

## Q-Stars

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## ABSTRACT

We construct a new class of non-topological soliton stars which appears in field theories with non-linear matter interactions coupled to classical Einstein gravity. If  $\phi_0$  ( $\sim 10^{-1} - 10^4$  GeV) is a free-particle inverse Compton wavelength and  $m_{pl}$  is the Planck mass, their energy density  $\varepsilon \sim \phi_0^4$ , radius  $\rho \sim m_{pl} / \phi_0^2$ , global charge  $Q \sim m_{pl}^3 / \phi_0^3$  and mass  $M \sim m_{pl}^3 / \phi_0^2$  obey a generalized Chandrasekar limit.

(submitted to Nuclear Physics B)

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\*Work supported in part by grants National Science Foundation PHY-86-12280 and the Department of Energy DE-AC03-76SF00515.

In 1986, Friedberg, Lee and Pang[1] gave a complete formulation of scalar and fermion non-topological solitons where Einstein gravity played an important role: soliton stars. The prototypical solutions examined were of two sorts:

- i) Very small with radius  $\rho \sim \phi_o^{-1}$  ( $\phi_o$  is a free-particle inverse compton wavelength which we take to lie in the range  $\sim 10^{-1} - 10^4$  GeV and  $m_{pl}$  is the Planck mass) with very large scalar field strengths  $\phi \sim m_{pl}$ : mini-soliton stars. These were first studied by Ruffini and Bonnazola and Breit, Gupta and Zaks [2]. The (very weakly) interacting case was first studied by Colpi, Shapiro and Wasserman [3].
- ii) Very large radius  $\rho \sim m_{pl}^2 / \phi_o^3$  with huge mass  $M \sim m_{pl}^4 / \phi_o^3$ :  $M \sim 10^{12}$  solar masses and  $\rho \sim 10^{-1}$  light years for  $\phi_o \sim 300$  GeV. We will call these very large soliton stars.

We regard the soliton stars appearing in the literature [1,2,3] thus far as either too large or too small for interacting matter fields. Mini-soliton stars have scalar field strength  $\phi \sim m_{pl}$  and energy density  $\varepsilon \sim \phi_o^2 m_{pl}^2$ . If an interaction  $\lambda\phi^4$  with  $\lambda \sim 1$  is introduced, it would induce an energy density  $\sim \lambda m_{pl}^4$  if the field strength remained  $\phi \sim m_{pl}$  and this would completely destabilize the solutions. Mini-soliton stars with tiny self-interaction have been considered [3] but with  $\lambda \sim \phi_o^2 / m_{pl}^2 \sim 10^{-32}$  for  $\phi_o \sim 10^2$  GeV they suffer a fine-tuning problem (why is  $\lambda$  so small?). Even if we were to find solutions to the classical coupled Einstein and scalar field equations for soliton stars with  $\phi \sim m_{pl}$  and  $\varepsilon \sim m_{pl}^4$  at the classical level, one has to wonder what role quantum gravity would play for such huge energy densities; to answer this we would need to know the correct theory of quantum gravity.

'Very large soliton stars' are in our opinion too large because their average energy density  $\varepsilon \sim \phi_o^6 / m_{pl}^2$  is so small. How do such large objects, light-years across, get into causal self-contact and establish coherence in the matter field  $\phi$  end-to-end? If they are to be formed in the early universe, how could they be larger than the particle horizon?

In this paper we introduce a new class of non-topological soliton stars which avoids the above problems: Q-Stars. They consist of an interior (I) with radius  $\rho \sim m_{pl} / \phi_o^2$ , a surface region (S) of thickness  $\sim \phi_o^{-1}$  and an exterior (E) where there is no matter so that, in the spherically symmetric case, the metric is Schwarzschild. The energy densities are low  $\varepsilon \sim \phi_o^4$  so quantum gravity plays no role. We call them 'Q-Stars' because the surface region is a specific sort of non-topological soliton or Q-ball [4,5].

In order to see clearly the structure of Q-Stars, we start with a simple example. Consider the case of one complex scalar field  $\Phi$  in a spherically symmetric metric.

$$ds^2 = -e^{2u} dt^2 + e^{2v} d\rho^2 + \rho^2 (d\alpha^2 + \sin^2 \alpha d\beta^2) \quad (1)$$

The metric  $u(\rho)$ ,  $v(\rho)$  is time-independent while the complex field  $\Phi$  has the time-dependence of a rigid rotator.

$$\Phi = e^{i\omega t} \frac{\phi(\rho)}{\sqrt{2}} \quad (2)$$

and has a potential energy density  $U(\Phi^\dagger \Phi)$ . The theory has an unbroken global  $U(1)$  (or  $SO(2)$  if we use a two-component real field) symmetry

$$\Phi \rightarrow e^{i\theta} \Phi \quad (3)$$

so there is a conserved current

$$j^\mu; \quad \mu = \frac{1}{\sqrt{-g}} (\sqrt{-g} j^\mu), \quad \mu = 0 \quad (4)$$

$$j^\mu = -i(\Phi^\dagger \Phi,{}^\mu - \Phi^\dagger,{}^\mu \Phi)$$

and a conserved charge

$$Q = \int \sqrt{-g} dx^1 dx^2 dx^3 j^0 \quad (5)$$

$$= \frac{8\pi}{\omega} \int W e^{u+\bar{v}} \rho^2 d\rho \quad (6)$$

$$W = \frac{\omega^2}{2} e^{-2u} \phi^2 \quad (7)$$

The conserved global charge  $Q$  stabilizes the Q-Star against decay into the vacuum; hence the name. There is a kinetic energy density

$$V = \frac{1}{2} e^{-2\bar{v}} \left( \frac{d\phi}{d\rho} \right)^2 \quad (8)$$

so that the equation of motion for the matter field  $\phi$  is

$$e^{-2\bar{v}} \left[ \frac{d^2\phi}{d\rho^2} + \left( \frac{2}{\rho} + \frac{d(u-\bar{v})}{d\rho} \right) \frac{d\phi}{d\rho} \right] = \frac{\delta(U-W)}{\delta\phi} \quad (9)$$

