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THE PHYSICS OF HEAVY FLAVORS*

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ABSTRACT

Various aspects of the physics of heavy flavors and of CP violation are reviewed.

1. INTRODUCTION

The primary aim of many of the theorists and experimentalists doing research in the area of heavy quark flavors is to search for physics that is beyond the standard model or which at least requires significant extensions of the standard model. In looking for such new physics there are two avenues:

- The high energy route involves the direct observation of new quarks, new leptons, particles predicted by supersymmetry, . . . Of necessity, this involves accelerators which are at the high energy frontier and now entails examining physics at the weak scale.
- The low energy route also can involve the direct observation of new particles such as additional neutrinos, axions, . . . The confirmation of non-zero neutrino mass and mixing falls in this category as well. However, much of the work at low energy aims to be sensitive to new physics through the indirect effects of virtual, *heavy* particles. These occur through amplitudes which receive contributions from one–loop Feynman diagrams and, through precision measurements, give us a window on the high energy world which others attack directly.

In this latter mode, we search for physics beyond the standard model through:

- Processes forbidden in the standard model, such as would be induced by lepton-flavor changing neutral currents.

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- Indications that CP violating phenomena have an origin other than from the non-trivial phase in the quark flavor mixing matrix.
- Deviations from expected rates, especially for rare processes which are forbidden at tree level in the electroweak interactions. These can be sensitive to heavy virtual particles (from a fourth generation, supersymmetry, left-right electroweak gauge symmetry, etc.) This is especially true of CP violating amplitudes, which, when they involve one loop amplitudes, arise first at momentum scales due to second and third generation quarks rather than those characteristic of Λ_{QCD} or light quarks.
- Theoretical relations between masses and mixing angles. The values of masses and angles are put into the standard model by hand, and hence originate from physics outside of it.

While less exciting, we also look at heavy flavor phenomena from inside the standard model to study:

- The interplay of strong and electroweak interactions in the weak decays of hadrons.
- The parameters of the standard model (masses and mixing angles). Eventually we will pin down these parameters, permitting us to calculate the standard model contributions to these processes unambiguously.

By making these measurements, we go on to complete the cycle. As each *a priori* free parameter of the standard model is pinned down and measured, we use these numbers, together with our improved calculational skills in evaluating matrix elements where strong and weak interactions overlap, to obtain updated predictions. Then we return to the former perspective of looking for physics beyond the standard model by comparing these predictions with all previous data and by pointing to further experiments which are yet more sensitive to new physics.

2. THE KOBAYASHI–MASKAWA MATRIX

In the standard model with $SU(2) \times U(1)$ as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix connecting them has become known as the Kobayashi-Maskawa[1] (K–M) matrix since an explicit parametrization in the six-quark case was first given by them in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle.[2]

By convention, the three charge 2/3 quarks (u, c, and t) are unmixed, and all the mixing is expressed in terms of a 3×3 unitary matrix V operating on the charge $-1/3$ quarks (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{1}$$

There are several parametrizations of the K–M matrix. In the 1988 edition of the Review of Particle Properties a "standard" form is advocated:[3]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} . \tag{2}$$

This is the notation of Harari and Leurer[4] for a form generalizable to an arbitrary number of "generations" and also proposed by Fritzsch and Plankl.[5] The choice of rotation angles follows that of Maiani,[6] and the placement of the phase follows that of Wolfenstein.[7] The three "generation" form was proposed earlier by Chau and Keung.[8] Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with i and j being "generation" labels, $\{i,j = 1,2,3\}$. In the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle.[2] The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases. Then all s_{ij} and c_{ij} are positive, and $|V_{us}| = s_{12}c_{13}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}c_{13}$. As c_{13} deviates from unity only in the fifth decimal place (from experimental measurement of s_{13}), $|V_{us}| = s_{12}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \le \delta_{13} < 2\pi$, with non-zero values generally breaking CP invariance for the weak interactions.

The values of individual K–M matrix elements can in principle all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Our present knowledge of the matrix elements comes from the following sources:

(1) Nuclear beta decay, when compared to muon decay, gives[9,10]

$$|V_{ud}| = 0.9747 \pm 0.0011 . \tag{3}$$

(2) Analysis of K_{e3} decays yields[11] $|V_{us}| = 0.2196 \pm 0.0023$. The analysis of hyperon decay data has larger theoretical uncertainties because of first order $SU(3)$ symmetry breaking effects in the axial-vector couplings, but due account of symmetry breaking gives a consistent value[12] of $0.220 \pm 0.001 \pm 0.003$. The average of these two results is[3]

$$|V_{us}| = 0.2197 \pm 0.0019 . \tag{4}$$

(3) The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. When the dimuon production cross

sections of the CDHS group[13] are supplemented by more recent measurements of the semileptonic branching fractions and the production cross sections in neutrino reactions of various charmed hadron species, the value[14]

$$|V_{cd}| = 0.21 \pm 0.03 \ . \tag{5}$$

is extracted.

(4) Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an $SU(3)$ symmetric sea, leads to a lower bound,[13] $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression[15] that follows from the standard weak interaction amplitude. This gives:[3]

$$|f_+^D(0)|^2 |V_{cs}|^2 = 0.51 \pm 0.07 \quad .$$

With sufficient confidence in a theoretical calculation of $|f_+^D(0)|$ a value of $|V_{cs}|$ follows,[16,17] but even with the very conservative assumption that $|f_+(0)| < 1$ it follows that

$$|V_{cs}| > 0.66 \quad . \tag{6}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

(5) The ratio $|V_{ub}/V_{cb}|$ can be obtained from the semileptonic decay of B mesons by fitting to the lepton energy spectrum as a sum of contributions involving $b \to u$ and $b \to c$. The relative overall phase space factor between the two processes is calculated from the usual four-fermion interaction with one massive fermion (c quark or u quark) in the final state. The value of this factor depends on the quark masses, but is roughly one-half. The lack of observation of the higher momentum leptons characteristic of $b \to u\ell\bar{\nu}_\ell$ as compared to $b \to c\ell\bar{\nu}_\ell$ has resulted thus far only in upper limits which depend on the lepton energy spectrum assumed for each decay.[17,18,19] Using the lepton momentum region near the end-point for $b \to c\ell\bar{\nu}_\ell$ and taking the calculation[19] of the lepton spectrum that gives the least restrictive limit results in[20]

$$|V_{ub}/V_{cb}| < 0.20 \ . \tag{7}$$

A lower bound on $|V_{ub}|$ can be established from the observation[21] of exclusive baryonic B decays into $p\bar{p}\pi$ and $p\bar{p}\pi\pi$ which involve $b \to u + d\bar{u}$ at the quark level. A chain of assumptions on the relative phase space, the fraction of the quark level process which hadronizes into baryonic channels, and the fraction of those that occur in the observed modes is required. No other channels that reflect $b \to u$ at the quark level have been observed.[22] Given the branching fractions of the two observed modes, a reasonable lower limit is[21]

$$|V_{ub}/V_{cb}| > 0.07 \ . \tag{8}$$

(6) The magnitude of V_{cb} itself can be determined if the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction:[3]

$$|V_{cb}| = 0.046 \pm 0.010 \ . \tag{9}$$

Most of the error quoted in Eq. (9) is not from the experimental uncertainty in the value of the b lifetime, but in the theoretical uncertainties in choosing a value of m_b and in the use of the quark model to represent inclusively semileptonic decays which, at least for the B meson, are dominated by a few exclusive channels. We have made the error bars larger than they are sometimes stated to reflect these uncertainties. They then also include the central values obtained for $|V_{cb}|$ by using a model for the exclusive final states in semileptonic B decay and extracting $|V_{cb}|$ from the absolute width for one or more of them.[17,19,23]

From Eqs. (3) through (9) plus unitarity (assuming only three generations), the 90% confidence limits on the magnitude of the elements of the complete matrix are:[3]

$$\begin{pmatrix} 0.9748 \text{ to } 0.9761 & 0.217 \text{ to } 0.223 & 0.003 \text{ to } 0.010 \\ 0.217 \text{ to } 0.223 & 0.9733 \text{ to } 0.9754 & 0.030 \text{ to } 0.062 \\ 0.001 \text{ to } 0.023 & 0.029 \text{ to } 0.062 & 0.9980 \text{ to } 0.9995 \end{pmatrix} . \tag{10}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of the others. The ranges given in Eq. (10) are consistent with the one standard deviation errors on the input matrix elements.

The data do not preclude there being more than three generations. Of course, the constraints deduced from unitarity are loosened when the K–M matrix is expanded to accommodate more generations. Still, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude $|V_{ub'}| < 0.07$, and the known elements of the first column require that $|V_{t'd}| < 0.15$.

Further information on the angles requires theoretical assumptions. For example, $B_d - \bar{B}_d$ mixing, if it originates from short distance contributions to ΔM_B dominated by box diagrams involving virtual t quarks, gives information on $V_{tb}\, V_{td}^*$ once hadronic matrix elements and the t quark mass are known.[24] A similar comment holds for $V_{tb}\, V_{ts}^*$ and $B_s - \bar{B}_s$ mixing. Even at the present stage of knowledge, we may use the published data claiming the observation of $B - \bar{B}$ mixing[25] to obtain a significant lower bound on $|V_{td}|$ within the three generation standard model. This is because the magnitude of the mixing depends on m_t, an hadronic matrix element, and $|V_{td}|$. Taking $m_t < 180$ GeV,[26] and the relevant matrix element parametrized as $|B_B f_B^2|$ to be less than $(200\ \text{MeV})^2$, we obtain

$$|V_{td}| > 0.006 \quad . \tag{11}$$

This is a considerable improvement over the constraint provided by unitarity and the measured values of other matrix elements in Eq. (10).

Up to this point we have discussed only information on magnitudes of K–M matrix elements. In principle, such measurements of magnitudes could tell us about the phase, δ_{13}, as well as the "rotation angles" θ_{12}, θ_{23}, and θ_{13} in Eq. (2). This is most easily seen for the case at hand, where the "rotation angles" are small, by using the unitarity of the K–M matrix applied to the first and third columns to derive that (c_{ij} have been set to unity):

$$1 \cdot V_{ub}^* - s_{12} \cdot V_{cb}^* + V_{td} \cdot 1 \approx 0 \quad . \tag{12}$$

This equation is represented graphically in Fig. 1 in terms of a triangle in the complex plane, the length of whose sides is $|V_{ub}^*|$, $|s_{12} \cdot V_{cb}^*|$, and $|V_{td}|$.

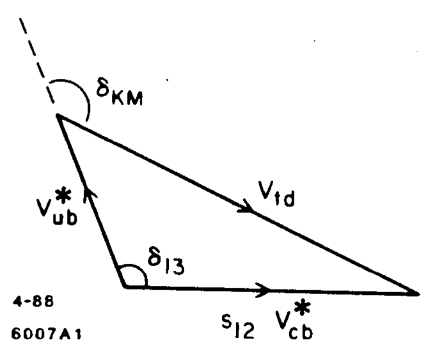

Fig. 1. Representation in the complex plane of the triangle formed by the K–M matrix elements V_{ub}^*, $s_{12} \cdot V_{cb}^*$, and V_{td}.

This triangle has been implicit, and even occasionally explicit, in many people's work on the constraints on the K–M matrix implied by various data involving mixing or CP violation, but has been particularly emphasized by Bjorken.[27,28]

With this representation of the unitarity of the K–M matrix, it is possible to see more directly the interplay of various pieces of experimental information. For example, an increase in the magnitude of the $b \to u$ transition obviously increases the side whose length is $|V_{ub}|$. The present upper bound on $|V_{ub}/V_{cb}|$ means that this side at most is as long as the side whose length is $|s_{12}V_{cb}^*|$. On the other hand, an increased magnitude for $B_d - \bar{B}_d$ mixing implies (keeping m_t and the appropriate hadronic matrix element, $B_B f_B^2$, fixed) stretching the side whose length is $|V_{td}|$. With the other sides set by independent measurements, the triangle gets flatter and flatter and eventually "breaks." At that point $B - \bar{B}$ mixing has become incompatible with other data plus assumed values of m_t and the hadronic matrix element. Hence the derivation of a lower bound on m_t from $B - \bar{B}$ mixing.[24,29]

In principle, accurate measurement of the lengths of all three sides could show that the triangle can not exist (and we must go beyond the three generation standard model), or cause the triangle to collapse to a line (and we must go beyond the standard model for an explanation of CP violation), or demand the existence of a nontrivial triangle with δ_{13} not equal to 0° or 180°. Unfortunately, given our present experimental knowledge and our limited theoretical ability to compute hadronic matrix elements, the three sides are not known with sufficient accuracy to discriminate between these situations, let alone determine the value of δ_{13}. To do this we are forced to consider a CP violating quantity and assume it can be understood within the three generation standard model.

