SLAC–PUB–4734
October 1988
T, AS

WORMHOLES AND COSMOLOGY*

IGOR KLEBANOV

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

and

LEONARD SUSSKIND†

Physics Department
Stanford University, Stanford, California 94309

ABSTRACT

We review Coleman's wormhole mechanism for the vanishing of the cosmological constant. We find a discouraging result that wormholes much bigger than the Planck size are generated. We also consider the implications of the wormhole theory for cosmology.

Invited talk delivered at the Meeting of
the Division of Particles and Fields of the APS,
Storrs, CT, August 15-18, 1988

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.
† Work supported by NSF PHY 812280

A wormhole is a microscopic contact between two otherwise smooth regions of space-time. It is small and costs little action but can connect arbitrarily distant regions. Evidently, wormholes provide a connection between the largest and the smallest distance scales encountered in physics. Such a connection may be necessary to solve the cosmological constant problem.[1] Recently, Coleman[2] and Giddings and Strominger[3] considered the effect of wormholes in the Euclidean path integral (EPI) of quantum gravity. Similar ideas were explored by Banks.[4] Remarkably, it was shown that the entire effect is to modify coupling constants and to provide a probability distribution for them. Coleman has advanced an even more remarkable claim[5] that the probability for the cosmological constant is overwhelmingly peaked at zero.[6] We are going to review Coleman's arguments for determination of the cosmological constant and other fundamental parameters and discuss the implications of his theory for physics of the early universe.

The basic assumption is that the EPI of quantum gravity is dominated by geometries which consist of some number of large universes connected by wormholes of Planck size. To begin with, we will treat the wormholes as dilute so that their emissions are independent. Let $\langle M \rangle_\lambda$ denote the expectation value of M in a large universe of spherical topology without wormholes and with parameters λ. Now consider the effects of wormholes. Suppose that the two points connected by a wormhole are x and x'. If $\phi_i(x)$ forms a basis of local operators at x, we assume that the effect of a wormhole is to insert the expression $\frac{1}{2}C_{ij}\phi_i(x)\phi_j(x')$ into the EPI, where $C_{ij} \sim \exp(-S_w)$ and S_w is the wormhole action. It is important to distinguish wormholes from the ordinary processes whose amplitudes fall off with distance: since wormholes 'short circuit' space-time, the coefficients C_{ij} do not depend on x and x', at least when the two points are distant. The sum over any number of wormholes attached to one large universe exponentiates to yield

$$\langle M \rangle \sim \int dg M e^{-I(g,\lambda)} \exp\left(\frac{1}{2}C_{ij}\int dx dx' \phi_i(x)\phi_j(x')\right) \tag{1}$$

where $\int dg$ denotes EPI over smooth metric and all other fields in one universe. If

$I(g,\lambda) = \lambda_i \int dx\phi_i$, this can be manipulated into the form

$$\langle M \rangle \sim \int \prod_k d\alpha_k \exp\left(-\frac{1}{2}D_{ij}\alpha_i\alpha_j\right) \int dg M\, e^{-I(g,\lambda+\alpha)} \tag{2}$$

where D_{ij} is the inverse of C_{ij}. Similarly, we can take into account processes involving additional large universes (figure 1). Each one gives a factor $X(\lambda+\alpha) = \int dg \exp(-I(g,\lambda+\alpha))$ in the α-integrand. The combinatorics again exponentiate giving

$$\langle M \rangle = \frac{1}{N} \int d\alpha \exp\left(-\frac{1}{2}D_{ij}\alpha_i\alpha_j\right) \exp\left(\int dg' e^{-I(g',\lambda+\alpha)}\right) \int dg M e^{-I(g,\lambda+\alpha)} \tag{3}$$

where N is a normalization factor. This can be written as

$$\langle M \rangle = \int d\alpha \rho(\alpha) \langle M \rangle_{\lambda+\alpha} \tag{4}$$

which implies that any expectation value is a weighted average over expectation values in ordinary universes without wormholes, with couplings $\lambda + \alpha$. The same formula would result for an ensemble of worlds with a statistical distribution of coupling constants. If ρ is not peaked sharply, we have lost power to predict measurable parameters.

Do wormholes create non-localities? Yes, but only the familiar ones, associated with space-time independence of all the coupling constants. Since Eq. (4) has a single overall integral over α_i, wormholes equalize the couplings in all regions of space-time including the large universes which would otherwise be disconnected.

