability distribution for the cosmological constant is determined by Planck scale physics
and is not peaked at 0 . This is in clear constrast to the quantity calculated by Coleman[4], which has a double exponential peak

$$
\begin{equation*}
\operatorname{cxp}\left(e^{2 / 3 \lambda_{e f f}}\right) \tag{28}
\end{equation*}
$$

In the sec. 4 we will discuss the Hilbert space interpretation of Coleman's calculation.

Let us next consider the total number of outgoing universes

$$
\begin{align*}
n= & \langle U| A^{\dagger} A|U\rangle \\
= & \int_{-\lambda}^{\infty} d \alpha \int_{-\infty}^{\infty} d \Phi \exp \left[\frac{2}{3(\lambda+\alpha)}\right]  \tag{29}\\
& \times \Psi_{U}^{*}(\Phi, \alpha ; 0)\left(c_{2}(\alpha) \Phi+c_{1}(\alpha) \partial_{\Phi}\right)^{2} \Psi_{U}(\Phi, \alpha ; 0)
\end{align*}
$$

We see that for a smooth matrix element this has a nonintegrable peak at $\lambda+\alpha=$ $0=\lambda_{e f f}$. Thus in a certain sense the overwhelming number of universes has $\lambda_{e f f}=0$. It is tempting to speculate that this in itself solves the cosmological constant problem. We do not think that this is so.

First of all the factor $e^{2 / 3(\lambda+\alpha)}$ in eq. (29) is not the probability for a given value of $\lambda$ in the same sense as Coleman's more singular double exponential. It represents the number of created universes, given that $\lambda_{\text {eff }}$ has a certain value. In Coleman's theory the number of created universes is also $e^{2 / 3(\lambda+\alpha)}$.

However, Coleman has an additional probability function for $\lambda+\alpha$ to have a given value, namely, the double exponential of eq. (28). No such additional $a$ priori probability appears in (29).

Nevertheless, the reader may be tempted to invoke some sort of "weak anthropic" ideas to say that, since so many more universes are created with $\lambda_{e f f}=0$, we are overwhelmingly likely to be in one of these. Unfortunately, when matter
is included one finds that the $e^{2 / 3 \lambda_{\text {eff }}}$ universes are all empty, cold and devoid of matter. A preliminary study of the theory with matter, presented in section 5 , indicates that the number of "habitable" universes is not peaked at $\lambda=0$ if the state of the googolplexus $|U\rangle$ is specified by a generic boundary condition at $t=0$.

Let us now give a somewhat better treatment of wormholes. It is not really satisfactory to introduce an independent baby universe field $\alpha$, since the intermediate states are just small $\Phi$ universes. We therefore try something different, adding a $\bar{\Phi}^{3}$ interaction $[5,17,19]$ :

$$
\begin{align*}
S_{\text {cubic }}= & \frac{1}{2} \int_{0}^{\infty} d t\left\{\dot{\Phi}^{2}(t)+\left(t^{2}-\lambda t^{4}\right) \Phi^{2}(t)\right\}  \tag{30}\\
& +\frac{g}{2} \int_{0}^{\infty} d t d t^{\prime} d t^{\prime \prime} \Phi(t) \Phi\left(t^{\prime}\right) \Phi\left(t^{\prime \prime}\right) \rho\left(t, t^{\prime}, t^{\prime \prime}\right) .
\end{align*}
$$

We have added a term which allows for the possibility that a universe of radius $a$ splits into universes of radii $a^{\prime}$ and $a^{\prime \prime}$. We now make the standard assumption that this is negligible unless one universe is microscopic, and that the amplitude for emission of this small universe is proportional to the local operator of lowest dimension, the cosmological constant:

$$
\begin{align*}
& S_{c u b i c}^{\prime}=\frac{1}{2} \int_{0}^{\infty} d t\left\{\dot{\Phi}^{2}(t)+t^{2} \Phi^{2}(t)\right.  \tag{31}\\
& \left.-t^{4} \Phi^{2}(t)\left(\lambda+g \int_{0}^{\infty} d t^{\prime} \mu\left(t^{\prime}\right) \Phi\left(t^{\prime}\right)\right)\right\} .
\end{align*}
$$

