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# CP Violation in the $K_L \to \pi^0 \ell^+ \ell^-$ Decay Amplitude for Large $m_t$ ⋆

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## ABSTRACT

We discuss CP violation in the amplitude for $K_L \to \pi^0 \ell^+ \ell^-$ when $m_t$ is large. Unlike the case of $K_L \to \pi\pi$, CP violation in the decay amplitude itself is comparable to that which comes from the mass matrix. We study the CP violating effects, including strong interaction (QCD) corrections to the amplitudes which arise from one-loop diagrams. Short-distance contributions from diagrams that involve a $W$ and a $Z$ or two $W$'s as well from those with a photon and a $W$ are important when $m_t \gtrsim M_W$.

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It is almost 25 years since the original observation of CP violation in long-lived neutral $K$ decays.[1] Until very recently, all experiments were consistent with this phenomenon originating in a "superweak" interaction,[2] whose one measurable manifestation was in the mass matrix of the neutral $K$ system. As a result, the long-lived neutral $K$ meson, $K_L \approx K_2 + \epsilon K_1$, is dominantly the CP odd state $K_2$, but contains a small admixture ($\propto \epsilon$) of the CP even state $K_1$.

A different, more definite origin of CP violation occurs in the three generation standard model through the presence of a single, non-trivial phase in the matrix which expresses the mixing of quark flavors under the weak interactions.[3] This model can explain CP violation in the mass matrix of neutral mesons and predicts CP violation in decay amplitudes as well.

In the past year the NA31 collaboration has presented statistically significant evidence[4] for a non-zero value of the parameter $\epsilon'$, which is a measure of CP violation in the $K \rightarrow \pi\pi$ decay amplitude. Experiments at Fermilab[5] and at CERN[4] are continuing with the aim of reducing the statistical and systematic errors to a level where, if the central value of the CERN experiment holds, a non-zero value of $\epsilon'$ will be firmly established and a "superweak" explanation made untenable. Such a value of $\epsilon'$ is consistent, within rather large uncertainties of the relevant hadronic matrix element, with the three generation standard model.

While the three generation standard model may explain CP violation as it is observed up to now in Nature, we would like to obtain additional evidence that points in this direction. If we could find several experimental processes which exhibit measurable CP violating effects and all could be fit by a <u>single</u> value of the *ab initio* free phase in the mixing matrix, then we will have gone a long way toward establishing this as the correct explanation.

$K_L \rightarrow \pi^0 \ell^+ \ell^-$ is one such process. If CP were conserved, the long-lived eigenstate would be the CP odd state, $K_2$. It would not decay to $\pi^0 \gamma_{\text{virtual}} \rightarrow \pi^0 \ell^+ \ell^-$, this being forbidden by CP invariance. Since Nature has chosen to break CP invariance, the decay can proceed through: (1) the small part, $\approx \epsilon K_1$, of the $K_L$

wave function that is CP even (we call this "indirect" CP violation); and (2) CP violating effects in the $K_2 \to \pi^0 \ell^+ \ell^-$ decay amplitude itself (we call this "direct" CP violation). In addition to these two CP violating amplitudes, the decay can proceed in a CP conserving manner *via* the decay chain $K_2 \to \pi^0 \gamma\gamma \to \pi^0 \ell^+ \ell^-$. Although higher order in $\alpha$, this latter amplitude is not necessarily negligible in comparison to either the "indirect" or "direct" CP violating amplitudes.

Naturally, we are most interested in the question of whether one can see the "direct" CP violation effects and especially to investigate if they can be the dominant amplitude contributing to the decay. This amplitude comes from "penguin" diagrams with a photon or $Z$ boson and also from box diagrams, as shown in Figure 1. For values of $m_t^2 << M_W^2$, it is the "electromagnetic penguin" that gives the dominant short-distance contribution to the amplitude. This was discussed, with estimates of the CP violating effects,[6] before evidence for the $b$ quark was found. A full analysis, including QCD corrections, was carried out in the case of six quarks,[7] building upon work done with four quarks.[8] A principal conclusion of that study was that the "direct" CP violation could be comparable to the "indirect" effects. However, the expected mass range for the $t$ quark has been pushed upward considerably since Ref. 7. As a consequence, the QCD corrections, which turned out to be quite important, need to be redone when $m_t^2/M_W^2$ can not be considered to be a small number. Also, the "$Z$ penguin" and "$W$ box" diagrams, which are "suppressed" by factors of $m_t^2/M_W^2$ and were neglected in old calculations, are important for large $m_t$. We need to consider the QCD corrections to them as well. A further motivation is that experiments at the required level of sensitivity are beginning to be considered.[9,10]

