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$K_L \to \pi^0 \ell^+ \ell^-$ Decays for Large m_t*

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ABSTRACT

We discuss the decays $K_L \to \pi^0 \ell^+ \ell^-$ when m_t is large. Unlike the case of $K_L \to \pi\pi$, CP violation in the decay amplitude itself is comparable to that which comes from the mass matrix. We study the CP violating effects, including strong interaction (QCD) corrections to the amplitudes which arise from one-loop diagrams. Short-distance contributions from diagrams that involve a W and a Z or two W's as well as from those with a photon and a W are important when $m_t \gtrsim M_W$.

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I. INTRODUCTION

It is almost 25 years since the original observation of CP violation in long-lived neutral K decays.[1] Until very recently, all experiments were consistent with this phenomenon originating in a "superweak" interaction,[2] whose one measurable manifestation was in the mass matrix of the neutral K system. As a result, the long-lived neutral K meson, $K_L \approx K_2 + \epsilon K_1$, is dominantly the CP-odd state K_2, but contains a small admixture ($\propto \epsilon$) of the CP-even state K_1.

A different, more definite origin of CP violation occurs in the three generation standard model where CP violating effects arise through the presence of a single, nontrivial phase in the matrix which expresses the mixing of quark flavors under the weak interactions.[3] For the K^0 mass matrix, the CP violating phase enters through "box" diagrams that involve heavy quarks and can connect the quarks in a K^0 ($d\bar{s}$) to those in a $\bar{K}^0$ ($s\bar{d}$), mimicking in this regard a "superweak" theory.

In the past year the NA31 collaboration has presented statistically significant evidence[4] for a nonzero value of the parameter ϵ', which is a measure of CP violation in the $K \to \pi\pi$ decay amplitude itself. Experiments at Fermilab[5] and at CERN[4] are continuing with the aim of reducing the statistical and systematic errors to a level where, if the central value of the CERN experiment holds, a nonzero value of ϵ' will be firmly established and a "superweak" explanation made untenable.

Such a value of ϵ' is consistent,[6–8] within rather large uncertainties of the relevant hadronic matrix element, with the three generation standard model. Indeed, it was suggested[9] 10 years ago that if CP violation originated in a phase of the three generation quark mixing matrix and if one-loop "penguin" diagrams give an important part of the $K \to \pi\pi$ decay amplitude, then a nonzero and measurable ϵ' would result.

While the three generation standard model plausibly explains CP violation as it is observed up to now in Nature, we would like to obtain additional evidence that points in this direction. If we could find several experimental processes which exhibit measurable CP violating effects and all could be fit by a single value of the *ab initio* free phase in the mixing matrix, then we will have gone a long way toward establishing this as the correct explanation. If along the way the standard model cannot account for the results of these experiments, so much the better – we'd have evidence for physics beyond the standard model.

There are several avenues toward accomplishing this; none of them is easy. One is to look for CP violating effects in the B meson system. Here the CP violating asymmetries potentially can be very large — of order 10^{-1} or more in some rare modes, rather than the order 10^{-3} effects in the neutral K mass matrix. The sheer numbers of B mesons estimated to be necessary to get a statistically significant effect put this exciting possibility many years in the future.[10] Another avenue is to consider other K decays where CP violating effects, although very small, may occur with a different weighting (from that in $K \to \pi\pi$) between effects originating in the mass matrix and in the decay amplitude. Although these experiments are also very difficult, there is the advantage of high intensity beams and sophisticated detectors already in existence to perform the measurements of ϵ' and search for rare K decays.

An example of such a process is $K_L \to \pi^0 \ell^+ \ell^-$. If CP were conserved, the long-lived eigenstate would be the CP-odd state, K_2. It would not decay to $\pi^0 \gamma_{\rm virtual} \to \pi^0 \ell^+ \ell^-$, this being forbidden by CP invariance.[11] Since Nature has chosen to break CP invariance, the decay can proceed through: (1) the small part, $\approx \epsilon K_1$, of the K_L wave function that is CP even (we call this "indirect" CP violation); and (2) CP violating effects in the $K_2 \to \pi^0 \ell^+ \ell^-$ decay amplitude itself (we call this "direct" CP violation). In addition to these two CP violating amplitudes, the decay can proceed in a CP conserving manner *via* the decay chain $K_2 \to \pi^0 \gamma\gamma \to \pi^0 \ell^+ \ell^-$, where the photons are either real or virtual. Although higher order in α, this latter amplitude is not necessarily negligible in comparison to either the "indirect" or "direct" CP violating amplitudes which are also suppressed precisely because they contain factors that are related to CP violation.

Naturally, we are most interested in the question of whether one can see the "direct" CP violation effects and especially to investigate if they can be the dominant amplitude contributing to the decay. This amplitude comes from "penguin" diagrams with a photon or Z boson and also from box diagrams, as shown in Fig. 1. For values of $m_t^2 << M_W^2$, it is the "electromagnetic penguin" that gives the dominant short-distance contribution to the amplitude. This was discussed, with estimates of the CP violating effects,[12] before evidence for the b quark was found. A full analysis, including QCD corrections, was carried out in the case of six quarks,[13] building upon work done with four quarks.[14,15] A principal conclusion of that study was that the "direct" CP violation could be comparable to the "indirect" effects.

Why do we reconsider this process now? First, the possible mass range for the t quark has been pushed upward considerably since Ref. 13. The QCD corrections, which turned out to be quite important, need to be redone when m_t^2/M_W^2 cannot be considered to be a small number. The successive steps of removing heavy particles from the theory and developing an effective Hamiltonian involving only the light quarks can no longer be carried out by first removing the W and then the t quark. Rather, they must be removed together. Second, the "Z penguin" and "W box" diagrams, which are "suppressed" by factors of m_t^2/M_W^2 and were neglected in old calculations, are important for large m_t. We need to consider the QCD corrections to them as well. Third, experiments at the required level of sensitivity are beginning to be considered.[16,17]

In what follows we consider matters in the reverse order of their fundamental (as we see it) interest, although not necessarily in reverse order of the magnitude of their contribution to the $K_L \to \pi^0 \ell^+ \ell^-$ decay rate. So, in Sec. II, we turn our attention to a brief review of the situation regarding the magnitude of the CP conserving amplitude. Then we discuss the contribution from "indirect" CP violation, followed by the main body of our work, which concerns the "direct" CP violation amplitudes when $m_t \sim M_W$. The final section puts the various pieces together.

II. THE CP CONSERVING AMPLITUDE

As noted in the Introduction, a CP conserving contribution to the process $K_2 \to \pi^0\ell^+\ell^-$ is induced through the chain $K_2 \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$, which is shown in Fig. 2. We give here a brief review of the checkered history of this amplitude, partly because it is of interest in and of itself, but mainly to set the stage for the treatment of the CP violating amplitude which follows.[17] In that regard, the main issue is whether the CP conserving contribution to $\Gamma(K_2 \to \pi^0\ell^+\ell^-)$ is comparable to the CP violating contribution or might even "drown out" the latter.

The absorptive part of Fig. 2 can be calculated with the two intermediate photons on shell. For the first part of this process, $K_2 \to \pi^0\gamma\gamma$, there are two invariant amplitudes.[18] If we take the momenta as p, p', q_1 and q_2, respectively, and define $x_{1,2} = p\cdot q_{1,2}/p\cdot p$, then they may be expressed in a gauge invariant way as:

$$
\begin{aligned}
<\pi\gamma\gamma|K_2> \;=\;& A(x_1,x_2)\big[q_2\cdot\epsilon_1\; q_1\cdot\epsilon_2 - q_1\cdot q_2\;\epsilon_1\cdot\epsilon_2\big] \;+\\
& B(x_1,x_2)\big[p^2\; x_1x_2\epsilon_1\cdot\epsilon_2 + q_1\cdot q_2\; p\cdot\epsilon_1\; p\cdot\epsilon_2/p^2 \\
& - x_1\; q_2\cdot\epsilon_1\; p\cdot\epsilon_2 - x_2\; q_1\cdot\epsilon_2\; p\cdot\epsilon_1\big] \quad .
\end{aligned}
\tag{1}
$$

with $\epsilon_{1,2}$ the polarization vectors of the two photons. When joined with the QED amplitude for $\gamma\gamma \to \ell^+\ell^-$ to form the amplitude for $K_2 \to \pi^0\ell^+\ell^-$, the contribution from the A amplitude gets a factor of m_ℓ in front of it. This is not hard to understand, as the total angular momentum of the $\gamma\gamma$ system that pertains to the A amplitude is zero; the same is then true of the final $\ell^+\ell^-$ system. However, the interactions, being electroweak, always match (massless) left-handed leptons to right-handed antileptons and *vice versa,* causing the decay of a J = 0 system to massless leptons and antileptons to be forbidden. Hence, the factor of m_ℓ in the overall amplitude for $K_2 \to \pi^0\;\ell^+\ell^-$, so that the A amplitude provides a negligible contribution for $K_2 \to \pi^0 e^+e^-$. A corollary of this theorem applies when the $K_2 \to \pi^0\;\gamma\gamma$ amplitude is calculated using traditional current algebra techniques in the limit of vanishing pion four-momentum. Only a nonvanishing A amplitude is predicted. The factor of m_e then found[19] to be produced in the absorptive part of the amplitude for $K_2 \rightarrow \pi^0 e^+e^-$ merely reflects the presence of only an A amplitude in the current algebra calculation. If this were the end of the story, the CP conserving contribution to $K_2 \rightarrow \pi^0 e^+ e^-$ would produce negligible branching ratios at the 10^{-13} level[19] or smaller.[18]

On the other hand, the contraction of the amplitude for $\gamma\gamma \rightarrow e^+e^-$ with the B amplitude produces no such factor of m_e. B does, however, contain a coefficient with two more powers of momentum, and one might hope for its contribution to be suppressed by angular momentum barrier factors. Because of the extra powers of momentum, in chiral perturbation theory this amplitude is put in by hand and its coefficient not predicted. An order-of-magnitude estimate may be obtained by pulling out the known dimensionful factors in terms of powers of f_π, and asserting that the remaining coupling strength should be of order one.[18] The branching ratio for $K_2 \rightarrow \pi^0 e^+e^-$ is then of order 10^{-14}. Again, the CP conserving amplitude would make a negligible contribution to the decay rate.

However, an old-fashioned vector dominance, pole model predicts[20] comparable A and B amplitudes in $K_2 \rightarrow \pi^0\gamma\gamma$ and a branching ratio for $K_2 \rightarrow \pi^0 e^+ e^-$ of order 10^{-11}, roughly at the level of that arising from the CP violating amplitudes (see below). The B amplitude is far bigger[21] than would be estimated[18] in chiral perturbation theory. The applicability of such a model, however, can be challenged on the grounds that the low-energy theorems and Ward identities of chiral perturbation theory are not being satisfied.[22] The consistent implementation of vector dominance with the chiral and other constraints may lead to an extra suppression factor, and to a smaller prediction than in the old fashioned model.

