SLAC–PUB–4854
February, 1989
(T/E)

Linear Plus Coulomb Potential or $\Upsilon(6S)$?*

JONG BUM CHOI†

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

The masses of the b-quarkonium system are analyzed relativistically in a linear plus Coulomb potential model in order to determine whether the observed $\Upsilon(11020)$ is the 6S $(b\bar{b})$ state. The quark mass and the potential parameters are determined by the least squares method, comparing the calculated and the observed masses of the system. It has been found that the $\Upsilon(11020)$ state must be assigned to a 4D $(b\bar{b})$ state in the relativistic linear plus Coulomb potential model.

Submitted to *Physical Review D.*

---

* Work supported in part by the Department of Energy, contract DE–AC03–76SF00515, and by the Korea Science and Engineering Foundation and the Ministry of Education of Korea.

† On leave from the Department of Physics Education, Chonbuk National University, Chonju, Seoul 560–756, KOREA.

I. INTRODUCTION

There now exist so many potential models to account for the observed heavy quarkonium systems that we cannot easily determine the form of confining potential.[1] The difficulty results from the fact that the number of parameters to be fixed by comparison with experimental data is not much smaller than the number of experimental data itself. Only qualitative arguments can be made to introduce a new potential form, and therefore it is better to consider simpler potentials than to explain experimental data and to find out the limits of the models. In this respect, the linear plus Coulomb potential model[2] is a suitable one to start with.

The form of the linear plus Coulomb potential is motivated from QCD asymptotic behaviors. For short distances, the running coupling constant becomes so small that one gluon exchange is expected to be a good approximation, and the Coulomb potential is given by this approximation. For large separation of a quark and an antiquark, the intermediate gluon fields are thought to form a linear tube so that the potential increases linearly with distance. It is well known that various phenomenological potential models accomodate these two features appropriately. However, since the experimental data of heavy quarkonium systems are mostly in the region from short distances to the intermediate distances,[3] it is uncertain whether the confining potential has the linear form. Moreover, the original potential forms must be modified if we consider relativistic corrections,[4] which are essential for the explanation of the experimental spectra. For low-lying quarkonium states, the spin-dependences cannot be neglected compared with the energy level differences between radially excited energy states. These spin-dependences are typical relativistic contributions to quarkonium spectra, and typically have been estimated with respect to the nonrelativistic wave functions obtained from assumed potentials. In this procedure, it is not clear whether the potential form alone plays the essential role to fix the energy levels or the relativistic spin-dependences have equally important roles. In order to figure out the right form of potential, we need to analyze the higher excited states with systematic treatment of relativistic contributions.

In this paper, we will calculate the spectra of the b-quarkonium system by using relativistic Hamiltonian, and determine the parameters by the least squares method without any averaging process. The quark mass and the potential parameters can be determined explicitly in this procedure, and it is possible to compare directly the results between different assignments of radially and orbitally excited states with the same quantum numbers to the observed states. Although the low-lying states can be assigned uniquely to the observed states, the higher states $\Upsilon(10860)$ and $\Upsilon(11020)$ can be assigned to either S-wave states or D-wave states. We will concentrate on the state $\Upsilon(11020)$ and compare the two assignments.

In Sec. II, calculational methods are presented. Calculated results are given in Sec. III. The final section is devoted to some discussions.

II. CALCULATIONAL METHODS

The relativistic free Hamiltonian of a system with masses m_1 and m_2

$$H_0 \;=\; \sqrt{\vec{p_1}^{\,2} + m_1^2} \;+\; \sqrt{\vec{p_2}^{\,2} + m_2^2} \tag{1}$$

can be approximated by

$$H_0 \;\cong\; m_1 + m_2 + \frac{\vec{p_1}^{\,2}}{2m_1} + \frac{\vec{p_2}^{\,2}}{2m_2}. \tag{2}$$