In this connection, note that the law of sines applied to the triangle gives:

$$\frac{\sin \delta_{KM}}{|s_{12}V_{cb}^*|} = \frac{\sin \delta_{13}}{|V_{td}|} \quad .$$

Setting cosines of small angles to unity and expressing V_{cb} as s_{23}, but V_{td} as $s_1 s_2$ in the original notation of Kobayashi and Maskawa[1], allows this equation to be converted to ($s_{12} \approx s_1$):

$$s_1^2 s_2 s_3 \sin \delta_{KM} \approx s_{12} s_{23} s_{13} \sin \delta_{13} \quad . \tag{13}$$

This is twice the area of the triangle, and, aside from cosines of small angles having been set to unity, is just proportional to the measure of CP violation in the three generation standard model proposed by Jarlskog.[30]

3. CP VIOLATION IN THE K° SYSTEM

As noted in the previous section, the standard model allows for CP violation in the form of phases originating in the quark mixing matrix when there are three or more generations of quarks and leptons. With just three generations, there is precisely one non-trivial CP violating phase.[31]

The computation of any difference of rates between a given process and its CP conjugate process (or of a CP violating amplitude) always has the form

$$\Gamma - \bar{\Gamma} \;\propto\; s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta_{KM} = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin\delta_{13}, \tag{14}$$

where we express things first in the original parametrization of the quark mixing matrix[1] and then in the "new" parametrization used in the previous section. Our present experimental knowledge assures us that the approximation of setting the cosines to unity induces errors of at most a few percent. In that case the true equality in Eq. (14), involving the invariant measure of CP violation,[30] becomes the highly accurate approximate statement in Eq. (13). The combination of factors characteristic of CP violation is (after removing s_1^2, whose value is accurately known)

$$s_2 s_3 \sin\delta_{KM} \;\equiv\; s_2 s_3 s_\delta \quad ,$$

where we have used the "old" parametrization.

An example of such a CP violating quantity is provided by the one well-measured CP violation parameter, ϵ, in the neutral K system. Assuming that ϵ arises from short distance effects, i.e., the box diagram with virtual c and t quarks shown in Fig. 2, gives the relation:

$$\begin{aligned} \epsilon \approx & \frac{e^{i\pi/4}}{\sqrt{2}} \frac{B G_F^2 f_K^2 m_K}{6\pi^2 \Delta M_K} \\ & \times s_1^2 s_2 s_3 s_\delta \left[-\eta_1 m_c^2 + \eta_2 s_2 (s_2 + s_3 c_\delta) m_t^2 + \eta_3 m_c^2 \, \ell n(m_t^2/m_c^2) \right] \quad . \end{aligned} \tag{15}$$

As stressed above, the factor $s_1^2 s_2 s_3 s_\delta$ must appear in Eq. (15) and it does.

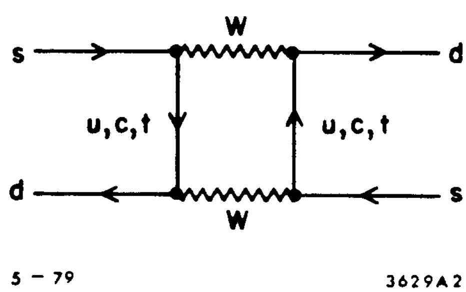

Fig. 2. Box diagram whose imaginary part contributes to the CP violation parameter ϵ in the neutral K system.

The quantities η_1, η_2, and η_3 are due to strong interaction (QCD) corrections. They are calculable and have the values 0.7, 0.6, and 0.4, respectively, given a renormalization scale of a few hundred MeV and typical quark masses.[32] Less well determined is the infamous parameter B, which is the ratio of the actual value of the matrix element between K^0 and $\overline{K}^0$ states of the operator composed of the product of two V-A neutral, strangeness changing currents divided by the value of the same matrix element obtained by inserting the vacuum between the two currents. The parameter B has a long history of calculation and re-calculation, but a reasonable range seems to be

$$1/3 < B < 1 \quad .$$

If we insert known experimental quantities, Eq. (15) becomes

$$|\epsilon| \approx \frac{0.314}{\text{GeV}^2}\ B\ s_2 s_3 s_\delta\ \left[-\eta_1 m_c^2 + \eta_2 s_2 (s_2 + s_3 c_\delta) m_t^2 + \eta_3 m_c^2\ \ell n(m_t^2/m_c^2) \right] \quad . \tag{16}$$

Eqs. (15) and (16), as written, are strictly valid when $m_t^2 << M_W^2$, but their forms are representative of the general character of the full expression[33] which we use in the analysis that follows. Numerically, even for $m_t \approx M_W$, the changes in the coefficients of the last two terms in brackets are not large.

Viewed from the direction of making a prediction, can we understand the magnitude of ϵ, i.e., why is CP violation in the neutral Kaon system so small? The answer is yes, we do understand its rough magnitude, for our present knowledge of the elements of the K–M matrix permits the placing of an upper bound on the quantity $s_2 s_3 s_\delta$ of about 2.5×10^{-3}. This plus the quark masses and other quantities on the right-hand side of Eq. (16) make $|\epsilon|$ be in the ballpark of 10^{-3}.

From the opposite direction, we can constrain the mixing angles by using the measured magnitudes of ϵ and $B - \bar{B}$ mixing. These constraints will depend on what we assume for the quantities B and m_t, as well as on the still uncertain value for $b \to u/b \to c$.

- If $m_t = 45$ GeV, the magnitude of ϵ and the "large" observed[25] $B - \bar{B}$ mixing push the K–M matrix elements into a corner: Such a "small" value of m_t together with the "large" mixing force V_{td} to be as large as possible. The long (V_{td}) side of the triangle in Fig. 1 is stretched as far as it will go, and in the process $|V_{ub}|$ is increased and the phase δ_{13} pushed toward 180°. The parameter B must be near the upper end of its allowed range as well, to obtain the experimental value of $|\epsilon|$ in Eq. (16). This is illustrated in Fig. 3 from Nir,[34] which indicates that for $m_t = 45$ GeV and $B = 1$ there is just barely an allowed region. It is centered around a phase $\delta_{13} \approx 150°$ and $b \to u/b \to c$ (s_{13}/s_{23} in Fig. 3) near its maximum allowed value. Correspondingly,

$$0.75 \times 10^{-3} \lesssim s_2 s_3 s_\delta \lesssim 1.25 \times 10^{-3}$$

is rather narrowly constrained as well.

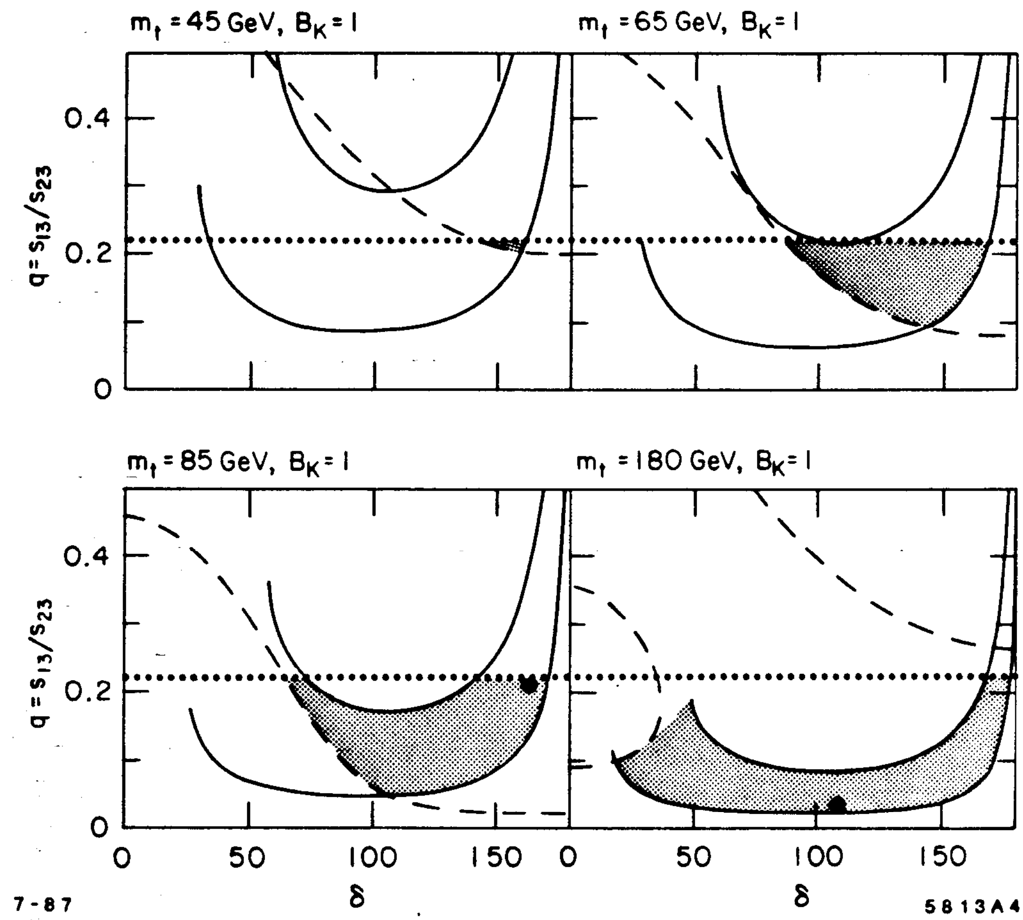

Fig. 3. The allowed range (shaded) of the K–M matrix parameter $q = s_{13}/s_{23}$ versus that for δ_{13} for top quark masses of 45, 65, 85, and 180 GeV when $B = 1$ as given in Ref. 34. The solid curves show the restrictions due to satisfying the constraint of imposing the experimental value of ϵ, while the dashed curves (the upper limit is off the scale of the first three subgraphs) do the same for $B - \bar{B}$ mixing. The horizontal dotted line gives the upper bound on q that follows from the bound on $b \to u$ using the inclusive lepton spectrum in B decay.

- As we go to larger values of m_t, a bigger range of angles is allowed. This is shown in Fig. 3, where the allowed range is shaded. The corresponding figures for other values of B show the same effect.[34]
- Very roughly, larger values of m_t generally favor smaller values of $s_2 s_3 s_\delta$, as the constraint imposed by the experimental value of $|\epsilon|$ inserted into Eq. (16) forces them to move in compensating directions if the other parameters (like B and m_c) are fixed.

- The region of K–M angles allowed by the constraints translates into a corresponding range for the amplitudes of processes induced at one loop which involve a virtual t quark. A sample of the resulting theoretical predictions is shown[34] in Fig. 4.

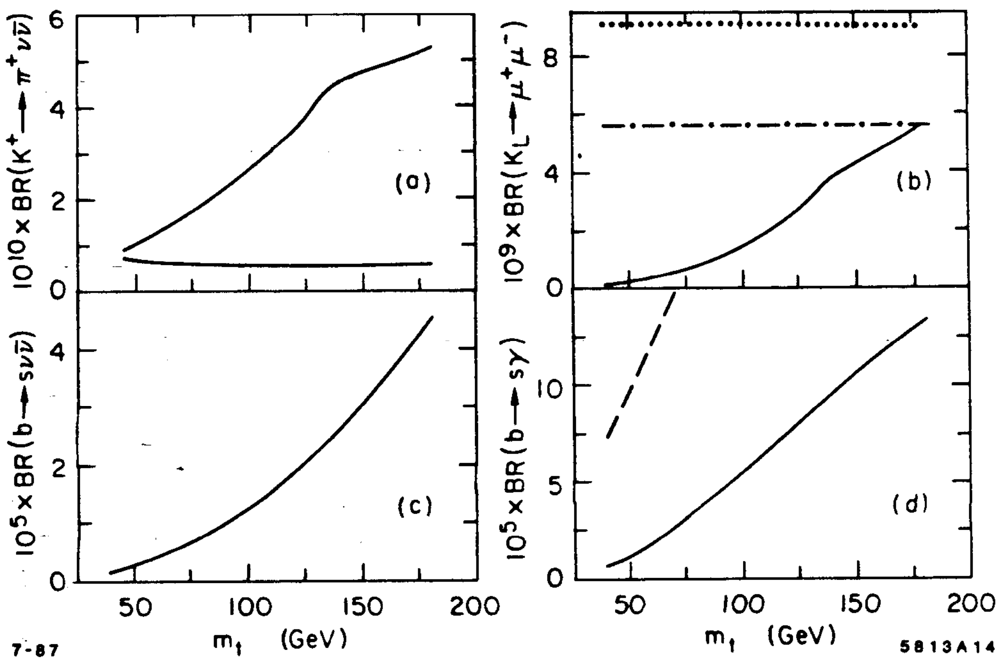

Fig. 4. The range of branching ratios allowed for several rare decays as a function of m_t from Ref. 34: (a) Upper and lower limits on $B(K^+ \to \pi^+\nu\bar{\nu})$; (b) Upper limit (solid curve) on the short distance contribution to $B(K_L \to \mu^+\mu^-)$. The dotted line gives the experimental branching ratio and the dot-dashed line an upper limit on the short distance contribution once the (long distance) contribution from the two photon intermediate state is subtracted from the experimental rate; (c) The branching ratio $B(b \to s\nu\bar{\nu})$; (d) The branching ratio $B(b \to s\gamma)$ without (solid curve) and with (dashed curve) QCD corrections.

A second quantity is provided by the parameter ϵ', which measures CP violation in the K decay amplitude itself, and arises in the standard model from diagrams involving heavy quarks in loops, the so-called "penguin" diagrams, an example of which is shown in Fig. 5.

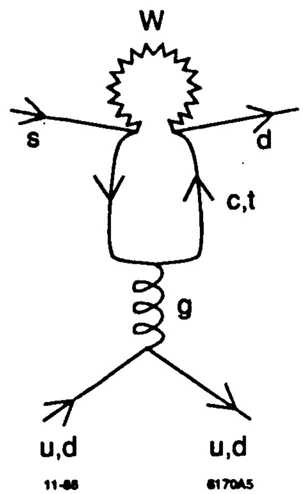

Fig. 5. "Penguin" diagram whose imaginary part contributes to the CP violation parameter ϵ' in the neutral K system.