At this point the burden is still on the theorists to show that the CP conserving contribution is truly negligible in $K_L \rightarrow \pi^0 e^+ e^-$. After a short period when factors of m_e^2 seemed to assure this, we are presently not able to claim it. In the longer run, it will be in the hands of experimentalists to measure $K_L \rightarrow \pi^0\gamma\gamma$ and eventually to separate the A and B amplitudes by measuring the Dalitz plot distributions, particularly the invariant mass of the two photons.[23]

III. THE CP VIOLATING AMPLITUDE FROM THE MASS MATRIX

As already noted in the Introduction, the presence of CP violation in the mass matrix of the neutral K system results in a small admixture of the CP-even K_1 state being found in the long-lived eigenstate:

$$K_L = \frac{K_2 + \epsilon\, K_1}{\left[1 + |\epsilon|^2\right]^{1/2}} \quad , \tag{2}$$

where the denominator is unity to an excellent approximation, as[24] $|\epsilon| = (2.275 \pm 0.021) \times 10^{-3}$. We define "indirect" CP violation as arising from the part of the K_L eigenstate which is proportional to ϵ in Eq. (2): CP is violated within the mass matrix itself, producing the K_1 admixture in the K_L, while the decay $K_1 \to \pi^0 \ell^+ \ell^-$ itself proceeds in a CP conserving manner.

So defined, the magnitude of "indirect" CP violation is dependent upon the choice of phase convention for the K^0 and $\bar{K}^0$ states, as the value of ϵ depends on this choice. We choose the commonly used convention where the weak interaction amplitude for $K^0 \to \pi\pi$ is real when the $\pi\pi$ system has isospin zero. As we do most calculations in a quark basis where this is not true (precisely because of CP violation in the decay amplitude for $K \to \pi\pi$), we will have to do a transformation

$$\begin{aligned} |K^0> &\to e^{-i\xi}|K^0> \\ |\bar{K}^0> &\to e^{+i\xi}|\bar{K}^0> \quad , \end{aligned} \tag{3}$$

with $15.6|\xi| = |\epsilon'/\epsilon|$ from strong interaction "penguin" effects, to get to the commonly used phase convention. This induces a term in the $K_L \to \pi^0 \ell^+ \ell^-$ amplitude proportional to $i \sin\xi \approx i\xi$ (which is about an order of magnitude smaller than that which is proportional to ϵ); we shall take due account of this term later, when we consider the total CP violating amplitude that includes both "indirect" and "direct" pieces. This net amplitude, being a physical quantity, is independent of phase convention.

With the above definition of "indirect" CP violation we may estimate its contribution to the decay rate from the identity:

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} \equiv B(K^+ \to \pi^+ e^+ e^-) \times \\ \frac{\tau_{K_L}}{\tau_{K^+}} \times \frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} \times \frac{\Gamma(K_L \to \pi^0 e^+ e^-)_{\text{indirect}}}{\Gamma(K_1 \to \pi^0 e^+ e^-)} \quad . \tag{4}$$

This allows us to relate the desired quantity to the known branching ratio for the CP conserving decay $K^+ \to \pi^+ e^+ e^-$. Experimental values[24] of 2.7×10^{-7} and 4.2 may be inserted for the first two factors on the right hand side, while the last factor is $|\epsilon|^2$ by the definition above of what we mean by "indirect" CP violation. The third factor can be measured directly one day. For the moment it is the subject of model dependent theoretical calculations, with a value of one if the transition between the K and the π is $\Delta I = 1/2$. This is the case for the short-distance amplitude which involves a transition from a strange to a down quark. For $\Delta I = 3/2$, the corresponding value is 4. With both isospin amplitudes present and interfering, any value is possible.[25] Using a value of unity for this factor makes

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} = 0.58 \times 10^{-11} \quad .$$

This is quite close to the previous estimate in Ref. 13, although the discussion is phrased in a different manner. Instead of relating the branching ratio back to that for $K^+ \to \pi^+ e^+ e^-$, one could proceed directly from the amplitude for $K_1 \to \pi^0 e^+ e^-$ using the theoretical, QCD corrected, short-distance contribution to the real part of this amplitude. This is dangerous; the QCD corrections to the real part of the short-distance contribution are so large as to change its sign, as pointed out in Ref. 13, and discussed in Sec. IV. As a result, its magnitude cannot be calculated reliably. It is too small to explain $\Gamma(K^+ \to \pi^+ e^+ e^-)$ and there is a high likelihood that long-distance contributions are important. Ultimately all of this discussion can be bypassed by an experimental measurement of $\Gamma(K_S \to \pi^0 e^+ e^-)$. This will provide a direct determination of the third factor on the right-hand side of Eq. (4), removing all the present uncertainty that stems from our theoretical inability to supply a precise prediction for this decay rate.

IV. CP VIOLATION FROM THE DECAY AMPLITUDE

We now turn to the principal part of our investigation, the calculation of the CP violating contributions to the $K_2 \to \pi^0 \ell^+ \ell^-$ amplitude itself. We work in the standard model with six quarks arranged in left-handed doublets with respect to weak isospin, quark weak eigenstates related to quark mass eigenstates by the Kobayashi–Maskawa matrix, and CP violating contributions to the decay amplitude possible because of the nontrivial phase present in that matrix.

We will express our calculation in the language of forming an effective Hamiltonian written in terms of the low mass quarks u, d, and s which are involved in the initial and final states of strange particle decays. The calculation proceeds by starting with the theory written in terms of the weak gauge boson and quark fields, and successively integrating out the heavy quanta from the theory. One starts at the largest momentum scale and moves to the lowest, at each stage making use of renormalization group equations to calculate the coefficients of the operators in the effective theory composed of those quarks still extant at that stage.[26]

In previous calculations applied to this process, the succession of scales was characterized by M_W, m_t, m_b, m_c, and finally μ, which represents the momentum scale relevant to the hadrons involved in the decay. In this paper we consider t quark masses comparable to or greater than that of the W, and the first step ceases to exist. Instead, we remove the t quark and W from the theory together.[27]

At each stage of the calculation, we will be left with an effective Hamiltonian in the form of a sum of Wilson coefficients times operators:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_i C_i(\mu^2) Q_i + h.\ c. \quad , \tag{5}$$

where, for example, at the last stage,

$$
\begin{aligned}
Q_1 &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{u}_\beta \gamma^\mu (1-\gamma_5) u_\beta) \\
Q_2 &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta)(\bar{u}_\beta \gamma^\mu (1-\gamma_5) u_\alpha) \\
Q_3 &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)[(\bar{u}_\beta \gamma^\mu (1-\gamma_5) u_\beta) \ + \ (\bar{d}_\beta \gamma^\mu (1-\gamma_5) d_\beta) \ + \ (\bar{s}_\beta \gamma^\mu (1-\gamma_5) s_\beta)] \\
Q_4 &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta)[(\bar{u}_\beta \gamma^\mu (1-\gamma_5) u_\alpha) \ + \ (\bar{d}_\beta \gamma^\mu (1-\gamma_5) d_\alpha) \ + \ (\bar{s}_\beta \gamma^\mu (1-\gamma_5) s_\alpha)] \\
Q_5 &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)[(\bar{u}_\beta \gamma^\mu (1+\gamma_5) u_\beta) \ + \ (\bar{d}_\beta \gamma^\mu (1+\gamma_5) d_\beta) \ + \ (\bar{s}_\beta \gamma^\mu (1+\gamma_5) s_\beta)] \\
Q_6 &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta)[(\bar{u}_\beta \gamma^\mu (1+\gamma_5) u_\alpha) \ + \ (\bar{d}_\beta \gamma^\mu (1+\gamma_5) d_\alpha) \ + \ (\bar{s}_\beta \gamma^\mu (1+\gamma_5) s_\alpha)] \\
Q_{7V} &= \frac{e^2}{4\pi}(\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{e}\gamma^\mu e) \\
Q_{7A} &= \frac{e^2}{4\pi}(\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{e}\gamma^\mu \gamma_5 e) \quad .
\end{aligned}
\tag{6}
$$

The color indices α and β are summed over the three colors, while the combination $V_{us}^* V_{ud}$ of Kobayashi–Maskawa matrix elements is the usual one involved in decays of strange particles. The quark fields appearing in the second factor in the definition of Q_3, Q_4, Q_5, and Q_6 generally include all those which have not yet been removed from the theory. At the last stage, where this includes only the u, d and s quarks, one of the operators in Eq. (6) is linearly dependent (this is usually taken to be Q_4). We have chosen the same operators as in Ref. 13, with the addition of Q_{7A}, whose presence is required now that we include the contributions from the "Z penguin" and "W box" graphs in Fig. 1.[28] We have neglected operators of the form $m_s \ \bar{s}\sigma_{\mu\nu} \ F^{\mu\nu} d$ as giving a very small contribution to the net amplitude.

If we think first about the situation in the absence of strong interactions, then the only one of the first six operators with a nonzero coefficient (to order g^2 in weak interactions) is Q_2, with $c_2 = 1$. To order g^2 in weak interactions and order e^2 in electromagnetic interactions, the diagrams in Fig. 1 generally give nonzero coefficients of Q_{7V} and Q_{7A}.[29] For example, if we consider $m_t \sim M_W$, then at the scale M_W, we have an "electromagnetic penguin" contribution [Fig. 1(a)] involving the t quark:

$$
C_{7V,t}^{(\gamma)}(M_W^2) \ = \ \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \ \widetilde{C}_{7V,t}^{(\gamma)}(M_W^2) \quad , \tag{7}
$$

where the coefficient with the Kobayashi–Maskawa factor removed is represented with a tilde over it:

$$\widetilde{C}^{(\gamma)}_{7V,i}(M_W^2) = \frac{(25-19x_i)x_i^2}{72\pi(x_i-1)^3} - \frac{(3x_i^4-30x_i^3+54x_i^2-32x_i+8)log(x_i)}{36\pi(x_i-1)^4} \quad , \tag{8}$$

and $x_i = m_i^2/M_W^2$. The "Z penguin" contributes [Fig. 1(b)]:

$$C^{(Z)}_{7V,t}(M_W^2) = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \, \widetilde{C}^{(Z)}_{7V,t}(M_W^2) \tag{9a}$$

$$C^{(Z)}_{7A,t}(M_W^2) = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \, \widetilde{C}^{(Z)}_{7A,t}(M_W^2) \quad , \tag{9b}$$

where

$$\widetilde{C}^{(Z)}_{7V,i}(M_W^2) = \frac{4\sin^2\theta_W - 1}{\sin^2\theta_W} \frac{x_i}{16\pi} \left[\frac{(x_i-6)(x_i-1)+(3x_i+2)log(x_i)}{(x_i-1)^2} \right] , \tag{10a}$$

and

$$\widetilde{C}^{(Z)}_{7A,i}(M_W^2) = \frac{1}{\sin^2\theta_W} \frac{x_i}{16\pi} \left[\frac{(x_i-6)(x_i-1)+(3x_i+2)log(x_i)}{(x_i-1)^2} \right] \quad . \tag{10b}$$