We calculate the CP violating contributions to the $K_2 \to \pi^0 \ell^+ \ell^-$ amplitude in the standard model with six quarks arranged in left-handed doublets with respect to weak isospin. Our calculation is expressed in the language of forming an effective Hamiltonian written in terms of the low mass quarks $u$, $d$, and $s$ which are involved in the initial and final states of strange particle decays. The calculation proceeds by starting with the theory written in terms of the weak gauge boson and quark fields, and successively integrating out the heavy quanta from the theory.[11] We consider $t$ quark masses comparable or greater than that of the $W$, and remove the $t$ quark and $W$ from the theory together.[12]

At each stage of the calculation we will be left with an effective Hamiltonian in the form of a sum of Wilson coefficients times operators:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\, V_{us}^* V_{ud} \sum_i C_i(\mu^2) Q_i \;+\; h.\ c.\ , \tag{1}$$

where $Q_1$ through $Q_6$ occur in the usual calculation of the strangeness-changing weak Hamiltonian,[11] while the operators

$$\begin{aligned} Q_{7V} &= \frac{e^2}{4\pi}(\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{e}\gamma^\mu e) \\ Q_{7A} &= \frac{e^2}{4\pi}(\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{e}\gamma^\mu \gamma_5 e) \end{aligned} \tag{2}$$

are new and explicitly of order $e^2$. The color indices $\alpha$ and $\beta$ are summed, while the combination $V_{us}^* V_{ud}$ of Kobayashi-Maskawa matrix elements is the usual one involved in decays of strange particles. We have chosen the same operators as in Ref. 7, with the addition of $Q_{7A}$, whose presence is required now that we include the contributions from the "Z penguin" and "W box" graphs in Figure 1.[13]

In the absence of strong interactions, the only one of the first six operators with a non-zero coefficient (to order $g^2$ in weak interactions) is $Q_2$, with $c_2 = 1$. To one loop order in electroweak interactions, the diagrams in Figure 1 generally give non-zero coefficients[12] of $Q_{7V}$ and $Q_{7A}$. For example, if we consider $m_t \sim M_W$, then at the scale $M_W$, we have a contribution involving the $t$ quark (with $i = t$):

$$\widetilde{C}^{(\gamma)}_{7V,i}(M_W^2) = \frac{(25-19x_i)x_i^2}{72\pi(x_i-1)^3} - \frac{(3x_i^4 - 30x_i^3 + 54x_i^2 - 32x_i + 8)log(x_i)}{36\pi(x_i-1)^4}\ , \tag{3}$$

$$\widetilde{C}^{(Z)}_{7V,i}(M_W^2) = \frac{4\sin^2\theta_W - 1}{\sin^2\theta_W}\,\frac{x_i}{16\pi}\left[\frac{(x_i-6)(x_i-1) + (3x_i+2)log(x_i)}{(x_i-1)^2}\right]\ , \tag{4}$$

$$\widetilde{C}^{(Z)}_{7A,i}(M^2_W) = \frac{1}{\sin^2\theta_W}\frac{x_i}{16\pi}\left[\frac{(x_i-6)(x_i-1)+(3x_i+2)log(x_i)}{(x_i-1)^2}\right] ; \tag{5}$$

$$\widetilde{C}^{(Box)}_{7V,i}(M^2_W) = -\,\widetilde{C}^{(Box)}_{7A,i}(M^2_W) = \frac{1}{\sin^2\theta_W}\frac{x_i}{8\pi}\left[\frac{1-x_i+log(x_i)}{(x_i-1)^2}\right] \tag{6}$$

where $x_i = m_i^2/M_W^2$ and the tilde over the coefficient means that the Kobayashi - Maskawa factor has been removed:

$$C_{7,i}(M_W^2) = \frac{V^*_{is}V_{id}}{V^*_{us}V_{ud}}\,\widetilde{C}_{7,i}(M_W^2)\ . \tag{7}$$

The full contribution of the $t$ quark to $\widetilde{C}^{(\gamma)}_{7V}$ at the scale $\mu$ is given by:

$$\widetilde{C}^{(\gamma)}_{7V,t}(\mu^2) = \widetilde{C}^{(\gamma)}_{7V,t}(M_W^2) - \frac{2}{9\pi}\int_{\mu^2}^{M_W^2}\frac{dq^2}{q^2}\left[C_2+3C_1\right]\ , \tag{8}$$

where, since we are considering $m_t \sim M_W$ and there are no large logarithms of the form $log(M_W^2/m_t^2)$, we take the full expression for $C_{7V,t}(M_W^2)$ as given in Eq. (3). Since in the absence of QCD the coefficients $C_2 = 1$ and $C_1 = 0$, the integral contributes the large logarithm in the problem,

$$-\frac{2}{9\pi}\ log\left(\frac{M_W^2}{\mu^2}\right)\ ,$$

to the right-hand-side of Eq. (8).