The Q-Star equations have a dimensionless quantity

$$\epsilon^2 = 8\pi G \phi_o^2 \quad (10)$$

which is very small;  $\epsilon \sim 10^{-16}$  for  $\phi_o \sim 300$  GeV. We will use the smallness of this quantity to great advantage working to lowest order in  $\epsilon$  throughout this paper. First, rescale all dimensionfull quantities with  $\phi_o$ .

$$\begin{aligned} \tilde{\rho} &= \rho \phi_o \\ \tilde{\phi} &= \phi / \phi_o, \tilde{\omega} = \omega / \phi_o \\ \tilde{W}, \tilde{V}, \tilde{U} &= W / \phi_o^4, V / \phi_o^4, U / \phi_o^4 \end{aligned} \quad (11)$$

In this paper, we will use the notation that all quantities with a 'twiddle' have been rescaled with respect to  $\phi_o$ .

The Q-Star ansatz is illustrated in Figure 1. It consists of three regions: an interior (I), surface (S) and exterior (E). We discuss each of these regions below.

## I. Interior Region

There is a thick interior region (I) with radius  $\rho_{s-} \sim m_{pl}/\phi_o^2$ . For  $\phi_o \sim 300$  GeV this will be  $\sim .01 - 1$  cm. The outer edge of (I) in units of  $\phi_o^{-1}$  (inner edge of the surface region S) is given by

$$\tilde{\rho}_{S-} = \epsilon^{-1} S \quad (12)$$

$$\tilde{\rho} = \tilde{\rho}_{S-} x \quad (13)$$

with  $S$  a scale of order 1 and  $0 \leq x \leq 1$  in the interior. In the interior, the metric and matter fields vary smoothly over distances  $\epsilon^{-1}$  so that

$$\begin{aligned} \frac{du}{d\tilde{\rho}}, \frac{d\tilde{v}}{d\tilde{\rho}}, \frac{d\tilde{\phi}}{d\tilde{\rho}} &\sim 0(\epsilon) \\ \frac{du}{dx}, \frac{d\tilde{v}}{dx}, \frac{d\tilde{\phi}}{dx} &\sim 0(1) \end{aligned} \quad (14)$$

$\tilde{U}$  and  $\tilde{W}$ , on the other hand, are 0 (1) in the interior so that we can immediately solve the matter equation (9) to lowest order in  $\epsilon$ :

$$\frac{\delta(\tilde{W} - \tilde{U})}{\delta\tilde{\phi}} = 0 \quad 0 \leq x \leq 1 \quad (15)$$

$$\tilde{V} = 0 \quad (16)$$

Now define the metric fields

$$\begin{aligned} A &= e^{-2\tilde{v}} \\ B &= e^{-2u} \\ \bar{B} &= \tilde{\omega}^2 e^{-2u} = \tilde{\omega}^2 B \end{aligned} \quad (17)$$

Equation (15) allows us to eliminate  $\tilde{\phi}$  with respect to  $\bar{B}$  and rewrite  $\tilde{U}, \tilde{W}$  in terms of  $\bar{B}$  alone in the interior. We must then solve the Einstein equations  $G_{\mu\nu} = 8\pi GT_{\mu\nu}$ .

$$\begin{aligned}
A - 1 + x \frac{dA}{dx} &= -S^2 x^2 (\tilde{W} + \tilde{U} + \tilde{V}) \\
A - 1 - x \frac{A d\bar{B}}{\bar{B} dx} &= S^2 x^2 (\tilde{W} - \tilde{U} + \tilde{V})
\end{aligned}
\tag{18}$$

(respectively the  $G_{tt}$  and  $G_{\rho\rho}$  equations) subject to the boundary condition that there is no matter for  $x < 0$ .

$$A(x=0) = 1 \tag{19}$$

We will get a second boundary condition on  $\bar{B}(x=1)$  by studying the interface of the interior region (I) with surface region(S).

#### S: Surface Region

The Q-Star ansatz has a very thin surface region S with interior (exterior) edges  $\tilde{\rho}_{S-}$  ( $\tilde{\rho}_{S+}$ ).

$$\frac{\tilde{\rho}_{S+} - \tilde{\rho}_{S-}}{\tilde{\rho}_{S-}} \sim 0(\epsilon) \tag{20}$$

The surface thickness is thus of order  $\phi_o^{-1}$ , the particle compton wavelength, which for  $\phi_o \sim 300$  GeV is about  $10^{-16}$  cm. The matter field varies rapidly within the surface but the metrics do not. For  $\tilde{\rho}_{S-} \leq \tilde{\rho} \leq \tilde{\rho}_{S+}$

$$\begin{aligned}
\frac{d\tilde{\phi}}{d\tilde{\rho}} &\sim 0(1) \\
\frac{d\tilde{u}}{d\tilde{\rho}}, \frac{d\tilde{v}}{d\tilde{\rho}} &\sim 0(\epsilon)
\end{aligned}
\tag{21}$$

We thus take the metrics to have constant values  $u_{surface}$  and  $\bar{v}_{surface}$  within the surface and write the matter equation (9) to lowest order in  $\epsilon$  (remember that

all dimensionful quantities have been rescaled with respect to  $\phi_0$ )

$$e^{-2\tilde{v}_{surface}} \frac{d^2 \tilde{\phi}}{d\tilde{\rho}^2} = - \frac{\delta(\tilde{W}_{surface} - \tilde{U})}{\delta \tilde{\phi}} \quad (22)$$

$$\tilde{W}_{surface} = \frac{1}{2} \tilde{\omega}^2 \tilde{\phi}^2 e^{-2u_{surface}}$$