The "W box" contributes [Fig. 1(c)]:

$$C^{(Box)}_{7V,t}(M_W^2) = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \, \widetilde{C}^{(Box)}_{7V,t}(M_W^2) \tag{11a}$$

$$C^{(Box)}_{7A,t}(M_W^2) = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \, \widetilde{C}^{(Box)}_{7A,t}(M_W^2) \quad , \tag{11b}$$

where

$$\widetilde{C}^{(Box)}_{7V,i}(M_W^2) = \frac{1}{\sin^2\theta_W} \frac{x_i}{8\pi} \left[\frac{1-x_i+log(x_i)}{(x_i-1)^2} \right] \tag{12a}$$

and

$$\widetilde{C}^{(Box)}_{7A,i}(M_W^2) = -\frac{1}{\sin^2\theta_W}\frac{x_i}{8\pi}\left[\frac{1-x_i+log(x_i)}{(x_i-1)^2}\right] \quad . \tag{12b}$$

The contribution of the t quark to $\widetilde{C}^{(\gamma)}_{7V}$ at the scale μ is given by:

$$\widetilde{C}^{(\gamma)}_{7V,t}(\mu^2) = \widetilde{C}^{(\gamma)}_{7V,t}(M_W^2) - \frac{2}{9\pi}\int_{\mu^2}^{M_W^2}\frac{dq^2}{q^2}[C_2+3C_1] \quad , \tag{13}$$

where, since we are considering $m_t \sim M_W$ and there are no large logarithms of the form $log(M_W^2/m_t^2)$, we take the full expression for $\widetilde{C}_{7V,t}(M_W^2)$ as given in Eq. (8). Since in the absence of QCD the coefficients $C_2 = 1$ and $C_1 = 0$, the integral contributes the large logarithm in the problem,

$$-\frac{2}{9\pi}\,log\left(\frac{M_W^2}{\mu^2}\right) \quad ,$$

to the right-hand side of Eq. (13).

Note that if we had considered the situation where $m_i^2 << M_W^2$, i.e., $x_i << 1$, then the full contribution from the quark i is generated at scales from m_i down to μ and the leading term is

$$\widetilde{C}_{7V,i}(\mu^2) \approx \widetilde{C}^{(\gamma)}_{7V,i}(\mu^2) \approx -\left(\frac{2}{9\pi}\right) log\frac{m_i^2}{\mu^2} \quad , \tag{14}$$

as in Ref. 13. The other contributions in Eqs. (10) and (12) due to the "Z penguin" and "W box" graphs, respectively, all vanish in comparison to Eq. (14) in the same limit by at least one power of x_i. In the limit $x_i \to 0$ such nonleading contributions are numerically small and therefore dropped, as are the nonleading terms in the "electromagnetic penguin" contribution.

Even though there is a μ dependence in the Wilson coefficient in Eq. (13), we know that there can be no dependence upon μ in the total amplitude, as it represents a physical observable. This μ dependence is cancelled by a corresponding dependence which occurs when we take the matrix elements of the effective Hamiltonian, $\mathcal{H}$, to order e^2. This occurs as follows: The contribution involving C_{7V} is of order e^2 from the operator itself, and of order e^0 from taking its matrix element. There also is a contribution from Q_2 involving "light" quarks, where the coefficient and operator is of order e^0, but the matrix element is of order e^2 by having a "light" quark and antiquark annihilate through a virtual photon into $\ell^+\ell^-$. This gives a term which has an exactly cancelling μ dependence. Note also that Eq. (8) may contain different (nonleading) constant terms, depending upon which renormalization scheme is used, but that in going from one scheme to another, changes in the coefficient of C_{7V} are compensated by corresponding changes in the matrix element of Q_2, as they must be.

Now let us introduce the strong interactions in the form of Quantum Chromodynamics (QCD). First, to order e^0, nonzero coefficients are generated for the first six operators as we move successively down from the weak scale to one quark mass and then another. The operators Q_3, Q_4, Q_5, and Q_6 arise from "penguin" diagrams involving gluons. The operators $Q_\pm = (1/2)[Q_2 \pm Q_1]$ are multiplicatively renormalized:

$$C_\pm(\mu^2) = \left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right]^{a_\pm} C_\pm(M_W^2) \quad , \tag{15}$$

with $C_\pm(M_W^2) = 1$, and where $a_+ = 6/(33 - 2N_f)$ and $a_- = -12/(33 - 2N_f)$ for N_f quark flavors in leading logarithmic approximation between the scale M_W and the scale μ. At the same time, to order e^2 the coefficients of the operators Q_{7V} and Q_{7A} are generated from their values at M_W plus mixing effects of the operators Q_1 and Q_2 with Q_{7V} or Q_{7A}. The "penguin" operators, Q_3, Q_4, Q_5, and Q_6, which arise only through QCD effects, have coefficients which start out at zero at the weak scale. They typically never grow to be more than an order of magnitude smaller than the coefficients for $Q_\pm$. So, while we in principle consider the whole 8×8 anomalous dimension matrix[30] which describes the mixing among all the operators in Eq. (6) as we go from one scale to another, it is an excellent approximation to consider the mixing only of $Q_{\pm}$ with Q_{7V} and Q_{7A}[13] and the renormalization of $Q_{\pm}$ as in Eq. (15). In the same spirit we neglect the effect of taking order e^2 matrix elements of the "penguin operators," which is also known to give a small effect.[31]

The derivation of the QCD corrected contributions when $m_t \sim M_W$ proceeds in a straightforward manner, if one follows the general method given in Ref. 13. This is outlined in Appendix A. Instead, we give here an account of the derivation following along the lines of the correct results for four quarks,[32] illustrating it for the case of the "electromagnetic penguin" contribution.

For this purpose, we start with the case of the contribution to $C_{7V}^{(\gamma)}$ from the t quark without QCD given in Eq. (13). The corrections to this due to QCD arise simply from the fact that $C_2 + 3C_1$ acquires a q^2 dependence:

$$\begin{aligned} C_2(q^2) + 3\, C_1(q^2) &= 2\, C_+(q^2) \;-\; C_-(q^2) \\ &= 2 \left[\frac{\alpha_s(M_W^2)}{\alpha_s(q^2)}\right]^{a_+} - \left[\frac{\alpha_s(M_W^2)}{\alpha_s(q^2)}\right]^{a_-} \quad . \end{aligned} \tag{16}$$

To carry out the integral in Eq. (13), we need only remember that

$$\frac{dq^2}{q^2} = -\frac{12\pi}{33 - 2N_f}\,\frac{d\alpha_s(q^2)}{\alpha_s^2(q^2)} \quad ,$$

and then to split the region of integration into subregions characterized by different numbers of active fermion species to obtain:

$$\begin{aligned} \widetilde{C}_{7V,t}^{(\gamma)}(\mu^2) \;=\; & \widetilde{C}_{7V,t}^{(\gamma)}(M_W^2) + \\ & - \frac{16}{99\alpha_s(m_c^2)}\left(1 - K_{\mu/c}^{-33/27}\right)K_{c/b}^{-6/25}K_{b/W}^{-6/23} - \frac{16}{93\alpha_s(m_b^2)}\left(1 - K_{c/b}^{-31/25}\right)K_{b/W}^{-6/23} \\ & - \frac{16}{87\alpha_s(M_W^2)}\left(1 - K_{b/W}^{-29/23}\right) + \frac{8}{45\alpha_s(m_c^2)}\left(1 - K_{\mu/c}^{-15/27}\right)K_{c/b}^{12/25}K_{b/W}^{12/23} \end{aligned}$$

$$+ \frac{8}{39\alpha_s(m_b^2)} \left(1 - K_{c/b}^{-13/25}\right) K_{b/W}^{12/23} + \frac{8}{33\alpha_s(M_W^2)} \left(1 - K_{b/W}^{-11/23}\right) \quad , \tag{17}$$

where $K_{b/W} = \alpha_s(m_b^2)/\alpha_s(M_W^2)$, $K_{c/b} = \alpha_s(m_c^2)/\alpha_s(m_b^2)$, and $K_{\mu/c} = \alpha_s(\mu^2)/\alpha_s(m_c^2)$ in effective five, four and three quark theories, respectively.

In the case of $\widetilde{C}_{7V,c}^{(\gamma)}$, the situation is much simpler since the relevant Wilson coefficient is only generated at scales between μ and m_c:

$$\widetilde{C}_{7V,c}^{(\gamma)}(\mu^2) = -\frac{2}{9\pi} \int_{\mu^2}^{m_c^2} \frac{dq^2}{q^2} \left[C_2 + 3C_1\right] \quad . \tag{18}$$

The result of putting in the QCD induced dependence of C_2 and C_1 on q^2 is then

$$\begin{aligned} \widetilde{C}_{7V,c}^{(\gamma)}(\mu^2) &= -\frac{16}{99\alpha_s(m_c^2)}(1 - K_{\mu/c}^{-33/27})K_{c/b}^{-6/25}K_{b/W}^{-6/23} \\ &+ \frac{8}{45\alpha_s(m_c^2)}(1 - K_{\mu/c}^{-15/27})K_{c/b}^{12/25}K_{b/W}^{12/23} \quad . \end{aligned} \tag{19}$$

In both these examples, the recovery of "free quarks" as the limiting case $\alpha_s \to 0$ is obtained trivially by looking back to the starting point in Eqs. (16) and (18). It also may be obtained from the final answer by expanding the factors of $K_{i/j}$ to order α_s, keeping the leading term as $\alpha_s \to 0$.

The situation with an experimentally reasonable $\alpha_s(q^2)$ is far from the free quark model, however. The QCD corrections to $\widetilde{C}_{7V,t}^{(\gamma)}$ are large. Those for $\widetilde{C}_{7V,c}^{(\gamma)}$ are enormous, for they can easily change not only the magnitude but the sign of this coefficient. As pointed out in Ref. 13, this is readily understandable by considering the right-hand side of Eq. (18), rewritten as

$$-\frac{2}{9\pi} \int_{\mu^2}^{m_c^2} \frac{dq^2}{q^2} \left[2\, C_+(q^2) - C_-(q^2)\right] \quad .$$

Before QCD effects are considered, the integrand is $[2 \times 1 - 1] = 1$. When QCD is included, the coefficient $C_+(q^2)$ decreases and $C_-(q^2)$ increases so that the cancellation between the terms in the integrand becomes more complete. In fact, over most or all of the region of integration from μ^2 to m_c^2 the second term overwhelms the first and the integrand is negative.