Here $\mu(t)$ is a function which falls rapidly for large $t$, such as $e^{-t^{2}}$. In order to make the theory simpler to analyze, we will make a slight modification: we will require $\mu(t)$ to vanish identically for $t>t_{c}$, where $t_{c}$ is some cutoff radius. Also,
we replace $t^{4}$ in eq. (31) with

$$
\begin{equation*}
\zeta(t)=t^{4} \theta\left(t-t_{c}\right) \tag{32}
\end{equation*}
$$

so that the support of $\mu(t)$ and $\zeta(t)$ does not overlap. The modification (32) should make no difference, since it amounts to neglecting the cosmological constant of very tiny universes. One may imagine $t_{c} \sim(1 \mathrm{ev})^{-1}$, so that wormholes which are large compared to the Planck scale are still included, while the modification (32) only affects universes which are very small in a cosmological sense. With a bit more effort one could analyze the theory without these modifications, and we are confident that the physics would be unchanged.

As before, we introduce an auxiliary field to make the action local in time and allow a Hilbert space interpretation:

$$
\begin{align*}
S_{c u b i c}^{\prime \prime}= & \int_{0}^{\infty} d t \frac{1}{2} \dot{\Phi}^{2}(t)+\frac{1}{2}\left\{t^{2}-\zeta(t)(\lambda+q(t))\right\} \Phi^{2}(t)  \tag{33}\\
& +p(t)(\dot{q}(t)-g \mu(t) \Phi(t))
\end{align*}
$$

In order to recover $S_{\text {cubic }}^{\prime}$ from $S_{\text {cubic }}^{\prime \prime}, q(t)$ must satisfy $q(0)=0$, while $q(\infty)$ is unconstrained. Intcrgrating over $p(t)$ then gives

$$
\begin{align*}
\zeta(t) q(t) & =g \zeta(t) \int_{0}^{t} \mu\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) d t^{\prime} \\
& =g \zeta(t) \int_{0}^{\infty} \mu\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) d t^{\prime} \tag{34}
\end{align*}
$$

the last equality following from the cutoffs on the support of $\zeta$ and $\mu$.

The theory (33) is solvable. The Heisenberg equations of motion are

$$
\begin{align*}
\partial_{t}^{2} \Phi(t)+\left\{t^{2}-(\lambda+q(t)) \zeta(t)\right\} \Phi(t)-g \mu(t) p(t) & =0  \tag{35}\\
\partial_{t} q(t)-g \mu(l) \Phi(t) & =0  \tag{36}\\
\partial_{t} p(t)+\frac{1}{2} \zeta(t) \Phi^{2}(t) & =0 \tag{37}
\end{align*} .
$$

Note that by (36), $q(t)$ is constant for $t>t_{c}$. Thus, in the large $-t$ analysis, $q$ is a constant $q(\infty)$, and the Hilbert space breaks up into $q(\infty)$ - sectors with independent dynamics and with cosmological constant $\lambda_{e f f}=\lambda+q(\alpha)$. Thus $q$ is almost identical to the earlier $\alpha$. We can form the same observables as in the $\alpha$ model: the probability distribution for $\lambda_{e f f}$,

$$
\begin{equation*}
P\left(\lambda_{e f f}\right)=\langle U| \delta\left(\lambda+q(\infty)-\lambda_{e f f}\right)|U\rangle \tag{38}
\end{equation*}
$$

and the total number of universes,

$$
\begin{equation*}
n=\langle U| A^{\dagger} A|U\rangle \tag{39}
\end{equation*}
$$

The state $|U\rangle$ is given by

$$
\begin{equation*}
\langle\Phi, q ; 0 \mid U\rangle=\Psi_{U}(\Phi) \delta(q) \tag{40}
\end{equation*}
$$

the $q$-dependence following from the remark below (33).
From (36) and (37) we have

$$
\begin{array}{lrl}
q(t) & =q(\infty), & t>t_{c}  \tag{41}\\
p(t) & =p(0), & t<t_{c}
\end{array}
$$

and so

$$
\begin{equation*}
-\partial_{t}^{2} \Phi(t)+\left\{t^{2}-(\lambda+q(\infty)) \zeta(t)\right\} \Phi(t)-g \mu(t) p(0)=0 \tag{42}
\end{equation*}
$$