To calculate $\rho(\alpha)$ in Eq. (4), we need to know $X(\lambda + \alpha)$, the EPI in a large universe without wormholes. Let us compute the effective action in a smooth universe with metric g_{ij} by integrating over fluctuations of all the fields:

$$\Gamma = \int d^4x\sqrt{g}\left(\Lambda - \frac{1}{16\pi G}R + aR_{abcd}R^{abcd} + bR_{ab}R^{ab} + cR^2 + \dots\right) \tag{5}$$

For simplicity, we assume that Λ, $1/G$, a, etc., are linear functions of the α's. If we approximate Γ by the first two terms, then the variational equation is $R_{ij} =$

$8\pi G\Lambda g_{ij}$. Its minimum action solution is the 4-sphere whose radius becomes large as $\Lambda \to 0$. Therefore, let us restrict Γ to large 4-spheres of radius r:

$$\Gamma(r) = \frac{8\pi^2}{3}\left(\Lambda r^4 - \frac{3}{4\pi G}r^2 + A_1 + \frac{A_2}{r^2} + \ldots\right) \tag{6}$$

For small Λ, Γ has a stationary point at $\bar{r}^2 \approx \frac{3}{8\pi G\Lambda}$ with $\Gamma(\bar{r}) \approx -\frac{3}{8G^2\Lambda} + \frac{8\pi^2}{3}A_1$. Coleman suggests that this saddle point dominates the EPI in one large universe. Then, $X \approx \exp(-\Gamma(\bar{r}))$ and

$$\rho \approx \frac{1}{N}\exp\left(-\frac{1}{2}D_{ij}\alpha_i\alpha_j\right)\exp\left(\exp\left(\frac{3}{8G^2\Lambda} - \frac{8\pi^2}{3}A_1\right) + \frac{3}{8G^2\Lambda} - \frac{8\pi^2}{3}A_1\right) \tag{7}$$

The cosmological constant problem is solved since the absolute maximum of this function occurs at $G^2\Lambda = 0$. This defines a surface in the α-space. On this surface the probability depends infinitely strongly on the value of A_1. Is there anything that prevents A_1 from being driven to $-\infty$? Let us suppose that each α_i is bounded by strong effects due to violation of the dilute wormhole approximation. Indeed, the shifts of parameters induced by wormholes of size a are proportional to their density in space-time. When density becomes comparable to $1/a^4$, wormholes pack space-time densely and we assume that further wormholes of this size cannot appear. This seems to put a bound on the shifts of parameters introduced by wormholes. However, there is a loophole: we have overlooked the contributions of the wormholes which are much bigger than the Planck size. One might think that they are suppressed by a large action: the eigenvalues of D_{ij} which correspond to large wormholes are enormous. However, the other terms in (7) are so singular as $\Lambda \to 0$ that no finite D_{ij} can restrict the variation of the α's. In addition, on purely geometrical grounds, a high density of small wormholes does not prevent the large ones from appearing (figure 2). Thus, if Planck-size wormholes pack space-time densely, then much bigger ones appear to further shift A_1 until they become dense, and so on. Eventually, we are forced into an unphysical conclusion that wormholes of macroscopic sizes must be generated. Although these effects are best addressed in a renormalization group framework, our qualitative discussion casts some doubt over the consistency of Coleman's saddle point analysis. Assuming the exponentially large contribution of each 4-sphere as $\Lambda \to 0$ solves the cosmological constant problem, but also leads to the unpleasant side effect of creating an infinite driving force on wormholes of all possible sizes. This suggests that the Euclidean de-Sitter space (the 4-sphere) is unstable with respect to wormhole fluctuations. Undoubtedly, we need a better understanding of the EPI in a large smooth universe. Perhaps, if this quantity has a power law rather than the exponential growth as $\Lambda \to 0$, the troublesome macroscopic wormholes can be avoided.