This equation is easily understood by an analogy first given by Freidberg, Lee, and Sirlin for non-topological solitons in the absence of gravity [4]. If we think of  $\tilde{\rho} e^{\tilde{v}_{surface}}$  as the ‘time’ and  $\tilde{\phi}$  as a ‘distance’ then eq. (22) is just Newton’s equation for a mechanical particle rolling around in a ‘potential’  $\tilde{W}_{surface} - \tilde{U}$ . There is no ‘friction’ because  $\frac{2}{\rho} + \frac{d(u-\tilde{v})}{d\rho} \sim 0(\epsilon)$  within the surface. Thus, a first integral of eq. (9) follows immediately; within the surface

$$\tilde{V} + \tilde{W}_{surface} - \tilde{U} = 0 \quad \tilde{\rho}_{S-} \leq \tilde{\rho} \leq \tilde{\rho}_{S+} \quad (23)$$

The (rescaled) ‘kinetic energy’  $\tilde{V}$  from eq. (8) plus the ‘potential energy’  $\tilde{W}_{surface} - \tilde{U}$  is conserved. We will assume that in the exterior region (E) that  $\tilde{V} = \tilde{W} = \tilde{U} = 0$  so eq. (23) follows. We also get a boundary condition on  $\bar{B}(x=1)$  ( $x=1$  is also the point  $\tilde{\rho} = \tilde{\rho}_{S-}$ ) because in order to match interior and surface solutions (see eq. 16 or 23)

$$\begin{aligned} \tilde{V}(1) &= 0 \\ [\tilde{W} - \tilde{U}]_{x=1} &= 0 \end{aligned} \quad (24)$$

which gives the boundary condition on  $\bar{B}(1) = \tilde{\omega}^2 e^{-2u_{surface}}$ . Together with eqs. (15, 16, 18, 19) this completely defines the interior region (I) as a solution of two coupled first-order equations in  $A$  and  $\bar{B}$  with boundary condition  $A(x=0)$  and  $\bar{B}(x=1)$ .

Remember that in the interior  $\delta(\tilde{W} - \tilde{U}) / \delta\tilde{\phi} = 0$  so in order to match solutions at  $x = 1$  the surface solutions must start ‘rolling’ with

$$\frac{\delta(\tilde{W} - \tilde{U})}{\delta\tilde{\phi}} \Big|_{x=1} = 0 \quad (25)$$

so that the ‘acceleration’ of  $\tilde{\phi}$  is zero at  $x = 1$  from eq. (22). Eqs. (24, 25) give a boundary condition on  $\bar{B}(1)$  and, together with eq. (22), define a certain kind of Q-ball or non-topological soliton [4, 5, 6]. For the Q-ball, and therefore the Q-star, to exist the potential  $\tilde{U}(\tilde{\phi}^2)$  must rise at least like  $\tilde{\phi}^2$  near  $\tilde{\phi}^2 = 0$ , go less quickly than  $\tilde{\phi}^2$  for some intermediate region, and then rise faster than  $\tilde{\phi}^2$  as  $\tilde{\phi}^2 \rightarrow \infty$ . This is illustrated in Fig. 2 with solid lines. The flatness condition  $\frac{d\tilde{\phi}}{d\tilde{\rho}}(\tilde{\rho}_{s+}) \sim 0(\epsilon)$  for the interface of the interior and surface regions in the Q-Star means that  $\tilde{W}_{surface} = \frac{\bar{B}(1)}{2} \tilde{\phi}^2$  intersects the curve  $\tilde{U}(\tilde{\phi}^2)$  at just one point as shown in Fig. 2 with solid lines. Thus, the matter field in the surface ‘rolls’ in the potential (solid lines Fig. 2)  $\tilde{W}_{surface} - \tilde{U}$  starting at the point  $\tilde{\phi}(\tilde{\rho}_{s-}) = \tilde{\phi}_{s-}$  and ending at the point  $\tilde{\phi}(\tilde{\rho}_{s+}) = \tilde{\phi}_{s+} = 0$ . It must end there for one complex field  $\Phi$  because we don’t want to break the global symmetry eq. (3) for the true vacuum at  $\tilde{\rho} = \infty$ ; that would destabilize the Q-Star by inducing Q-non-conservation. (An example of a Q-star with a true vacuum with a spontaneously broken symmetry is given in example B below). Thus, the Q-Star surface is (up to terms of  $0(\epsilon)$ ) a non-topological soliton which rolls without friction between degenerate maxima of  $\tilde{W}_{surface} - \tilde{U}$ .

The Einstein equations within the surface are (see eq. (18))

$$A - 1 + x \frac{dA}{dx} = -S^2 x^2 2\tilde{U} \quad (26)$$

$$A - 1 - x \frac{A}{\bar{B}} \frac{d\bar{B}}{dx} = 0 \quad (27)$$

since  $\tilde{W}_{surface} + \tilde{V} - \tilde{U} = 0$  within the surface. Note from eq. (27) that if A is constant within the surface,  $\bar{B}$  and  $d\bar{B}/dx$  will be also (as shown in Fig. 1).