We define a function $g\left(\lambda_{e f f}, t\right)$ which satisfies

$$
\begin{equation*}
-\partial_{t}^{2} g\left(\lambda_{e f f}, t\right)+\left\{t^{2}-\lambda_{e f f} \zeta(t)\right\} g\left(\lambda_{e f f}, t\right)=0 \tag{43}
\end{equation*}
$$

with the same boundary condition as the function $f(t)$ : the large $t$ behavior is
(18), with $\lambda \rightarrow \lambda_{\text {eff }}$ and $\operatorname{Re} \dot{g}\left(\lambda_{e f f}, 0\right)=0$. This is essentially the same as the function $f(t)$, except for the minor modification $t^{4} \rightarrow \zeta(t)$. Thus, at $t=0$,

$$
\begin{align*}
& g\left(\lambda_{e f f}, 0\right)=-c_{2}^{-1}\left(\lambda_{e f f}\right) e^{-1 / 3 \lambda_{e f f}}+i c_{1}\left(\lambda_{e f f}\right) e^{1 / 3 \lambda_{e f f}} \\
& \dot{g}\left(\lambda_{e f f}, 0\right)=i c_{2}\left(\lambda_{e f f}\right) e^{1 / 3 \lambda_{e f f}} \tag{44}
\end{align*}
$$

Also define $g_{1,2}(t)$ satisfying

$$
\begin{align*}
-\partial_{\iota}^{2} g_{1,2}(t)+t^{2} g_{1,2}(t) & =0 \\
g_{1}(0) & =1 \quad \dot{g}_{1}(0)=0  \tag{45}\\
g_{2}(0) & =0 \quad \dot{g}_{2}(0)=1
\end{align*}
$$

Comparing (45) and (43), one sees that

$$
\begin{equation*}
g\left(\lambda_{e f f}, t\right)=g\left(\lambda_{e f f}, 0\right) g_{1}(t)+\dot{g}\left(\lambda_{e f f}, 0\right) g_{2}(t) \text { for } t<t_{c} \tag{46}
\end{equation*}
$$

We can now proceed to solve eqs. $(35,36,37)$. The parts of the solution that we need are: from (42),

$$
\begin{gather*}
\Phi(t)=\Phi(0) g_{1}(t)+\Pi(0) g_{2}(t) \\
-g p(0) \int_{0}^{t} d t^{\prime} \mu\left(t^{\prime}\right)\left\{g_{1}\left(t^{\prime}\right) g_{2}(t)-g_{2}\left(t^{\prime}\right) g_{1}(t)\right\}  \tag{47}\\
\text { for } t<t_{c} .
\end{gather*}
$$

Using this in (36),

$$
q(\infty)=q(0)+\Phi(0) h_{1}+\Pi(0) h_{2}+p(0) h_{3}
$$

where $h_{i}$ are the constants

$$
\begin{align*}
h_{1,2} & =g \int_{0}^{t_{c}} d t^{\prime} \mu\left(t^{\prime}\right) g_{1,2}\left(t^{\prime}\right) \\
h_{3} & =g^{2} \int_{0}^{t_{c}} d t \int_{0}^{t_{c}} d t^{\prime} \mu(t) \mu\left(t^{\prime}\right) g_{1}(t) g_{1}\left(t^{\prime}\right) \operatorname{sign}\left(t-t^{\prime}\right) \tag{48}
\end{align*}
$$

Also,

$$
\begin{equation*}
A=i W\left(g^{*}(\lambda+q(\infty), t), \Phi(t)\right)-i g \int_{t}^{t_{c}} d t^{\prime} \mu\left(t^{\prime}\right) g\left(\lambda+q(\infty), t^{\prime}\right) p(0) \tag{49}
\end{equation*}
$$

where the second term appears because of the source in (42). Evaluating (49) at $t=0$ gives

$$
\begin{align*}
A=i g^{*}(\lambda & +q(\infty), 0)\left\{\Pi(0)-h_{1} p(0)\right\}  \tag{50}\\
& -i \ddot{g}^{*}(\lambda+q(\infty), 0)\left\{\Phi(0)+h_{2} p(0)\right\}
\end{align*}
$$