By inserting experimentally measured quantities, the contribution to ϵ' from the "penguin" operator contribution to $K \to \pi\pi$ can be written[35]

$$\epsilon'/\epsilon = 6.0\ s_2 s_3 s_\delta \left(\frac{Im\widetilde{C}_6}{-0.1}\right) \left(\frac{< \pi\pi|Q_6|K^0 >}{1.0\ \mathrm{GeV}^3}\right) (1 - \Omega_{\eta,\eta'} + \Omega_{em}) \quad , \qquad (17)$$

where Q_6 is the "penguin" operator in the short distance expansion of the strangeness-changing weak Hamiltonian,[36] $\mathrm{Im}\widetilde{C}_6$ is the imaginary part of the corresponding Wilson coefficient with the K–M factor taken out, and $\Omega_{\eta,\eta'}$ and Ω_{em} are corrections due to $\pi^0 - \eta$ and $\pi^0 - \eta'$ mixing, and to "electromagnetic penguins," respectively. These latter two contributions tend to cancel; the factor $(1 - \Omega_{\eta,\eta'} + \Omega_{em})$ may result[37] in anything between a $\sim 30\%$ decrease and a small increase in ϵ'/ϵ. The value of -0.1 for $\mathrm{Im}\widetilde{C}_6$ is relatively stable from calculation to calculation if the renormalization scale is taken as a few hundred MeV, since the imaginary part depends on momentum scales from m_c to m_t where the short distance expansion is well justified. The value of the matrix element of Q_6 is much less certain. If it is large enough to explain the experimental magnitude of $A(K \to \pi\pi)$, i.e., roughly 1 to 2 GeV^3, then, combined with the value of $s_2 s_3 s_\delta$ needed to fit $|\epsilon|$ (see above), it yields the prediction that ϵ'/ϵ is positive and of order 10^{-2}. This was the basic observation in Ref. 36.

Over the past ten years there have been many calculations of ϵ'/ϵ. The prediction depends directly on the value of the matrix element of Q_6, but also on that of m_t. The latter dependence is direct in that $\mathrm{Im}\widetilde{C}_6$ depends on m_t. In the calculations done until now, there is also an indirect dependence on m_t which is even more important: the constraint involving $|\epsilon|$ is used to determine $s_2 s_3 s_\delta$. One needs to be very careful in comparing the predictions from different calculations as the fashionable value of m_t has drifted higher and higher over the years. Sometimes nothing more fundamental than a change in the favorite value of m_t is the cause of a large change in the predictions. As m_t has risen, the predictions for ϵ'/ϵ have correspondingly gone down.[38]

As the last year has unfolded, the standard model "explanation" of CP violation has looked better and better. In particular, there have been two important new experimental results for ϵ'/ϵ. First came the result from a test run of the Fermilab experiment[39] which has been updated to:

$$\epsilon'/\epsilon = 3.2 \pm 2.8 \pm 1.2 \ \times 10^{-3}.$$

The full data set is now being analyzed. The result from the NA31 experiment[40] at CERN

$$\epsilon'/\epsilon = 3.3 \pm 1.1 \ \times 10^{-3},$$

is the first significant indication for CP violation in the decay amplitude itself. They are running again. Both experiments have the capability of eventually decreasing both their statistical and systematic error bars below the 10^{-3} level. We will have to wait and see if the central value of ϵ'/ϵ remains non-zero by many standard deviations when the combined error bars shrink to this level.

While we wait, we can ask in any case whether the present central value, if it persists, is consistent with the standard model. The answer is yes, particularly if the value of m_t is large. One perspective on this is obtained by taking whatever knowledge we have of the matrix element, m_t, and $s_2 s_3 s_\delta$ and predicting ϵ'/ϵ in the usual way.[41] A different perspective is gained by turning the situation around and assuming a value for ϵ'/ϵ, and then asking what combined Wilson coefficient, "penguin" matrix element, and electromagnetic corrections would produce such a result. In the future, when the experimental situation settles down with small error bars, this will be more typical; we will take the experimental value of ϵ'/ϵ as an input and use it to measure the magnitude of the "penguin" operator contribution to K decay. Hopefully the theory will have progressed sufficiently that there will then be a significant comparison between this and lattice gauge theory calculations of the same quantity.

So, let us assume that $\epsilon'/\epsilon = 3.5 \times 10^{-3}$. When $m_t = 45$ GeV, there is not too much room to maneuver and still satisfy the constraint of getting the correct value of $|\epsilon|$. Our previous discussion, together with Eq. (17), makes

$$\left(\frac{Im\widetilde{C}_6}{-0.1}\right)\left(\frac{<\pi\pi|Q_6|K^0>}{1.0 \text{ GeV}^3}\right)\left(1 - \Omega_{\eta,\eta'} + \Omega_{em}\right) = 0.47$$

for the biggest value allowed for $s_2 s_3 s_\delta$, and

$$\left(\frac{Im\widetilde{C}_6}{-0.1}\right)\left(\frac{<\pi\pi|Q_6|K^0>}{1.0\ \mathrm{GeV}^3}\right)(1-\Omega_{\eta,\eta'}+\Omega_{em}) = 0.78$$

for the smallest. The corresponding values for $m_t = 100$ GeV are 0.4 and 2.1, respectively.

The outcome of this exercise, recalling that a value for the matrix element of the "penguin" operator of 1 to 2 GeV^2 is large enough to make it a plausible explanation for the $\Delta I = 1/2$ rule, is that the "penguin" contribution to the $K \to \pi\pi$ amplitude is unlikely to be negligible when compared with an "interesting" value. "Penguins" could well play an important part in K decay.

Another decay in which it is possible to observe CP violation in the K° system is $K_L \to \pi^0 e^+ e^-$. If we define K_1 and K_2 to be the even and odd CP eigenstates, respectively, of the neutral K system, then

$$K_2 \to \pi^0\ \gamma_v \to \pi^0 e^+ e^-$$

is CP violating, while

$$K_1 \to \pi^0\ \gamma_v \to \pi^0 e^+ e^-$$

and

$$K_2 \to \pi^0\ \gamma\gamma \to \pi^0 e^+ e^-$$

are CP conserving. Therefore $K_L \to \pi^0 e^+ e^-$ has three contributions:

- Through the small (proportional to ϵ) part of the K_L which is K_1 due to CP violation in the mass matrix. We call this "indirect" CP violation.
- Through the large part of the K_L which is K_2 due to CP violation in the decay amplitude. We call this "direct" CP violation.
- Through a two photon intermediate state. This is higher order in α, but is CP conserving.

The question before us is the relative magnitude of these three contributions. Let us take them one at a time.

- We may estimate the contribution to the decay rate from the amplitude induced by "indirect" CP violation by using the identity:

$$\begin{aligned} B(K_L \to \pi^0 e^+ e^-)_{\mathrm{indirect}} &\equiv B(K^+ \to \pi^+ e^+ e^-) \times \\ \frac{\tau_{K_L}}{\tau_{K^+}} \times \frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} &\times \frac{\Gamma(K_L \to \pi^0 e^+ e^-)_{\mathrm{indirect}}}{\Gamma(K_1 \to \pi^0 e^+ e^-)} \quad . \end{aligned} \tag{18}$$

  Experimental values[42] of 2.7×10^{-7} and 4.2 may be inserted for the first two factors on the right hand side. The last factor is $|\epsilon|^2$ by the definition of what we mean by "indirect" CP violation in the convention where $A_0(K \to \pi\pi)$ is real. The third factor can be measured directly one day. For the moment it is the subject of model dependent theoretical calculations, with a value of 1 if the transition between the K and the π is $\Delta I = 1/2$. This is the case for the short-distance amplitude which involves a transition from a strange to a down quark. For $\Delta I = 3/2$, the corresponding value is 4. With both isospin amplitudes present and interfering, any value is possible.[43] Using a value of unity for this factor makes

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} = 0.58 \times 10^{-11} \quad .$$

- The amplitude for "direct" CP violation comes from penguin diagrams with a photon or Z boson replacing the usual gluon and also from box diagrams with quarks (of charge 2e/3), leptons (neutrinos) and W bosons as sides, shown in Fig. 6.

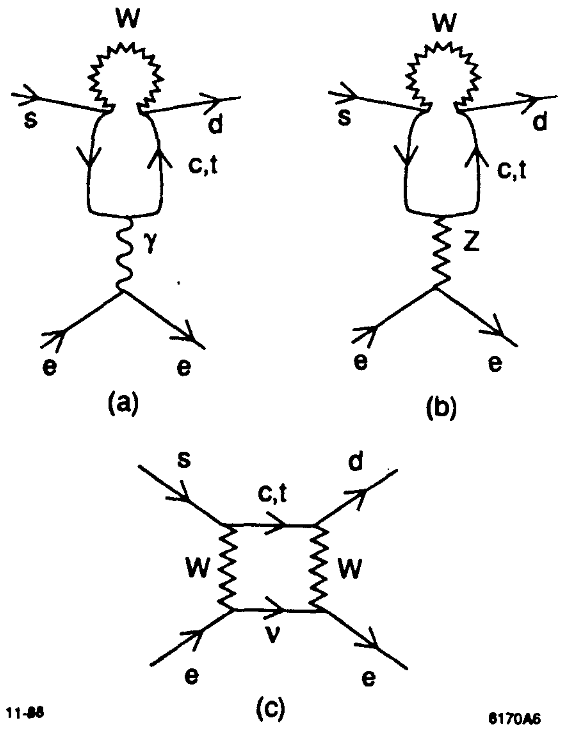

Fig. 6. Three diagrams giving a short distance contribution to the process $K \to \pi e^+ e^-$: (a) the "electromagnetic penguin;" (b) the "Z penguin;" (c) the "W box."

For values of $m_t << M_W$, it is the "electromagnetic penguin" that gives the dominant short-distance contribution to the amplitude, which behaves like $\ell n(m_t^2/m_c^2)$. A full analysis, including QCD corrections, has been carried out in the case of six quarks,[44] building upon work done with four quarks.[45] The CP violating amplitude from the "electromagnetic penguin" is summarized in the Wilson coefficient of the appropriate operator,

$$Q_7 \;=\; \alpha \; \big(\bar{s}\gamma_\mu(1-\gamma_5)d\big)\big(\bar{e}\gamma^\mu e\big) \;,$$

with the K–M factor $s_2 s_3 s_\delta$ factored out. This coefficient is $\mathrm{Im}\widetilde{C}_7$ in the notation of Ref. 44, and typically has a value of 0.1 to 0.2 for $m_t^2 << M_W^2$, with QCD corrections included. The Z penguin and W box graph contributions are suppressed in amplitude by a power of m_t^2/M_W^2. This is no longer a suppression when we contemplate values of $m_t \sim M_W$. In fact, the "Z penguin" and "W box" contributions add another operator involving $\bar{e}\gamma_\mu\gamma_5 e$ and together become comparable to that of the "electromagnetic penguin" in this region. We may relate the hadronic matrix element of the relevant operator and the phase space to that which occurs in K_{e3} decay. Then we find that[46]

$$B(K_L \to \pi^0 e^+ e^-) = 1.0 \times 10^{-5}\; (s_2 s_3 s_\delta)^2 \; \left[(\mathrm{Im}\widetilde{C}_7)^2 + (\mathrm{Im}\widetilde{C}_{7A})^2\right] \quad .$$

With QCD corrections and m_t between 50 and 200 GeV, the last factor ranges[46] between about 0.1 and 1.0, so that the corresponding branching ratio induced by this amplitude alone for $K_L \to \pi^0 e^+ e^-$ is around 10^{-11}. It appears that the contribution from the "direct" CP violating amplitude is at least comparable to that from the "indirect" amplitude, reinforcing an earlier conclusion[44] that they gave comparable contributions (at a time when the favorite values used for m_t were 15 and 30 GeV). A full analysis of the "direct" contribution is in preparation.[46]

- The CP conserving amplitude is interesting, if only for its checkered history of theoretical ups and downs. There are two invariant amplitudes[47] for the CP conserving subprocess $K_2 \to \pi^0\;\gamma\gamma$. If we take the momenta as p, p', q_1 and q_2, respectively, and define $x_{1,2} = p\cdot q_{1,2}/p\cdot p$, then they may be expressed in a gauge invariant way as:

$$\begin{aligned} <\pi\gamma\gamma|K_2> \;=\;& A(x_1,x_2)\big[q_2\cdot\epsilon_1\; q_1\cdot\epsilon_2 - q_1\cdot q_2\;\epsilon_1\cdot\epsilon_2\big] \;+ \\ & B(x_1,x_2)\big[p^2\; x_1 x_2 \epsilon_1\cdot\epsilon_2 + q_1\cdot q_2\; p\cdot\epsilon_1\; p\cdot\epsilon_2/p^2 \\ & \qquad - x_1\; q_2\cdot\epsilon_1\; p\cdot\epsilon_2 - x_2\; q_1\cdot\epsilon_2\; p\cdot\epsilon_1\big] \end{aligned}$$

with $\epsilon_{1,2}$ the polarization vectors of the two photons. When joined with the QED amplitude for $\gamma\gamma \to e^+e^-$ to form the amplitude for $K_2 \to \pi^0 e^+ e^-$, the contribution from the A amplitude gets a factor of m_e in front of it. This is not hard to understand, as the total angular momentum of the $\gamma\gamma$ system that pertains to the A amplitude is zero; the same is then true of the final e^+e^- system. However, the interactions, being electroweak, always match (massless) left-handed electrons to right-handed positrons and *vice versa*, causing the decay of a J = 0 system to massless electrons and positrons to be forbidden. Hence the factor of m_e in the overall amplitude for $K_2 \to \pi^0\ e^+e^-$, so that the A amplitude provides a negligible contribution. A corollary of this theorem applies to the $K_2 \to \pi^0\ \gamma\gamma$ amplitude calculated using current algebra low energy theorems. In the limit of vanishing pion four-momentum, a non-vanishing A amplitude is predicted. The factor of m_e found[48] in the resulting amplitude for $K_2 \to \pi^0\ e^+e^-$ is then no surprise. On the other hand, the contraction of $\gamma\gamma \to e^+e^-$ with the B amplitude produces no such factor of m_e. B does however contain a coefficient with two more powers of momentum, and one might hope for its contribution to be suppressed by angular momentum barrier factors. Because of the extra powers of momentum, in chiral perturbation theory this amplitude is put in by hand with its coefficient not predicted. An order of magnitude estimate may be obtained by pulling out the known dimension-full factors in terms of powers of f_π, and asserting that the remaining coupling strength should be of order one.[47] The branching ratio for $K_2 \to \pi^0\ e^+e^-$ is then of order 10^{-14}. If so, the CP conserving amplitude makes a negligible contribution to the decay rate. However, an old fashioned vector dominance, pole model predicts[49] comparable A and B amplitudes and a branching ratio of order 10^{-11}, comparable to that from the CP violating amplitudes. The applicability of such a model, however, can be challenged on the grounds that the low energy theorems and Ward identities of the overall theory are not being satisfied.[50] The consistent implementation of vector dominance with all the other constraints leads to extra powers of momentum in some of the couplings, and possibly to a smaller prediction than in the old fashioned model.