Although the present version of the theory may be incomplete, we find the basic set of ideas very attractive. Ignoring the possible difficulties outlined above, we are tempted to test these ideas on other issues, such as cosmology. It will be disappointing if the theory truly predicts nothing rather than something: a cold universe devoid of matter and energy. We must hope that there is some number of universes which have undergone an interesting cosmological development. To study generation of heat, we include a scalar field ϕ with a double-well potential $V(\phi)$. Now there are 2 Euclidean saddle points: the bigger (smaller) 4-sphere is relevant to nucleation of the universe in the lower (higher) well ϕ_a (ϕ_b). Eventually, tunneling from ϕ_b to ϕ_a, accompanied by generation of heat, takes place in the classically allowed region. Therefore, a warm (cold) universe can be recognized in the EPI as the smaller (bigger) 4-sphere. In analogy with figure 1, we assume that the EPI is dominated by networks of large and small bubbles connected by wormholes. One likely outcome of the theory is that wormholes drive both $V(\phi_a)$ and $V(\phi_b)$ to zero. Then there can only be cold universes. We have argued[7] that there should also be models where only $V(\phi_a)$ is sent to zero. Under these circumstances, there is a finite number of warm universes in contact with an infinity of cold ones. Due to this contact, $V(\phi_a) = 0$, which implies that the cosmological constant in the warm universes vanishes. The details of this scenario may vary depending on the specific mechanism for inflation. However, the idea that the cosmological constant in our warm universe is driven to zero by contact with an infinity of cold universes can be quite general.

Acknowledgements: This talk is based on joint work with T. Banks[7] where the difficulty with the large wormholes was first mentioned. It was suggested to us by V. Kaplunovsky. The present form of the argument was inspired by conversations with S. Weinberg. Conversations with M. Wise are also gratefully acknowledged.