We now determine the observables (38) and (39). The following identities, obtained using the integral representation of the delta function and the Campbell-Baker-Hausdorf Lemma, are useful:

$$
\begin{align*}
& \langle q=0| \delta(q+x p+y)|q=0\rangle=1 / x \\
& \langle q=0| \delta(q+x p+y) p|q=0\rangle=-y / x^{2}  \tag{51}\\
& \langle q=0| p \delta(q+x p+y) p|q=0\rangle=y^{2} / x^{3}
\end{align*}
$$

Then

$$
\begin{equation*}
P\left(\lambda_{e f f}\right)=\frac{1}{\left|h_{3}\right|} \tag{52}
\end{equation*}
$$

and

$$
\begin{align*}
n= & \int_{-\infty}^{\infty} d \lambda_{e f f}\langle U| \delta\left(\lambda+q(\infty)-\lambda_{e f f}\right) A^{\dagger} A|U\rangle \\
\simeq & \int_{0}^{\infty} d \lambda_{e f f} e^{2 / 3 \lambda_{e f f}} \int_{-\infty}^{\infty} d \Phi  \tag{53}\\
& \frac{1}{\left|h_{3}\right|} \Psi_{U}^{*}(\Phi)\left(c_{2}^{\prime}\left(\lambda_{e f f}\right) \Phi+i c_{1}^{\prime}\left(\lambda_{e f f}\right) \partial_{\Phi}\right)^{2} \Psi_{U}(\Phi) .
\end{align*}
$$

where

$$
\begin{align*}
& c_{1}^{\prime}\left(\lambda_{e f f}\right)=c_{1}\left(\lambda_{e f f}\right)\left(1+\frac{h_{1} h_{2}}{h_{3}}\right)+c_{2}\left(\lambda_{e f f}\right) \frac{h_{2}^{2}}{h_{3}}  \tag{54}\\
& c_{2}^{\prime}\left(\lambda_{e f f}\right)=c_{2}\left(\lambda_{e f f}\right)\left(1-\frac{h_{1} h_{2}}{h_{3}}\right)+c_{2}\left(\lambda_{e f f}\right) \frac{h_{2}^{2}}{h_{3}}
\end{align*}
$$

As in the case of the $\alpha$-model, the probability distribution for $\lambda_{e f f}$, eq. (52), is smooth. There is one pathology here: the integrated probability is not normalized, diverging as $\lambda_{e f f} \rightarrow \pm \infty$. This traces back to the non-normalizable wavefunction for $q$, eq. (40). We assume that this is an artifact of our model, since we do not trust it in any case for $\left|\lambda_{e f f}\right| \gtrsim 1$ and we will proceed on the assumption that the model is reasonable for $\lambda_{e f f} \ll 1$. The number of universes, eq. (53), is almost identical to that found in the $\alpha$-model, and has the same non-integrable singularity at $\lambda_{c f f} \rightarrow 0$. The matrix elements in eqs. (29) and (53) differ slightly because the wormhole physics has gotten mixed up with the small- $t$ evolution of $\Phi$, but the physics is the same. The agreement between the $\Phi^{3}$ model and the $\alpha$-model is the Hilbert space analog of the clean separation between large spheres and small wormholes in the euclidean path integral. Incidentally, the $\Phi^{3}$ model can also be analyzed by shifting $\Phi$ so as to eliminate the linear $\mu p \Phi$ term from $S_{c u b i c}^{\prime \prime}$, leaving a model very similar to the $\alpha$-model.

To conclude this section, we summarize the results: in both models of wormholes, the probability distribution for $\lambda_{\text {eff }}$ is smooth, with no preference for $\lambda_{e f f}=$ 0 . One universe observables, on the other hand, are dominated by $\lambda_{e f f}=0^{+}$, the divergence being the single exponential peak of Baum [11] and Hawking [12]. Again we emphasize that this is not a solution of the cosmological constant problem, for reasons that will be explained in sec. 5 .