In a center-of-mass system, we have $\vec{p_1} = -\vec{p_2} \equiv \vec{p}$ and the first order spin-dependent potential is given by[4]

$$
\begin{aligned}
V(r) &= \epsilon(r) + \frac{1}{2}\left(\frac{\vec{s_1}}{m_1^2} + \frac{\vec{s_2}}{m_2^2}\right)\cdot\vec{L}\ \left(-\frac{1}{r}\frac{d\epsilon(r)}{dr} + \frac{8}{3}\,\alpha_s\,\frac{1}{r^3}\right) \\
&+ \frac{1}{m_1 m_2}\,(\vec{s_1} + \vec{s_2})\cdot\vec{L}\ \frac{4}{3}\,\alpha_s\,\frac{1}{r^3} \\
&+ \frac{1}{3m_1 m_2}\,(3\vec{s_1}\cdot\hat{r}\vec{s_2}\cdot\hat{r} - \vec{s_1}\cdot\vec{s_2})\,\frac{4}{3}\,\alpha_s\,\frac{3}{r^3} \\
&+ \frac{2}{3m_1 m_2}\,\vec{s_1}\cdot\vec{s_2}\ \frac{4}{3}\,\alpha_s\,4\pi\delta(\vec{r}) \quad ,
\end{aligned}
\tag{3}
$$

where $\epsilon(r)$ is the static potential, $\vec{s_1}$ and $\vec{s_2}$ are the spin angular momenta, and the orbital angular momentum is defined by $\vec{L} \equiv \vec{L_1} = -\vec{L_2}$. For the b-quarkonium system, the running coupling constant α_s is assumed to be small, so that the first order calculations give us a sufficient picture of the whole structure. In this paper, we assume that

$$
\epsilon(r) = \frac{r}{a^2} - \frac{4}{3}\,\alpha_s\,\frac{1}{r} + b \quad ,
\tag{4}
$$

with free parameters a and b. We want to solve the equation

$$
\Big[H_0 + V(r)\Big]\Psi = E\Psi
\tag{5}
$$

by numerical method. There is no problem for nonzero orbital angular momentum states; however, for S-wave states the equation becomes

$$
\left[2m + \frac{\vec{p}^{\,2}}{m} + \frac{8\alpha_s}{9m_1 m_2}\vec{s_1}\cdot\vec{s_2}\ 4\pi\delta(\vec{r})\right]\Psi = E\Psi \quad .
\tag{6}
$$

It is impossible to solve the above equation because of the δ-function term. In order to resolve this problem, we introduce the smeared δ-function[5]

$$f(r) = \frac{1}{r_0^3} \exp\left(- \frac{\pi r^2}{r_0^2} \right) \tag{7}$$

with r_0 another parameter to be determined from experimental data. In fact, the calculated results turn out to be nearly independent of r_0 for a wide range of values. This is due to the fact that the spin splittings of the b-quarkonium system are not comparable to the splittings between radially excited states.

Now there are all five parameters m, a, b, α_s, and r_0. These parameters can be fixed by minimizing the root mean square value

$$\Delta m \equiv \left[\frac{1}{N} \sum_i \left(E_i^{cal} - E_i^{obs} \right)^2 \right]^{\frac{1}{2}} \quad , \tag{8}$$

where N is the total number of observed $(b\bar{b})$ states and E_i^{cal} and E_i^{obs} are the masses of calculated and observed states.

III. CALCULATED RESULTS

There are 12 observed $(b\bar{b})$ states in the table of the Particle Data Group.[6] Of the 12 states, 10 states have been assigned to S- and P-waves and the other 2 states, $\Upsilon(10860)$ and $\Upsilon(11020)$, have not been assigned to any particular wave states. These two states can be assigned to either S- or D-waves. However, the leptonic decay width Γ_{ee} of $\Upsilon(10860)$ is 0.31 KeV — much larger than 0.24 KeV of $\Upsilon(4S)$ — so that it is reasonable to assume that the wave function at the origin of $\Upsilon(10860)$ state is larger than that of $\Upsilon(4S)$, implying that $\Upsilon(10860)$ is an S-wave state. Thus, we can safely assign $\Upsilon(10860)$ to a 5S triplet state. Then there remain two possibilities: one with $\Upsilon(11020)$ assigned to a 6S triplet state, and the other with $\Upsilon(11020)$ assigned to a 4D triplet state.