For the real (CP conserving) part of the short-distance generated amplitude, the contribution from the top quark is negligible because of the Kobayashi–Maskawa factor in Eq. (7). The charm quark gives the important short-distance contribution to the real part of the amplitude for $K \to \pi \ell^+ \ell^-$, and the possibilities for making a precise theoretical prediction are nil because of the situation we have just described: The QCD corrections typically change not just the magnitude but even the sign of the coefficient of Q_{7V}. Aside from this explicit indication of danger from delicate cancellations in the calculation, a comparison of the magnitude of the resulting amplitude with that required from the measured rate for $K^+ \to \pi^+ e^+ e^-$ shows that the theoretical calculation gives a result that is much too small to explain the data. Long-distance contributions, not unexpectedly, are necessary to understand the magnitude of the real part of the amplitude.

This is entirely different than the situation with regard to the imaginary (CP violating) part of the amplitude. The Kobayashi–Maskawa factors for charm and top are the same, up to a sign:

$$Im \, \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \;=\; -Im \, \frac{V_{cs}^* V_{cd}}{V_{us}^* V_{ud}} \;=\; \frac{Im \, [V_{ts}^* V_{td} V_{us} V_{ud}^*]}{|V_{us} V_{ud}|^2} \quad . \tag{20}$$

These quantities are all invariant under a (quark field) rephasing,[33,34] and in Eq. (20) have been kept in a form to exhibit that fact. The numerator on the right-hand side is just a form of the invariant measure of CP nonconservation proposed by Jarlskog[33] for three generations. In the original parametrization of Ref. 3, the quantities in Eq. (20) are expressible as $\sin\theta_2 \sin\theta_3 \sin\delta = s_2 s_3 s_\delta$, with cosines of small angles set equal to unity. We shall use this shorthand to refer to the rephase invariant quantity in Eq. (20) in what follows.

Because of the Kobayashi–Maskawa factors, it is momentum scales from m_c^2 to m_t^2 that contribute to the imaginary part. This can be seen, for example, by combining the charm and top quark leading logarithmic contributions to the imaginary part of $C_{7V}^{(\gamma)}$ in the absence of QCD:

$$\begin{aligned} ImC_{7V}^{(\gamma)} &= s_2 s_3 s_\delta \left[\widetilde{C}_{7V,t}^{(\gamma)}(\mu^2) - \widetilde{C}_{7V,c}^{(\gamma)}(\mu^2)\right] \\ &= s_2 s_3 s_\delta \left[-\frac{2}{9\pi}\int_{\mu^2}^{m_t^2}\frac{dq^2}{q^2} + \frac{2}{9\pi}\int_{\mu^2}^{m_c^2}\frac{dq^2}{q^2}\right] \\ &= -s_2 s_3 s_\delta \left[\frac{2}{9\pi}\int_{m_c^2}^{m_t^2}\frac{dq^2}{q^2}\right] \quad . \end{aligned} \tag{21}$$

Thus, the dependence on the scale μ cancels out. There is every reason to expect the short-distance contributions to give the dominant part of the "direct" CP violating amplitude.[35]

Once QCD corrections are applied, the integrand is reduced, but over most or all of the range of integration it does not change sign (from that for free quarks). Thus, while the QCD corrections are nonnegligible, they are fairly insensitive to changes in parameters and reliably calculable for the imaginary part. This is shown in Fig. 3, where the QCD corrected $\widetilde{C}_{7V}^{(\gamma)} = \widetilde{C}_{7V,t}^{(\gamma)} - \widetilde{C}_{7V,c}^{(\gamma)}$, calculated from Eqs. (17) and (19), is indicated with solid curves for $\Lambda_{QCD} = 100$ and 250 MeV as a function of the top quark mass. The result is independent of μ^2. While about a factor of two smaller than the result without QCD (dashed curve), the result does not depend strongly on Λ_{QCD} or top quark mass.

To assemble the full coefficient, C_{7V}, we need to add the "Z penguin" and "W box" contributions. For those involving the t quark, they may be taken directly from Eqs. (10a) and (12a), respectively. When $m_t \sim M_W$ there are no QCD corrections to be applied, as these contributions are generated at momentum scales from m_t to M_W where there are no large logarithms.[36] For those contributions involving the c quark, there are important QCD corrections. However, these contributions, being proportional to $x_c = m_c^2/M_W^2$, are themselves so small as to be negligible.

The total coefficient $\widetilde{C}_{7V}$ and the contributions from each of its components is shown in Fig. 4. Even after being reduced by QCD corrections, the contribution from the "electromagnetic penguin," $\widetilde{C}_{7V}^{(\gamma)}$, is the largest of the three. This is in good part due to the smallness of the vector coupling of the Z to charged leptons (which is proportional to $1 - 4\sin^2\theta_W$). Otherwise the contribution of the "Z penguin" would dominate for large values of m_t.

The dominance of the "Z penguin" contributions at large m_t can be seen in Fig. 5, where the total and component parts of the coefficient $\widetilde{C}_{7A}$ are shown. As $m_t \to \infty$, $\widetilde{C}_{7A,t}^{(Z)}$ grows as m_t^2, while $\widetilde{C}_{7A,t}^{(box)}$ goes to a constant. In $\widetilde{C}_{7A}$ the "box" contribution is less than that from the "Z penguin" for $m_t \gtrsim M_W$.

Note that in the opposite situation where $m_t^2 << M_W^2$, both these contributions behave as $x_t\ log(x_t)$ and are nonleading when compared to the "electromagnetic penguin" contribution (to $\widetilde{C}_{7V}$), which behaves in the same limit as $log(x_t)$. QCD provides corrections to such large logarithms, which can arise when there is a large ratio of momentum scales. Our philosophy here, with $m_t \sim M_W$, has been to keep the leading and nonleading contributions at the scale M_W, and to also carry out the QCD corrections to the large logarithms that arise from integration over scales with a big ratio. In fact, the contribution from $\widetilde{C}_{7V,t}^{(\gamma)}(M_W^2)$ is a small part of the full $\widetilde{C}_{7V}^{(\gamma)}$. Some of the ambiguities in the charm or top quark contributions by themselves (*e.g.,* at a low scale μ) also cancel out in the imaginary part of the amplitude where the different sign in Kobayashi–Maskawa factors for charm and top makes the resulting amplitude arise from scales larger than m_c.

To proceed to actual branching ratios or decay rates, we may avoid some arithmetic by relating the hadronic matrix element of the operator, $(\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)$, which occurs in Q_{7V} and Q_{7A}, to that of the corresponding charged current operator, $(\bar{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha)$, which occurs in $K_{\ell 3}$ decay. Then the form factors and phase space involved in this latter decay are automatically entered by Nature into the measured branching ratio for that mode. Using this, we find from the measured[24] branching ratio for K_{e3} decay that

$$B(K_2 \to \pi^0 e^+ e^-) \;=\; 1.0 \times 10^{-5}\,(s_2 s_3 s_\delta)^2\,\left[(\widetilde{C}_{7V})^2 + (\widetilde{C}_{7A})^2\right] \quad . \qquad (22)$$

The factor in square brackets is shown in Fig. 6. With QCD corrections, and with m_t between 50 and 200 GeV, it ranges between about 0.1 and 1.0. While the combination $s_2 s_3 s_\delta$ enters other CP violating quantities such as ϵ and ϵ', imprecisely known hadronic matrix elements and m_t presently allow a broad range of values of this combination. From measurement of Kobayashi–Maskawa matrix elements, $s_2 s_3 s_\delta \leq 2.5 \times 10^{-3}$. For m_t at the low end of the acceptable range (as constrained by $B^0 - \bar{B}^0$ mixing), the allowed region of Kobayashi–Maskawa parameters contracts and $s_2 s_3 s_\delta$ must be quite close[7] to 10^{-3}. More generally, a typical value is in this neighborhood. Putting this information into Eq. (22), we see that the branching ratio for $K_L \to \pi^0 e^+ e^-$ from CP violation in the decay amplitude alone is around 10^{-11}.

V. CONCLUSIONS

From the results of the previous three sections, it appears that from our present knowledge, the three contributions to the process $K_L \to \pi^0 \ell^+ \ell^-$ could each give rise to a branching ratio in the 10^{-11} range. With further theoretical and/or experimental work, discussed in Sec. II, it is possible that the CP conserving contribution might yet be shown to be well below this level.

This is not the case for the effects of CP violation in the mass matrix and in the decay amplitude. Their contributions are comparable, roughly at the 10^{-11} level in branching ratio, and in general will interfere in the expression for the total decay rate.

Some care must be exercised about phase conventions in calculating this interference. We have been calculating the CP violation in the decay amplitude in terms of what happens at the quark level, where strong interaction "penguin" diagrams induce a $\Delta I = (1/2)$ $K \to \pi\pi$ transition which has a CP violating phase.

The standard convention, on the other hand, where $\epsilon \approx (2.275 \times 10^{-3})e^{i\pi/4}$, starts from making the amplitude for $K \to \pi\pi$ real when the final state has $I = 0$ [as it would from a $\Delta I = (1/2)$ transition]. To get to the standard convention from the quark basis requires absorbing a phase ξ proportional to ϵ' into the neutral K field, as described in Sec. III. As a result, in the amplitude for "indirect" CP violation, $\epsilon \to \epsilon - i\xi$, if $|\xi|$ is small. A somewhat abbreviated expression for the branching ratio from all CP violating effects is then,

$$B(K_L \to \pi^0 e^+ e^-) \approx \left\{ \left| 0.76 \left(e^{i\pi/4} - i\frac{\xi}{|\epsilon|} \right) \left[\frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} \right]^{1/2} + i\left(\frac{s_2 s_3 s_\delta}{10^{-3}} \right) \widetilde{C}_{7V} \right|^2 + \left| \left(\frac{s_2 s_3 s_\delta}{10^{-3}} \right) \widetilde{C}_{7A} \right|^2 \right\} \cdot 10^{-11} \quad , \tag{23}$$

where we have taken into account the phase conventions mentioned above. In the last term of Eq. (23) we have neglected the contribution from ϵ times the real part of C_{7A}, which is less than 1% of the imaginary part of C_{7A}. Eq. (23) indicates the interference of amplitudes coming from "indirect" and "direct" CP violation. Neglected is the fact that the two interfering amplitudes (which involve vector coupling to the lepton pair) can have a different dependence on the pair invariant-mass and the interference can then vary with this quantity. If both amplitudes came from short-distance effects (which we have indicated is very unlikely for the "indirect" CP violation), then $[\Gamma(K_1 \to \pi^0 e^+ e^-)/\Gamma(K^+ \to \pi^+ e^+ e^-)]^{1/2}$ is negative, the interference is the same for all values of the pair invariant-mass, and Eq. (23) stands as written.