The dust is not yet settled. The burden is still on the theorists to show that the CP conserving amplitude is very much smaller than the CP conserving one, so that the experimental observation of the decay $K_L \to \pi^0\ e^+e^-$ could be interpreted as another example of CP violation in the neutral K system, this time with comparable effects from the mass matrix and the decay amplitude itself.

4. NEW DEVELOPMENTS IN HEAVY FLAVORS

This is a field that is reaching maturity. Many of the questions that remain open are quantitative rather than qualitative ones. To complement this kind of question, there are high statistics data samples available from electron-positron annihilation and, more recently, from the use of vertex detectors in fixed target experiments at hadron machines. For the tau lepton and for states containing charm quarks, this has meant thousands of events in major decay channels. Correspondingly, we are beginning to probe some of the rarer decays, or to establish significant limits thereon. What follows is a brief, personal view of some topics in the physics of heavy flavors.

4.1 Weak Decays of Heavy Quarks

We have a solid general framework within which to calculate the weak decays of heavy quarks. Starting with the electroweak interactions and their gauge group, $SU(2) \times U(1)$, we add the corrections due to the strong interactions through the use of the renormalization group equations for the coefficients of the operators, with anomalous dimensions computed from QCD.[51]

These calculations are carried out at the quark level. A first stage in their application to actual hadrons is to consider weak decays inclusively and to simply neglect any other constituent of the decaying hadron aside from the heavy quark. In such a spectator model, as it is called, one directly carries over the quark level calculation as the hadron level result; spectator quarks and gluons are assumed to arrange themselves into final state particles, together with the quarks or leptons coming from the heavy quark, at no cost or benefit in the overall rate. This simple spectator model in fact gives a qualitative, if not semi-quantitative, account of the known data.

It is clear, however, that there are corrections to this picture since the lifetimes of different species of charmed particles, which are all the same in the spectator model, differ by a factor of two or so. The data on charmed particle lifetimes have recently undergone a qualitative improvement. The Fermilab photoproduction experiment E691, using silicon strip vertex detection, provides clean data with high statistics; the result is precise lifetime measurements for different charmed species.[52] The results on charm lifetimes from E691 are shown in Table 1.

Table 1.
Charmed particle lifetime measurements from Ref. 52.

Mode	Signal	Background	Lifetime (psec)
D^+	2992	1354	$1.090 \pm 0.030 \pm 0.025$
D^0	4212	975	$0.422 \pm 0.008 \pm 0.010$
D_s^+	228	75	$0.47 \pm 0.04 \pm 0.02$
Λ_c	93	85	$0.22 \pm 0.03 \pm 0.02$

If we want to do better than the factor of two level of agreement, then we need to go beyond the spectator model. Final state interactions, annihilation diagrams, interference between different amplitudes, and color (mis)matching have all entered the discussion. All have a role to play.[53] We discuss here some recent developments.

Final state interactions. Final state interactions must be present. The question is their importance. Direct evidence of their magnitude is provided by the Mark III data on $D \to \bar{K}\pi$ modes: $D^0 \to K^-\pi^+$, $D^0 \to \bar{K}^0\pi^0$, and $D^+ \to \bar{K}^0\pi^+$. The $K\pi$ final state can occur with isospin 1/2 and 3/2, and so there is one triangle relation between the three amplitudes. Without final state interactions the amplitudes should be relatively real. The experimentally measured branching fractions demand that the isospin 1/2 and isospin 3/2 amplitudes have a large phase between them.[54] Similar comments hold for the $D \to \bar{K}^*\pi$ channel, but, on the contrary, $D \to \bar{K}\rho$ shows only a small phase difference. Thus, at least in some cases, final state interactions are very important.

We note in passing that the absolute D meson nonleptonic branching ratios from Mark III have been revised downward[55] by 19% to 25%, thereby removing the "charm deficit" in B decays.[56] The newest Mark III branching ratio for $D^0 \to K^-\pi^+$ of $4.2 \pm 0.4 \pm 0.4\%$ has also been confirmed in an independent manner by the HRS, which obtains[57] $4.0 \pm 0.6\ ^{+0.7}_{-0.6}\%$.

Annihilation diagrams. Quark-antiquark annihilation must occur in the decay of a meson if only leptons are present in the final state. In particular, the decay $D_s^+ \to \tau^+\nu_\tau$ is expected with a roughly 2% branching ratio. The bound[58]

$$B(D^+ \to \mu^+\nu_\mu) < 6.2 \times 10^{-4}$$

from the Mark III may be used to assert that $f_D < 290$ MeV, where the probability of the c and $\bar{d}$ quarks in the D^+ to annihilate has been summarized in the pseudoscalar decay constant, f_D.

The question which remains outstanding is again a quantitative one: What is the magnitude of quark-antiquark annihilation or of W-exchange in nonleptonic decays? The observation[59] of the decay $D^0 \to \phi\bar{K}^0$ at the 1% level in branching ratio looks, on the face of it, to be evidence for W exchange. (One needs to get rid of both the c and $\bar{u}$ quarks in the initial state since neither appears in the final hadrons.) It can be argued, however, that such a final state can be generated without W exchange by final state interactions.[60] As this one decay mode is not decisive, we seek more evidence. This comes from observing that if W exchange is important in D^0 decay, annihilation should play at least as important a role in D_s^+ decay, and final state interactions should be different there as well. However, the results of recent experiments[52] indicate that the lifetime of the D_s^+ appears to be at least as long as the D^0 (see Table 1), and the search for the specific mode $D_s \to \rho\pi$ has turned up only upper limits. The best of these limits is[61]

$$B(D_s^+ \to \rho^0\pi^+)/B(D_s^+ \to \phi\pi^+) < 0.08 \quad .$$

Also striking is that while[61] $B(D_s^+ \to \pi^+\pi^+\pi^-)/B(D_s^+ \to \phi\pi^+) = 0.44 \pm 0.10 \pm 0.04$, a non-negligible part is due to $D_s^+ \to f_0(980)\pi^+$ where the $f_0(980)$, which contains strange quarks, decays to two pions because of the very restricted phase space for its decay to $K\bar{K}$. Thus, interpreted at the quark level, a substantial part of the three pion decays of the D_s actually involve strange quarks and should not be counted as due to annihilation of the initial quark and antiquark. As of now, the conclusion is that while annihilation and W exchange are surely present, they are not very important.

That, however, leaves us with another question: Where have all the strange quarks gone in D_s decay? For if the $\bar{s}$ quark in a D_s^+ does not annihilate, it must appear in the final state together with an s quark from (Cabibbo-allowed) charm decay. However the major known modes that satisfy this criterion, like $D_s \to \phi\pi$, $D_s \to K\bar{K}$, and $D_s \to K\bar{K}^*$, do not appear to have branching ratios that would allow us to account for the majority of D_s decays. An answer to the question has now come from the Mark II and Mark III collaborations, which observe[62,63] very substantial modes involving eta's and eta prime's (which contain $s\bar{s}$ valence quarks). The data[63] from Mark III shows an $\eta\pi^+$ mode of the D_s^+, while Mark II has signals[62] for both $D_s \to \eta\pi$ and $D_s \to \eta'\pi$. respectively. These results correspond to[63] $B(D_s \to \eta\pi)/B(D_s \to \phi\pi) \approx 2.5$ and to[62] $B(D_s \to \eta'\pi)/B(D_s \to \eta\pi) \geq 1$ Modes involving eta's and eta prime's are large. The strange quarks resulting from D_s decay are found a good faction of the time in eta's and eta prime's; any problem with modes involving strange quarks may well be solved.

4.2 Onium

Bound states of heavy quarks and heavy antiquarks provide us with the showcase of our understanding of strong interactions spectroscopy. The same flavor-independent potential can fit the spectra of both charmonium and bottomonium.[51] In this area also we are asking detailed, quantitative questions, and have much beautiful data to supply us with answers.

An example is provided by the three χ_b' (*i.e.*, $2^3P_{0,1,2}$) states which were recently clearly separated by the CUSB collaboration.[64] Now that we have both χ_b and χ_b' states and their mass splitting, it becomes interesting to ask if we understand this theoretically.

The splitting of these states, which is due to spin-orbit and tensor terms in the non-relativistic potential, can be expressed in terms of one absolute mass difference and one ratio,

$$R_\chi = \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)} = \frac{2a - \frac{12}{5}b}{a + 6b} \quad , \tag{19}$$

where a and b are the matrix elements of the spin-orbit and tensor terms, respectively. In a picture where one thinks of Lorentz vector and scalar exchanges as giving rise to the effective potential between the heavy quark and antiquark, an expansion in powers of v^2/c^2 gives the spin-independent potential $v(r) + s(r)$, plus various spin-dependent pieces which yield[65]

$$a = \frac{1}{2m^2}\left\langle -\frac{ds}{rdr} + 3\frac{dv}{rdr}\right\rangle \quad , \tag{20a}$$

$$b = \frac{1}{12m^2}\left\langle \frac{dv}{rdr} - \frac{d^2v}{dr^2}\right\rangle \quad , \tag{20b}$$

where m is the mass of the heavy quark. If only a Coulomb-like vector part of the potential, $v(r) \propto 1/r$, is present, $R_\chi = 0.8$. As the strength of the scalar term, $s(r)$, is increased, there is more cancellation between the two terms on the right-hand-side of Eq. (20a); the matrix element a decreases, and R_χ drops below 0.8.

The most recent experimental results[64] are $R_\chi = 0.67 \pm 0.06$ and $R_{\chi'} = 0.69 \pm 0.05$ for bottomonium. A rather simple model accounts for these numbers.[66,67]

We take the Cornell potential,[68]

$$V(r) = \frac{-\beta}{r} + kr = \frac{-0.52}{r} + \frac{r}{(2.34\ \mathrm{GeV}^{-1})^2} \quad , \tag{21}$$

where the two coefficients have been adjusted to fit the charmonium spectrum (although the model does a quite adequate job in describing bottomonium as well). The Schrodinger equation can be put in dimensionless form by using the variables $\rho = \mu\beta r = r/r_B$ and $K = k/(\beta^3\mu^2)$, where μ is the reduced mass and $r_B = 1/\beta\mu$ is the Bohr radius of the corresponding purely Coulomb potential problem. The assumption is then made that the $-\beta/r$ piece of the potential is a Lorentz four-vector, and the kr piece is a Lorentz scalar for all values of r.

Figure 7 then shows[66] the ratio of mass splittings R_χ for the 1P and 2P levels of the Cornell potential as a function of the scaled variable K. The arrows indicate the values of K corresponding to charmonium and bottomonium. The agreement with experiment is quite good for both charmonium (where experimentally, $R_\chi = 0.48 \pm 0.01$) and bottomonium, considering that nothing about spin-dependent effects was used as an input in the choice of parameters. For charmonium, however, the absolute magnitude of the χ_c splittings is about a factor of two smaller than experiment.

For quark masses above ~ 13 GeV, which corresponds to $K = 1/34$, $R_\chi > R_{\chi'}$, opposite to the situation for bottomonium. Even with very high mass quarks, very high radial excitations (which "live" primarily in the confining part of the potential) revert back to the situation for bottomonium: One gets larger values of R_χ as we go up in principal quantum number. More generally, the behavior of R_χ as we go from the lowest P states to their radial excitations is sensitive to the radial dependence of the Lorentz character of the effective interaction between heavy quarks, and can be used as a tool to understand this more detailed feature of the potential.

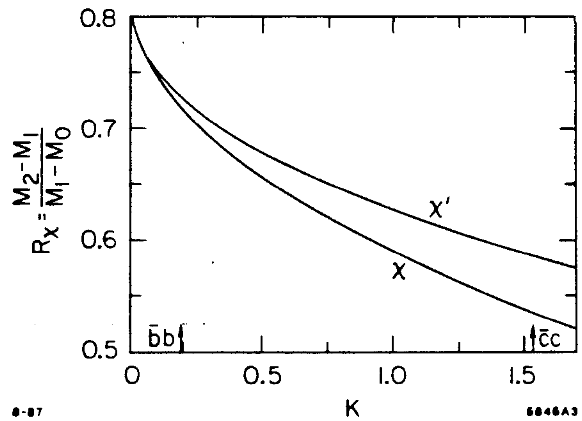

Fig. 7. The ratio R_χ for the Cornell potential as a function of the scaled, dimensionless variable K for the χ and χ' states, respectively. The arrows indicate the values of K corresponding to charmonium and bottomonium with $m_c = 1.84$ GeV and $m_b = 5.17$ GeV, respectively.

