SLAC – PUB – 4719 (Rev.)
March 1989
(A)

TRANSIENT ANALYSIS OF MULTICAVITY KLYSTRONS*

T. L. LAVINE, R. H. MILLER, P. L. MORTON, AND R. D. RUTH
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

ABSTRACT

We describe a model for analytic analysis of transients in multicavity klystron output power and phase. Cavities are modeled as resonant circuits, while bunching of the beam is modeled using linear space-charge wave theory. Our analysis has been implemented in a computer program which we use in designing multicavity klystrons with stable output power and phase. We present as examples transient analyses of a relativistic klystron using a magnetic pulse compression modulator, and of a conventional klystron designed to use phase shifting techniques for RF pulse compression.

INTRODUCTION

Large linear electron colliders require high power short pulsed RF sources in order to attain accelerating gradients of 100–200 MV/m. Two techniques being developed to supply this RF are relativistic klystrons with modulators using magnetic pulse compression, and conventional klystrons using phase shifting techniques for RF pulse compression. The RF power and frequency range being explored is 100–500 MW at 11–17 GHz. The RF output pulse length desired is 50–100 nsec, making the transient behavior of the multicavity klystrons employed in both approaches important. This paper describes a model for analytic analysis of transients in multicavity klystron output power and phase. Cavities are modeled as resonant circuits, while bunching of the beam is modeled using linear space-charge wave theory. The model has been implemented in a computer program which is used in designing multicavity klystrons with stable output power and phase.

RF CAVITIES

Each beam loaded klystron cavity is modeled as a parallel network of cavity and beam loading impedances as shown in Fig. 1. External resistance R_e includes additional resistive loading by iris-coupled waveguides. The RF driver connected to the input cavity typically consists of a power source, isolator, waveguide, and coupling iris. The driver is modeled as a generator of alternating current $I_g = \hat{I}_g e^{i\omega t}$ with shunt resistance R_e attached to the beam loaded input cavity (Fig. 1). Downstream cavities are driven by the bunched beam current.

The RF voltage on a cavity is $V(t) = \hat{V}(t)e^{i\omega t}$ where $\hat{V}(t)$ is the transient modulation of the RF oscillation $e^{i\omega t}$. The transient behavior of $V(t)$ is calculated from the circuit equation

$$\frac{d^2}{dt^2}CV + \frac{d}{dt}\frac{V}{R} + \frac{V}{L} = \dot{I}$$

which can be rewritten as

$$\ddot{V} + \left(\frac{1}{RC} + 2\frac{\dot{C}}{C}\right)\dot{V} + \left(\frac{1}{LC} - \frac{1}{RC}\frac{\dot{R}}{R} + \frac{\ddot{C}}{C}\right)V = \frac{\dot{I}}{C} \quad (1)$$

where L, R, and C are the beam loaded cavity inductance, resistance, and capacitance, respectively. L, R, and C may be time dependent due to resistive and reactive loading of the cavity by beam pulses with finite risetime. The current I flowing in the circuit model is the sum of generator current I_g for the input

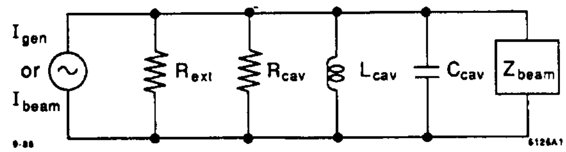

FIG. 1. Each beam loaded klystron cavity is modeled as a parallel network of cavity and beam impedances. External resistance includes additional resistive loading by iris coupled waveguides. The input cavity is driven by an RF generator current. Downstream cavities are driven by the bunched beam current.

cavity, RF beam current I_1 for intermediate and output cavities, and DC beam current I_0 for all cavities. Time dependence of I_g (for the input cavity) is $e^{i\omega t}$ where ω is the RF angular frequency. Time dependence of I_1 (for intermediate and output cavities) is $\hat{I}_1(t)e^{i\omega t}$ where $\hat{I}_1(t)$ is the transient modulation of the RF oscillation $e^{i\omega t}$. Time dependence of I_0 is due to finite risetime.

