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BEAM DETERMINATION OF QUADRUPOLE MISALIGNMENTS AND BEAM POSITION MONITOR BIASES IN THE SLC LINAC*

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ABSTRACT

Misalignments of magnetic quadrupoles and biases in beam position monitors (BPMs) in the Stanford Linear Collider (SLC) linac can lead to a situation in which the beam is off-center in the disk-loaded waveguide accelerator structure. The off-center beam produces wakefields which can limit SLC performance by causing unacceptably large emittance growth. We present a general method for determining quadrupole misalignments and BPM biases in the SLC linac by using beam trajectory measurements. The method utilizes both electron and positron beams on opposite RF cycles in the same linac lattice to determine simultaneously magnetic quadrupole misalignments and BPM biases. The two-beam trajectory data may be acquired without interrupting SLC colliding beam operations.

INTRODUCTION

The lattice of the SLC linac in the region traversed by both electron and positron beams consists of many sets of quadrupole magnets for focusing, dipole magnets for steering correction, and stripline BPMs for measuring both horizontal and vertical beam positions.[1] In each set the BPM is mounted in the bore of the quadrupole and these elements are positioned 35–65 cm upstream from the dipole. The sets are separated from one another by spaces of between three and 12 meters containing RF sections. Misalignments of quadrupoles relative to the beam trajectories steer the beams. The BPMs are used to measure the trajectories and dipoles are used to correct the steering. The quadrupole misalignments and biases in the BPMs (due to both electronics and BPM misalignment) can lead to a situation in which the beam is off-center in the disk-loaded waveguide accelerator structure. The off-center beam produces wakefields which can limit SLC performance by causing unacceptably large emittance growth and beam breakup at currents below the design value. In the past, an optical system has been used to align the linac quadrupoles to high precision.[2] Beam-based surveying techniques such as the one described here provide a complimentary alignment tool which we are using to improve the mechanical alignment of the SLC linac.

THEORY

Figure 1 illustrates schematically a linac lattice segment containing a biased BPM and a misaligned quadrupole. Beam transport equations in a lattice with quadrupole misalignments and BPM biases can be formulated using the following definitions. We denote by 0, ..., $N+1$ the $N+2$ sets of BPMs, quadrupoles, and dipoles in a linac lattice segment. The accelerator axis is defined to pass through the centers of the endpoint BPMs (0 and $N+1$). For either transverse coordinate (labeled x) let

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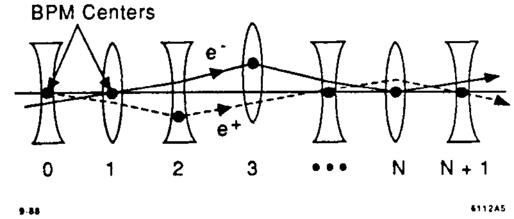

FIG. 1. Linac lattice containing a biased BPM (2) and a misaligned quadrupole (3). The electron and positron beams have been steered using correctors near each quadrupole to minimize the BPM measurements.

d_i = misalignment of ith quadrupole relative to the axis

b_i = bias of ith BPM relative to center of ith quadrupole

$m_i^{\pm}$ = measured displacement of the $e^{\pm}$ bunch at ith BPM

$x_i^{\pm}$ = $e^{\pm}$ trajectory displacement off axis at ith quadrupole

$\theta_i^{\pm}$ = $e^{\pm}$ trajectory slope relative to axis at ith quadrupole

l_i = drift length preceding the ith quadrupole

D_i = integrated kick angle of the ith dipole

($D > 0$ kicks e^- in the direction of increasing x)

Q_i = integrated gradient of the ith quadrupole

($Q > 0$ focuses e^- in the horizontal plane)

Trajectory displacement off axis at a quadrupole is $d+b+m$ as shown in Fig. 2. The integrated dipole kicks D_i and quadrupole gradients Q_i are inversely proportional to the beam momentum. which is modified at each RF section of the linac.