## 4. Correspondence with the Path Integral

Coleman [4] found that the Euclidean path integral has a double exponential peak (28) as $\lambda_{e f f} \rightarrow 0$. In this section we discuss the Hilbert space interpretation of Coleman's calculation.

In general, path integrals represent transition amplitudes. In the special case of a time-independent Hamiltonian, the initial and final ground states are the same, and so the ground statc to ground state transition amplitudes are also ground state expectation values. In a time-dependent background, as we have here, there is no correspondence between expectation values and the path integral (unless the latter is generalized in a rather complicated way). Thus, Coleman's expression should represent something of the form $\langle$ out $| \cdots|i n\rangle$. The $\mid$ out $\rangle$ vacuum is given by eq. (10). The $\mid$ in $\rangle$ state depends on the $t=0$ boundary conditions on the path integral; we assume that this is to be identified with $|U\rangle$.

Given a quantum theory in Hilbert space, it is a standard exercise to derive the corresponding path integral representation. In the present case, this is complicated because a contour rotation is necessary, and because this rotation can be done in several ways [21]. We will therefore not discuss the path integral directly, but will calculate $\langle o u t| \cdots|U\rangle$ expectation values and compare with Coleman's result.

We need to discuss $|U\rangle$ somewhat more explicitly now. In the case that the trilinear universe coupling $g$ is small, a connected matrix element $\langle$ out $| \Phi\left(t_{1}\right) \cdots \Phi\left(t_{n}\right)|U\rangle_{c}$ is of order $g^{n-2}$. Neglecting $n \geq 3$, since these go as positive powers of $g$, the wavefunction $|U\rangle$ is therefore a gaussian with width of order 1 , centered on a value of order $g^{-1}$ :

$$
\begin{equation*}
|U\rangle \simeq e^{i\left(J \Phi(0)+J^{\prime} \Pi(0)\right) / g}|\eta\rangle \tag{55}
\end{equation*}
$$

where $J$ and $J^{\prime}$ are $0(1)$ and

$$
\begin{align*}
B_{\eta}|\eta\rangle & =0  \tag{56}\\
B_{\eta} & \equiv \Pi(0)+i \eta \Phi(0) .
\end{align*}
$$

For convenience we ignore wormhole effects and work at fixed $\lambda$; afterwards, one
can add the $\alpha$-field and introduce an arbitrary smooth $\alpha$-dependence into $|U\rangle$. Then (we set $J^{\prime}=0$ for simplicity - the more general case works out the same),

$$
\begin{equation*}
\langle o u t \mid U\rangle=\langle o u t \mid \eta\rangle e^{-J^{2} G(0,0) / 2 g^{2}} \tag{57}
\end{equation*}
$$

where

$$
\begin{equation*}
G\left(t_{1}, t_{2}\right)=\frac{\langle o u t| T \Phi\left(t_{1}\right) \Phi\left(t_{2}\right)|\eta\rangle}{\langle o u t \mid \eta\rangle} \tag{58}
\end{equation*}
$$

From eqs. (11) and (56),

$$
\begin{equation*}
B_{\eta}=\alpha A+\beta A^{\dagger} \tag{59}
\end{equation*}
$$

where

$$
\begin{align*}
\alpha & =\dot{f}(0)+i \eta f(0)  \tag{60}\\
\beta & =\dot{f}^{*}(0)+i \eta f^{*}(0)
\end{align*}
$$