For the conventional assignment of $\Upsilon(11020)$ to a 6S triplet state, the variations of Δm as functions of m are shown in Fig. 1, where three cases with a = 2.0, 2.5, and 3.0 GeV^{-1} have been presented. In each case, the running coupling constant α_s has been set to three values — 0.2, 0.3, and 0.4 — which are in a reasonable range of variation for b-quarkoniums. The value of r_0 is taken to be 1.0 GeV^{-1} and it has been found that the final result is not very dependent on the value of r_0. Finally, in Fig. 1 the value of b is taken in such a way that the calculated mass of the 1S triplet state coincides with that of $\Upsilon(9460)$. For a = 2.0 GeV^{-1}, the fits become better as the value of α_s decreases and the b-quark mass m increases. However, in Fig. 1(b) we find that the best fit comes with $\alpha_s = 0.3$ for the case of the a = 2.5 GeV^{-1}, and when a = 3.0 GeV^{-1}, the situation changes between the $\alpha_s = 0.2$ and $\alpha_s = 0.4$ cases. The minimum Δm occurs near m = 12 GeV, with $\alpha_s = 0.3$ and $b = -14.352$ GeV. In order to find out precise values of m and a, we use these values of α_s and b. The determined values of m and a are

$$m = 12.061 \ \mathrm{GeV} \quad , \qquad a = 2.26 \ \mathrm{GeV}^{-1} \quad . \tag{9}$$

For these values, the dependences on α_s and b are shown in the upper portions of Figs. 2(a) and (b). The final results are

$$\alpha_s = 0.297 \quad , \qquad b = -14.356 \ \mathrm{GeV} \quad , \qquad \text{and} \quad \Delta m = 50.6 \ \mathrm{MeV} \quad . \tag{10}$$

The predicted and the observed masses are shown in Table 1. It can be seen that the difference between the calculated and the observed values of $\Upsilon(11020)$ is more than 100 MeV, and $\Upsilon(10860)$ is closer to a 4D triplet state than to a 5S state. One unconventional point is that the determined mass m and displacement b have very large values compared with the values determined mainly by the lowest lying states. The value of the running coupling constant α_s agrees well with other calculations.

The other choice with $\Upsilon(11020)$ assigned to a 4D triplet state can be calculated in the same way as for the above 6S case. The calculated results are shown in Fig. 3 with a = 2.0, 2.5, and 3.0 GeV^{-1} and with three α_s values. Since the minimum occurrs with $\alpha_s = 0.3$, $r_0 = 1.0$ GeV^{-1}, and $b = -9.864$ GeV in Fig. 3(b), we fix these parameters and vary m and a to find out the minimum value of Δm. The determined values are

$$m = 9.772 \text{ GeV} \quad , \qquad a = 2.22 \text{ GeV}^{-1} \quad , \tag{11}$$

and for these parameters the dependences on α_s and b are shown in the lower part of Figs. 2(a) and (b). The final minimum is given by

$$\alpha_s = 0.299 \quad , \qquad b = -9.865 \text{ GeV} \quad , \qquad \text{and} \quad \Delta m = 27.8 \text{ MeV} \quad . \tag{12}$$

The calculated masses are shown in Table 2.

In both cases, the determined b-quark mass m is very large and this is related to the large value of the parameter b. When the quark mass becomes larger, the first order spin-dependent potential gives a better approximation since the spin-dependent terms have been obtained by expansion in inverse powers of m. To compensate for this large value of m, we need large negative displacement in the potential. At first it seems arbitrary to vary quark mass m and the potential parameter b; however, we can determine the values explicitly by requiring the best fit to the observed spectra. Moreover, we can directly compare different assignments of observed states to various radially and orbitally excited states of quarkoniums. The value of $\Delta m = 27.8$ MeV is much better than $\Delta m = 50.6$ MeV for the b-quarkonium system.

IV. DISCUSSIONS

In this paper, we have calculated the mass spectra of b-quarkonium system in a relativistic linear plus Coulomb potential model with the $\Upsilon(11020)$ assigned to 6S and 4D triplet states, respectively. The quark mass and the potential parameters have been determined by the least squares method, comparing the calculated and the observed masses. The root mean square value of the differences between calculated and observed spectra becomes much smaller when $\Upsilon(11020)$ is taken to be a 4D triplet state than when taken to be a 6S state. In this method, the quark mass has been determined to be very large compared with the conventional values obtained by fitting to lowest-lying states. The large quark mass gives a better approximation of spin-dependent potentials and is related to the large displacement parameter in static potential. If we accept the determined parameters and use the linear plus Coulomb potential, the conventional assignment to the 6S state must be changed to a 4D state.