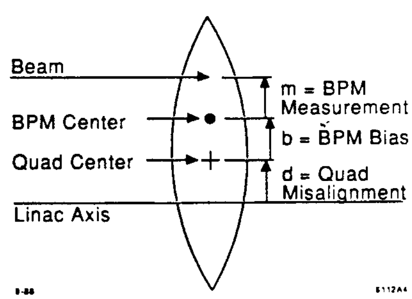

FIG. 2. Trajectory displacement off axis at a quadrupole is $d+b+m$.

In the approximation that all lenses are thin and that the BPM, quadrupole, and dipole of each set are superimposed at the same axial position, the beam transport equations are

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$$
\begin{aligned}
x_i^{\pm} &= m_i^{\pm}, \quad i = 0 \text{ and } N+1, \\
x_i^{\pm} - b_i - d_i &= m_i^{\pm}, \quad i = 1, \ldots, N, \\
\theta_i^{\pm} - \theta_{i-1}^{\pm} \mp (x_i^{\pm} - d_i)Q_i &= \mp D_i, \quad i = 1, \ldots, N, \\
x_i^{\pm} - x_{i-1}^{\pm} - l_i\theta_{i-1}^{\pm} &= 0, \quad i = 1, \ldots, N+1.
\end{aligned}
\tag{1}
$$

These equations can be rewritten in matrix form as

$$
\begin{pmatrix}
A_{1,1} & \cdots & A_{1,6N+6} \\
 & & \\
\vdots & & \vdots \\
 & & \\
A_{6N+6,1} & \cdots & A_{6N+6,6N+6}
\end{pmatrix}
\begin{pmatrix} d_1 \\ \vdots \\ d_N \\ b_1 \\ \vdots \\ b_N \\ x_0^+ \\ \vdots \\ x_{N+1}^- \\ \theta_0^+ \\ \vdots \\ \theta_N^- \end{pmatrix}
=
\begin{pmatrix} m_0^+ \\ \vdots \\ m_{N+1}^+ \\ m_0^- \\ \vdots \\ m_{N+1}^- \\ D_1 \\ \vdots \\ D_N \\ 0 \\ \vdots \\ 0 \end{pmatrix}
\tag{2}
$$

where A matrix elements are the coefficients of the d_i, b_i, x_i, and θ_i in Eq. (1). These $6N+6$ equations may be solved for the quadrupole misalignments (d_1, …, d_N), BPM biases (b_1, …, b_N), trajectory displacements ($x_0^{\pm}$, …, $x_{N+1}^{\pm}$), and trajectory slopes ($\theta_0^{\pm}$, …, $\theta_N^{\pm}$). Calculated values of d_i, b_i, x_i, and θ_i at any lattice point i are linear combinations of BPM measurements and dipole kicks. Schematically, the solution for the jth unknown in the column vector on the left side of Eq. (2) is

$$
\sum_{k=1}^{N+2} A_{jk}^{-1} m_k^+ + \sum_{k=N+3}^{2N+4} A_{jk}^{-1} m_k^- + \sum_{k=2N+5}^{3N+4} A_{jk}^{-1} D_k \,.
$$

The square of the statistical error in the calculated value of the jth unknown therefore is approximately

$$
\sum_{k=1}^{2N+4} (A_{jk}^{-1})^2 \sigma^2 \tag{3}
$$

where A_{jk}^{-1} is the kth element of the jth row of the inverse of matrix A in Eq. (2) and σ is the BPM resolution, which typically is 50 μm for single-pulse measurements and can be reduced by averaging measurements made on several consecutive pulses. Uncertainties in the dipole kicks D_k and in the quadrupole gradients Q_k are neglected in Eq. (3) because their effect is small compared to the BPM resolution.

ANALYSIS

A computer program has been written to implement the two-beam method. The program acquires BPM measurements $m_i^{\pm}$, quadrupole gradients Q_i, and dipole kicks D_i from previously acquired data files, solves Eq. (2) numerically, and computes statistical errors using Eq. (3). The program may be applied to any segment of the SLC linac for which BPM measurements exist for electrons and positrons at all N quadrupoles and at both endpoints.

The precision of the two-beam method as an alignment tool depends on statistical and systematic errors. Statistical errors have been studied by applying the method to segments of varying length at different regions of the SLC linac. Systematic errors have been studied by testing the reproducibility of the method for different endpoints and different beam trajectories. These studies are described below.