However, differentiating (26) once with respect to  $\rho$  we have

$$\frac{d}{d\tilde{\rho}} \left( \frac{dA}{dx} \right) = \frac{d}{d\tilde{\rho}} (-2S^2 x \tilde{U}) + \frac{\epsilon}{S} \frac{A - 1 - x \frac{dA}{dx}}{x^2} \quad (28)$$

so that dropping the  $O(\epsilon)$  term and integrating across the thin surface  $1 \leq x \leq 1 + O(\epsilon)$  or  $\tilde{\rho}_{S-} \leq \tilde{\rho} \leq \tilde{\rho}_{S+}$  we get the surface discontinuity in  $dA/dx$  (see Fig. 1)

$$\left. \frac{dA}{dx} \right|_{\rho = \rho_{S+}} = \left[ \frac{dA}{dx} + 2S^2 \tilde{U} \right]_{x=1} \quad (29)$$

We of course assume that there is no matter potential energy outside the Q-Star so  $U(\tilde{\rho} \geq \tilde{\rho}_{S+}) = 0$

#### E: Exterior Region

Outside the surface, there is no matter so the metrics are Schwarzschild

$$A = B^{-1} = 1 - \frac{2GM}{\rho} = 1 - \frac{\tilde{M}}{4\pi S x} \quad (30)$$

with rescaled  $\tilde{M} = M/\phi_o$  the Q-Star mass in units of  $\phi_o$ . Note that the use of eqns. (26, 27) and (29) is crucial in order that the metric derivatives be Schwarzschild as well. We neglect the mass stored in the thin surface and write

$$M = \frac{4\pi S^3 \phi_o}{\epsilon^3} \int_0^1 dx x^2 (\tilde{W} + \tilde{U}) \quad (31)$$

Note that the dimensionful factor  $\phi_o^3 \epsilon^{-3} = (8\pi G)^{-3/2} \phi_o^{-2} \sim m_{pl}^3 / \phi_o^2 \sim 10^{50}$  GeV for  $\phi_o \sim 300$  GeV means that Q-Stars obey a generalized Chandrasekar limit

[7]. An equivalent formula for the mass is

$$M = \frac{4\pi S^3 \phi_o}{\epsilon^3} \int_0^1 dx x^2 (4\tilde{W} - 2\tilde{U}) e^{u+\bar{v}} \quad (32)$$

Eqs. (15, 16, 18, 19 and 24) completely define the interior region in terms of  $A$  and  $\bar{B} = \tilde{\omega}^2 e^{-2u}$  so we know  $\tilde{W}$  and  $\tilde{U}$  as functions of  $A$  and  $\bar{B}$ . Thus since eqs. (31) and (32) are equivalent, we may calculate the frequency  $\omega$  and the charge  $Q$

$$\tilde{\omega} = \frac{\int_0^1 dx x^2 (\tilde{W} + \tilde{U})}{\int_0^1 dx x^2 \left( \frac{4\tilde{W} - 2\tilde{U}}{\sqrt{A\bar{B}}} \right)} \quad (33)$$

$$Q = \frac{4\pi S^3}{\epsilon^3} \int_0^1 dx x^2 \left( \frac{2\tilde{W}}{\sqrt{A\bar{B}}} \right) \quad (34)$$

Note from eqs. (31) and (34) that the mass and charge each grow roughly as the Q-Star volume (generalized Chandrasekar limit).

Recall that for general Q-balls without gravity (not Q-Stars) the acceleration  $\frac{d^2 \tilde{\phi}}{d\rho^2}$  ( $x=1$ ) is not zero. This comes from having an effective frequency ('sc' for 'supercritical')  $\tilde{\omega}_{sc}^2$  as illustrated in Fig. 2 with dotted lines. A particle 'rolling' on the dotted hill without friction would start at the circled point with non-zero 'acceleration' and zero velocity and roll to  $\tilde{\phi}_{S_+} = 0$ . Thus the Q-ball solutions without gravity have a parametric constraint  $\tilde{\omega}_{sc}^2 \geq \tilde{\omega}_{critical}^2 = \bar{B}(1)$  with the equal sign corresponding to the infinite volume case. The introduction of gravity allows stars to form with  $\tilde{\omega}^2 < \tilde{\omega}_{critical}^2$  because  $\bar{B}(1)$  is fixed by the Q-ball surface common to all Q-Stars with a given potential  $\tilde{U}$ .

$$\tilde{\omega}^2 = \bar{B}(1) e^{2u_{surface}} \quad (38)$$

As  $M \rightarrow M_{black\ hole}$ , (the Q-star becomes a black hole at some critical mass  $M_{black\ hole}$ )  $e^{2u_{surface}} \rightarrow 0$  and  $\tilde{\omega}^2 \rightarrow 0$

The complex field has effective time-dependence within the interior

$$\Phi = \frac{\phi(\rho)}{\sqrt{2}} \exp [i\sqrt{\bar{B}(\rho)} e^{u(\rho)} t\phi_o] \quad (39)$$

$\bar{B} = \tilde{\omega}^2 e^{-2u}$  does not go to zero as  $M \rightarrow M_{black\ hole}$  since  $\bar{B}(1)$  is fixed by the surface region of the Q-Star. An observer far away from the Q-Star sees a slowing of the frequency of the field as the Q-Star accretes mass until it stops altogether when  $M = M_{black\ hole}$

Since  $\delta\tilde{M}/dQ = \tilde{\omega}$  we have the evolution of M vs Q depicted in Fig. 3. for particles  $\phi$  with free particle mass  $\phi_o$ . This is a generic feature of this sort of Q-ball which, when Einstein gravity becomes important, becomes, as Q is increased, a Q-Star.

### Two Examples

A) We start with the simple example of one complex field  $\Phi$  with  $\phi(\rho \rightarrow \infty) = 0$  unbroken symmetry. Choose the non-renormalizable potential in Fig. 4.

$$\begin{aligned} \tilde{U} &= \frac{\tilde{\phi}^2}{2} (1 - \tilde{\phi}^2 + \frac{\tilde{\phi}^4}{3}) \\ \tilde{W} &= \frac{1}{2} \tilde{\omega}^2 e^{-2u} \tilde{\phi}^2 = \frac{\bar{B}}{2} \tilde{\phi}^2 \\ \tilde{W}_{surface} &= \frac{\bar{B}(1)}{2} \tilde{\phi}^2 \end{aligned} \quad (35)$$

(The existence of a reflection point at  $\tilde{\phi}^2 = 1$  is unrelated to the structure of Q-Stars; we have chosen the coefficients in  $\tilde{U}$  in eq. (35) so that we will have nice, simple numbers in our result).