Since $\epsilon'/\epsilon = -15.6\xi \approx 3 \times 10^{-3}$, the extra piece from the change of basis is small, but interferes constructively with that from ϵ. The terms coming from "direct" CP violation are comparable to those from the mass matrix ("indirect" CP violation), and we can't give a definitive conclusion as to their relative magnitudes without further knowledge of $A(K_1 \to \pi^0 \ell^+ \ell^-)$, $s_2 s_3 s_\delta$, and m_t. Nor can we give a statement as to constructive or destructive interference without a model for the long-distance effects which we suspect are inherent in the "indirect" CP violation amplitude. As m_t becomes larger, more of the "direct" CP violation comes through Q_{7A} (see Figs. 4, 5 and 6). As a result, the theoretical predictions become more definitive, as the QCD corrections to C_{7A} are very small and this contribution does not interfere in the expression for the decay rate with that from "indirect" CP violation. Even for large m_t, however, it is hard to get a branching ratio that is more than a few times 10^{-11}.

We have a major advantage over calculations of other CP violating effects in the K^0 system in that the hadronic matrix element of the relevant operators (Q_{7V} and Q_{7A}) from the short-distance physics is given to us from $K_{\ell 3}$ decay. There is no uncertainty here. Nevertheless, we would assign an uncertainty from the QCD corrections, the neglect of nonleading QCD terms, and possible "direct" CP violating contributions from order e^2 matrix elements of Q_1 to Q_6, of 10 to 20% for $\widetilde{C}_{7V}$, even if we knew m_t precisely along with all the Kobayashi–Maskawa parameters. Conversely, if there were both a precise measurement of m_t and of the $K_L \to \pi^0 \ell^+ \ell^-$ branching ratio that resulted in an isolation of the amplitude for "direct" CP violation, there would be an uncertainty of this magnitude in the extracted value of $s_2 s_3 s_\delta$. While not as precise as one might like, this would be far better than the determination from ϵ and ϵ', where nontrivial hadronic matrix elements enter.

There are a number of experimental observations which would help to sort out various contributions and their magnitudes. We conclude by briefly discussing some of them:

- The short-distance generated amplitudes have a dependence on the kinematic variables of the final state which is identical to that in $K_{\ell 3}$ decay, with obvious substitutions of particle names. This allows an easy calculation of decay rates with cuts on final state kinematic variables, *e.g.,* restrictions on $m_{\ell\bar{\ell}}$. Comparison with observations of $K^+ \to \pi^+ \ell^+ \ell^-$, $K_S \to \pi^0 \ell^+ \ell^-$, and $K_L \to \pi^0 \ell^+ \ell^-$, would help to sort out long-distance contributions from short-distance ones.

- The relative rates for $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ are sensitive as well to the CP conserving two-photon contribution, with the factor of m_ℓ that accompanies the A amplitude (see Sec. II) acting to enhance its contribution in the latter reaction in comparison to the former.

- The direct measurement of $K_L \to \pi^0 \gamma\gamma$ can be used as an input to calculations of the two-photon, CP conserving contribution to $K_L \to \pi^0 \ell^+ \ell^-$. In particular, one could separate the A and B amplitudes by measuring the Dalitz plot distributions, such as the invariant mass distribution of the two photons.[23]

- If both CP conserving and CP violating amplitudes are present with even roughly comparable strengths, they will in general interfere on the Dalitz plot, giving rise to a large lepton–antilepton energy asymmetry.[20]

- The "indirect" CP violating amplitude can be obtained from a measurement of $K_S \to \pi^0 \ell^+ \ell^-$. Any deviation in the then measured rate for $K_L \to \pi^0 \ell^+ \ell^-$ from the straightforward prediction involving multiplication of the former rate by $|\epsilon - i\xi|^2$ is then evidence for "direct" CP violation in the decay amplitude (assuming the CP conserving contribution has been shown experimentally or theoretically to be small).

- One can imagine a full interference pattern being measured, as was done for the $\pi\pi$ mode, where one sees both the regime of $K_S \to \pi^0 \ell^+ \ell^-$ decay followed by that for $K_L \to \pi^0 \ell^+ \ell^-$, with an interference region between the two regimes of exponential decay. This would permit not only the measurement of the two rates, but the phase between the "indirect" and "direct" amplitudes whose interference is indicated in Eq. (23).

As of now, we have a long way to go experimentally. While recent upper limits[37,38] are around 4×10^{-8}, and are improvements by orders of magnitude on earlier limits,[24] we have about three orders of magnitude further improvement in sensitivity needed to see the standard model signal.

Finally, we note that in the large m_t regime all the decays $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \ell^+ \ell^-$, and $K_L \to \pi^0 \nu\bar{\nu}$ have amplitudes which are dominated by contributions from the "Z penguin" and "W box" graphs. The latter two, which are CP violating, have comparable rates in this regime. The decay $K_L \to \pi^0 \nu\bar{\nu}$ arises almost entirely from "direct" CP violation.[39] The first of the three offers a different combination of Kobayashi–Maskawa parameters and the ratio of its branching ratio to that for the third process is a function purely of Kobayashi–Maskawa angles which is especially clean and free of theoretical ambiguities.

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APPENDIX

In Sec. IV, the leading logarithmic QCD corrections to the "electromagnetic penguin" were derived following the approach of Ref. 32. Here we carry out this calculation using an effective Hamiltonian formalism, as in Ref. 13, where the heavy fields are removed from the theory in successive steps, and the coefficients of the operators appearing in the effective Hamiltonian are determined by means of renormalization group equations.

At the scale of M_W or above, the terms in the Hamiltonian are taken to be those in a free (no strong interactions), six quark theory.[29] Below M_W, the effects of QCD are included through the mixing of the effective operators using the machinery of the renormalization group. We first assume a succession of scales characterized by the "old" hierarchy of scales: M_W, m_t, m_b, m_c and finally μ. At the end of the Appendix we remove the W boson and the top quark together in order to treat the case $m_t \gtrsim M_W$.

After the W is treated as heavy and removed from the theory, the effective Hamiltonian can be expressed as:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c,t} V_{qs}^* V_{qd} \sum_{i=+,-,7} A_i{}^{(q)} O_i{}^{(q)} + h.\,c. \quad , \tag{A.1}$$

where

$$\begin{aligned} O_\pm^{(q)} &\equiv \frac{1}{2}\left([\bar{s}_\alpha \gamma_\mu (1-\gamma_5) q_\alpha]\,[\bar{q}_\beta \gamma^\mu (1-\gamma_5) d_\beta]\right. \\ &\quad \left.\pm [\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha]\,[\bar{q}_\beta \gamma^\mu (1-\gamma_5) q_\beta]\right) \quad . \end{aligned} \tag{A.2}$$

Note that each of the operators $O_\pm^{(q)}$ only involves the charge 2e/3 quark, q; we do not follow the more traditional procedure[13] of using the unitarity of the Kobayashi–Maskawa matrix to rewrite Eq. $(A.1)$ in terms of operators $O_\pm^{(q)} - O_\pm^{(u)}$ and a sum over only charm and top. Instead, we follow the evolution of the contribution from each quark to lower scales, imposing unitarity only at the end when just u, d, and s quarks are left in the theory.

We recognize that $O_\pm^{(u)} \equiv Q_\pm$, and that the appropriate operators $O_7^{(q)}$ for $q = u, c, t$ are

$$O_7^{(q)} = O_7 \equiv \frac{1}{\alpha_s} Q_{7V} \quad . \tag{A.3}$$

A factor of $1/\alpha_s$ is absorbed in the normalization of O_7 to make all the elements of the anomalous dimension matrix be of the same order in α_s (see below). At the end of the calculation the effective Hamiltonian will be expressed in terms of the operators $Q_\pm$ and Q_{7V}, and the factor $1/\alpha_s$ put back into the coefficient of the latter operator. The operators $O_\pm^{(q)}$ appear only at scales above m_q where the quark q is still extant in the theory and where they mix with $O_7^{(q)}$ through one-loop corrections. The operator O_7 appears at all scales, and its coefficient contains leading logarithmic QCD corrections as well as nonleading terms coming from the free quark theory above M_W. As discussed in the text, the mixing of the strong interaction "penguin" operators [Q_3 to Q_6 in Eq. (6)] with Q_{7V} has been neglected, as their effects are small. This has allowed us to truncate Eq. $(A.1)$ with just the three operators $O_\pm$ and O_7 (rather than seven operators).

As we go to scales μ below M_W, these operators satisfy a set of coupled renormalization group equations of the form:

$$[\mathcal{D}\delta_{ij} \ + \ \gamma_{ij}] \, O_j^{(q)} \ = \ 0 \quad , \tag{A.4}$$

with a summation over j implicit, and

$$\mathcal{D} \ \equiv \ \mu\frac{\partial}{\partial\mu} \ + \ \beta(g)\frac{\partial}{\partial g} \ + \ \gamma_m(g) \, m_q\frac{\partial}{\partial m_q} \quad . \tag{A.5}$$

Since the Hamiltonian is μ-independent, the coefficients $A_i^{(q)}$ must satisfy the equation:

$$[\mathcal{D}\delta_{ij} \ - \ {\gamma^T}_{ij}] \, A_j^{(q)}\left(\frac{M_W}{\mu}\right) \ = \ 0 \quad , \tag{A.6}$$

with the boundary conditions at $\mu = M_W$ given by $A_+^{(q)}(1) = A_-^{(q)}(1) = 1$. The value of $A_7^{(q)}(1)$ corresponds to the coefficient of O_7 in an effective free quark theory at the scale M_W. In the case where all quarks are much lighter than the W boson, the coefficient of $O_7^{(q)}$ at M_W is negligibly small compared to the leading logarithmic contributions to it from mixing with $O_\pm^{(q)}$. It can be taken to be zero, as was done in Ref. 13. However, for the case where $m_t \gtrsim M_W$, discussed at the end of this Appendix, $A_7^{(q)}(1)$ receives important nonleading-logarithmic contributions, which should not be neglected.