Then

$$
\begin{equation*}
|\eta\rangle=e^{-\beta A^{\dagger} A^{\dagger} / 2 \alpha}|o u t\rangle\left(1-\left|\beta^{2} / \alpha^{2}\right|\right)^{1 / 4} \tag{61}
\end{equation*}
$$

and

$$
\begin{equation*}
G\left(t_{1}, t_{2}\right)=f\left(t_{>}\right)\left\{f^{*}\left(t_{<}\right)-\frac{\beta}{\alpha} f\left(t_{<}\right)\right\} \tag{62}
\end{equation*}
$$

where $t_{>}$is the larger of $t_{1}$ and $t_{2}$ and $t_{<}$is the smaller.
The interesting part of eq. (57) is the exponential factor. This factor is given by the sum of graphs shown in figure 1. This is a sum over disconnected spheres: each line represents a universe which is created with zero radius by the source, propagates for a while, shrinks to zero radius, and is annihilated by the source (figure 1b)). Thus,

$$
\begin{equation*}
\langle o u t \mid U\rangle \sim e^{-J^{2} G(0,0) / 2 g^{2}} \tag{63}
\end{equation*}
$$

represents the same graphs that give rise to the double exponential (28) in the work of Coleman [4]. Naively it appears that we must have $G(0,0) \sim e^{2 / 3 \lambda}$. This
is not generically the case, however. Rather,

$$
\begin{align*}
G(0,0) & =\frac{-i f(0)}{\dot{f}(0)+i \eta f(0)} \\
& =-i \frac{c_{1}+i c_{2}^{-1} e^{-2 / 3 \lambda}}{c_{2}+i \eta c_{1}-\eta c_{2}^{-1} e^{-2 / 3 \lambda}}  \tag{64}\\
& =-\frac{i c_{1}}{c_{2}+i \eta c_{1}}+0\left(e^{-2 / 3 \lambda}\right)
\end{align*}
$$

This result, which at first sight seems to disagree with Coleman, was found previously from an analysis of the minisuperspace path integral in [21]. It can be interpreted as follows. The minisuperspace model, even in the free case (no $\alpha \Phi^{2}$ or $\Phi^{3}$ interaction) already contains a subset of wormhole configurations. These are configurations, in the path integral over $a(\tau)$, in which $a$ vanishes at one or more values of $\tau$ : these are zero-size wormholes attached to the north and south poles of large 4 -surfaces. Thus, $G\left(t_{1}, t_{2}\right)$ gets contributions from all of the paths shown in figure 2. Note that eq. (64) can be formally rewritten

$$
\begin{equation*}
G(0,0)=\left(-\frac{1}{\eta}+i \frac{c_{1} c_{2}}{\eta} e^{2 / 3 \lambda}\right)\left(1+\left[i c_{1} c_{2}+\frac{c_{2}^{2}}{\eta}\right] e^{2 / 3 \lambda}+\cdots\right) \tag{65}
\end{equation*}
$$

the series being geometric. One sees roughly the correspondence between eq. (65) and figure 2 , with each large arc representing a euclidean four-sphere which gives a contribution of order $e^{2 / 3 \lambda}$. The formal sum of these multiple bounces is the $\mathcal{O}(1)$ result of eq. (64)[21].

However, this result is not consistent. It applies neither to the theory without wormholes nor to the theory with all wormholes turned on. Indeed when wormholes are accounted for, let us say by introducing the wormhole field $\alpha$, we will overcount the worm holes connecting north to south poles.

Is there a way to turn off these minisuperspace wormholes? Evidently. Varying $\eta$ in the initial state changes the weighting given to bounces at $a=0$. Setting
$\eta=i c_{2} / c_{1}$, we see from eq. (64) that

$$
\begin{align*}
& G(0,0) \approx c_{1}^{2} e^{2 / 3 \lambda} \\
& \langle\text { out } \mid U\rangle \sim \exp \left(-J^{2} c_{1}^{2} e^{2 / 3 \lambda} / 2 g^{2}\right) . \tag{66}
\end{align*}
$$

This is rather close to the double exponential of rcf. [4]: only the minus sign in the exponent differs. It has already been noted [10] that the plus sign assumed in ref. [4] was not justified, but the result (66) does not agree with ref. [10] either: the phase found for the exponent was $(-i)^{d+2}$, which happens to be -1 in $d=4$, but the minus sign in eq. (66) is $d$-independent. It seems that we are on the right track in identifying the path integral with $\langle o u t| \cdots|U\rangle$ amplitudes, but some details remain to be sorted out.