The new assignment is also inferred from the leptonic decay width Γ_{ee} of $\Upsilon(11020)$. For highly excited states, the linear potential becomes more important and the wave function at origin is not reduced for higher S-waves. This can be seen for $\Upsilon(4S)$ and 5S $\Upsilon(10860)$ because Γ_{ee} is even larger for $\Upsilon(10860)$ than for $\Upsilon(4S)$. This occurs because the Coulomb part changes into a linear part in this region. However, Γ_{ee} for $\Upsilon(11020)$ is much smaller than that of $\Upsilon(10860)$, implying that $\Upsilon(11020)$ is not a higher S-wave state.

We need more study on the impact of the large quark mass and the implications of the large negative value of b. It may be related to more complicated spin-independent potentials,[7] or the least squares method is not adequate.

REFERENCES

1. For example, see D. B. Lichtenberg *et al.,* DFTT 20/88.
2. E. Eichten *et al., Phys. Rev. Lett.* **34**, 369 (1975).
3. For example, see X. Song and H. Lin, *Z. Phys. C* **34**, 223 (1987).
4. E. Eichten and F. Feinberg, *Phys. Rev. D* **23**, 2724 (1981).
5. S. Ono and F. Schöberl, *Phys. Lett.* **118B**, 419 (1982).
6. Particle Data Group, *Phys. Lett.* **204B**, 1 (1988).
7. M. G. Olsson and C. J. Suchyta III, *Phys. Rev. D* **36**, 1459 (1987).

FIGURE CAPTIONS

Fig. 1(a) : Root mean square values Δm as functions of the quark mass for $a = 2.0$ GeV^{-1} and $\Upsilon(11020)$ assigned to a 6S triplet state. The unit of Δm is MeV, and that of m is GeV. Three cases are shown with $\alpha_s = 0.2$, 0.3, and 0.4 and the value of r_0 is fixed to be 1.0 GeV^{-1} and b chosen to fit the $\Upsilon(9460)$.

Fig. 1(b) : Three 6S cases for $a = 2.5$ GeV^{-1}.

Fig. 1(c) : Curves for $a = 3.0$ GeV^{-1}.

Fig. 2(a) : Δm as functions of the running coupling constant α_s. The upper graph represents the case of $\Upsilon(11020)$ assigned to a 6S triplet state and the lower graph corresponds to a 4D triplet state. The unit of Δm is MeV.

Fig. 2(b) : Root mean square values in MeV as functions of the displacement parameter b. The parameter b is varied between $-14.40 \le b \le -14.30$ (GeV) for a 6S case and $-9.90 \le b \le -9.80$ (GeV) for a 4D case.

Fig. 3(a) : Δm in MeV as functions of m for $a = 2.0$ GeV^{-1} with $\Upsilon(11020)$ assigned to a 4D triplet state. The unit of m is GeV.

Fig. 3(b) : Curves for 2.5 GeV^{-1} corresponding to 4D assignment.

Fig. 3(c) : 4D cases for 3.0 GeV^{-1}.

TABLE CAPTIONS

Table 1. The calculated masses with $\Upsilon(11020)$ assigned to a 6S triplet state. The parameters are $m = 12.061$ GeV, $a = 2.26$ GeV^{-1}, $\alpha_s = 0.297$, $r_0 = 1.00$ GeV^{-1}, $b = -14.356$ GeV, and $\Delta m = 50.6$ MeV. Experimental values are shown in the right columns.

Table 2. The calculated and observed masses with $\Upsilon(11020)$ assigned to a 4D triplet state. The parameters are $m = 9.772$ GeV, $a = 2.22$ GeV^{-1}, $\alpha_s = 0.297$, $r_0 = 1.00$ GeV^{-1}, $b = -9.865$ GeV, and $\Delta m = 27.8$ MeV.

Table 1.