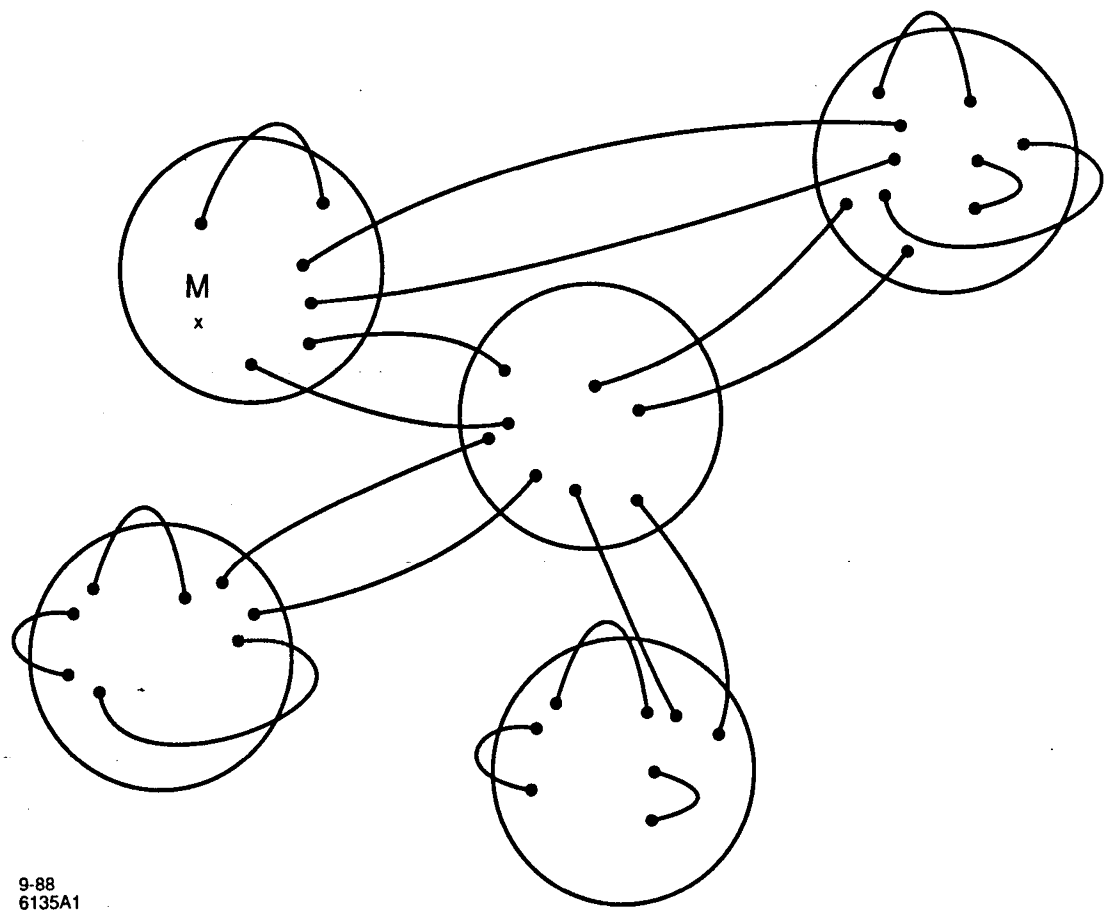
REFERENCES

1. A. Linde, *Phys. Lett.* **200B** (1988), 272
2. S. Coleman, *Nucl. Phys.* **B307** (1988), 867
3. S. Giddings and A. Strominger, *Nucl. Phys.* **B307** (1988), 854
4. T. Banks, Santa Cruz preprint SCIPP-88/09
5. S. Coleman, Harvard preprint HUTP-88/A022
6. The idea that the probability for the cosmological constant is peaked at zero appears in an earlier paper by S. Hawking, *Phys. Lett.* **134B** (1984), 403
7. I. Klebanov, L. Susskind and T. Banks, SLAC-PUB-4705

FIGURE CAPTIONS

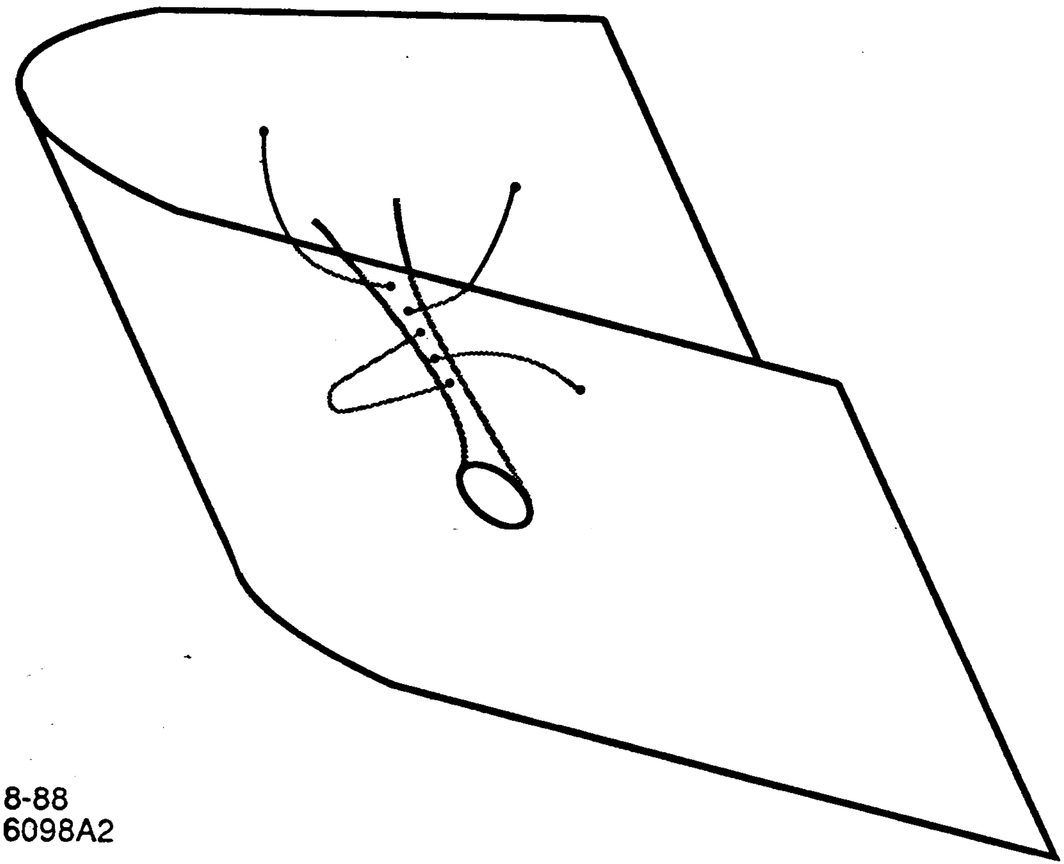
1) A number of large Euclidean universes connected by wormholes.

2) A large wormhole with small wormholes attached to it.



9-88
6135A1

Fig. 1



8-88
6098A2

Fig. 2