The lumped circuit elements in Eq. (1) can be expressed in terms of measurable quantities through the definitions

$$\omega_r{}^2 = \frac{1}{LC}, \quad Q = \omega_r \frac{\frac{1}{2}CV^2}{\frac{1}{2}V^2/R} = \omega_r RC, \quad \tau = \frac{2Q}{\omega_r} = 2RC \quad (2)$$

where R/Q is constant. Inserting definitions (2) into Eq. (1) gives

$$\ddot{V} + 2\left(\frac{1}{\tau} - \frac{\dot{\omega}_r}{\omega_r}\right)\dot{V} + \left[\omega_r{}^2 - \frac{2}{\tau}\left(\frac{\dot{\tau}}{\tau} + \frac{\dot{\omega}_r}{\omega_r} + 2\frac{\dot{\omega}_r{}^2}{\omega_r{}^2} - \frac{\ddot{\omega}_r}{\omega_r}\right)\right]V = \omega_r\frac{R}{Q}\dot{I} \quad (3)$$

which we solve for the transient behavior of $V(t)$.

The total quality factor Q for resistive beam loading of the cavity is composed of contributions from cavity walls (Q_0), from beam loading (Q_b), and from coupling to external waveguides (Q_e). Total Q is given by

$$Q = (Q_0^{-1} + Q_b^{-1} + Q_e^{-1})^{-1}.$$

Beam loading Q_b is related to the beam parameters by

$$Q_b = \gamma_0(\gamma_0{}^2 - 1)K_1/I_0 \quad (4)$$

where K_1 depends on the spatial distribution of the beam, $\gamma_0 = 1 + eV_0/m_ec^2$, and V_0 is the accelerating voltage. The resonance angular frequency of the cavity is

$$\omega_r = \omega_0 + \delta$$

where ω_0 is the resonance frequency without beam, and δ is the detuning by reactive beam loading. The detuning is related to beam parameters by

$$\omega_0/2\delta = \gamma_0(\gamma_0{}^2 - 1)K_2/I_0 \quad (5)$$

where K_2 depends on the spatial distribution of the beam. Time dependence of Q_b and δ results from finite risetime of the beam current I_0 and of the beam energy $\gamma_0 m_e c^2$ in Eq. (4) and (5).

* Work supported by the Department of Energy under contract DE-AC03-76SF00515.

Presented at the Particle Accelerator Conference, Chicago, Illinois, March 20–23, 1989

Initial Conditions

Velocity modulation is initiated by an external RF driver coupled by a waveguide to the klystron input cavity. The driver in general is not matched perfectly to the input cavity impedance which is time dependent due to the transient beam loading. If the driver is switched on long before the beam enters the input cavity, the input cavity voltage builds up to an asymptotic value which can be determined from the available drive power and the impedance mismatch as follows: The driver is modeled as a generator of alternating current $I_g = \hat{I}_g e^{i\omega t}$ with shunt resistance $R_e = (R/Q)Q_e$ attached to the beam loaded input cavity (Fig. 1). The beam loaded input cavity has impedance

$$Z_L = (R/Q)[Q_0{}^{-1} + Q_b{}^{-1} + i(\omega/\omega_r - \omega_r/\omega)]^{-1}.$$

The fraction of available drive power that enters the input cavity is $1 - |\Gamma|^2$ where $\Gamma = (Z_L - R_e)/(Z_L + R_e)$ is the voltage wave reflection coefficient. The asymptotic peak voltage on the input cavity produced by available rms drive power P has the same phase (relative to $\hat{I}_g$) as the total impedance through which I_g flows. The total impedance is $Z_T = (Z_L{}^{-1} + R_e{}^{-1})^{-1}$. The phase of the asymptotic peak voltage on the input cavity produced by the generator then is

$$\phi = \tan^{-1}\frac{\mathrm{Im} Z_T}{\mathrm{Re} Z_T} = \tan^{-1}\frac{-(\omega/\omega_r - \omega_r/\omega)}{Q_0{}^{-1} + Q_b{}^{-1} + Q_e{}^{-1}}.$$

The asymptotic peak voltage on the input cavity then is

$$V_d = \sqrt{2P(1 - |\Gamma|^2)(R/Q)(Q_0{}^{-1} + Q_b{}^{-1})^{-1}}\, e^{i\phi}.$$

Assuming the fields produced in the input cavity by the RF generator have reached equilibrium before the beam turns on, the voltage $V(t)$ on the input cavity at $t = 0$ just before the beam enters is $V(0) = V_d$ and $\dot{V}(0) = i\omega V_d$.

Alternatively, the transient due to switching on the generator is analyzed using the input cavity initial condition $V(0) = \dot{V}(0) = 0$.

Each downstream cavity is driven only by the beam and has $V = 0$ and $\dot{V} = 0$ before the beam enters.