	Cal.	Exp.		Cal.	Exp.
$1\,^3S_1$	9.411	9.460	$1\,^3P_0$	9.914	9.860
$2\,^3S_1$	10.005	10.023	$1\,^3P_1$	9.934	9.892
$3\,^3S_1$	10.304	10.355	$1\,^3P_2$	9.946	9.913
$4\,^3S_1$	10.547	10.580	$2\,^3P_0$	10.225	10.235
$5\,^3S_1$	10.810	10.865	$2\,^3P_1$	10.240	10.255
$6\,^3S_1$	11.141	11.019	$2\,^3P_2$	10.249	10.269
$7\,^3S_1$	11.544		$3\,^3P_0$	10.466	
$1\,^3D_1$	10.162		$3\,^3P_1$	10.480	
$2\,^3D_1$	10.405		$3\,^3P_2$	10.488	
$3\,^3D_1$	10.629		$4\,^3P_0$	10.704	
$4\,^3D_1$	10.892		$4\,^3P_1$	10.721	
$5\,^3D_1$	11.223		$4\,^3P_2$	10.732	

Table 2.

	Cal.	Exp.		Cal.	Exp.
$1\,^3S_1$	9.436	9.460	$1\,^3P_0$	9.896	9.860
$2\,^3S_1$	10.002	10.023	$1\,^3P_1$	9.919	9.892
$3\,^3S_1$	10.315	10.355	$1\,^3P_2$	9.932	9.913
$4\,^3S_1$	10.591	10.580	$2\,^3P_0$	10.220	10.235
$5\,^3S_1$	10.921	10.865	$2\,^3P_1$	10.239	10.255
$6\,^3S_1$	11.338		$2\,^3P_2$	10.249	10.269
$7\,^3S_1$	11.843		$3\,^3P_0$	10.484	
$1\,^3D_1$	10.150		$3\,^3P_1$	10.504	
$2\,^3D_1$	10.413		$3\,^3P_2$	10.514	
$3\,^3D_1$	10.675		$4\,^3P_0$	10.771	
$4\,^3D_1$	11.005	11.019	$4\,^3P_1$	10.798	
$5\,^3D_1$	11.420		$4\,^3P_2$	10.812	