4.3 Mixing

As in the neutral K system, the neutral D and neutral B systems are capable of exhibiting mixing between, for example, an initial $B_d^0(\bar{b}d)$ and its charge conjugate state, $\bar{B}_d^0(\bar{d}b)$. A typical signature (although hardly the only one) arises from the ensuing semileptonic decay involving a negatively charged lepton instead of the positively charged one which would come from a B_d^0. Calling the eigenstates of the mass matrix B_1 and B_2, with $\Delta M = M_1 - M_2$ and $\Delta\Gamma = \Gamma_1 - \Gamma_2$, the relationship to experiment is made through the quantity

$$r = \frac{(\Delta M)^2 + (\Delta\Gamma/2)^2}{2\Gamma^2 + (\Delta M)^2 - (\Delta\Gamma/2)^2} \approx \frac{(\Delta M/\Gamma)^2}{2 + (\Delta M/\Gamma)^2}, \tag{22}$$

where the last approximation follows when $\Delta\Gamma << \Delta M$, as should be the case for the $B - \bar{B}$ system. When the initial B is tagged as to being a B^0 rather than $\bar{B}^0$, $r = \ell^-/\ell^+$, the number of "wrong" to "right" sign leptons in its semileptonic decay. For uncorrelated $B^0 + \bar{B}^0$ pairs it follows that $2r/(1+r^2) = \ell^\pm\ell^\pm/\ell^+\ell^-$, but for correlated pairs produced at the $\Upsilon(4S)$, $r = \ell^\pm\ell^\pm/\ell^+\ell^-$.

For the $D^0 - \bar{D}^0$ system we expect that r is of order 10^{-3} or so.[69] The tightest upper limit,[70]

$$r < 5 \times 10^{-3}$$

comes from E691. On the other hand, the Mark III has three events from operating at the ψ'' which have kaons of the same sign in nonleptonic decays of the final pair of D mesons.[54] These events also could arise from the doubly Cabibbo-suppressed decay of one of the D mesons and the Cabibbo-allowed decay of the other, and only "look like" mixing. This is also expected[69] at the level of a few times 10^{-3}, while the observed events, if real correspond to a signal at the 10^{-2} level. It will take more experiments to decide what is the level of mixing and of doubly Cabibbo-suppressed decays in the D system.

As already noted, we expect mixing in the neutral B system to be due to ΔM. Before the fact, a theoretical guess on the high end for $(\Delta m/\Gamma)_{B_d} = x_d$ was ~ 0.2. In 1987 the ARGUS collaboration found[25] $x_d = 0.73 \pm 0.18$ (corresponding to $r = 0.21 \pm 0.08$); the mixing time is not so different from the lifetime. This has now been confirmed by the CLEO collaboration, which finds a value of $r = 0.182 \pm 0.055 \pm 0.056$, which is compatible within the statistical or systematic errors.[71] For theorists this has meant an upward adjustment in the combination of a hadronic matrix element, a K–M matrix element, and, most importantly, in the value of m_t. For experimentalists, this together with the b lifetime means that in some situations not only will B_d mesons live long enough to leave a measurable gap, but that in this time there is a non-negligible chance that they will oscillate into the corresponding antiparticle state. The B_s meson must have large mixing in the three generation standard model, which has important consequences for observing CP violation for the B_s system as we will see later.

4.4 b → u

As noted in our discussion of the K–M matrix, the ARGUS collaboration claims evidence for the existence of a $b \to u$ transition from the observation of charmless final decay products that include baryons.[21] Specifically, they find $B(B^- \to p\bar{p}\pi^-) = 5.2 \pm 1.4 \pm 1.9 \times 10^{-4}$ and $B(B^0 \to p\bar{p}\pi^+\pi^-) = 6.0 \pm 2.0 \pm 2.2 \times 10^{-4}$ and have performed numerous checks to eliminate the possibility of a systematic error in the analysis or to have another interpretation of the origin of these final states.

The CLEO collaboration, however, on the basis of an enlarged data sample sees no evidence for these modes; they only quote upper limits. They do see[71] other decay modes, like $B \to D\pi \to K\pi\pi$, whose product branching ratios are at or below the ARGUS numbers, so there is no lack of sensitivity at the desired level. The conflict between the two experiments is several standard deviations.

With more data and more data analysis coming soon, we wait for a resolution of the difference between experimental results. A quantitative result for V_{ub} may have to await the observation of the semileptonic decays $B \to \pi e\nu$ and $B \to \rho e\nu$.

4.5 Penguins in B Decay

As we saw for K decays, in addition to changing the strength of the usual four-fermion effective weak interaction, there are additional operators introduced by QCD, the "penguins". In bottom decay it is possible to have particular quark level processes which are suppressed by K–M angles and are such that "penguin" diagrams give rise to contributions comparable to, or maybe even larger than, those of ordinary tree level graphs.[72] Figure 8 shows a possible example. The "penguin" diagram contributes an effective Hamiltonian density:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{12\pi}\, V_{tb}\, V_{ts}^*\, \ln(m_t^2/m_c^2)\; \bar{s}\gamma_\mu(1-\gamma_5)\lambda^a\, b\quad \bar{u}\gamma^\mu\, \lambda^a\, u \quad , \tag{23}$$

whereas the usual spectator diagram (aside from short-distance QCD correction factors, $c_\pm$, which are close to unity) corresponds to

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\, V_{ub}\, V_{us}^*\; \bar{u}\gamma_\mu(1-\gamma_5)b\;\; \bar{s}\gamma^\mu(1-\gamma_5)u \quad . \tag{24}$$

The "penguin" loses to the spectator graph because of the $\frac{\alpha_s}{12\pi}\ln(m_t^2/m_c^2)$ that arises from having one loop and the couplings due to the presence of the gluon, but it wins by at least a factor of 20 in amplitude because of the K–M factor $V_{tb}V_{ts}^*$ (which involves a combination of zero and one generation jumps), as compared to $V_{ub}V_{us}$, (which involves a combination of two and one generation jumps). Depending in part on the matrix elements in particular processes, it could well be that the spectator graph gives the lesser of the two contributions. Then, for example, in the decays $B_d \to K^+\pi^-$ or $B_s \to \phi\rho$ the "penguin" contribution may be dominant.[73]

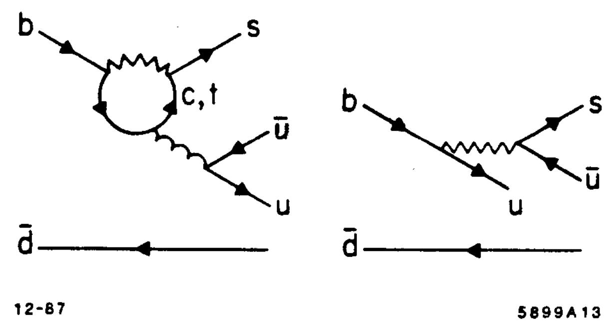

Fig. 8. Diagrams for the K–M suppressed decay $b \to s\bar{u}u$

It is possible that the inclusive rate for $b \to s\bar{q}q$ is of order 1% and exclusive modes a few percent of that. The experimental limits on exclusive channels[74,75] are beginning to reach the appropriate level. In particular, the CLEO limit[75] on the branching ratio for $B^0 \to K^+\pi^-$ is 0.9×10^{-4} and a number of the limits on other decay channels of this type from both ARGUS and CLEO are at the several times 10^{-4} level. The next few years could well see the observation of some of these decays.

There are also processes in the B system induced by "electromagnetic penguin" diagrams. The benchmark process of this type is $B \to K\mu\bar{\mu}$. In the standard model the decay $b \to se^+e^-$ should occur with a branching ratio[76,77] of several times 10^{-6}; the associated exclusive modes should be roughly an order of magnitude smaller. The presence of a fourth generation[78] could increase the branching ratio by perhaps an order of magnitude. The measurement of such small branching ratios still seems a way off.

The same basic one-loop diagram can lead to a real photon and result in the decay $b \to s+\gamma$ at the quark level, or $B \to K^*+\gamma$, $B \to K^{**}+\gamma$, etc. at the hadron level. Here QCD corrections are absolutely critical: They change the GIM suppression in the amplitude from being in the form of a power law, $(m_t^2 - m_c^2)/M_W^2$, to the softer form of a logarithm, $ln(m_t^2/m_c^2)$. This corresponds to an enhancement, depending on m_t, of one order of magnitude or more[79–81] over the rate expected from the simplest one-loop electroweak graph.[82] The inclusive process at the quark level, $b \to s\gamma$, should then occur with a branching ratio of roughly[79,80,81] several times 10^{-4}; exclusive modes like $B \to K^*\gamma$ and $B \to K^{**}\gamma$ are estimated at 5 to 10% of this.[79] Again a fourth generation could enhance this rate by an order of magnitude or so.[83] The extension to a supersymmetric world is more interesting. The obvious new diagrams come from putting the supersymmetric partners of the quarks and the W in the loop of the "electromagnetic penguin" diagram. Much more important,[84] however, is the transition from a "penguin" to a "penguino," — the "penguin" diagram involving a gluino and a squark. Because it involves strong interaction couplings rather than weak ones, it competes (and interferes) with the QCD enhanced "electromagnetic penguin" and produces an inclusive branching ratio that could be of order 10^{-3}.

Here again experiment is beginning to probe to the level of sensitivity needed to test theory. The ARGUS limit[74] on the branching ratio for $B \to K^*\gamma$ is now 2.4×10^{-4} and the limits on several other exclusive radiative B decay channels are close to this level. One can already say that these processes cannot be enhanced far beyond the standard model predictions.

5. CP VIOLATION IN B DECAY

When we form a CP violating asymmetry we divide a difference between the rate for a given process and the rate for its CP conjugate by their sum:

$$\text{Asymmetry} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \quad . \tag{25}$$

If we do this for K decays, the decay rates for the dominant hadronic and leptonic modes all involve a factor of s_1^2, i.e., essentially the Cabibbo angle squared. A CP violating asymmetry will then have the general dependence on K–M factors:

$$\text{Asymmetry}_{K\ Decay} \propto s_2 s_3 s_\delta \quad . \tag{26}$$

The right-hand-side is of order 10^{-3}. This is both a theoretical plus and an experimental minus. The theoretical good news is that CP violating asymmetries in the neutral K system are naturally at the 10^{-3} level, in agreement with the measured value of $|\epsilon|$. The experimental bad news is that, no matter what the K decay process, it is always going to be at this level, and therefore difficult to get at experimentally with the precision necessary to sort out the standard model explanation of the origin of CP violation from other explanations.

Note also that because CP violation must involve all three generations while the K has only first and second generation quarks in it (and its decay products only involve first generation quarks), CP violating effects must come about through heavy quarks in loops. There is no CP violation arising from tree graphs alone.

This is not the case in B decay (or B mixing and decay). First, the decay rate for the leading decays is very roughly proportional to s_2^2, which happens to be much smaller than the corresponding quantity (s_1^2) in K decay. But more importantly, we can look at decays which have rates that are K–M suppressed by factors of $(s_1 s_2)^2$ or $(s_1 s_3)^2$, just to choose two examples. By choosing particular decay modes, it is even possible to have asymmetries which behave like

$$\text{Asymmetry}_{B\ Decay} \propto s_\delta \quad . \tag{27}$$

With luck, this could be of order unity! Note, though, that we have to pay the price of CP violation somewhere. That price, the product $s_1^2 s_2 s_3 s_\delta$, is given in the CP violating difference of rates in Eq. (25). The K–M factors either are found in the basic decay rate, resulting in a very small branching ratio, or they enter the asymmetry, which is then correspondingly small. This is a typical pattern: the rarer the decay, the bigger the potential asymmetry. The only escape from this pattern comes from outside of K–M factors. A good example of this is provided by $B - \bar{B}$ mixing, which can be big because of a combination of the values of a hadronic matrix element and m_t, as well as a particular combination of K–M matrix elements.

The fact that asymmetries in K and B decay can be different by orders of magnitude is part and parcel of the origin of CP violation in the standard model. It "knows" about the quark mass matrices and can tell the difference between a b quark and an s quark. This is entirely different from what we expect in general from explanations of CP violation that come from very high mass scales, as in the superweak model or in left-right symmetric gauge theories. Then, all quark masses are negligible compared to the new, very high mass scale. Barring special provisions, there is no reason why such theories would distinguish one quark from another; we expect all CP violating effects to be roughly of the same order, namely that already observed in the neutral K system.

The possibilities for observation of CP violation in B decays are much richer than for the neutral K system. The situation is even reversed, in that for the B system the variety and size of CP violating asymmetries in decay amplitudes far overshadows that in the mass matrix.[85]

To start with the familiar, however, it is useful to consider the phenomenon of CP violation in the mass matrix of the neutral B system. Here, in analogy with the neutral K system, one defines a parameter ϵ_B. It is related to p and q, the coefficients of the B° and $\bar{B}^\circ$, respectively, in the combination which is a mass matrix eigenstate by

$$\frac{q}{p} = \frac{1-\epsilon_B}{1+\epsilon_B} \quad .$$

The charge asymmetry in $B^\circ\bar{B}^\circ \to \ell^\pm\ell^\pm + X$ is given by[86]

$$\frac{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) - \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)}{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) + \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)} = \frac{|\frac{p}{q}|^2 - |\frac{q}{p}|^2}{|\frac{p}{q}|^2 + |\frac{q}{p}|^2} \tag{28}$$

$$= \frac{\mathrm{Im}(\Gamma_{12}/M_{12})}{1+\frac{1}{4}|\Gamma_{12}/M_{12}|^2} \tag{29}$$

where we define $< B^\circ|H|\bar{B}^\circ >= M_{12} - \frac{i}{2}\Gamma_{12}$. The quantity $|M_{12}|$ is measured in $B - \bar{B}$ mixing and we may estimate Γ_{12} by noting that it gets contributions from B° decay channels which are common to both B° and $\bar{B}^\circ$, *i.e.*, K–M suppressed decay modes. This causes the charge asymmetry for dileptons most likely to be in the ballpark of a few times 10^{-3}, and at best 10^{-2}. For the foreseeable future, we might as well forget it experimentally.