How are we to interpret the fine-tuning needed to produce the double exponential form (66)? We offer three possibilities:
(i) It is possible that the correct answer for $G(0,0)$ is eq. (64), of order 1 , and that the path integral for euclidean quantum gravity does not pass over the spherical saddle of large negative action. This is plausible: there are contour rotations which make the euclidean action positive [21] and eliminate the large saddle point. In this interpretation the fine-tuning which gives eq. (66) happens to correspond to a contour which does pass over the saddle, but this finely tuned object is uninteresting.
(ii) The condition $\eta=i c_{2} / c_{1}$ may be necessary to eliminate minisuperspace artifacts and give the correct correspondence with the full theory. Thus, this determines in part the state $|U\rangle$. We note that this choice of $\eta$ can be described in another way. It corresponds to the boundary condition $\dot{\Phi}(0)=c_{2} \Phi(0) / c_{1}$ in the second quantized path integral. With this boundary condition, the WheelerDeWitt equation has a normalizable solution at $\lambda=0$.
(iii) One problem with a purely imaginary $\eta$ is that the state $|U\rangle$ is not normal-
izable. This can be cured by adding an exponentially small real part:

$$
\begin{equation*}
\eta=i c_{2} / c_{1}+\mathcal{O}\left(e^{-2 / 3 \lambda}\right) \tag{67}
\end{equation*}
$$

An example of this kind of fine tuning is $\eta=i \dot{f}^{\star}(0) / f^{\star}(0)$ which sets $\beta=0$ in eq. (60). According to eq. (61), this results in $|\eta\rangle=|o u t\rangle$ : as the source $J$ is turned off, the fine-tuned state of the googolplex $|U\rangle$ contains no outgoing universes. In this case a "fine-tuned" boundary condition at $t=0$ is equivalent to a "natural" boundary condition at late times.

Further analysis, probably going beyond minisuperspace, is needed to decide between these alternatives.

## 5. Matter and Heat

Let us return to eq. (7) which describes universes with some number of matter fields and restrict it to the case of one scalar field $x$. For simplicity let us choose $V(x)=0$ in which case we have $x$-translation invariance. As usual, we can Fourier expand $\Phi(x)$ and find that the dynamics of the Fourier modes $\tilde{\Phi}(k)$ decouple. Thus a given universe can be characterized by a "wave number" $k$ and the Wheeler DeWitt equation for such a universe is

$$
\begin{equation*}
\left\{\frac{\partial^{2}}{\partial t^{2}}+\left(\frac{k^{2}}{t^{2}}-t^{2}+t^{4} \lambda\right)\right\} \tilde{\Phi}(t ; k)=0 \tag{68}
\end{equation*}
$$

We see that for $k \neq 0$ there are two oscillatory regions of $t$. In addition to the de Sitter region of large $t \quad(t \gtrsim 1 / \sqrt{\lambda})$, a second classically allowed region appears with $t \lesssim \sqrt{k}$. This is the rcgion of Friedman-Robertson-Walker expanding and recollapsing universes. These universes have enough matter to cause them to recollapse, never appearing as outgoing universes at large $t$. We will assume that these are the "interesting" universes which are similar to our own.

In the FRW region creation and annihilation operators can be introduced [18]. To do this we find two oscillatory solutions of the Wheeler-De Witt equation which behave like

$$
\begin{equation*}
h_{ \pm}(t ; k)=\frac{\sqrt{t}}{\sqrt{k}} \exp ( \pm i k \log t) \tag{69}
\end{equation*}
$$

and expand $\tilde{\Phi}(t ; k)$ as

$$
\begin{equation*}
\tilde{\Phi}(t ; k)=B(k) h_{+}+B^{\dagger}(k) h_{-} \tag{70}
\end{equation*}
$$

$B(k)$ and $B^{\dagger}(k)$ can be regarded as annihilation and creation operators for FRW universes.

Since the FRW universes typically collapse before $\lambda$ becomes important, the evolution of the wave function from $t=0$ to $t \approx \sqrt{k}$ will not significantly depend on $\lambda$ as $\lambda \rightarrow 0$. Therefore, no Baum-Hawking enhancement will occur if the wave function of the googolplexus is specified by a generic boundary condition at $t=0$. We are thus led to the negative conclusion that the probability for an interesting universe is not peaked as $\lambda_{e f f} \rightarrow 0$.