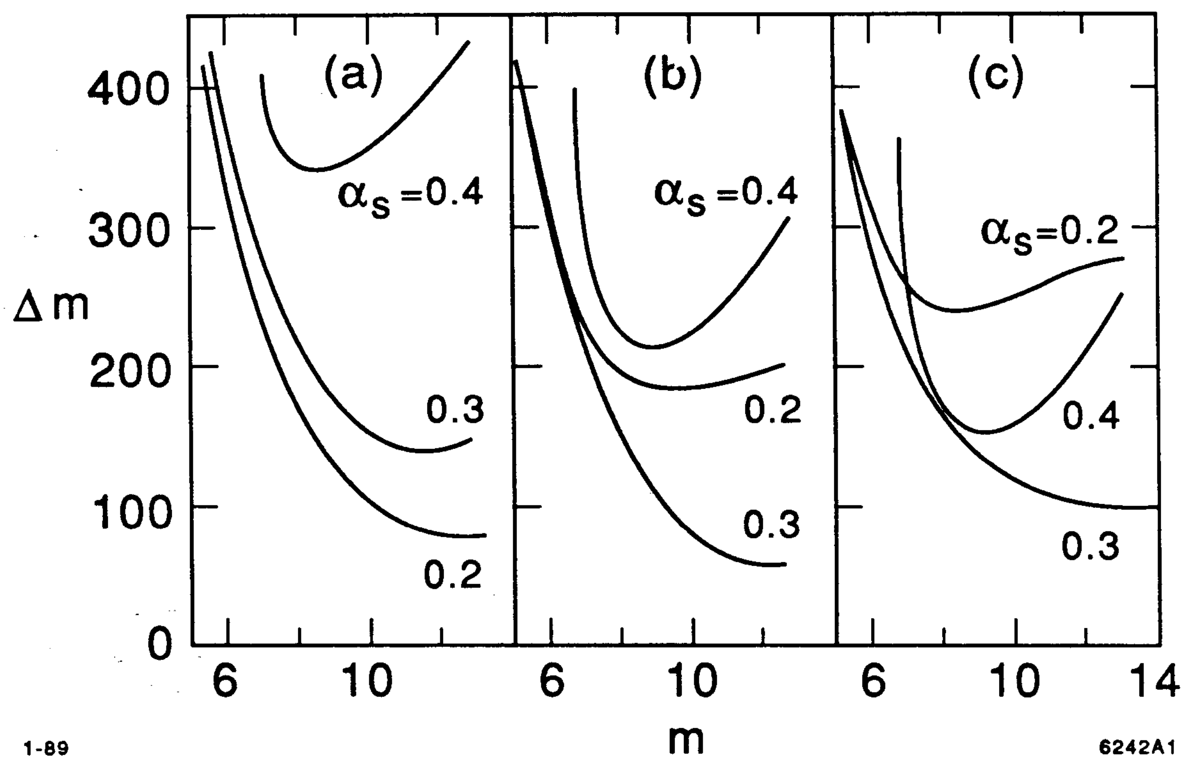

(a)
(b)
(c)
400
300
200
100
0
Δm
α_s =0.4
0.3
0.2
α_s =0.4
0.2
0.3
α_s=0.2
0.4
0.3
6
10
6
10
6
10
14
m
1-89
6242A1