If all the elements of the anomalous dimension matrix γ are of the same order in the strong coupling g and the quark masses m_q, the solution to Eq. $(A.4)$ can be readily found by first transforming to a basis where γ is diagonal, solving a set of uncoupled differential equations, and finally transforming the solution back to the original basis. Although numerical values change from one region to another, the anomalous dimension matrices, in the basis $O_+^{(q)}$, $O_-^{(q)}$ and $O_7^{(q)}$ and above m_q, have the general form:

$$\gamma = g^2 \begin{pmatrix} \gamma_+ & 0 & \gamma_{+7} \\ 0 & \gamma_- & \gamma_{-7} \\ 0 & 0 & \gamma_7 \end{pmatrix} \quad . \tag{A.7}$$

Below m_q, all entries are zero except γ_7. The transformation matrices that diagonalize the matrix γ in Eq. $(A.7)$ are of the form:

$$T = \begin{pmatrix} (\gamma_+ - \gamma_7)/\gamma_{+7} & 0 & 0 \\ 0 & (\gamma_- - \gamma_7)/\gamma_{-7} & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad . \tag{A.8}$$

For the "electromagnetic penguin" the anomalous dimension matrices to order g^2 are:

$$\gamma = \begin{pmatrix} g^2/4\pi^2 & 0 & -2g^2/9\pi^2 \\ 0 & -g^2/2\pi^2 & g^2/9\pi^2 \\ 0 & 0 & -(33-2N_f)g^2/24\pi^2 \end{pmatrix} \quad . \tag{A.9}$$

If we let $\gamma(\alpha)$ denote the eigenvalues of γ/g^2, then the solution of Eq. $(A.4)$ takes the form

$$A_i^{(q)}\left(\frac{M_W}{\mu}\right) = \sum_{\alpha,j} (T)_{i,\alpha} \, K_{W/\mu}^{r(\alpha)} \, (T^{-1})_{\alpha,j} \, A_j^{(q)}(1) \quad , \tag{A.10}$$

with Greek indices for the diagonal basis, the factor $K_{W/\mu} \equiv \alpha_s(M_W)/\alpha_s(\mu)$, $r(\alpha) \equiv 24\pi^2\gamma(\alpha)/(33-2N_f)$, and N_f denoting the number of quark flavors operative at the scale under consideration. For scales above the top quark mass, $N_f = 6$.

Similar steps in scale to that outlined for M_W to m_t, allow us to move from m_t to m_b, from m_b to m_c, and finally from m_c to μ, a somewhat ill-defined scale characteristic of a typical momentum in K decay. In consequence, the effective Hamiltonian at a scale μ below m_c is given by an expression of the form:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[V_{us}^* V_{ud}\Big(C_+ Q_+ + C_- Q_-\Big) + \sum_{q=u,c,t}\Big(V_{qs}^* V_{qd}\,\widetilde{C}_{7,q}\Big)Q_{7V}\right] \quad (A.11)$$

where

$$C_+ = K_{W/t}^{6/21}\,K_{t/b}^{6/23}\,K_{b/c}^{6/25}\,K_{c/\mu}^{6/27} \quad (A.12)$$

$$C_- = K_{W/t}^{-12/21}\,K_{t/b}^{-12/23}\,K_{b/c}^{-12/25}\,K_{c/\mu}^{-12/27} \quad (A.13)$$

$$\begin{aligned}\widetilde{C}_{7,u} = {} & A_7^{(u)}(1) + \left\{\left(1-K_{W/t}^{27/21}\right)\left(\frac{16}{81}\right) + \left(1-K_{W/t}^{9/21}\right)\left(-\frac{8}{27}\right)\right\}\frac{1}{\alpha_s(M_W)}\\ & + \left\{K_{W/t}^{6/21}\left(1-K_{t/b}^{29/23}\right)\left(\frac{16}{87}\right) + K_{W/t}^{-12/21}\left(1-K_{t/b}^{11/23}\right)\left(-\frac{8}{33}\right)\right\}\frac{1}{\alpha_s(m_t)}\\ & + \left\{K_{W/t}^{6/21}K_{t/b}^{6/23}\left(1-K_{b/c}^{31/25}\right)\left(\frac{16}{93}\right)\right.\\ & \qquad \left. + K_{W/t}^{-12/21}\,K_{t/b}^{-12/23}\left(1-K_{b/c}^{13/25}\right)\left(-\frac{8}{39}\right)\right\}\frac{1}{\alpha_s(m_b)}\\ & + \left\{K_{W/t}^{6/21}K_{t/b}^{6/23}\,K_{b/c}^{6/25}\left(1-K_{c/\mu}^{33/27}\right)\left(\frac{16}{99}\right)\right.\\ & \qquad \left. + K_{W/t}^{-12/21}K_{t/b}^{-12/23}\,K_{b/c}^{-12/25}\left(1-K_{c/\mu}^{15/27}\right)\left(-\frac{8}{45}\right)\right\}\frac{1}{\alpha_s(m_c)}\end{aligned} \quad (A.14)$$

$$\begin{aligned}\widetilde{C}_{7,c} = {} & A_7^{(c)}(1) + \left\{\left(1-K_{W/t}^{27/21}\right)\left(\frac{16}{81}\right) + \left(1-K_{W/t}^{9/21}\right)\left(-\frac{8}{27}\right)\right\}\frac{1}{\alpha_s(M_W)}\\ & + \left\{K_{W/t}^{6/21}\left(1-K_{t/b}^{29/23}\right)\left(\frac{16}{87}\right) + K_{W/t}^{-12/21}\left(1-K_{t/b}^{11/23}\right)\left(-\frac{8}{33}\right)\right\}\frac{1}{\alpha_s(m_t)}\\ & + \left\{K_{W/t}^{6/21}K_{t/b}^{6/23}\left(1-K_{b/c}^{31/25}\right)\left(\frac{16}{93}\right)\right.\\ & \qquad \left. + K_{W/t}^{-12/21}\,K_{t/b}^{-12/23}\left(1-K_{b/c}^{13/25}\right)\left(-\frac{8}{39}\right)\right\}\frac{1}{\alpha_s(m_b)}\end{aligned} \quad (A.15)$$

$$\widetilde{C}_{7,t} = A_7^{(t)}(1) + \left\{ \left(1 - K_{W/t}^{27/21}\right)\left(\frac{16}{81}\right) + \left(1 - K_{W/t}^{9/21}\right)\left(-\frac{8}{27}\right) \right\} \frac{1}{\alpha_s(M_W)} \quad . (A.16)$$

In order to fix the boundary conditions $A_7^{(q)}(1)$ at the scale of M_W, we require that the Hamiltonian of the free electroweak theory[29] coincide with the $\alpha_s \to 0$ limit of the effective Hamiltonian:

$$\begin{aligned} \mathcal{H}_{eff} \to \frac{G_F}{\sqrt{2}} \Big\{ & V_{us}^* V_{ud} \left[O_+^{(u)} + O_-^{(u)} + \left\{ A_7^{(u)}(1) - 2\pi(\gamma_{+7} + \gamma_{-7})\alpha_s \log\left(\frac{M_W^2}{\mu^2}\right) \right\} O_7 \right] \\ & + V_{cs}^* V_{cd} \left[A_7^{(c)}(1) - 2\pi(\gamma_{+7} + \gamma_{-7})\alpha_s \log\left(\frac{M_W^2}{m_c^2}\right) \right] O_7 \\ & + V_{ts}^* V_{td} \left[A_7^{(t)}(1) - 2\pi(\gamma_{+7} + \gamma_{-7})\alpha_s \log\left(\frac{M_W^2}{m_t^2}\right) \right] O_7 \Big\} \quad . \qquad (A.17) \end{aligned}$$

In addition, we must consider the matrix element of $O_{\pm}^{(u)}$ to one loop order, which is:

$$< O_+^{(u)} + O_-^{(u)} > = 2\pi(\gamma_{+7} + \gamma_{-7})\alpha_s \log\left(\frac{m_u^2}{\mu^2}\right) < O_7 > \quad . \qquad (A.18)$$

Equations $(A.17)$ and $(A.18)$, while written for the $\alpha_s \to 0$ limit, are illustrative of general properties with respect to μ dependence, renormalization-scheme-dependent matrix elements, and subleading terms in $\mathcal{H}_{eff}$. Although these points are only of academic interest (see below) for our calculation of the contributions to "direct" CP violation, we note them here for completeness. First, the μ dependence explicitly cancels between Eqs. $(A.17)$ and $(A.18)$, as it should. Second, there are possible subleading terms on the right-hand side of Eq. $(A.18)$ which depend on the renormalization scheme, as do subleading terms in $\mathcal{H}_{eff}$. Since we use the anomalous dimensions and beta function calculated in leading order we do not consistently predict subleading terms in the expansion of $\mathcal{H}_{eff}$; consequently only the leading logarithmic terms in Eq. $(A.18)$ are meaningful. The subleading terms are introduced only as boundary conditions in $A_7^{(q)}(1)$, which are obtained by comparing the free Hamiltonian with the limit of the effective Hamiltonian in Eq. $(A.17)$:

$$A_7^{(q)}(1) \;=\; \widetilde{C}_{7V,q}^{(\gamma)}(M_W^2) + \frac{2}{9\pi} log\ x_q \quad , \qquad (A.19)$$

where the $\widetilde{C}_{7V,q}^{(\gamma)}(M_W^2)$ are given in Eq. (8).

While the Hamiltonian separates into three pieces, for $q = u, c$ and t, there are cancellations among these terms due to unitarity of the Kobayashi–Maskawa matrix. In particular, $V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$ implies that the "electromagnetic penguin" contributions to O_7 due to mixing from $O_\pm$ cancel in the region between M_W and m_t. Finally, but very importantly, the contributions to "direct" CP violation in $K_L \to \pi^0 \ell^+ \ell^-$ come from $Im\mathcal{H}$. Since $V_{us}^* V_{ud}$ is real, the contribution from the "electromagnetic penguin" are restricted to the region between m_t (or M_W, if $m_t \gtrsim M_W$) and m_c, and the matrix elements of $O_\pm^{(u)}$ are irrelevant [as are subleading, renormalization-scheme-dependent constants in $A_7^{(q)}(1)$].

The above expressions were developed for $m_t^2 << M_W^2$. The case where $m_t \gtrsim M_W$ can be easily obtained by simply letting $K_{W/t} \to 1$, $K_{t/b} \to K_{W/b}$, and $\alpha_s(m_t) \to \alpha_s(M_W)$ in Eqs. $(A.11)$ to $(A.16)$ and dropping the second term on the right-hand side of Eq. $(A.19)$ for $q = t$. Using the unitarity relation of the Kobayashi–Maskawa matrix, the terms involving $V_{us}^* V_{ud}$ in Eq. $(A.11)$ can be absorbed into the other terms, casting the expression for the effective Hamiltonian into a form identical to Eq. (5) in the text; then the coefficients $\widetilde{C}_{7V,i}^{(\gamma)}(\mu^2) = \widetilde{C}_{7,i} - \widetilde{C}_{7,u}$ can be read off and proven to agree with the expressions in Eqs. (17) and (19) of Sec. IV.

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FIGURE CAPTIONS

1) Three diagrams giving a short distance contribution to the process $K \to \pi\ell^+\ell^-$: (a) the "electromagnetic penguin;" (b) the "Z penguin;" (c) the "W box."