Note that if we had considered the situation where $m_i^2 << M_W^2$, *i.e.*, $x_i << 1$, then the contribution from the quark $i$ is generated at scales from $m_i$ down to $\mu$ and the leading term is

$$\widetilde{C}_{7V,i}(\mu^2) \approx \widetilde{C}^{(\gamma)}_{7V,i}(\mu^2) \approx -\left(\frac{2}{9\pi}\right)\ log\frac{m_i^2}{\mu^2}\ , \tag{9}$$

as in Ref. 7. The other contributions in Eqs. (4) - (6) due to the "$Z$ penguin" and "$W$ box" graphs, respectively, all vanish in comparison to Eq. (9) in the same limit by at least one power of $x_i$. In the limit $x_i \to 0$ such non-leading contributions are numerically small and therefore dropped, as are the non-leading terms in the "electromagnetic penguin" contribution.

Even though there is a $\mu$ dependence in the Wilson coefficient in Eq. (1), we know that there can be no dependence upon $\mu$ in the total amplitude, as it represents a physical observable. This $\mu$ dependence is cancelled by a corresponding dependence which occurs when we take the matrix elements of the effective Hamiltonian, $\mathcal{H}$, to order $e^2$.

Now let us introduce the strong interactions in the form of Quantum Chromodynamics (QCD). First, to order $e^0$, non-zero coefficients are generated for the first six operators as we move successively down from the weak scale to one quark mass and then another. The operators $Q_3$, $Q_4$, $Q_5$, and $Q_6$ arise from "penguin" diagrams involving gluons. The operators $Q_\pm = \frac{1}{2}[Q_2 \pm Q_1]$ are multiplicatively renormalized:

$$C_\pm(\mu^2) = \left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right]^{a_\pm} C_\pm(M_W^2) \tag{10}$$

where $a_+ = 6/(33-2N_f)$ and $a_- = -12/(33-2N_f)$ for $N_f$ quark flavors in leading logarithmic approximation between the scale $M_W$ and the scale $\mu$. At the same time, to order $e^2$ the coefficients of the operators $Q_{7V}$ and $Q_{7A}$ are generated from their value at $M_W$ plus mixing effects of the operators $Q_1$ and $Q_2$ with $Q_{7V}$ or $Q_{7A}$. The "penguin" operators, $Q_3$, $Q_4$, $Q_5$, and $Q_6$, which arise only through QCD effects, have coefficients which start out at zero at the weak scale. They typically never grow to be more than an order of magnitude smaller than the coefficients for $Q_\pm$. So, it is an excellent approximation[7] to consider the mixing only of $Q_\pm$ with $Q_{7V}$ and $Q_{7A}$ and the renormalization of $Q_\pm$ as in Eq. (10). In the same spirit we neglect the effect of taking matrix elements of the "penguin operators" to order $e^2$, which also give a small effect.

The derivation of the QCD corrected contributions when $m_t \sim M_W$ proceeds in a straightforward manner, if one follows the general method given in Ref. 7. We have also derived the same result following along the lines of Ref. 8 to obtain:

$$
\begin{aligned}
\widetilde{C}^{(\gamma)}_{7V,t}(\mu^2) = \; & \widetilde{C}^{(\gamma)}_{7V,t}(M_W^2) \\
-\frac{16}{99\alpha_s(m_c^2)}\left(1-K_{\mu/c}^{-33/27}\right)K_{c/b}^{-6/25}K_{b/W}^{-6/23} & - \frac{16}{93\alpha_s(m_b^2)}\left(1-K_{c/b}^{-31/25}\right)K_{b/W}^{-6/23} \\
-\frac{16}{87\alpha_s(M_W^2)}\left(1-K_{b/W}^{-29/23}\right) & + \frac{8}{45\alpha_s(m_c^2)}\left(1-K_{\mu/c}^{-15/27}\right)K_{c/b}^{12/25}K_{b/W}^{12/23} \\
+\frac{8}{39\alpha_s(m_b^2)}\left(1-K_{c/b}^{-13/25}\right)K_{b/W}^{12/23} & + \frac{8}{33\alpha_s(M_W^2)}\left(1-K_{b/W}^{-11/23}\right) ,
\end{aligned}
\tag{11}
$$