Within the surface we have from eq. (22)

$$A(x=1) \frac{d^2\tilde{\phi}}{d\tilde{\rho}^2} = - \frac{\delta(\tilde{W}_{surface} - \tilde{U})}{\delta\tilde{\phi}} = \tilde{\phi}[(\tilde{\phi}^2 - 1)^2 - \bar{B}(1)] \quad (36)$$

and  $\tilde{W}_{surface} + \tilde{V} - \tilde{U} = 0$ . In the interior region  $\delta(\tilde{W} - \tilde{U})/\delta\tilde{\phi} = 0$  so the

$\phi$  field is related to the time metric by

$$\begin{aligned}\tilde{\phi}^2 &= 1 + \sqrt{\bar{B}} \\ \tilde{W} &= \frac{\bar{B}(1 + \sqrt{\bar{B}})}{2} \\ \tilde{U} &= \frac{1 + \bar{B}^{3/2}}{6}\end{aligned}\tag{37}$$

The boundary condition on  $\bar{B}(1)$  is gotten from  $[\tilde{W} - \tilde{U}]_{x=1} = 0$ .

$$\bar{B}(1) = \frac{1}{4}\tag{38}$$

Further  $A(0)=1$  so eqs. (18) are completely defined for  $0 \leq x \leq 1$

$$\begin{aligned}A - 1 + x \frac{dA}{dx} &= -\frac{S^2 x^2}{6} (1 + 3\bar{B} + 4\bar{B}^{3/2}) \\ A - 1 - x \frac{A}{\bar{B}} \frac{d\bar{B}}{dx} &= \frac{S^2 x^2}{6} (-1 + 3\bar{B} + 2\bar{B}^{3/2})\end{aligned}\tag{39}$$

The interior solutions of course match up beautifully at  $\tilde{\rho} = \tilde{\rho}_{S-}$  (also the point  $x = 1$ ) with the surface region Q-ball solution to eq. (36) with its two boundary conditions  $\frac{d\tilde{\phi}}{d\rho} |_{\rho_{S-}} = 0$ ,  $\tilde{\phi}^2(\rho_{S-}) = 1 + \sqrt{\bar{B}(1)} = \frac{3}{2}$ . Since  $2\tilde{U}(x=1) = 3/8$ , the discontinuity in  $dA/dx$  is just

$$\frac{dA}{dx} |_{\rho_{S+}} = \frac{dA}{dx} |_{x=1} + \frac{3S^2}{8}\tag{40}$$

The metric solutions and their derivatives are then Schwarzschild at  $\tilde{\rho} = \tilde{\rho}_{S+}$ .

We have solved eq. (39) numerically using the computer program COLSYS [8] and have plotted in Fig. 1 the solution for this example when  $S = 1.9$ ,  $\tilde{\omega} = .336$ ,  $\tilde{M} = 13.1\epsilon^{-3}$  and  $Q = 32.3\epsilon^{-3}$  with the surface region magnified. Some results for other solutions with the potential  $\tilde{U}$  in eq. (35) are given in Table 1.

B) We next examine the case of one complex field  $\Phi$  and one real scalar  $\sigma$  where the true vacuum breaks spontaneously the discrete symmetry  $\sigma \rightarrow -\sigma$ . A generalized version of these solitons and discussion of their properties is in preparation [6]. First rescale  $\tilde{\sigma} = \sigma/\phi_0$  and choose the potential

$$\begin{aligned}\tilde{U} &= \frac{\tilde{\phi}^2 \tilde{\sigma}^2}{2} + \frac{\tilde{\phi}^4}{4} + \frac{\lambda}{4} (\tilde{\sigma}^2 - 1)^2 \\ \tilde{W} &= \frac{\bar{B}}{2} \tilde{\phi}^2 \\ \tilde{V} &= \frac{e^{-2\tilde{v}}}{2} \left[ \left( \frac{d\tilde{\phi}}{d\tilde{\rho}} \right)^2 + \left( \frac{d\tilde{\sigma}}{d\tilde{\rho}} \right)^2 \right]\end{aligned}\tag{41}$$

The presence of the  $\tilde{\phi}^4$  (repulsive) term is crucial to the existence of the Q-Star; without it we cannot satisfy eqs. (24, 25) so that the solitons examined by Freidberg, Lee and Sirlin [4] have no associated Q-Stars. Within the surface we have constant metrics,  $\tilde{W}_{surface} + \tilde{V} + \tilde{U} = 0$  and

$$\begin{aligned}A(1) \frac{d^2 \tilde{\phi}}{d\tilde{\rho}^2} &= \frac{\delta(\tilde{U} - \tilde{W}_{surface})}{\delta \tilde{\phi}} = \tilde{\phi} [\tilde{\sigma}^2 + \tilde{\phi}^2 - \bar{B}(1)] \\ A(1) \frac{d^2 \tilde{\sigma}}{d\tilde{\rho}^2} &= \frac{\delta \tilde{U}}{\delta \tilde{\sigma}} = \tilde{\sigma} [\tilde{\phi}^2 + \lambda(\tilde{\sigma}^2 - 1)]\end{aligned}\tag{42}$$