Turning now to CP violation in decay amplitudes, in principle this can occur whenever there is more than one path to a common final state. For example, let us consider decay to a CP eigenstate, f, like ψK_s°. Since there is substantial $B^\circ - \bar{B}^\circ$ mixing, one can consider two decay chains of an initial B° meson:

$$\begin{array}{ll} B^\circ \to B^\circ & \searrow \\ & \quad f \quad , \\ B^\circ \to \bar{B}^\circ & \nearrow \end{array}$$

where f is a CP eigenstate. The second path differs in its phase because of the mixing of $B^\circ \to \bar{B}^\circ$, and because the decay of a $\bar{B}$ involves the complex conjugate of the K–M factors involved in B decay. The strong interactions, being CP invariant, give the same phases for the two paths. The amplitudes for these decay chains can interfere and generate non-zero asymmetries between $\Gamma(B^\circ(t) \to f)$ and $\Gamma(\bar{B}^\circ(t) \to f)$. Specifically,

$$\Gamma(\bar{B}^\circ(t) \to f) \sim e^{-\Gamma t}\left(1 - \sin[\Delta m\ t]\mathrm{Im}\left[\frac{p}{q}\rho\right]\right) \tag{30a}$$

and

$$\Gamma(B^\circ(t) \to f) \sim e^{-\Gamma t}\left(1 + \sin[\Delta m\ t]\mathrm{Im}\left(\frac{p}{q}\rho\right)\right) \quad . \tag{30b}$$

Here we have neglected any lifetime difference between the mass matrix eigenstates (thought to be very small) and set $\Delta m = m_1 - m_2$, the difference of the eigenstate masses, and $\rho = A(B \to f)/A(\bar{B} \to f)$, the ratio of the amplitudes, and we have used the fact that $|\rho| = 1$ when f is a CP eigenstate in writing Eqs. (30). From this we can form the asymmetry:

$$A_{\mathrm{CP\ Violation}} = \frac{\Gamma(B) - \Gamma(\bar{B})}{\Gamma(B) + \Gamma(\bar{B})} = \sin[\Delta m\ t]\mathrm{Im}\left(\frac{p}{q}\rho\right) \quad . \tag{31}$$

In the particular case of decay to a CP eigenstate, the quantity Im $(p/q\rho)$ is given entirely by the K–M matrix and is independent of hadronic amplitudes. However, to measure the asymmetry experimentally, one must know if one starts with an initial B° or $\bar{B}^\circ$, *i.e.*, one must "tag."

We can also form asymmetries where the final state f is not a CP eigenstate. Examples are $B_d \to D\pi$ compared to $\bar{B}_d \to \bar{D}\bar{\pi}$; $B_d \to \bar{D}\pi$ compared to $\bar{B}_d \to D\bar{\pi}$; or $B_s \to D_s^+ K^-$ compared to $\bar{B}_s \to \bar{D}_s^- K^+$. These is a decided disadvantage here in theoretical interpretation, in that the quantity $Im(p/q\rho)$ is now dependent on hadron dynamics.

It is instructive to look not just at the time - integrated asymmetry between rates for a given decay process and its CP conjugate, but to follow the time dependence,[87] as given in Eqs. (30a) and (30b). As a first example, Figs. 9, 10 and 11 show[88] the time dependence for the process $\bar{b} \to \bar{c}u\bar{d}$ (solid curve) in comparison to that for $b \to c\bar{u}d$ (dashed curve).

At the hadron level this could be, for example, $B_d \to \bar{D}^-\pi^+$ in comparison to $\bar{B}_d \to D^+\pi^-$. The direct process is very much K–M favored over that which is introduced through mixing, and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much greater than unity. Figs. 9, 10 and 11 show[89] the situation for $\Delta m/\Gamma = 0.2$ (at the high end of theoretical prejudice before the ARGUS result, Ref. 25, for B_d mixing), $\Delta m/\Gamma = \pi/4$ (near the central value from ARGUS), and $\Delta m/\Gamma = 5$ (roughly the minimum value expected for the B_s in the three generation standard model, given the central value of ARGUS for B_d). In none of these cases

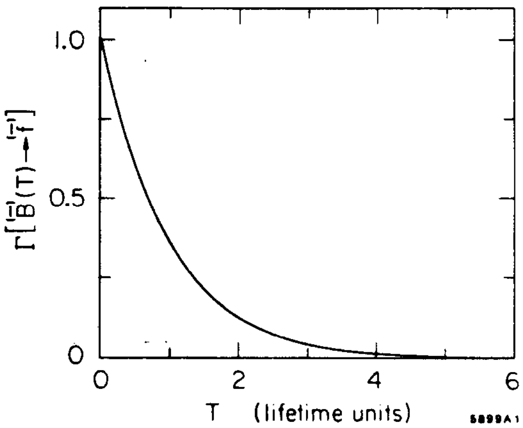

Fig. 9. The time dependence for the quark level process $\bar{b} \to \bar{c}u\bar{d}$ (solid curve) in comparison to that for $b \to c\bar{u}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to \bar{D}^-\pi^+$ in comparison to $\bar{B}_d \to D^+\pi^-$. $\Delta m/\Gamma = 0.2$.

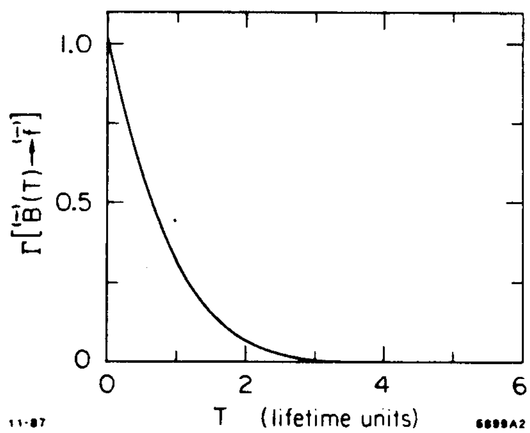

Fig. 10. Same as Fig. 9, but with $\Delta m/\Gamma = \pi/4$.

are the dashed and solid curves distinguishable within "experimental errors" in drawing the graphs. This is simply because $|\rho|$ is so large that even with "big" mixing the second path to the same final state has a very small amplitude, and hence not much of an interference effect.

A much more interesting case is shown in Figs. 12, 13 and 14 for the time dependence at the quark level for the process $\bar{b} \to \bar{c}c\bar{s}$ (solid curve) in comparison to that for $b \to c\bar{c}s$ (dashed curve).

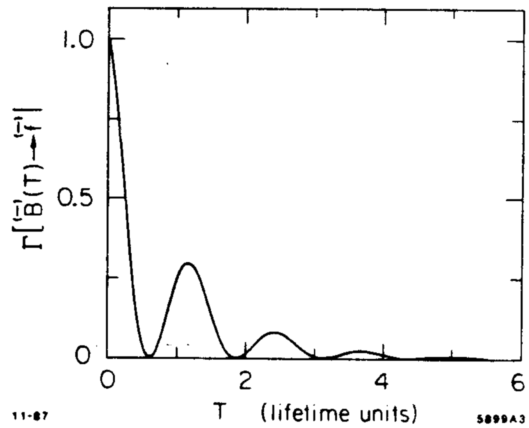

Fig. 11. Same as Fig. 9, but with $\Delta m/\Gamma = 5$.

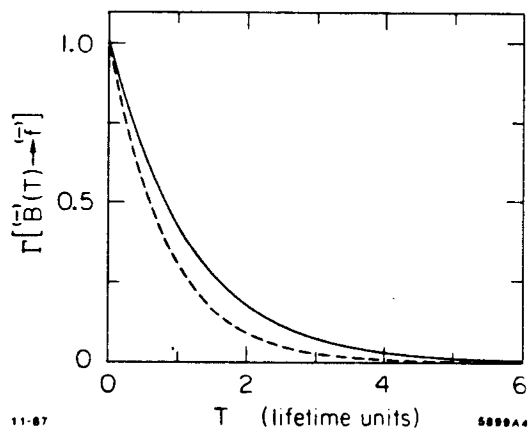

Fig. 12. The time dependence for the quark level process $\bar{b} \rightarrow \bar{c}c\bar{s}$ (solid curve) in comparison to that for $b \rightarrow c\bar{c}s$ (dashed curve). At the hadron level this could be, for example, $B_d \rightarrow \psi K_s^\circ$ (dashed curve) in comparison to $\bar{B}_d \rightarrow \psi K_s^\circ$ (solid curve). (The curves are interchanged for the ψK_s° final state because it is odd under CP.) $\Delta m/\Gamma = 0.2$.

At the hadron level this could be, for example, B_d in comparison to $\bar{B}_d$ decaying to the same, (CP self-conjugate) final state, ψK_s°. As discussed before, $|\rho| = 1$ in this case. The advantages of having $\Delta m/\Gamma$ for the B_d° system as suggested by ARGUS (Fig. 13) rather than previous theoretical estimates (Fig. 12) are very apparent. When we go to mixing parameters expected for the B_s° system (Fig. 14), the effects are truly spectacular.

Figures 15, 16 and 17 illustrate the opposite situation to that in Figs. 9–11; mixing into a big amplitude from a small one.

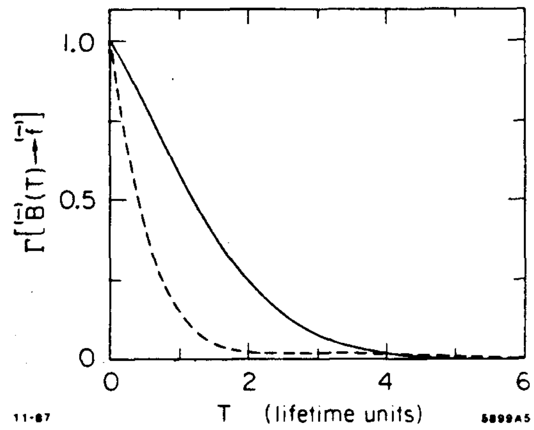

Fig. 13. Same as Fig. 12, but with $\Delta m/\Gamma = \pi/4$.

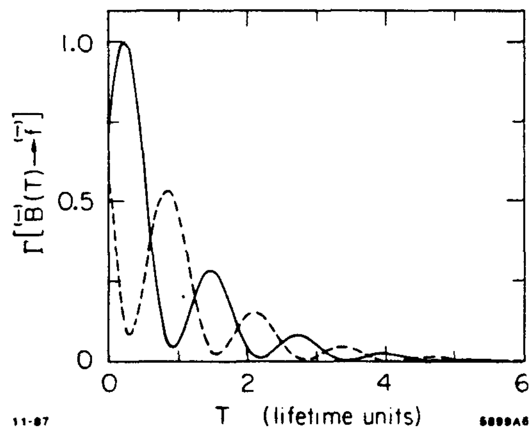

Fig. 14. Same as Fig. 12, but with $\Delta m/\Gamma = 5$.

We are explicitly comparing the quark level process $\bar{b} \to \bar{u}c\bar{d}$ (solid curve) to $b \to u\bar{c}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to D^+\pi^-$ in comparison to $\bar{B}_d \to \bar{D}^-\pi^+$. The direct process is very much K–M suppressed compared to that which occurs through mixing and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much less than unity. Here we have an example where too much mixing can be bad for you! As the mixing is increased (going from Figs. 15 to 17), the admixed amplitude comes to completely dominate over the original amplitude, and their interference (leading to an asymmetry) becomes less important in comparison to the dominant term.

A more likely example of the situation for B_s mixing is shown[90] in Fig. 18c. The oscillations are so rapid that even with a very favorable difference in the time dependence for an initial B_s *versus* an initial $\bar{B}_s$, the time-integrated asymmetry

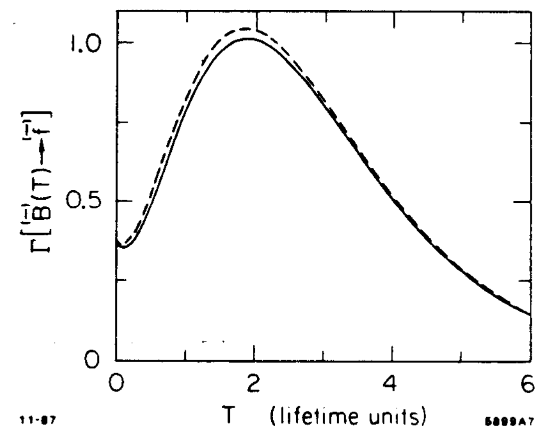

Fig. 15. The time dependence for the quark level process $\bar{b} \to \bar{u}c\bar{d}$ (solid curve) in comparison to that for $b \to u\bar{c}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to D^+\pi^-$ in comparison to $\bar{B}_d \to \bar{D}^-\pi^+$. $\Delta m/\Gamma = 0.2$.