where we use $K_{b/W} = \alpha_s(m_b^2)/\alpha_s(M_W^2)$, $K_{c/b} = \alpha_s(m_c^2)/\alpha_s(m_b^2)$, and $K_{\mu/c} = \alpha_s(\mu^2)/\alpha_s(m_c^2)$ in effective five, four and three quark theories, respectively, and

$$
\begin{aligned}
\widetilde{C}^{(\gamma)}_{7V,c}(\mu^2) = & -\frac{16}{99\alpha_s(m_c^2)}\left(1-K_{\mu/c}^{-33/27}\right)K_{c/b}^{-6/25}K_{b/W}^{-6/23} \\
& + \frac{8}{45\alpha_s(m_c^2)}\left(1-K_{\mu/c}^{-15/27}\right)K_{c/b}^{12/25}K_{b/W}^{12/23} .
\end{aligned}
\tag{12}
$$

For $K_2 \to \pi^0 \ell^+ \ell^-$ it is only the imaginary (CP violating) part of the amplitude which contributes. The rephase invariant Kobayashi–Maskawa factors for charm and top are the same, up to a sign:

$$
Im\ \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = -Im\ \frac{V_{cs}^* V_{cd}}{V_{us}^* V_{ud}} = \frac{Im\ [V_{ts}^* V_{td} V_{us} V_{ud}^*]}{|V_{us} V_{ud}|^2} . \tag{13}
$$

In the original parametrization of Ref. 3, the quantities in Eq. (13) are expressible as $\sin\theta_2 \sin\theta_3 \sin\delta = s_2 s_3 s_\delta$, with cosines of small angles set equal to unity. Although scales down to $\mu$ enter the $c$ and $t$ quark contributions separately, it is only momentum scales from $m_c$ to $m_t$ that contribute to the imaginary part. The dependence on the scale $\mu$ cancels out and while the QCD corrections are non-negligible, they are fairly insensitive to changes in parameters. This is shown in Figure 2, where the QCD corrected $\widetilde{C}^{(\gamma)}_{7V} = \widetilde{C}^{(\gamma)}_{7V,t} - \widetilde{C}^{(\gamma)}_{7V,c}$, calculated from Eqs. (11) and (12), is indicated with solid curves for $\Lambda_{QCD} = 100$ and 250 MeV as a function of the top quark mass. While about a factor of two smaller than the result without QCD (dashed curve), the result does not depend strongly on $\Lambda_{QCD}$ or top quark mass.

To assemble the full coefficient, $C_{7V}$, we need to add the "$Z$ penguin" and "$W$ box" contributions. Those involving the $t$ quark may be taken directly from Eqs. (4) and (6), respectively, as these contributions are generated at momentum scales from $m_t$ to $M_W$ where there are no large logarithms to which QCD corrections are applied. For those contributions involving the $c$ quark, there are important QCD corrections. However, these contributions, being proportional to $x_c = m_c^2/M_W^2$, are themselves so small as to be negligible. Even after being reduced by QCD corrections, the contribution to $\widetilde{C}_{7V}$ from the "electromagnetic penguin," $\widetilde{C}_{7V}^{(\gamma)}$, is the largest, because the "$Z$ penguin" is suppressed by the small $Z$ vector coupling to charged leptons. The "Z penguin" contribution does dominate at large $m_t$ in the coefficient $\widetilde{C}_{7A}$ where the axial-vector coupling of the $Z$ to charged leptons enters. $\widetilde{C}_{7A}$ overtakes $\widetilde{C}_{7V}$ in magnitude when $m_t \approx 150$ GeV.

To proceed to actual branching ratios, we may avoid some arithmetic by relating the hadronic matrix element of the operator, $\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha$, which occurs in $Q_{7V}$ and $Q_{7A}$, to that of the corresponding charged current operator, $\bar{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha$, which occurs in $K_{\ell 3}$ decay, so that form factors and phase space are automatically taken care of by measurement of the latter decay.[14] Using this, we find:

$$B(K_2 \to \pi^0 e^+ e^-) = 1.0 \times 10^{-5}\ (s_2 s_3 s_\delta)^2\ \left[(\widetilde{C}_{7V})^2 + (\widetilde{C}_{7A})^2\right] . \qquad (14)$$