The obvious generalization of the interior matter solutions with more than one field

$$\delta(\tilde{W} - \tilde{U}) / \delta \tilde{\phi}_i = 0 \quad i = 1, \dots, n\tag{43}$$

allows us to eliminate all the  $\phi_i$  with respect to  $\bar{B}$  in the interior. For this example

$$\begin{aligned}\tilde{\phi}^2 &= \bar{B} \\ \tilde{\sigma} &= 0 \\ \tilde{W} &= \frac{\bar{B}^2}{2} \\ \tilde{U} &= \frac{\bar{B}^2 + \lambda}{4}\end{aligned}\tag{44}$$

in the interior while at the boundary with the surface region

$$\begin{aligned}\bar{B}(1) &= \lambda^{1/2} \\ 2\tilde{U}(1) &= \lambda\end{aligned}\tag{45}$$

which match perfectly the boundary condition in eq. (42) for the soliton surface solution  $d\tilde{\phi}/d\tilde{\rho}(\tilde{\rho}_{S-}) = 0$ ;  $\tilde{\phi}^2(\tilde{\rho}_{-}) = \lambda^{1/2}$ . The discontinuity in the space metric derivative is

$$\left.\frac{dA}{dx}\right|_{\rho_{S+}} = \left.\frac{dA}{dx}\right|_{\rho_{S-}} + \lambda S^2\tag{46}$$

so that the metrics and their derivatives match perfectly the Schwarzschild metric calculated with the mass  $M$  in eq. (31) or (32). The metric in the interior of the Q-Star satisfies

$$\begin{aligned}A - 1 + x \frac{dA}{dx} &= -\frac{S^2 x^2}{4} (3\bar{B}^2 + \lambda) \\ A - 1 - x \frac{A}{\bar{B}} \frac{d\bar{B}}{dx} &= \frac{S^2 x^2}{4} (\bar{B}^2 - \lambda)\end{aligned}\tag{46}$$

We have solved eq. (46) numerically. A Q-Star solution with the potential eq. (41) is plotted in Fig. 5 with  $S = 2.985$ ,  $\tilde{\omega} = .4277$ ,  $\tilde{M} = 16.9\epsilon^{-3}$ , and  $Q = 34.3\epsilon^{-3}$ ,  $\lambda = 1/9$ . Some results for other Q-Star solutions with this potential are given in Table 2. Note that the lower limit for  $\tilde{\omega}$  for this type of Q-ball without gravity would have been  $\tilde{\omega}_{critical} = \lambda^{1/4} \simeq .577$  for  $\lambda = 1/9$

### Conclusions

The generic behavior of Q-balls of the sort discussed in this paper which then evolve, as  $Q$  is increased, into Q-Stars is depicted in Figures 2, 3 and 6. For very small  $Q$ , the Q-ball rolls (with the 'friction' term  $d\tilde{\phi}/d\tilde{\rho}$  from  $\tilde{\nabla}^2 = d^2/d\tilde{\rho}^2 + \frac{2}{\tilde{\rho}}d/d\tilde{\rho}$  it starts above the circled point) on the dotted hill in Fig. 2 and the dotted wave form on Fig. 6 results. If the free particle mass of the  $\phi$  field is taken to be 1 (in units of  $\phi_o$ ), this corresponds to the  $\tilde{\omega} < 1$  case. For

larger  $Q$ , but small enough so that gravity is negligible, we have  $1 > \tilde{\omega} > \tilde{\omega}_{critical}$ . Near  $\tilde{\omega}_{critical}$  this almost corresponds to the situation of the solid lines in Fig. 2 and has the waveform of dashes in Fig. 6. For  $Q$  sufficiently large that gravity is important we have  $\tilde{\omega} < \tilde{\omega}_{critical}$ . The Q-star surface now does correspond to the solid lines in Fig. 2 and the waveform is given by the solid lines in Fig. 6. Thus, Q-Stars are the natural continuation of this certain class of Q-ball, depicted in Fig. 2, 3 and 6 when  $Q \sim m_{pl}^3 / \phi_o^3$  is large enough so that gravity becomes important.

We now discuss Q-Star stability. Since  $\delta\tilde{M}/\delta Q = \tilde{\omega} < 1$  (this follows by considering only finite energy solutions) all Q-stars lying on the solid line in Fig. 3 below the dashed line  $\tilde{M} = Q$  are stable against break up and dispersion into free particles. (We have taken the free-particle mass here to be  $\phi_o$ ). Further, their radius  $\rho \sim m_{pl} / \phi_o^2 \gg \phi_o^{-1}$  is large, so they are stable against Hawking radiation [7].

The real danger to boson Q-star stability lies in the possible decay of  $\Phi$  to other particles. If  $\Phi \rightarrow$  bosons were an available channel we would expect exponential decay in time and the lifetime of the Q-Star would be essentially the single  $\Phi$  lifetime. If on the other hand  $\Phi \rightarrow$  two fermions are the only  $\Phi$  decay channels, the situation is quite different and the Q-Star lifetime is proportional to  $Q^{1/3}$ . Cohen, Coleman, Georgi and Manohar [9] have shown that if  $\Phi \rightarrow \bar{\psi}\psi$  is the only decay channel, large Q-balls 'evaporate'; their decay rate is proportional to the Q-ball surface area  $\alpha$ . They place an upper bound

$$\frac{dQ}{dt} < \frac{\omega_{critical}^3}{192\pi^2} \alpha \quad (47)$$

which is independent of the  $\Phi\bar{\psi}\psi$  coupling. Let us estimate the Q-Star lifetime. Neglecting gravity, we have  $Q \sim$  volume (Chandrasekar limit) so

$$Q \simeq \frac{4\pi}{3} \left(\frac{\alpha}{4\pi}\right)^{3/2} \bar{B}^{1/2}(1) \tilde{\phi}_S^2 \phi_o^3 \quad (48)$$

Integrating eqs. (47, 48) we get an approximate boson Q-Star lifetime

$$\tau > 10^{-7} \text{sec} \left( \frac{300 \text{ GeV}}{\phi_o} \right)^2 \left[ Q^{1/3} \epsilon \frac{\tilde{\phi}_{S-}^{4/3}}{\bar{B}^{7/6}(1)} \right] \quad (49)$$

where we have replaced  $\tilde{\omega}_{critical} \rightarrow \bar{B}^{1/2}(1)$  in eq. (47). The factor in eq. (49) in square brackets is  $O(1)$  (see Tables 1,2) so boson Q-Stars may be very long-lived relative to unstable elementary particles but short-lived with respect to the lifetime of the universe. This estimate of the boson Q-Star lifetime should still be reasonably accurate when gravity is included because the ‘evaporation’ occurs at the surface where gravitational effects are minimal.