2) Diagrams involving $K_2 \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$ which give a CP conserving contribution to $K_L \to \pi^0\ell^+\ell^-$.

3) $\widetilde{C}_{7V}^{(\gamma)} = \widetilde{C}_{7V,t}^{(\gamma)} - \widetilde{C}_{7V,c}^{(\gamma)}$ as a function of m_t without (dashed curve) and with (solid curves) QCD corrections for $\Lambda_{QCD} = 100$ and 250 MeV.

4) Contributions to the coefficient $\widetilde{C}_{7V}$ from each of its components, the "electromagnetic penguin," the "Z penguin" and the "box" diagrams and the total $\widetilde{C}_{7V}$ with QCD corrections (solid curves) with $\Lambda_{QCD} = 150$ MeV, and the total coefficient without QCD corrections (dashed curve) as a function of m_t.

5) Contributions to the coefficient $\widetilde{C}_{7A}$ from the "Z penguin" and "box" diagrams as a function of m_t.

6) The quantities $(\widetilde{C}_{7V})^2$ and $(\widetilde{C}_{7A})^2$ as a function of m_t, and their sum, $(\widetilde{C}_{7V})^2 + (\widetilde{C}_{7A})^2$, with (solid curve, $\Lambda_{QCD} = 150$ MeV) and without (dashed curve) QCD corrections, which enters the branching ratio induced for $K_L \to \pi^0\ell^+\ell^-$ by CP violation in the decay amplitude.

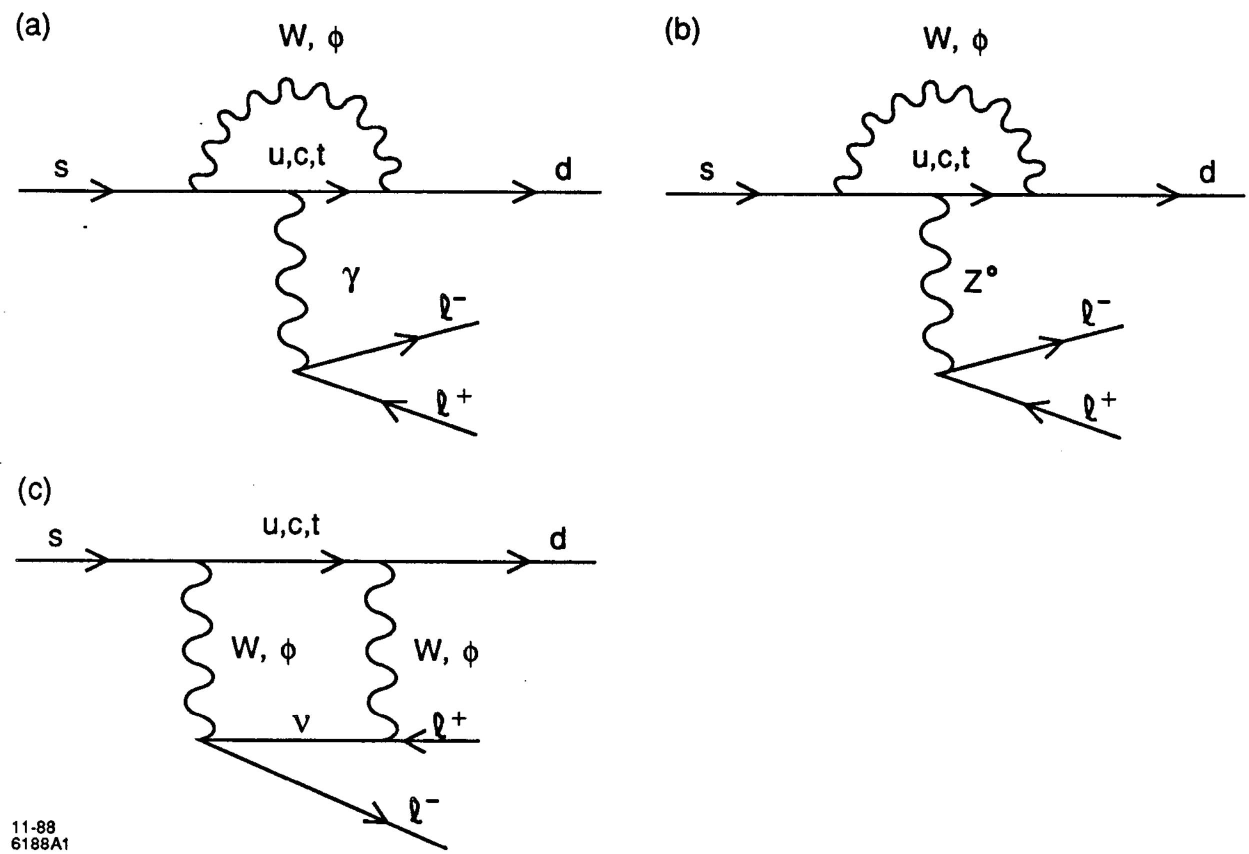


Fig. 1

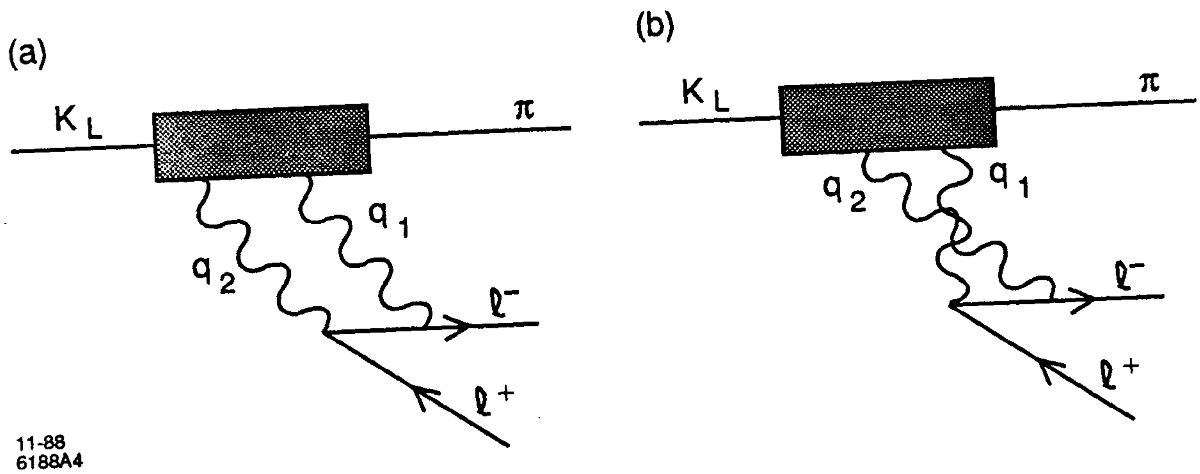
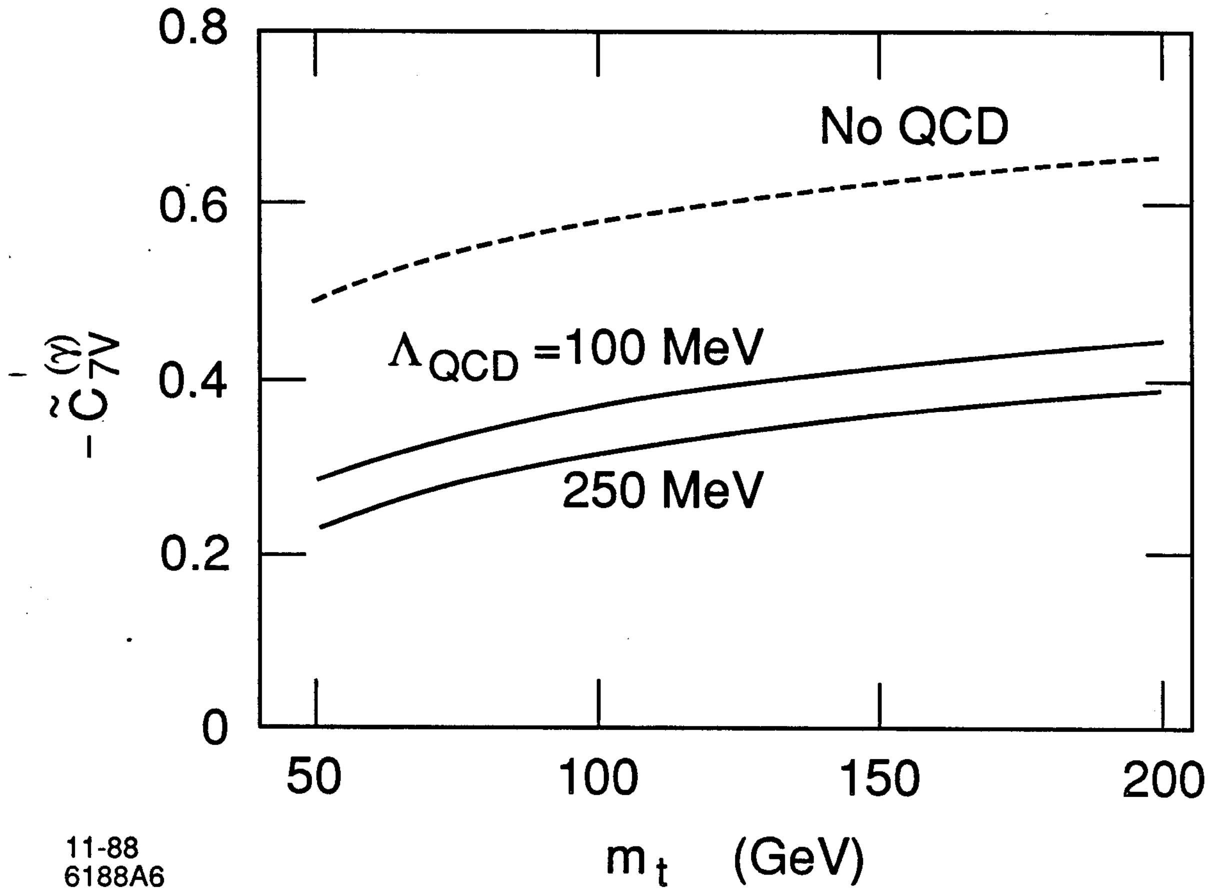
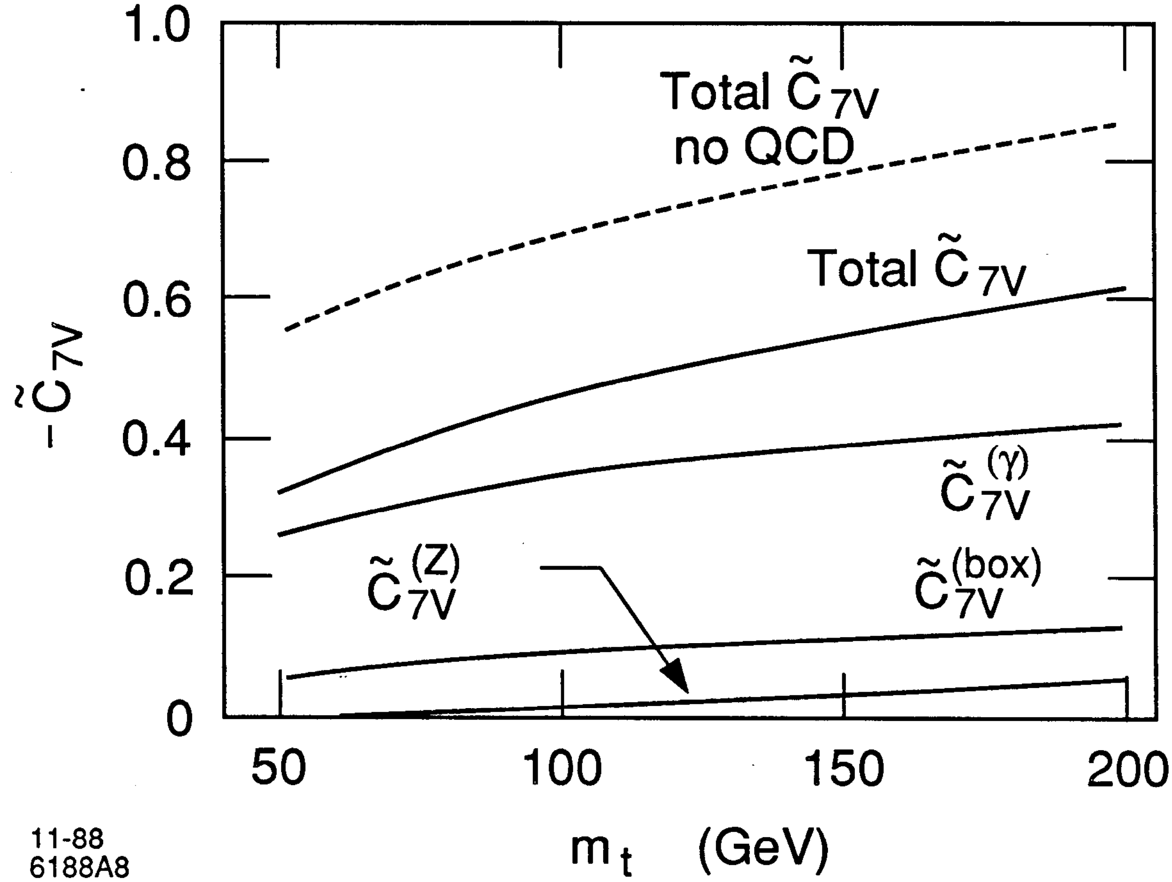