The factor in square brackets is shown in Figure 3. With QCD corrections, and with $m_t$ between 50 and 200 GeV, it ranges between about 0.1 and 1.0. From measurements of Kobayashi–Maskawa matrix elements, $s_2 s_3 s_\delta \leq 2.5 \times 10^{-3}$. While the combination $s_2 s_3 s_\delta$ enters other CP violating quantities such as $\epsilon$ and $\epsilon'$, imprecisely known hadronic matrix elements and $m_t$ presently allow a broad range of values of this combination. For $m_t$ at the low end of the acceptable range (as constrained by $B^0 - \bar{B}^0$ mixing), the allowed region of Kobayashi - Maskawa parameters contracts and $s_2 s_3 s_\delta$ must be quite close to $10^{-3}$. More generally, a typical value is in this neighborhood. Putting this information into Eq. (14) we see that the branching ratio for $K_L \to \pi^0 e^+ e^-$ from CP violation in the decay amplitude alone is around $10^{-11}$.

From our present knowledge, the three contributions to the process $K_L \to \pi^0 \ell^+ \ell^-$ could each give rise to a branching ratio in the $10^{-11}$ range. With further theoretical and/or experimental work, it is possible that the CP conserving contribution might yet be shown to be well below this level.[10] CP violation in the mass matrix and the decay amplitude, however, give comparable contributions, roughly at the $10^{-11}$ level in branching ratio. In general, they will interfere in the expression for the total decay rate.

Some care must be exercised about phase conventions in calculating this interference. In the standard phase convention, where the $K \to \pi\pi(I=0)$ amplitude is chosen to be real, the small CP-even admixture in $K_L$ is $\epsilon \approx (2.275 \times 10^{-3}) e^{i\pi/4}$. However, at the quark level, penguin diagrams induce a small phase $\xi$ into $K \to \pi\pi(I=0)$. As a result, in the amplitude for "indirect" CP violation, $\epsilon \to \epsilon - i\xi$, if $|\xi|$ is small. A somewhat abbreviated expression for the branching ratio in $K_L \to \pi^0 e^+ e^-$ from all CP violating effects is then,

$$B(K_L \to \pi^0 e^+ e^-) \approx \Bigg[\Bigg|0.76 \left(e^{i\pi/4} - i\frac{\xi}{|\epsilon|}\right)\left(\frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)}\right)^{1/2} + \left(\frac{s_2 s_3 s_\delta}{10^{-3}}\right)\widetilde{C}_{7V}\Bigg|^2 + \Bigg|\left(\frac{s_2 s_3 s_\delta}{10^{-3}}\right)\widetilde{C}_{7A}\Bigg|^2\Bigg] \cdot 10^{-11} , \tag{15}$$

where we have taken into account the phase conventions mentioned above. Eq. (15) indicates the interference of amplitudes coming from "indirect" and "direct" CP violation. Neglected is the fact that the two interfering amplitudes (which involve vector coupling to the lepton pair) can have a different dependence on the pair invariant-mass and the interference can then vary with this quantity. If both amplitudes came from short-distance effects (which is very unlikely[7] for the "indirect" CP violation), then the interference is the same for all values of the pair invariant-mass and Eq. (15) stands as written.

Since $\epsilon'/\epsilon \approx 3 \times 10^{-3} = -15.6\xi$, the extra piece from the change of basis is small, but interferes constructively with that from $\epsilon$. A definitive conclusion as to the relative magnitudes of the contributions from "indirect" and "direct" CP violation depends on further knowledge of $A(K_1 \to \pi^0 e^+ e^-)$, $s_2 s_3 s_\delta$, and $m_t$. Similarly, constructive or destructive interference between these terms requires a model for the long-distance effects which are inherent in the $K_1 \to \pi^0 e^+ e^-$ amplitude. As $m_t$ becomes larger, more of the "direct" CP violation comes through $Q_{7A}$. As a result, the theoretical predictions become more definitive, as the QCD corrections to $C_{7A}$ are very small and this contribution does not interfere in the expression for the decay rate with that from "indirect" CP violation. Even for large $m_t$, however, it is hard to get a branching ratio that is more than a few times $10^{-11}$.