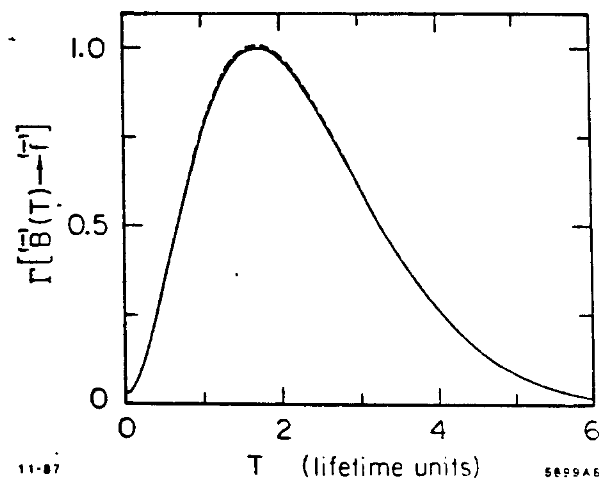

Fig. 16. Same as Fig. 15, but with $\Delta m/\Gamma = \pi/4$.

is quite small. Measurement of the time dependence becomes a necessity for CP violation studies.

A second path to the same final state could arise in several other ways besides through mixing. For example, one could have two cascade decays that end up with the same final state, such as:

$$B_u^- \to D^\circ K^- \to K_s^\circ \pi^\circ K^-$$

and

$$B_u^- \to \bar{D}^\circ K^- \to K_s^\circ \pi^\circ K^- \quad .$$

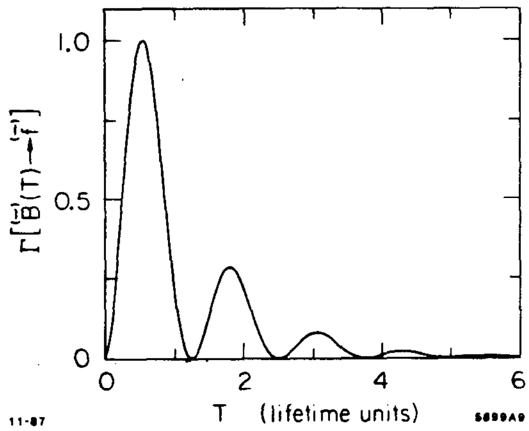

Fig. 17. Same as Fig. 15, but with $\Delta m/\Gamma = 5$.

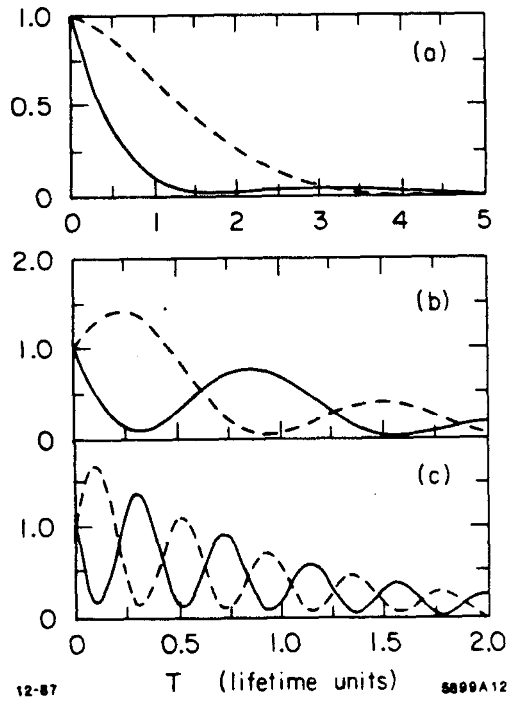

Fig. 18. The time dependence for the quark level process $\bar{b} \to \bar{u}u\bar{d}$ (dashed curve) in comparison to that for $b \to u\bar{u}d$ (solid curve). At the hadron level this could be, for example, $B_s \to \rho K_s^0$ (solid curve) in comparison to $\bar{B}_s \to \rho K_s^0$ (dashed curve) (the curves are interchanged for the ρK_s^0 final state because it is odd under CP) for values of (a) $\Delta m/\Gamma = 1$, (b) $\Delta m/\Gamma = 5$, and (c) $\Delta m/\Gamma = 15$, from Ref. 90.

Another possibility is to have spectator and annihilation graphs contribute to the same process.[91] Still another is to have spectator and "penguin" diagrams interfere. This latter possibility is the analogue of the origin of the parameter ϵ' in neutral K decay, but as discussed previously, there is no reason to generally expect a small asymmetry here. Indeed, with a careful choice of the decay process, large CP violating asymmetries are expected.

Note that not only do these routes to obtaining a CP violating asymmetry in decay rates not involve mixing, but they do not require one to know whether one started with a B or $\bar{B}$, *i.e.*, they do not require "tagging." These decay modes are in fact "self-tagging" in that the properties of the decay products (through their electric charges or flavors) themselves fix the nature of the parent B or $\bar{B}$.

Even with potentially large asymmetries, the experimental task of detecting these effects is a monumental one. When the numbers for branching ratios, efficiencies, etc. are put in, it appears that 10^7 to 10^8 produced B mesons are required to end up with a significant asymmetry (say, 3σ), depending on the decay mode chosen.[85] This is beyond the samples available today (of order a few times 10^5) or in the near future ($\sim 10^6$). The exciting prospect of being able to do this physics, but needing at least an order of magnitude more B's to have even a reasonable chance to see a statistically significant effect, has led to a series of studies (and even proposals) of high luminosity electron-positron machines ("B factories"), of detectors for hadron colliders, and of the possibilities in fixed target experiments.[92]

I look at the next several years as being analogous to reconnaissance before a battle: We are looking for the right place and manner to attack CP violation in the B meson system. We need:

- Information on branching ratios of "interesting" modes down to the $\sim 10^{-5}$ level in branching ratio. For example, we would like to know the branching ratios for $B_d \rightarrow \pi\pi, p\bar{p}, K\pi, \psi K, D\bar{D}$ + three body modes + . . . and for $B_s \rightarrow \psi\phi, K\bar{K}, D\pi, \rho K, \ldots$
- Accurate $B\bar{B}$ mixing data, first for B_d, but especially verification of the predicted large mixing of B_s.
- A look at the "benchmark" process of rare decays, $B \rightarrow K\mu\bar{\mu}$.
- Experience with triggering, secondary vertices, tertiary vertices, "tagging" B *versus* $\bar{B}$, distinguishing B_u from B_d, distinguishing B_d from B_s, . . .
- Various "engineering numbers" on cross sections, x_F dependence, B *versus* $\bar{B}$ production in hadronic collisions, . . .

Many of these things are worthy, lesser goals in their own right, and may reveal their own "surprises." But the major goal is to observe CP violation. With all the possibilities, plus our past history of getting some "lucky breaks," over the next few years we ought to be able to find some favorable modes and a workable trigger and detection strategy. While the actual observation of CP violation may well be five or more years away, this is a subject whose time has come.

6. THE t QUARK

We define the t quark to be the partner of the left-handed b quark in a weak isospin doublet. Such a partner must exist, i.e., the b quark cannot be in a singlet, because of the non-zero front-back asymmetry exhibited in the reaction $e^+e^- \to \bar{b}b$ in experiments at PEP and PETRA.[93] As this asymmetry is generated by an interference of the vector couplings of the photon with the axial-vector couplings of the Z, its magnitude can be interpreted in terms of the coupling, $g_{A,b}$, of the Z. This in turn is proportional to $I_{3,b}$. The data[93] indicates that this is non-zero, and to nobody's surprise, consistent with the value $-1/2$ that corresponds to the lower member of a weak isospin doublet. Therefore the b quark has non-zero weak isospin; it must share the same weak multiplet with another quark. An earlier argument[94] to the same end had shown that if the b was in a weak singlet, then there would be flavor changing neutral currents inducing processes like $b \to se^+e^-$ at tree level, in contradiction to experiment.

The discovery and elucidation of the properties of the t quark and its bound states is interesting from a number of aspects, aside from just being the completion of the task of finding all the fermions of the three generation standard model. In accordance with the reasons for studying heavy flavor physics which we outlined at the beginning of these lectures:

- The t quark mass and the K-M matrix elements connecting it to other quarks will allow us to check for possible relations between masses and mixing angles which follow from various proposals advanced until now. Perhaps it will suggest a new one.
- Its properties could also be an indication for physics beyond the standard model, for example by indicating mixing with a fourth generation or by decaying into a charged scalar.
- Its mass is a key input into many of the one loop calculations which we have discussed and an important part of the present uncertainty in theoretical predictions for the magnitude of these amplitudes will thereby be eliminated.
- The top quark is a very useful tool for the discovery of other particles. For example, a heavy Higgs boson should have a prominent decay to $t\bar{t}$ and toponium, if light enough, should have an appreciable decay to a photon plus a light Higgs boson.
- The weak decays of the top quark will involve the electroweak and the strong interactions in a regime where the strong coupling is truly small. This simpler context may well permit a quantitative, perturbative understanding of weak nonleptonic decays of the t quark, which is interesting in itself, and which

can be extrapolated back to lower mass scales and a better understanding of the b and c quark decays.

- The strong interaction spectroscopy of toponium probes the effective non-relativistic potential between quarks as the distance between them becomes very small. This is a region where we might hope perturbative QCD would give us information, with the potential eventually behaving as $-4/3\alpha_s/r$ as $r \to 0$ from one gluon exchange. In this case the spectroscopy is sensitive to other possible short distance effects due to new physics as well.

What is our present knowledge on the mass of the top quark? First, the t quark mass is constrained to be above 27.3 GeV from TRISTAN,[95] above 44 GeV from UA1,[96] and above about 50 GeV from theoretical considerations[24,29] based on the ARGUS result[25] for $B - \bar{B}$ mixing. In fact, the $B - \bar{B}$ mixing results, interpreted within the standard model and with nominal values for the relevant K–M and hadronic matrix elements, would have one entertain t quark masses in the vicinity of 100 GeV.

On the other end there is an upper limit on the t quark mass from the ρ parameter, defined as

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \quad . \tag{32}$$

The value of ρ is constrained to be unity when $SU(2)_L$ symmetry is exact. If we do not define $\cos\theta_W$ by imposing[97] Eq. (32) with $\rho = 1$ and the physical W and Z masses, then the value of ρ will deviate from unity due to the one loop contributions of quark-antiquark pairs to the vector boson masses, once the quarks in an $SU(2)_L$ doublet no longer have the same mass. In particular, the most relevant doublet is that consisting of the t and b quarks. Since the analysis[98,99] of the experimental data shows that[98]

$$\rho = 0.998 \pm 0.009 \ ,$$

the splitting between the t and b quark masses cannot be arbitrarily large. Specifically, Ref. 98 finds that $m_t < 180$ GeV. While one may quibble with some of the input or analysis that leads to either the lower to upper limits, a range of about 40 to 200 GeV now seems to be the relevant hunting ground for t quarks.

Where and how can we expect to find such a t quark? At $p\bar{p}$ colliders, the lowest-order production processes are from creation of a W followed by the decay $W \to t\bar{b}$ (if the t is light enough) and from gluon - gluon fusion, $gg \to t\bar{t}$. At the CERN collider the total cross section is about 1 nb for $m_t = 50$ GeV, and comes dominantly from W decay.[96] At the TEVATRON collider, the cross sections are bigger - something like 3 nb for a 50 GeV top mass, but dropping to roughly 100 pb when $m_t = 100$ GeV.[100] The data planned for collection in the coming year should suffice for detection of the t quark if its mass is less than about 100 GeV. To get truly spectacular cross sections we can go to the SSC, where even a 200 GeV top quark, the limiting case, is expected to be pair produced at the 10 nb level.[101]

At electron-positron colliders, the asymptotic cross section (for $\sqrt{s} >> m_t, M_Z$) is

$$\sigma(e^+e^- \to t\bar{t}) \sim 2.1\sigma_{\text{pt.}}(s) ,$$

where $\sigma_{\text{pt.}}(s) = 4\pi\alpha^2/3\ s$ is the cross section for $e^+e^- \to \mu^+\mu^-$ with just the one photon intermediate state. If m_t is as low as 40 GeV, so that $Z \to t\bar{t}$, the electron - positron annihilation cross section jumps at the Z peak to more than a nanobarn (more than a hundred times $\sigma_{\text{pt.}}$ at that energy).

The actual detection of the t quark is somewhat similar for both high energy $p\bar{p}$ and e^+e^- colliders. One looks for one t quark to decay in a semileptonic mode, (i.e., the real or virtual W decays to $\ell\bar{\nu}_\ell$), while the other decays in a hadronic mode (i.e., quark jets). The semileptonic decay can result in an isolated lepton with both high momentum and high transverse momentum with respect to the associated quark jet, giving a distinctive signature. One tries to "reconstruct" the W's, and then the t and $\bar{t}$. The demand that both the t and the $\bar{t}$ in the event yield the same mass is a non-trivial constraint which can be used to reject background.

How does the t quark decay? In the three generation standard model a great deal of the physics of t decays is fixed. Let us consider the semileptonic decay of t to b:

$$t \to b\ +\ W^+\ \to b\ e^+\nu_e\ ,$$

with the W^+ being either real for virtual, depending on the t mass. The tree-level width, for any value of m_t, is given by[102]

$$\Gamma(t \to b\ e^+\nu_e) =$$

$$\frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4\ |\vec{Q}\,|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \left[2|\vec{Q}\,|^2 + 3\ Q^2\left(1 - \frac{Q_0}{m_t}\right)\right] , \tag{33}$$

where Γ_W is the total width of the W and the integration variable Q^2 is the square of the four-momentum which it carries, with the associated quantities $Q_0 = (m_t^2 + Q^2 - m_b^2/2m_t)$ and $|\vec{Q}\,|^2 = Q_0^2 - Q^2$. In general, the right-hand side of Eq. (33) should contain the square of the relevant K–M matrix element, $|V_{tb}|^2$. In the case of three generations this is 0.997, i.e., equal to one to high accuracy.