The statistical error of a calculated quadrupole misalignment d, calculated using Eq. (3), grows with the number of quadrupoles N. We observe that the error in the calculated d_i values in the segment range from a maximum of $\Delta d_i \approx N^{3/2}\sigma$ near the center of the segment to a minimum of $\Delta d_i \approx N^{1/2}\sigma$ near the endpoints. The error in a calculated BPM bias b is $\Delta b \approx 2\sigma$, independent of N.

Systematic errors result from the assumption that the endpoint BPMs (0 and $N+1$) define the accelerator axis. The effect of these systematic errors can be observed in the sensitivity of the calculated misalignments to the particular choice of endpoints. Each quadrupole misalignment d_j is related to all BPM measurements $m_k^{\pm}$ ($k = 0, \ldots, N+1$) because all the A^{-1} matrix elements relating d_j to $m_k^{\pm}$ are non-zero. Therefore, calculated quadrupole misalignments are sensitive to the particular choice of endpoints and to N. In contrast, each BPM bias b_j is determined only from measurements $m_j^{\pm}$ by that BPM, and by measurements $m_{j\pm1}^{\pm}$ by its immediate neighbors, because the A^{-1} matrix elements relating b_j to all non-neighboring BPM measurements are zero. Therefore, calculated BPM biases are insensitive to the endpoints and to N.

Additional systematic errors result from energy errors, from transverse RF kicks, and from the BPM readout electronics which permits BPM biases to be different for electrons than for positrons. From the data, we can check the magnitude of these systematic effects.

The magnitude of systematic effects have been checked by calculating quadrupole misalignments and BPM biases separately for two overlapping sets of 16 quadrupoles each, with eight quadrupoles in the overlap region. One set of calculated quadrupole misalignments in the overlap region, $d_i(1)$, and the differences between the two sets, $d_i(1) - d_i(2)$, are shown in Fig. 3. The distribution of differences is approximately 200 μm wide, consistent with the statistical variation expected due to the BPM resolution. The variation between sets expected for each d_i is

$$
\Delta\big(d_i(1) - d_i(2)\big)^2 = \Delta d_i(1)^2 + \Delta d_i(2)^2 - 2\sum_{j'} A_{ij'}^{-1}(1) A_{ij'}^{-1}(2)\sigma^2
$$

where $\Delta d_i(1)$ and $\Delta d_i(2)$ are calculated using Eq. (3), and the j' sum includes only coefficients of BPM measurements in the overlap region where the same BPM data are used in both analyses. The calculated BPM biases b_i, also shown in Fig. 3, are identical in both sets, consistent with the fact that the calculation of b_i involves only measurements by the associated BPM and its immediate neighbors.

Another check of the magnitude of systematic effects has been performed by calculating quadrupole misalignments and BPM biases separately for two different two-beam trajectories recorded at different times through the same set of eight quadrupoles. One set of calculated quadrupole misalignments and BPM biases, $d_i(1)$ and $b_i(1)$, and the differences between the two sets, $d_i(1) - d_i(2)$ and $b_i(1) - b_i(2)$, are shown in Fig. 4. The distributions of differences in Fig. 4 are approximately 200 μm wide, consistent with the statistical variation expected due to the BPM resolution. The variation between sets expected for each d_i is

$$
\Delta\big(d_i(1) - d_i(2)\big)^2 = \Delta d_i(1)^2 + \Delta d_i(2)^2 \tag{4}
$$

where $\Delta d_i(1)$ and $\Delta d_i(2)$ are calculated using Eq. (3), and no correlation terms are present because the two trajectories have independent BPM measurements. The variation between sets expected for each b_i is expressed as in Eq. (4) with d replaced by b.

Figure 5 shows the distribution of quadrupole misalignments and BPM biases calculated in the horizontal and vertical planes of SLC Linac Sector 27.