Driving Term

The rate of change in current $\dot{I}$ that drives the voltage on a cavity appears on the right side of Eq. (3). For the input cavity, which is driven by I_g and by the risetime of I_0, $\dot{I} = i\omega I_g + \dot{I}_0$. Maximum rms drive power P is transferred when the input cavity is matched to the driver. In this case, $P = \frac{1}{2}(\frac{1}{2}I_g)^2 R_e$ so $I_g = \sqrt{8P/R_e}$. For the intermediate and output cavities, which are driven by I_1 and by the risetime of I_0, $\dot{I} = i\omega I_1 + \dot{I}_0$. Calculation of I_1 from linear space-charge wave theory is discussed below.

LINEAR SPACE-CHARGE WAVE THEORY OF BUNCHING

Bunching of the beam is modeled by linear space-charge wave theory. The transient cavity voltage $\hat{V}(t)e^{i\omega t}$ modulates the velocity of the beam. Longitudinal space-charge forces then produce space-charge waves which bunch the beam downstream.

Space-Charge Force

The space-charge potential of a long bunch of charge density ρ and radius a in a beam tube of radius b is

$$V(r < a < b) = -\frac{\rho a^2}{4\epsilon_0}\left(1 + 2\ln\frac{b}{a} - \frac{r^2}{a^2}\right)$$

assuming the longitudinal dimension (z) of the bunch is long compared to the radial dimension (r) so that end effects can be neglected. The longitudinal space-charge force in the frame moving with the beam at velocity $v_0 = \beta_0 c$ is

$$F_z(r) = -e\frac{\partial V(r)}{\partial z} = \frac{ea^2}{4\epsilon_0\gamma_0{}^2}\left(1 + 2\ln\frac{b}{a} - \frac{r^2}{a^2}\right)\frac{\partial\rho}{\partial z} \qquad (6)$$

where $\gamma_0 = (1 - \beta_0{}^2)^{-1/2}$. Note that when averaged over a uniform radial current distribution

$$\langle F_z\rangle = \frac{\int_0^a F_z(r)\, r\, dr}{\int_0^a r\, dr} = \frac{1}{2}\left[\frac{1 + 4\ln(b/a)}{1 + 2\ln(b/a)}\right]F_z(0). \qquad (7)$$

Wave Equation

The longitudinal space-charge force produces space-charge waves on the beam. The space-charge wave equation can be derived from the linearized continuity equation in the coordinate frame moving with the beam at velocity v_0 with respect to the klystron,

$$\rho_0\partial v_1/\partial z = -\partial\rho_1/\partial t, \qquad (8)$$

where the charge density ρ and the velocity v are both sums of a constant term and a small modulation: $\rho(z, t) = \rho_0 + \rho_1(z, t)$ and $v(z, t) = v_0 + v_1(z, t)$. In the beam frame, the beam velocity is $v_1(z, t)$.

For small velocity modulation, $v_1 = c(\gamma - \gamma_0)/\beta_0\gamma_0{}^3$ where $\gamma = (1 - \beta^2)^{1/2}$. The acceleration is

$$\dot{v}_1 = \dot{\gamma}c/\beta_0\gamma_0{}^3 = F_z(r)/m\gamma_0{}^3 \qquad (9)$$

because $\dot{\gamma} = F_z\beta_0 c/m_e c^2$. Substituting $\dot{v}_1$ from Eq. (9) into the time derivative of Eq. (8) and using F_z from Eq. (6), gives the wave equation

$$\frac{\partial^2\rho_1}{\partial z^2} = \frac{1}{v_\phi{}^2}\frac{\partial^2\rho_1}{\partial t^2}$$

where the phase velocity v_ϕ of space-charge waves is given by

$$\left(\frac{v_\phi}{c}\right)^2 = \frac{I_0}{17\,\mathrm{kA}}\frac{1}{\beta_0\gamma_0{}^5}\left(1 + 2\ln\frac{b}{a} - \frac{r^2}{a^2}\right). \qquad (10)$$

The beam current is $I_0 = \rho_0\beta_0 c\pi a^2$.

Space-Charge Wavelength

Space-charge wavenumber and wavelength are computed from the phase velocity (10) by averaging, as in Eq. (7), over a uniform radial beam current distribution. The average space-charge wavenumber is

$$\begin{aligned}\langle k_p\rangle &= \omega\,\langle v_\phi\rangle/v_0{}^2\\ &= \frac{2}{3}\left[(1+g)^{3/2} - g^{3/2}\right]\left[\frac{I_0}{17\,\mathrm{kA}}\frac{1}{(\beta_0\gamma_0)^5}\right]^{1/2}\frac{\omega}{c}\end{aligned} \qquad (11)$$

where $g = 2\ln(b/a)$. The average space-charge wavelength is

$$\begin{aligned}\langle\lambda_p\rangle &= \frac{2\pi v_0{}^2}{\omega}\left\langle\frac{1}{v_\phi}\right\rangle\\ &= 2\left[(1+g)^{1/2} - g^{1/2}\right]\left[\frac{17\,\mathrm{kA}}{I_0(t)}(\beta_0\gamma_0)^5\right]^{1/2}\frac{2\pi c}{\omega}.\end{aligned} \qquad (12)$$