We have a major advantage over calculations of other CP violating effects in the $K^0$ system in that the hadronic matrix element of the relevant operators ($Q_{7V}$ and $Q_{7A}$) from the short-distance physics is given to us from $K_{\ell 3}$ decay. There is no uncertainty here. Nevertheless, we would assign an uncertainty from the QCD corrections, the neglect of non-leading QCD terms, and possible "direct" CP violating contributions from order $e^2$ matrix elements of $Q_1$ to $Q_6$, of 10 to 20% for $C_{7V}$, even if we knew $m_t$ precisely along with all the Kobayashi–Maskawa parameters. Conversely, if there were both a precise measurement of $m_t$ and of the $K_L \to \pi^0 \ell^+ \ell^-$ branching ratio that resulted in an isolation of the amplitude for "direct" CP violation, there would be an uncertainty of this magnitude in the extracted value of $s_2 s_3 s_\delta$. While not as precise as one might like, this would be far better than the determination from $\epsilon$ and $\epsilon'$, where non-trivial hadronic matrix elements enter.

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Snowmass provided stimulating questions and comments. We thank J. Donoghue and M. Savage for discussions, and B. Grinstein and M. Wise for sharing their expertise on the intricacies of calculating QCD corrections.

## FIGURE CAPTIONS

1) Three diagrams giving a short distance contribution to the process $K \to \pi\ell^+\ell^-$: (a) the "electromagnetic penguin;" (b) the "$Z$ penguin;" (c) the "$W$ box."

2) $\widetilde{C}_{7V}^{(\gamma)} = \widetilde{C}_{7V,t}^{(\gamma)} - \widetilde{C}_{7V,c}^{(\gamma)}$ as a function of $m_t$ without (dashed curve) and with (solid curves) QCD corrections for $\Lambda_{QCD} = 100$ and 250 MeV.

3) The quantities $(\widetilde{C}_{7V})^2 = (\widetilde{C}_{7V,t} - \widetilde{C}_{7V,c})^2$ and $(\widetilde{C}_{7A})^2 = (\widetilde{C}_{7A,t} - \widetilde{C}_{7A,c})^2$ as a function of $m_t$, and their sum, $(\widetilde{C}_{7V})^2 + (\widetilde{C}_{7A})^2$, with (solid curve) and without (dashed curve) QCD corrections, which enters the branching ratio induced for $K_L \to \pi^0\ell^+\ell^-$ by CP violation in the decay amplitude.

## REFERENCES

1. J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).

2. L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

3. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

4. H. Burkhardt *et al.*, Phys. Lett. 206B, 169 (1988).

5. M. Woods *et al.*, Phys. Rev. Lett. 60, 1695 (1988).

6. J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B109, 213 (1976).

7. F. J. Gilman and M. B. Wise, Phys. Rev. D21, 3150 (1980).

8. A. I. Vainshtein *et al.*, Yad. Fiz. 24, 820 (1976) [Sov. J. Nucl. Phys. 24, 427 (1976)]; V. A. Novikov *et al.*, Phys. Rev. D16, 223 (1977); V. V. Flambaum, Yad. Fiz. 22, 661 (1975) [Sov. J. Nucl. Phys. 22, 340 (1975)]; D. V. Nanopoulos and G. G. Ross, Phys. Lett. 56B, 279 (1975); L. E. Ibanez *et al.*, Phys. Rev. D21, 1428 (1980).

9. See, for example, the report on $K$ physics by the group led by R. Bock and L. Littenberg, in *Proceedings of the Summer Study on High Energy Physics in the 1990s*, Snowmass, June 27 - July 15, 1988 (unpublished).

10. A preliminary report of this work was given by C. Dib at the Summer Study on High Energy Physics in the 1990's, Snowmass, June 27 - July 15, 1988 (unpublished) and F. J. Gilman, lectures at the Heavy Flavor Physics Symposium, Beijing, August 10-20, 1988 and SLAC preprint SLAC-PUB-4736, 1988 (unpublished). A full account of our work and references to previous work will be given in C. O. Dib, I. Dunietz, and F. J. Gilman, to be published. We have recently received a paper by J. Flynn and L. Randall on this subject.

11. We use the technique and notation of F. J. Gilman and M. B. Wise, Phys. Rev. D20, 2392 (1979).

12. This was done previously for the closely related process $b \rightarrow s e^+ e^-$ by B. Grinstein, M. J. Savage, and M. B. Wise, LBL preprint LBL-25014, 1988 (unpublished).

13. The relation of our choice of operators to those in Ref. 12, after an appropriate change of quark labels to account for the different processes under consideration, is $C_{7V} = c_8^{(GSW)}/2\pi$ and $C_{7A} = c_9^{(GSW)}/2\pi$. For the "electromagnetic penguin" contribution, the coefficient $F_1$ of T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981) and *erratum*, 65, 1772 (1981) is related by $F_1(x_i) = 2\pi \widetilde{C}_{7V,i}$. We have repeated this calculation and are in agreement with it and the corresponding one in Ref. 12.