## 6. Conclusion

We have given a Hilbert space analysis of quantum gravity with topology change, using a second quantized minisuperspace model whose Feynman diagrams are wormhole-connected specetimes. This theory has a natural probability interpretation which results in a smooth probability distribution for the cosmological constant.

Path integrals in this theory represents transition amplitudes, not expectation values. In particular, the double exponential of Coleman appears only in amplitudes of the form $\langle o u t| \cdots|U\rangle$. This is an exclusive amplitude: the amplitude for $|U\rangle$ to contain exactly zero, (or, with insertions, some small number) of outuniverses. There is a close analogy with the soft-photon divergence of QED. With
a small photon mass $m$, an external time-dependent current will produce a number $n_{\gamma}$ of soft photons which diverges as $m \rightarrow 0$. Any exclusive amplitude then vanishes as $e^{-n_{\gamma}}$. The large cold universes produced by the mechanism of sec. 2 are much like the soft photons of QED, with $n_{\gamma} \rightarrow e^{2 / 3 \lambda}$.

However, these "soft" universes are as uninteresting as the divergent cloud of soft photons produced by an accelerated charge in QED. When matter is included, say by eq. (7), then warm excited universes will be like hard photons. A meaningful question will go something like this: for each value of $\lambda_{e f f}$, what is the number of universes with a given amount of heat (and other relevant properties) summed over all possible numbers of the unobserved cold empty de Sitter universes. By analogy with QED, the suppression factor (66) will not appear in this expression.

In conclusion, we seem to be left in the following unhappy predicament. Worm holes do influence coupling constants and give rise to a probability distribution as claimed by Coleman. However this probability is not given by either the BaumHawking or Coleman function but is defined by unknown short distance physics which has no reason to prefer $\lambda_{e f f}=0$. Thus, even if $\lambda$ were tuned to zero, worm holes, if they occur, would still make $\lambda_{\text {eff }}$ a probabilistic quantity with no peak at $\lambda_{e f \int}=0$.

Is there an escape? Onc possibility is that worm holes do not exist at all and $\lambda=0$ for other reasons. We have nothing new to say about this option. A second possibility is that we have the boundary conditions on the wave function of the googolplexus all wrong. To see how this might affect the conclusion, let us suppose that the boundary conditions on $|U\rangle$ are given at $t \rightarrow \infty$ instead of $t \rightarrow 0$. Specifically we assume that, for each $k$, the wave function at late times is the out vacuum $\mid$ out $\rangle$ or any other wave packet whose width has no sharp dependence on $\lambda_{e f f}$. Reversing the argument in section 2 we then find that the Fock space of FRW universes must be highly excited. Indeed, for each $k$ the average number of FRW universes is $\sim \exp \left(2 / 3 \lambda_{\text {eff }}\right)$.

Such a speculation represents a radical departure from the usual thinking about
naturalness. We usually assume that the laws of nature are specified at small distances (early times). Here we have done the reverse. At the moment we see no way to decide if this is reasonable.

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## FIGURE CAPTIONS

1) a) The sum of Feynman diagrams representing the exponential of eq. (63). The vertical axis is the scale factor $a$, the horizontal is parameter time $\tau$. Each line represents $G(0,0)$, the sum over all paths from $a=0$ back to $a=0$. b) $\mathrm{G}(0,0)$ is a sum over minisuperspace geometries with the topology of a sphere.
2) The euclidean trajectories of the form $a(\tau)=|\sin (\sqrt{\lambda} \tau)| / \sqrt{\lambda}$, with $0<$ $\tau<n \pi / \sqrt{\lambda}$, that need to be included in the semiclassical approximation for $G(0,0)$. The reflections off the barrier at $a=0$ are the minisuperspace wormholes that attach to the north and south poles of the large four-spheres.


Fig. 1


Fig. 2


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[^1]:    * The googolplex is the largest finite integer with a special name. It is equal to $10^{10^{100}}$. (Websters New Collegiate Dictionary.)