For beam energy $eV_0 = (\gamma_0 - 1)m_e c^2 = 1.2\,\mathrm{MeV}$, current $I_0 = 1\,\mathrm{kA}$, and filling factor $a/b = 0.7$, the space-charge wavelength calculated from Eq. (12) is $\langle\lambda_p\rangle = 184\,\mathrm{cm}$, in good agreement with simulations by the electromagnetic particle-in-cell code, Mask.[1]

RF Current

Velocity modulation by a cavity produces RF modulation of the beam current downstream. We approximate each klystron cavity as a narrow gap with voltage $\hat{V}(t)e^{i\omega t}$. After drifting a distance d, the beam develops an RF current given in the linear approximation by

$$\hat{I}_1(t) = -\frac{4\pi i V(t)}{Z_0}\sqrt{\frac{I_0(t)}{17\,\mathrm{kA}}\frac{1}{(1+g)\beta\gamma}}\sin\left(\langle k_p\rangle d\right)e^{i\omega d/\beta c} \qquad (13)$$

where $Z_0 = 377\Omega$ and $\langle k_p\rangle$ is computed from Eq. (11).

In the drift downstream from a cavity, space-charge waves evolve on the beam according to Eq. (13) with boundary conditions given by the cavity voltage. Bunching evolves on the scale of $\lambda_p/4$. In klystron designs, intercavity spacings are somewhat less than $\lambda_p/4$, making the overall lengths of klystrons scale with λ_p times the number of cavities.

MULTICAVITY KLYSTRONS

The transient calculation for a multicavity klystron is outlined in Fig. 2. For each cavity in sequence from input to output, the time dependent RF cavity voltage produced by the RF generator (for the input cavity) or the bunched beam (for the intermediate and output cavities) is calculated from Eq. (3) using the appropriate initial condition and driving term as discussed above. The voltage solution is used to compute from Eq. (13) the resulting RF modulation of the beam current at the downstream cavities. The RF current used to drive each cavity in the calculation is the phasor sum of RF currents from all cavities upstream. The output RF power is computed from the output cavity RF voltage as $|V(t)|^2/2R_e$. The output RF phase relative to the RF generator is $\tan^{-1}(\mathrm{Im}V(t)/\mathrm{Re}V(t))$.

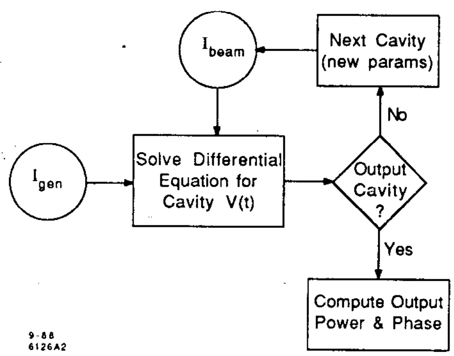

FIG. 2. Flowchart for klystron transient calculations.

EXAMPLES

Relativistic klystrons under development at the Stanford Linear Accelerator Center (SLAC) and Lawrence Livermore National Laboratory (LLNL) are designed to extract hundreds of megawatts of RF power at 11.4 GHz from electron beam pulses of 1 kA current, 1 MeV kinetic energy, and 50 nsec duration.[2] These beams are produced by a linear induction accelerator driven by modulators using magnetic pulse compression at LLNL. One relativistic klystron, known as SL4,[2] is a six cavity tube designed to operate at 11.4 GHz with 1 kA beam current and 1.2 MV beam kinetic energy. Figure 3 shows our analysis of transients in klystron output power and phase due to a pulsed relativistic beam entering the SL4 klystron after the fields produced in the input cavity by the RF generator have reached equilibrium.

Phase shifting techniques for RF pulse compression are under development at SLAC.[3] A 100-MW conventional klystron at 11.424 GHz has been designed for use in these experiments.[4] Figure 4 shows our analysis of the transients due to switching on the RF generator in the presence of a DC beam, and then due to reversing the generator phase after the switch-on transient has subsided.

SUMMARY

We have described a model for analytic linear analysis of transients in multicavity klystron output power and phase. We have presented as examples our transient analyses of two multicavity klystrons designed for applications in which fast risetime and stable output power and phase are important.