In the limit that $m_t \ll M_W$, the momentum dependence of the W propagator can be neglected and the expression simplifies to

$$\begin{aligned}\Gamma(t \to b\, e^+\nu) =& \frac{G_F^2 m_t^5}{24\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2\, |\vec{Q}| \Big[2|\vec{Q}|^2 + 3\, Q^2(1 - Q_0/m_t)\Big] \\ =& \frac{G_F^2 m_t^5}{6\pi^3} \int\limits_0^{(m_t-m_b)^2} dQ^2\, |\vec{Q}|^3 \\ =& \frac{G_F^2 m_t^5}{192\pi^3}\Big[1 - 8\Delta^2 + 8\Delta^6 - \Delta^8 - 24\Delta^4\, ln\Delta\Big]\,,\end{aligned} \tag{34}$$

where $\Delta = m_b/m_t$.

In the other limit, where $m_t >> M_W$, we may integrate over the Breit–Wigner for producing a "real" W, and using

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\pi\sqrt{2}}\,, \tag{35}$$

rewrite Eq. (33) as

$$\Gamma(t \to b{+}W \to b\, e^+\nu_e) = B(W \to e\nu)\cdot\frac{G_F|\vec{Q}|}{2\pi\sqrt{2}}\left[2|\vec{Q}|^2{+}3M_W^2\left(1 - \frac{Q_0}{m_t}\right)\right]\,, \tag{36}$$

where now $Q^2 = M_W^2$ so that $Q_0 = (m_t^2 + M_W^2 - m_b^2/2m_t)$ and $|\vec{Q}|^2 = Q_0^2 - M_W^2$. For very large values of m_t, the width in Eq. (36) behaves as $B(W \to e\nu)\cdot G_F m_t^3/8\pi\sqrt{2}$, to be contrasted with Eq. (34).

The finite width of the W determines the behavior of the rate as we cross the threshold for producing a real W. Once we are several full widths of the W above threshold, the much larger width given in Eq. (36) for producing a "real" W dominates the total t decay rate. This is seen in Fig. 19, where[103] the $t \to be^+\nu_e$ decay rate is plotted *versus* m_t. The dashed curve is the result in Eq. (36) which would hold for production of a real, infinitely narrow W, while the solid curve gives the result of integrating Eq. (33) numerically.[104] For smaller values of m_t the width is less than $G_F^2 m_t^5/192\pi^3$ because of the finite value of m_b [here taken to be 5 GeV, see Eq. (34)], but then is enhanced by the W propagator as m_t increases. Even for values of $M_t \approx 50$ GeV, the finite mass of the W results in a $\approx 25\%$ increase in the t decay width over the value calculated with the point (infinite M_W) Fermi interaction; The exact result quickly matches that for an infinitely narrow W once we are several W widths above threshold. The finite W width simply provides a

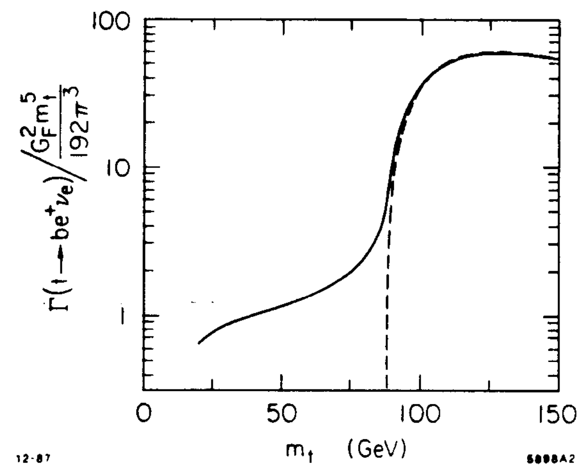

Fig. 19. $\Gamma(t \to b\ e^+\nu_e)/(G_F^2 m_t^5/192\pi^3)$ *as a function of* m_t *from the full expression in Eq. (1) for* $M_W = 83$ *GeV,* $\Gamma_W = 2.25$ *GeV and* $m_b = 5$ *GeV (solid curve), and from Eq. (4) for decay into a real, infinitely narrow W (dashed curve).*

smooth interpolation as the decay rate jumps by over an order of magnitude in crossing the threshold.

When m_t is in the present experimentally acceptable range, the rate for weak decay of the constituent t quarks within possible hadrons becomes comparable with that for electromagnetic and strong decays. Weak decays become a major fraction of, for example, the decays of the $J^P = 1^-$ toponium ground state, and even for the $T^*(t\bar{q})$ vector meson, weak decays can dominate the radiative magnetic dipole transition to its hyperfine partner, the T meson $J^P = 0^-$ ground state.[105] By the stage that $m_t = 100$ GeV, the total decay width of the t quark is ≈ 80 MeV. The weak decays of the constituent t and $\bar{t}$ quarks completely overshadow the usual electromagnetic and strong interaction decays of toponium. In fact, with a slightly higher mass the t decays so fast that it disappears before hadronic bound states can form.[106]

We now examine the transition region where $m_t \approx m_b + M_W$ in more detail.[103] Ordinarily the weak transition $t \to s$ is suppressed relative to $t \to b$ by the ratio of the relevant K–M matrix elements squared, $|V_{ts}|^2/|V_{tb}|^2 \approx 1/500$. However, we have seen that $\Gamma(t \to b\ e^+\nu_e)$ increases sharply as m_t crosses the W threshold, changing from being proportional to G_F^2 to being proportional to G_F. Thus we expect $\Gamma(t \to se^+\nu_e)$ to be enhanced relative to $\Gamma(t \to be^+\nu_e)$ when m_t lies between the two thresholds: $M_W + m_s < m_t < M_W + m_b$. The question is whether the threshold enhancement "wins" over the K–M suppression.

To examine this quantitatively we consider the ratio of the widths with the K–M factors divided out:

$$\frac{\Gamma(t \to b\ e^+\nu_e)/|V_{tb}|^2}{\Gamma(t \to se^+\nu_e)/|V_{ts}|^2},$$

Either well below or well above threshold for a "real" W this ratio should be near unity. For an infinitely narrow W the denominator is strongly enhanced, but the numerator is not, when $M_W + m_s < m_t < M_W + m_b$. The ratio indeed drops dramatically near $t \to s + W$ threshold, as shown in Fig. 20, for $\Gamma_W =$ 0.0225 GeV (dotted curve) and even for $\Gamma_W = 0.225$ GeV (dashed curve). However, the expected W width of 2.25 GeV (solid curve) smears out the threshold effect over a mass range that is of the same order as $m_b - m_s$, and gives only a modest dip (to ≈ 0.6) in the ratio. This is hardly enough to make $t \to s$ comparable to $t \to b$.

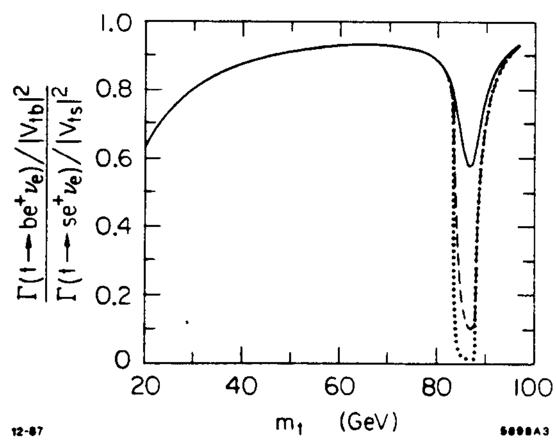

Fig. 20. The ratio of decay rates with K–M factors taken out, $\left(\Gamma(t \to b\ e^+\nu_e)/|V_{tb}|^2\right)/\left(\Gamma(t \to se^+\nu_e)/|V_{ts}|^2\right)$ *with* $m_b =$ *5 GeV and* $m_s = 0.5$ *GeV and* Γ_W *equal to fictitious values of 0.0225 GeV (dotted curve) and 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve).*

We now turn to the possible exclusive decay channels of hadrons which contain a t quark. In decays of heavy flavor mesons the branching ratios for typical exclusive channels scale like $(f/M_Q)^2$, where f is a meson decay constant (like f_π or f_K), of order 100 MeV, and M_Q is the mass of the heavy quark. For D mesons individual channels have branching ratios of a few percent; for B mesons they are roughly ten times smaller; and for T (or T^*) mesons they should be a hundred or more times smaller yet. It should be possible to treat T decays in terms of those of the constituent t quark, $t \to b + W^+$, with the b quark appearing in a b jet not so different from those already observed at PEP and PETRA.

There is one possible exception to these last statements, and that is when $m_t \approx m_b + M_W$, the situation we are examining in more detail here. In this case there is a premium on giving as much energy to the W as possible, *i.e.*, keeping as far above threshold for "real" W production as possible, and hence on keeping the invariant mass of the hadronic system containing the b quark small. Then we expect the T and T^* to decay dominantly into a few exclusive channels: a "real" W plus a B or a "real" W plus a B^*.

Furthermore, this is one place where the use of the non-relativistic quark model is *a priori* well-justified. The t quark and final W are very heavy. When $m_t \approx m_b + M_W$, the final heavy b quark is restricted to have a few GeV or less of kinetic energy if the W is to be as "real" as possible. The accompanying light quark in the T hadron is very much a spectator which simply becomes part of the final B or B^* hadron.

We need only match up the matrix elements of the weak currents taken between hadron states (and expressed in terms of form factors) with the matrix elements of those same currents taken between quark states in the appropriate spin and flavor configurations found in the hadrons. The details of all this are found in Ref. 103.

Within the scenario of discovery of the top quark at a hadron collider, it would be useful to have several handles on the value of m_t. An indirect method would be to measure a quantity in top decays which depends strongly on the top mass. For m_t in the vicinity of $M_W + m_b$, such a quantity is the ratio of the production of longitudinal W's to that of transverse W's in top decay.

The decay widths into longitudinal and transverse W's are defined by decomposing the numerator of the W propagator as

$$g_{\mu\nu} - Q_\mu Q_\nu / M_W^2 = \sum_\lambda \epsilon_\mu(\lambda)\epsilon_\nu^\star(\lambda) = \epsilon_\mu^{(+)}\epsilon_\nu^{(+)\star} + \epsilon_\mu^{(0)}\epsilon_\nu^{(0)\star} + \epsilon_\mu^{(-)}\epsilon_\nu^{(-)\star} \,, \tag{37}$$

where the superscripts give the helicity of the W, whether virtual or real. In calculating the t decay rate in Eq. (33), we define $\Gamma_L = \Gamma^{(0)}$, originating from W's with helicity zero, and $\Gamma_T = \Gamma^{(+)} + \Gamma^{(-)}$, originating from W's with helicity ± 1. Separating in this way the portions of Eq. (33) that originated from longitudinal and transverse W's, we find

$$\Gamma_L \equiv \frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \, |\vec{Q}\,|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \left[2|\vec{Q}\,|^2 + Q^2\left(1 - \frac{Q_0}{m_t}\right)\right] , \tag{38a}$$

$$\Gamma_T = \frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \, |\vec{Q}\,|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \left[2\, Q^2\left(1 - \frac{Q_0}{m_t}\right)\right] . \tag{38b}$$

In the case $m_t << M_W$ the integrals can be done with the result

$$\frac{\Gamma_L}{\Gamma_T} = 2\ . \tag{39}$$

Sufficiently far above the W threshold we need only calculate the relative production of longitudinal and transverse real W's:

$$\frac{\Gamma_L}{\Gamma_T} = \frac{1}{2} + \frac{m_t|\overrightarrow{Q}_W|^2}{E_b M_W^2}\ . \tag{40}$$

Precisely at threshold where we have an s-wave decay with two transverse W polarization states and one longitudinal state, $\Gamma_L/\Gamma_T = 1/2$. The value of Γ_L/Γ_T near the threshold is shown in Fig. 21 for $\Gamma_W = 0.0225$ GeV (dotted curve), 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve). In this case we see that even for the expected value of Γ_W the ratio varies rapidly with m_t, especially just below the threshold.

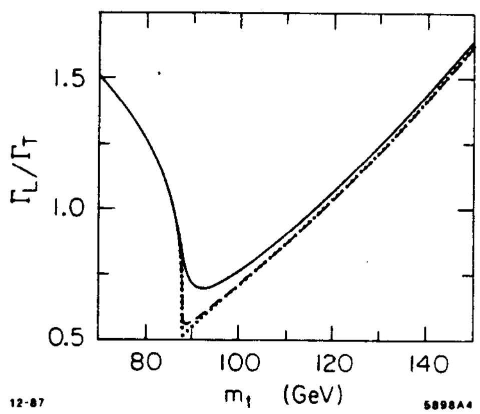

Fig. 21. The ratio Γ_L/Γ_T of $t \to b + W \to b\ e^+\nu_e$ decay widths into longitudinal compared to transverse W's as a function of m_t for Γ_W equal to fictitious values of 0.0225 GeV (dotted curve) and 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve).

The ratio of longitudinal to transverse W's is reflected in the angular distribution of the electrons[107] from its decay. With the final b quark direction as a polar axis,

$$\frac{d\Gamma}{d\cos\theta} = 1 + \alpha\ \cos^2\theta\ , \tag{41}$$

where

$$\alpha = \frac{\Gamma_T - \Gamma_L}{\Gamma_T + \Gamma_L} \quad . \tag{42}$$

Thus a measurement of α gives a value for Γ_L/Γ_T and indirectly a value for m_t. In particular, α changes rapidly and becomes positive only a few GeV below the threshold; this may provide a useful lower bound on m_t.

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