Fig. 2



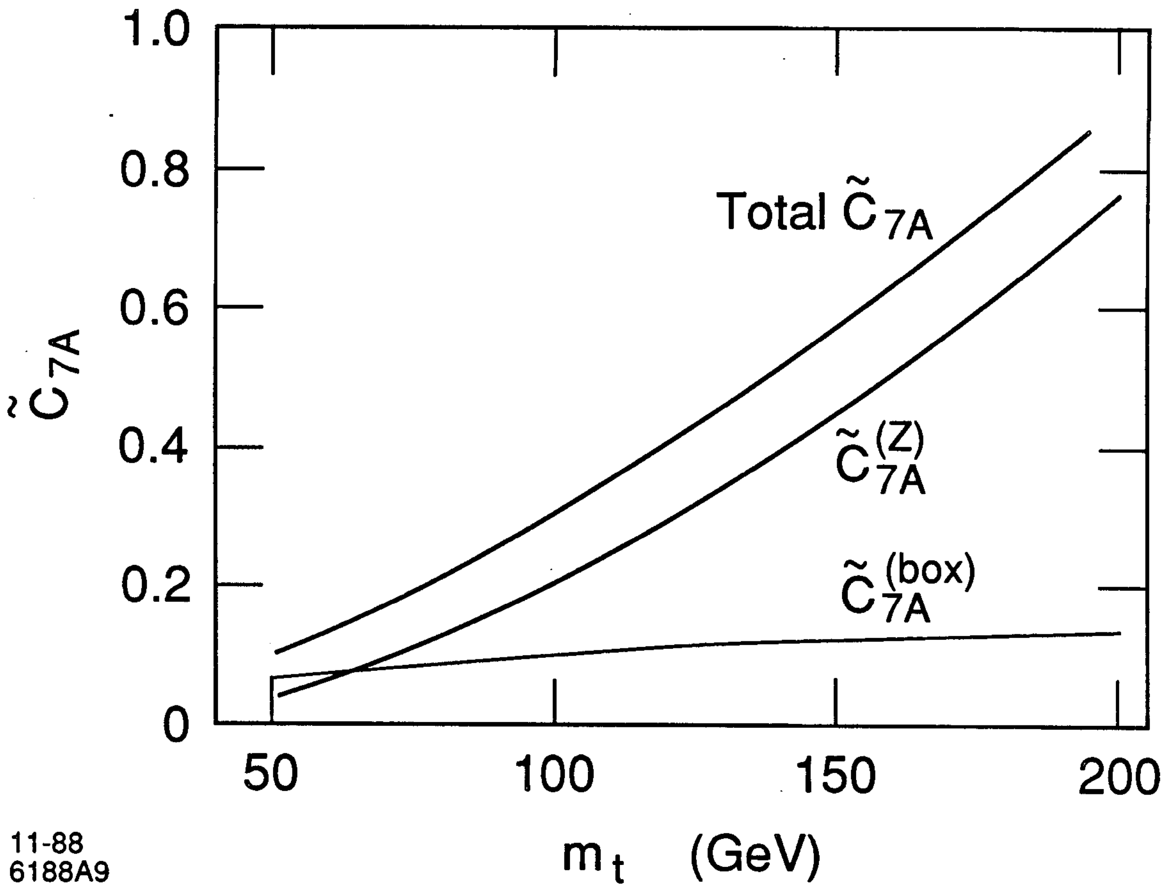
0.8
0.6
0.4
0.2
0
No QCD
Λ_{QCD} =100 MeV
250 MeV
$-\tilde{C}_{7V}^{(\gamma)}$
50
100
150
200
m_t (GeV)
11-88
6188A6

Fig. 3



1.0
0.8
0.6
0.4
0.2
0
$-\tilde{C}_{7V}$
Total $\tilde{C}_{7V}$
no QCD
Total $\tilde{C}_{7V}$
$\tilde{C}_{7V}^{(\gamma)}$
$\tilde{C}_{7V}^{(Z)}$
$\tilde{C}_{7V}^{(box)}$
50
100
150
200
m_t (GeV)
11-88
6188A8

Fig. 4



1.0
0.8
0.6
0.4
0.2
0
$\tilde{C}_{7A}$
Total $\tilde{C}_{7A}$
$\tilde{C}_{7A}^{(Z)}$
$\tilde{C}_{7A}^{(box)}$
50
100
150
200
m_t (GeV)
11-88
6188A9

Fig. 5

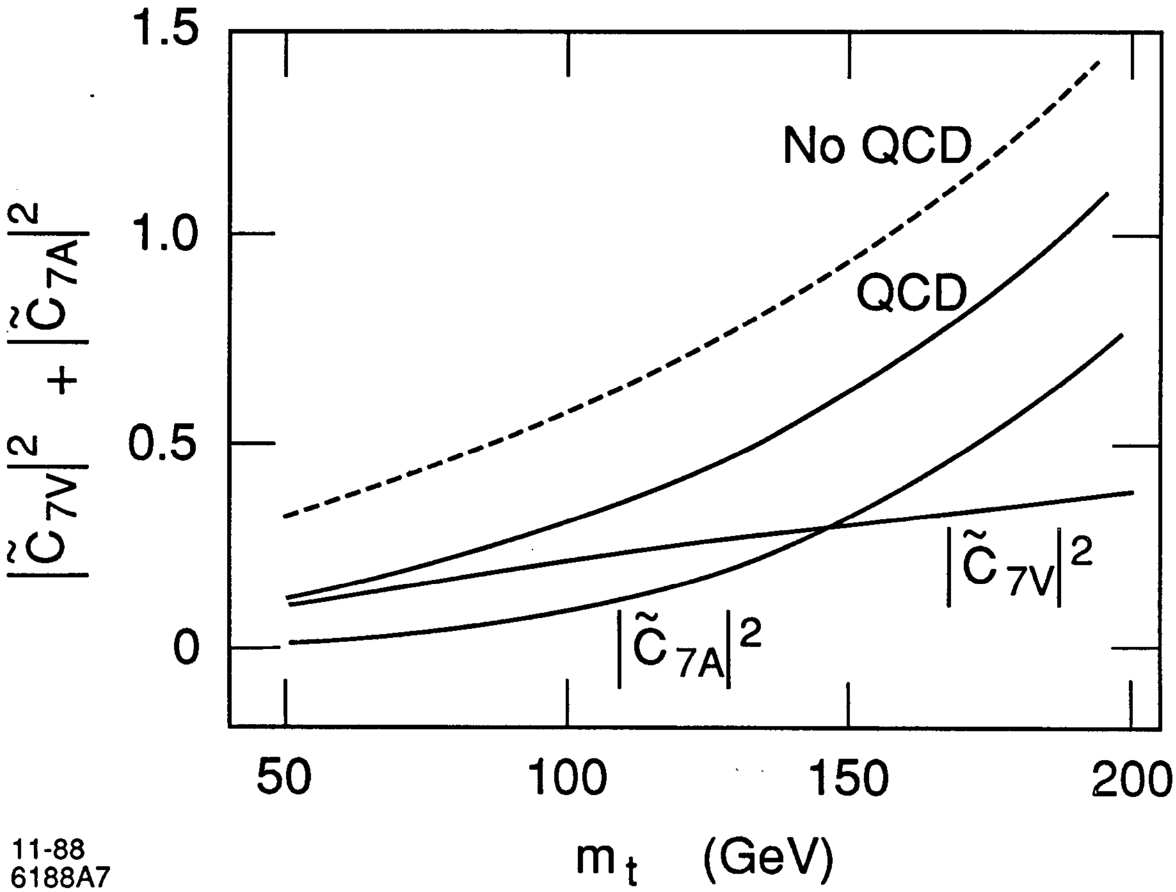

1.5
1.0
0.5
0
$|\tilde{C}_{7V}|^2 + |\tilde{C}_{7A}|^2$
No QCD
QCD
$|\tilde{C}_{7V}|^2$
$|\tilde{C}_{7A}|^2$
50
100
150
200
m_t (GeV)
11-88
6188A7

Fig. 6