It is easy to read off the equation of state relating the energy density  $\epsilon$  to the (isotropic) pressure  $P$ . Since  $\epsilon = W + U$  and  $P = W - U$ , we find immediately for the two-scalar field (renormalizable potential) example B above  $\epsilon = 3P + \lambda \phi_o^4$ . This simple result is by no means generic; example A (non-renormalizable potential) has an equation  $\epsilon(P)$  which is complicated and unilluminating. Nevertheless because of eqs. (23,24)  $P = 0$  at the surface of all Q Stars as it should be.

One can imagine adding a conformal coupling  $L_{conformal} \sim R\phi^2$ , with  $R$  the Ricci scalar, to the Q-Star Lagrangian but this changes nothing to lowest order in  $\epsilon = (8\pi G\phi_o^2)^{1/2}$  since  $R \sim \epsilon^2 \phi_o^2$  for time-independent metrics. Thus the Q-Star is unaffected by the addition of  $L_{conformal}$ .

We will explore the formation, hydrodynamics and decay of Q-Stars [10-11] as well as extend the Q-Star idea to other sorts of field theories [12-17] in later publications. The main points are that the energy densities  $\epsilon \sim \phi_o^4$ , radii  $\rho \sim m_{pl}/\phi_o^2$  and masses  $M \sim m_{pl}^3/\phi_o^2$  remain relatively low and that the surface region is a certain sort of non topological soliton or Q-ball.

To summarize, we have constructed a new class of non-topological soliton stars appearing in field theories with non-linear matter interactions coupled to Einstein gravity. We have named these ‘Q-Stars’.



### Acknowledgements

I would like to thank the Theoretical Division and the ALEPH Collaboration at CERN where this work was done for their warm hospitality during the summer of 1988.

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### Figure Captions

1. Q-Star example A: matter field  $\tilde{\phi}$  and metrics  $e^{2u}$  and  $e^{-2\bar{v}}$  with surface region magnified.
2. Q-balls with  $\tilde{\omega}_{sc}^2 > \tilde{\omega}_{critical}^2$  (dashes) and Q-star surface (solid)
3. Mass vs. charge for Q-balls and Q-stars. The free-particle boson mass is taken to be one. Thus, all solitons lying below the dashed line  $\tilde{M} = Q$  are stable against break up to free particles. The point  $\delta\tilde{M}/\delta Q = \tilde{\omega}_{critical}$  is the Q-ball limit without gravity. The point  $\delta\tilde{M}/\delta Q = 0$  is the onset of black holes.
4. Potential and surface of Q Star in example A
5. Q-Star of example B: Matter fields  $\tilde{\sigma}$ ,  $\tilde{\phi}$  and metrics  $e^{2u}$  and  $e^{-2\bar{v}}$ .
6. Field strength  $\tilde{\phi}$  for Q-balls evolving into Q-Stars with increasing Q: Q very small (dots), large but gravity unimportant (dashes), larger with gravity important (solid).

$\tilde{\rho}_{S-} (\epsilon^{-1})$	$\tilde{M} (\epsilon^{-3})$	$Q (\epsilon^{-3})$	$\tilde{\omega}$	$\tilde{\epsilon}_o$
0.100	0.002	0.003	0.499	0.375
0.250	0.024	0.049	0.499	0.375
0.500	0.196	0.397	0.492	0.379
1.400	4.515	9.837	0.431	0.423
1.620	7.212	16.31	0.401	0.457
1.700	8.479	19.52	0.388	0.479
1.800	10.39	24.59	0.367	0.521
1.910	13.52	33.54	0.330	0.661

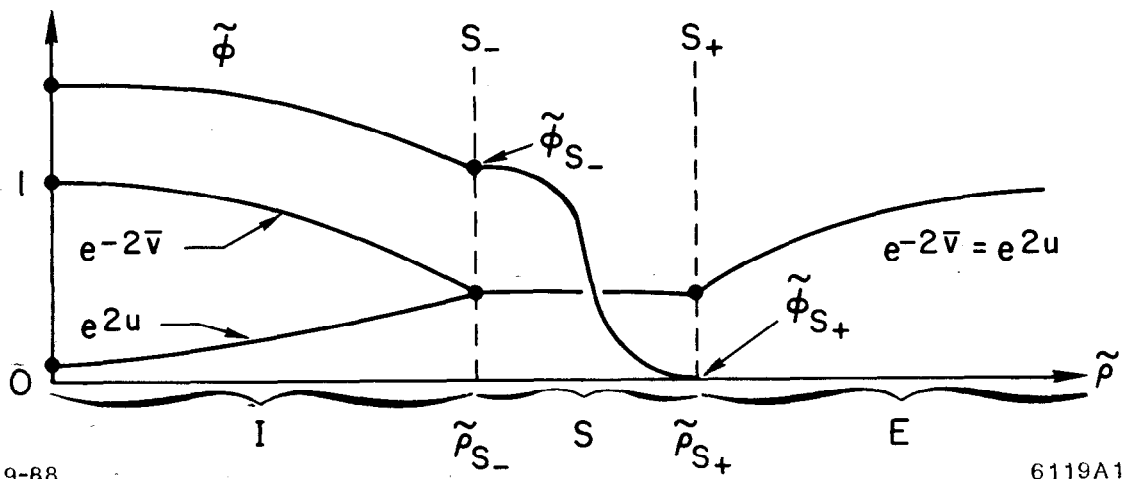
Table 1

Q-Stars in example A: Radius, mass, charge, frequency and central energy density  $\tilde{\epsilon}_o = [\tilde{W} + \tilde{U}]_{\tilde{\rho}=o}$  rescaled with  $\phi_o$ . Units of  $\tilde{M}$ ,  $Q$  are  $\epsilon^{-3} = [8\pi G\phi_o^2]^{-3/2} \sim 10^{48}$  for  $\phi_o \sim 300$  GeV. Units of  $\tilde{\rho}_{S-}$  are  $\epsilon^{-1}$ .