ACKNOWLEDGEMENTS

We thank K. R. Eppley, Z. D. Farkas, W. R. Fowkes, W. E. Gabella, W. B. Herrmannsfeldt, T. G. Lee, A. E. Vlieks, P. B. Wilson, and S. S. Yu for interesting and useful discussions.

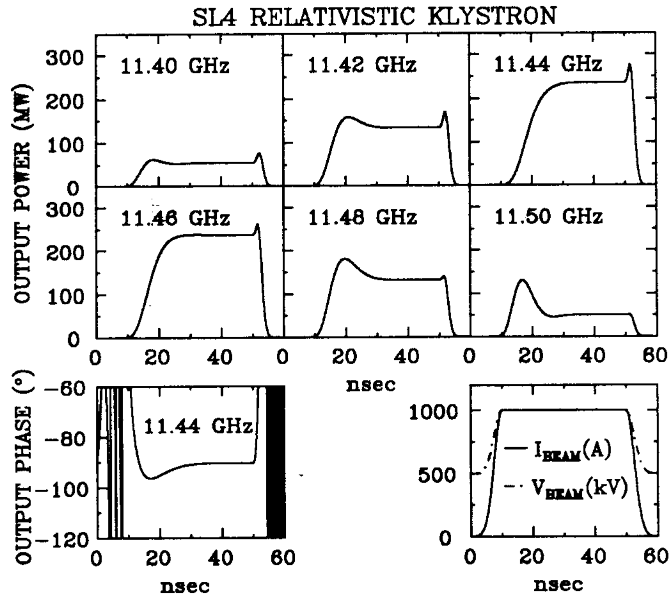

FIG. 3. Transient analysis of SL4, an 11.4-GHz six-cavity relativistic klystron. In this linear calculation, the fields produced in the input cavity by the RF generator have reached equilibrium before the beam turns on. The klystron cavity frequencies without beam are tuned to 0, 31, 23, 45, 425, and 6 MHz, respectively, above 11424 MHz. The pulsed beam detunes the cavities by 25 MHz and loads them with $Q_{beam} = 230$ when beam current and voltage are at their maximum values, 1 kA and 1 MV. The gain cavities are loaded externally with $Q_e = 120$ for faster risetime. Input $Q_e = 300$. Output $Q_e = 20$. For the six cavities, $R/Q = 27, 27, 27, 27, 60$, and 51Ω, respectively. Intercavity spacings are 28, 14, 21, 21, and 14 cm.

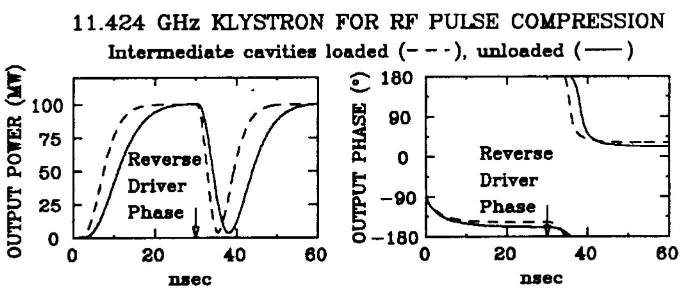

FIG. 4. Transient analysis of a 100-MW, 11.424-GHz five-cavity conventional klystron design for RF pulse compression. In this linear calculation, the RF generator is switched on at 0 nsec in the presence of a DC beam. The generator phase is reversed at 30 nsec, after the switch-on transient has subsided. Dashed curves show that improved risetime is obtained by adding external loads with $Q_e = 150$ to the gain cavities. Input $Q_e = 190$. Output $Q_e = 26$. The klystron cavity frequencies without beam are tuned to 21, 18, 40, 500, and 20 MHz, respectively, above 11424 MHz. The DC, 510 A, 440 kV beam detunes the cavities by −21 MHz and loads them with $Q_b = 210$. For the five cavities, $R/Q = 36, 36, 36, 39$, and 20Ω, respectively. Intercavity spacings are 6, 6, 8, and 3 cm.

REFERENCES

1. K. R. Eppley, private communication.
2. M. A. Allen *et al.*, "Relativistic Klystrons" (SLAC-PUB-4861, February 1989), 1989 PAC IEEE Conference Records 89CH2669-0 (these proceedings).
3. Z. D. Farkas, "Binary Peak Power Multiplier and its Application to Linear Accelerator Design," *IEEE Trans. MTT-34*, 1036 (1986).
4. T. G. Lee, private communication.