14. Particle Data Group, Phys. Lett. 170B, 1 (1986).

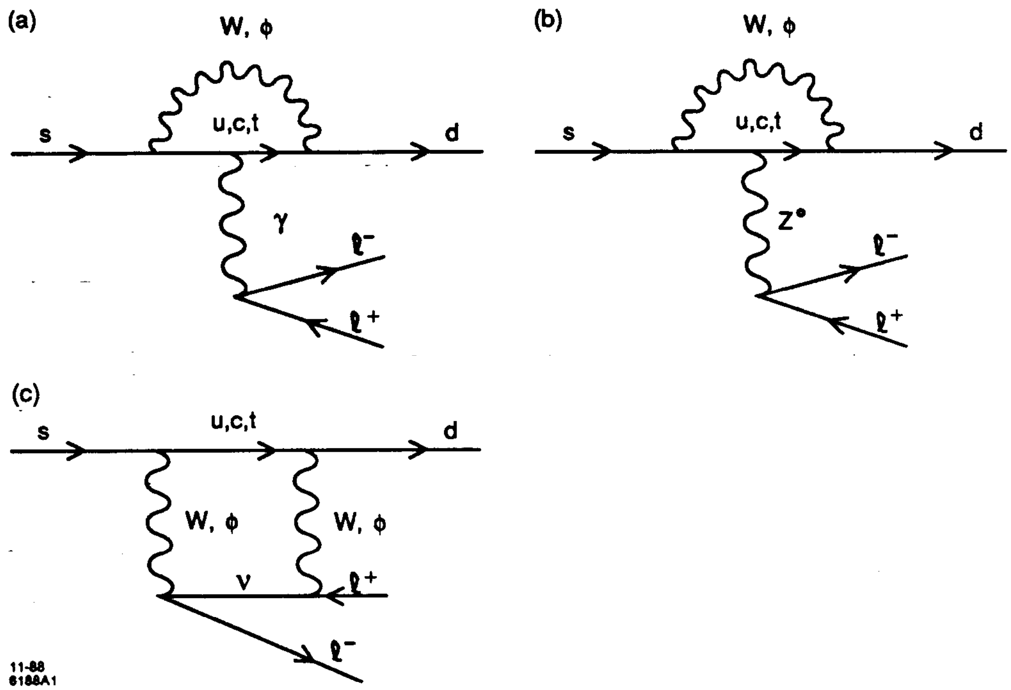



Fig. 1

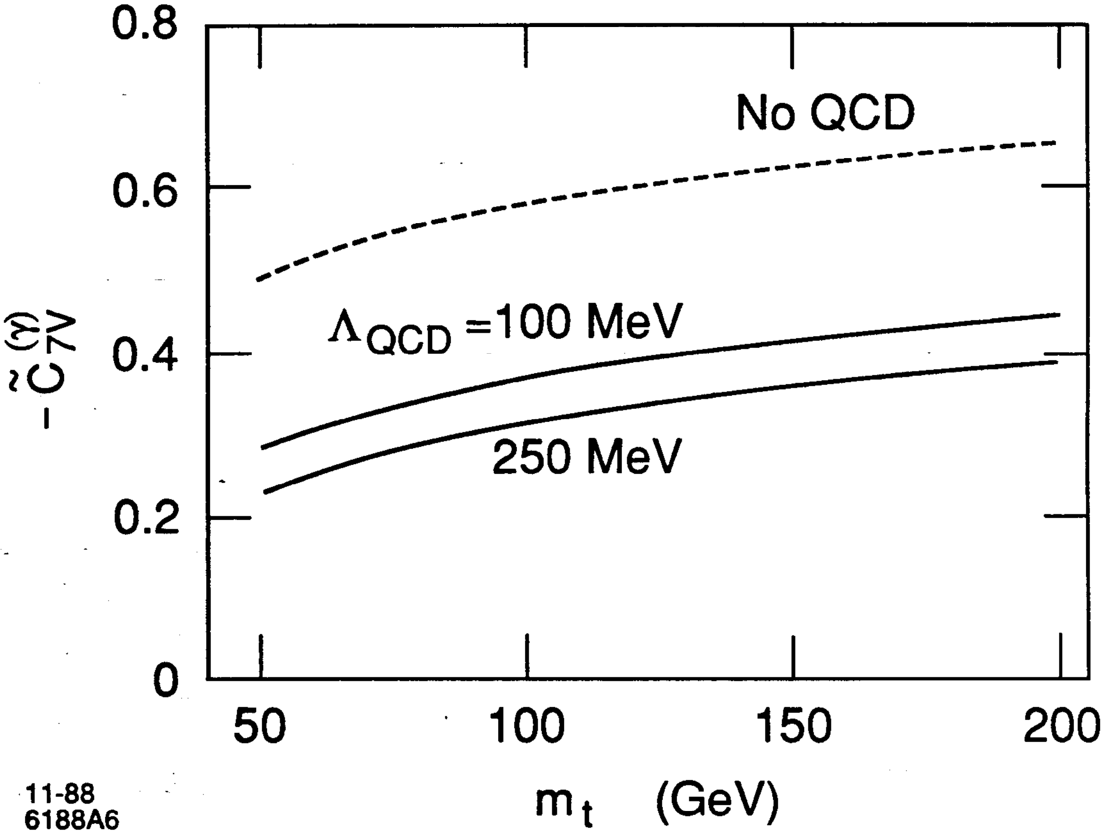
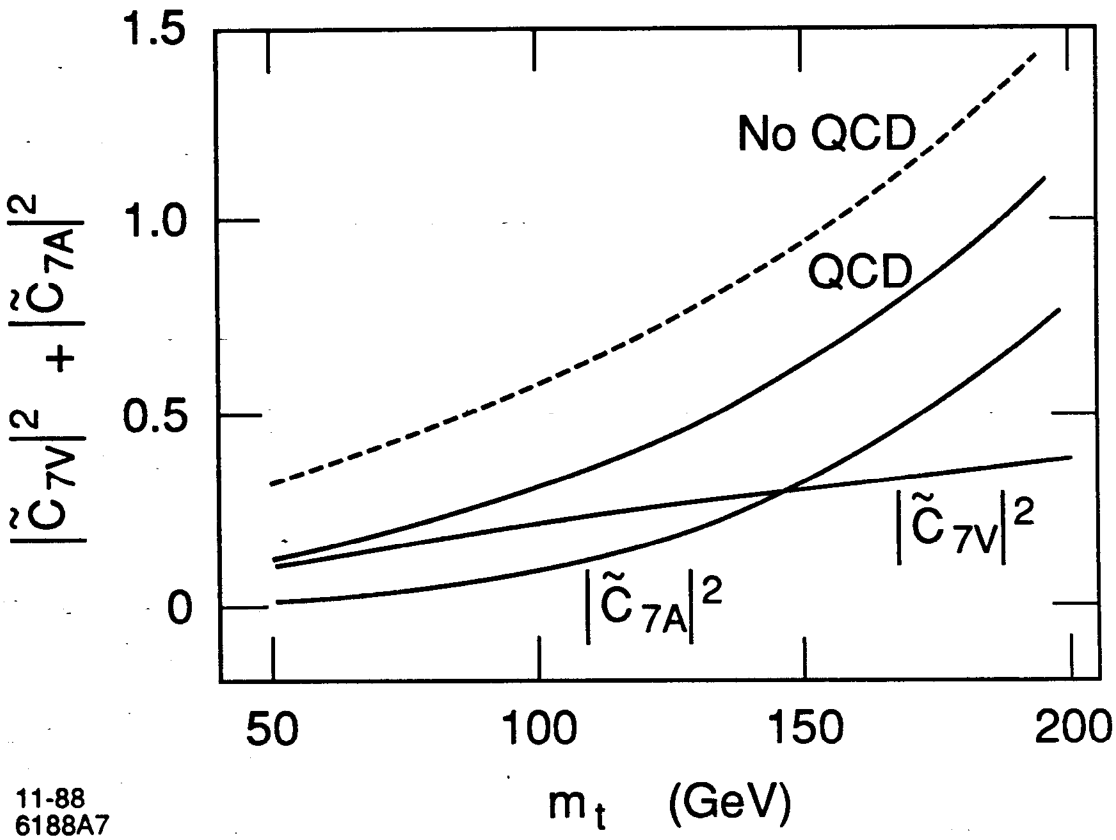

No QCD
$\Lambda_{QCD}$ =100 MeV
250 MeV
$-\tilde{C}_{7V}^{(\gamma)}$
0
0.2
0.4
0.6
0.8
50
100
150
200
$m_t$ (GeV)
11-88
6188A6

Fig. 2



1.5
1.0
0.5
0
$|\tilde{C}_{7V}|^2 + |\tilde{C}_{7A}|^2$
No QCD
QCD
$|\tilde{C}_{7V}|^2$
$|\tilde{C}_{7A}|^2$
50
100
150
200
$m_t$ (GeV)
11-88
6188A7


Fig. 3