Fig. 1

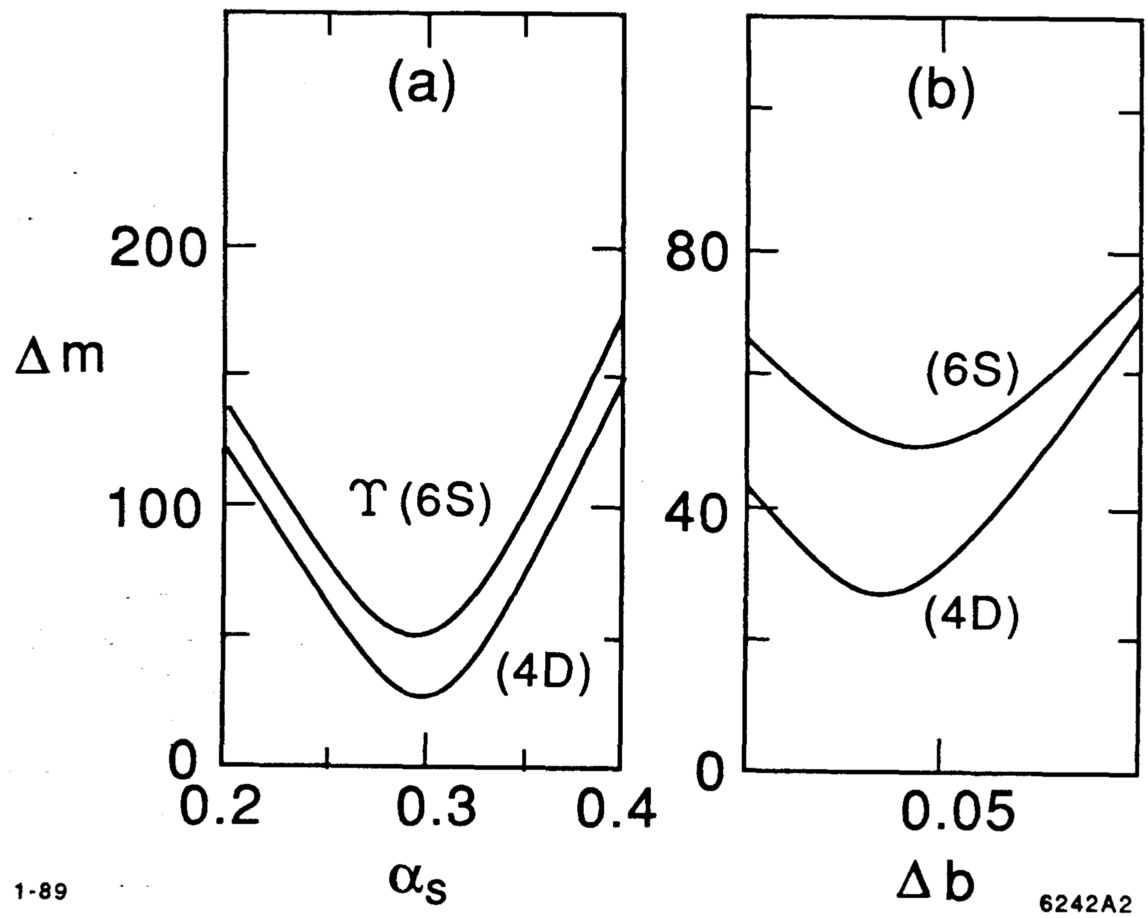
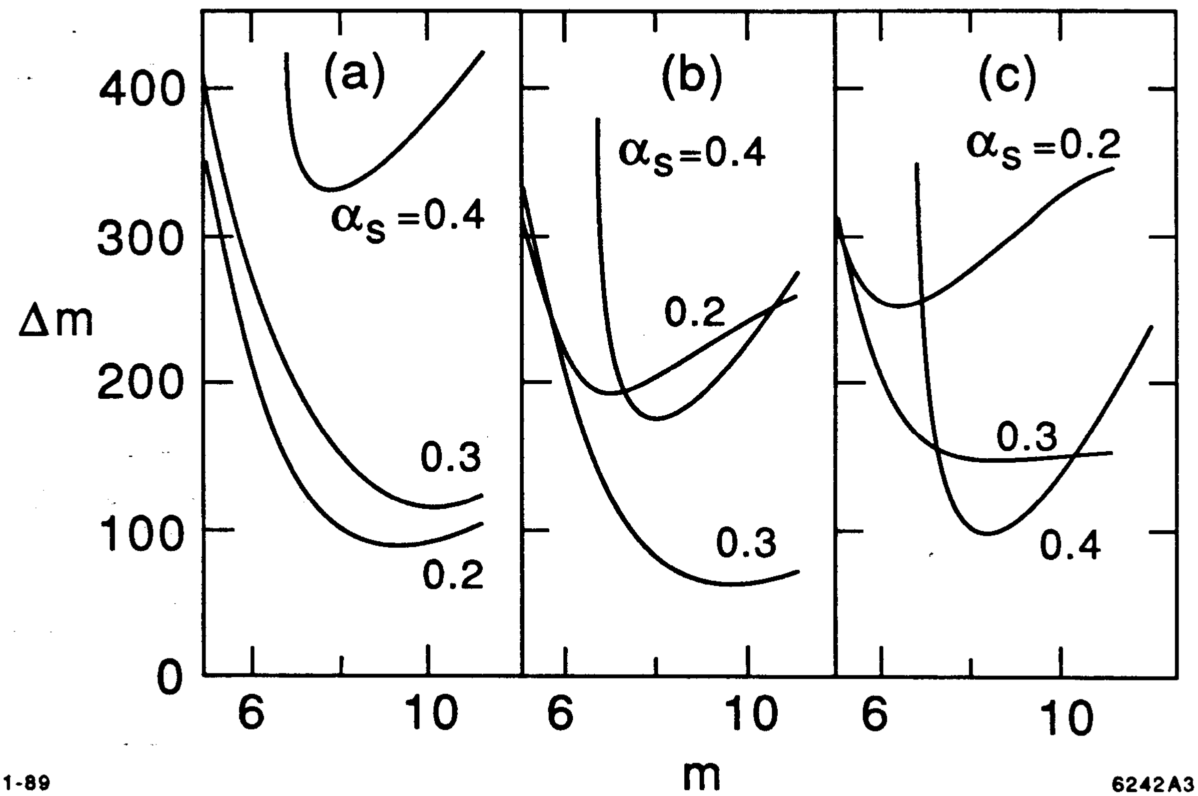

(a)
(b)
200
100
0
80
40
0
Δm
ϒ (6S)
(4D)
(6S)
(4D)
0.2
0.3
0.4
0.05
α_s
Δb
1-89
6242A2

Fig. 2



(a)
(b)
(c)
400
300
200
100
0
Δm
$\alpha_s = 0.4$
0.3
0.2
$\alpha_s = 0.4$
0.2
0.3
$\alpha_s = 0.2$
0.3
0.4
6
10
6
10
6
10
m
1-89
6242A3

Fig. 3