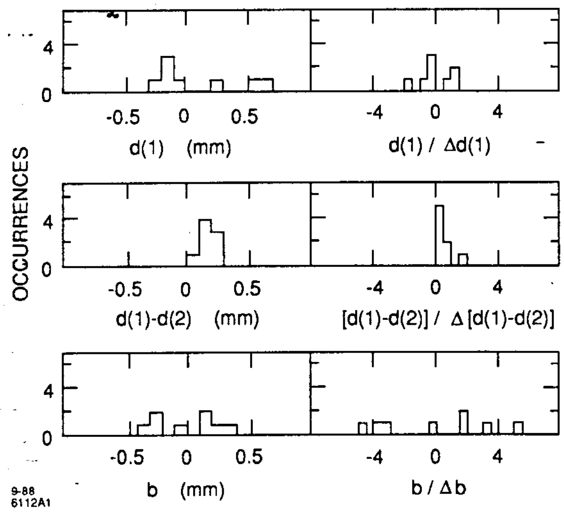

FIG. 3. Quadrupole misalignments d and BPM biases b calculated for eight quadrupoles in the overlap region between two overlapping sets, (1) and (2), of 16 quadrupoles each. The data plotted are from the vertical plane of SLC Linac Sector 2.

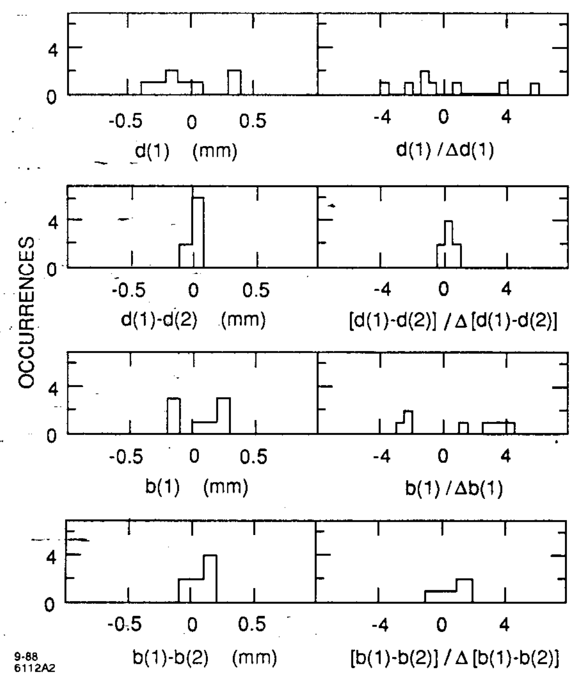

FIG. 4. Quadrupole misalignments d and BPM biases b calculated from two different two-beam trajectories, (1) and (2), through the same eight quadrupoles in the vertical plane of SLC Linac Sector 2.

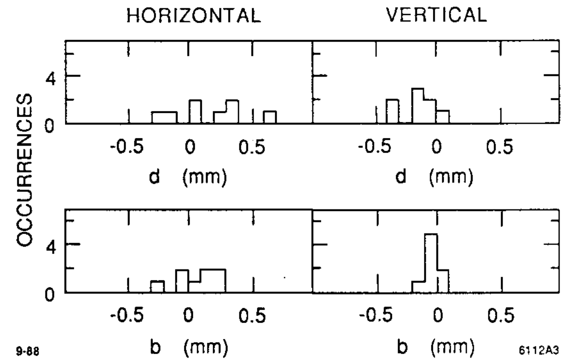

FIG. 5. Quadrupole misalignments d and BPM biases b calculated in SLC Linac Sector 27.

CONCLUSION

Figures 3–5 demonstrate that the two-beam method as described is not sensitive to systematic errors in quadrupole misalignments and BPM biases at approximately the 200 μm level. We have calculated some misalignments significantly greater than this level and are using this information to improve the mechanical alignment of the SLC linac.

The sensitivity of the method can be improved by reducing the BPM measurement resolution by averaging measurements over many beam pulses. The two-beam method may be improved to include two biases for each BPM, different for electrons than for positrons due to differences between different BPM electronics channels. This modification requires additional constraints which may be obtained by introducing a third trajectory measurement for an electron beam coasting without focusing or steering correction. This three-beam modification interferes with normal accelerator operations, but may provide a better measure of quadrupole misalignments than the two-beam method by modeling BPM biases more realistically.

ACKNOWLEDGEMENTS

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