$\tilde{\rho}_{S-} (\epsilon^{-1})$	$\tilde{M} (\epsilon^{-3})$	$Q (\epsilon^{-3})$	$\tilde{\omega}$	$\tilde{\epsilon}_o$
0.100	0.001	0.001	0.577	0.111
0.250	0.007	0.013	0.577	0.111
0.500	0.058	0.101	0.575	0.112
1.000	0.470	0.823	0.566	0.114
2.000	3.940	7.174	0.530	0.128
2.660	10.10	19.37	0.482	0.160
2.930	14.97	29.87	0.444	0.210
2.985	16.90	34.29	0.428	0.251

Table 2

Q-Stars in example B:  $\lambda = 1/9$ . Radius, mass, charge, frequency and central energy density scaled with units as in Table 1.



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Figure 1

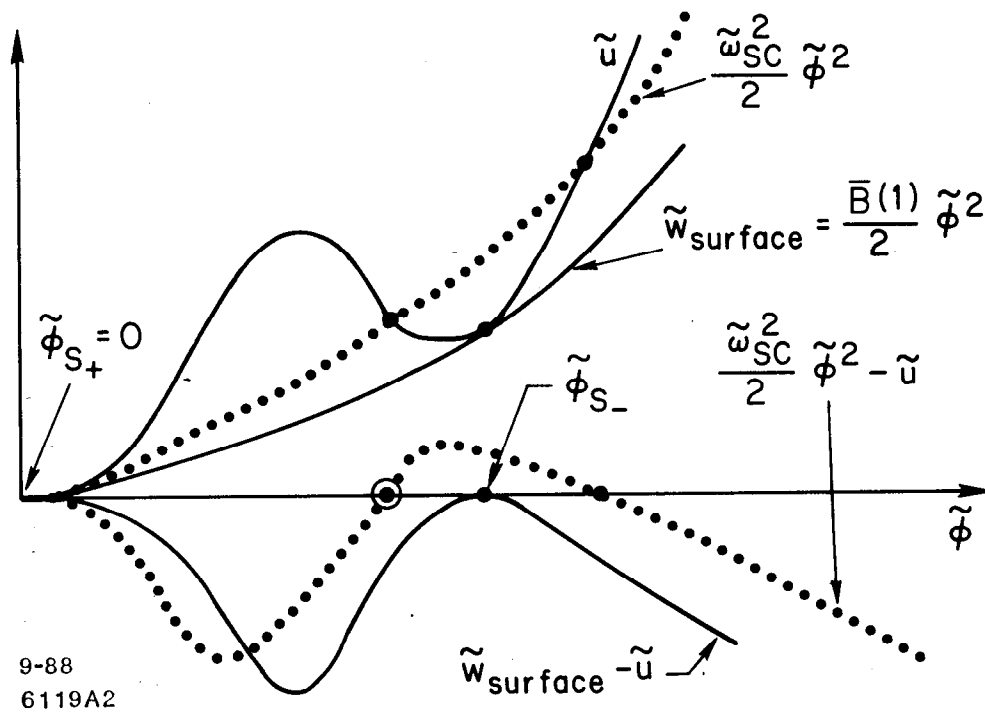
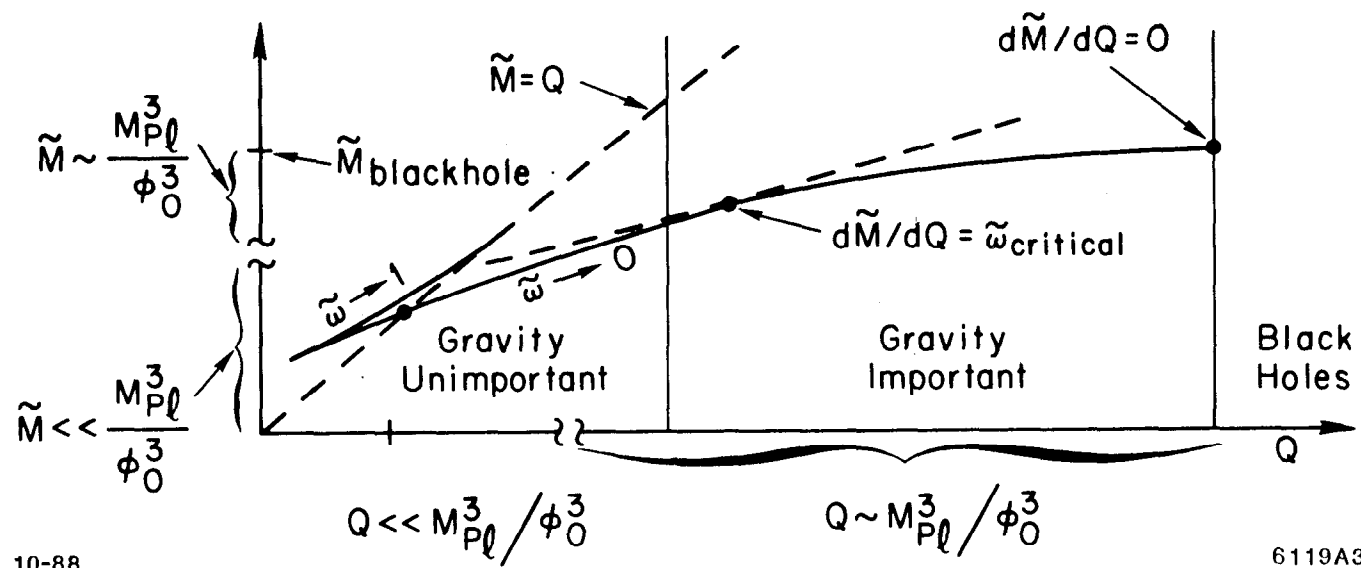


Figure 2

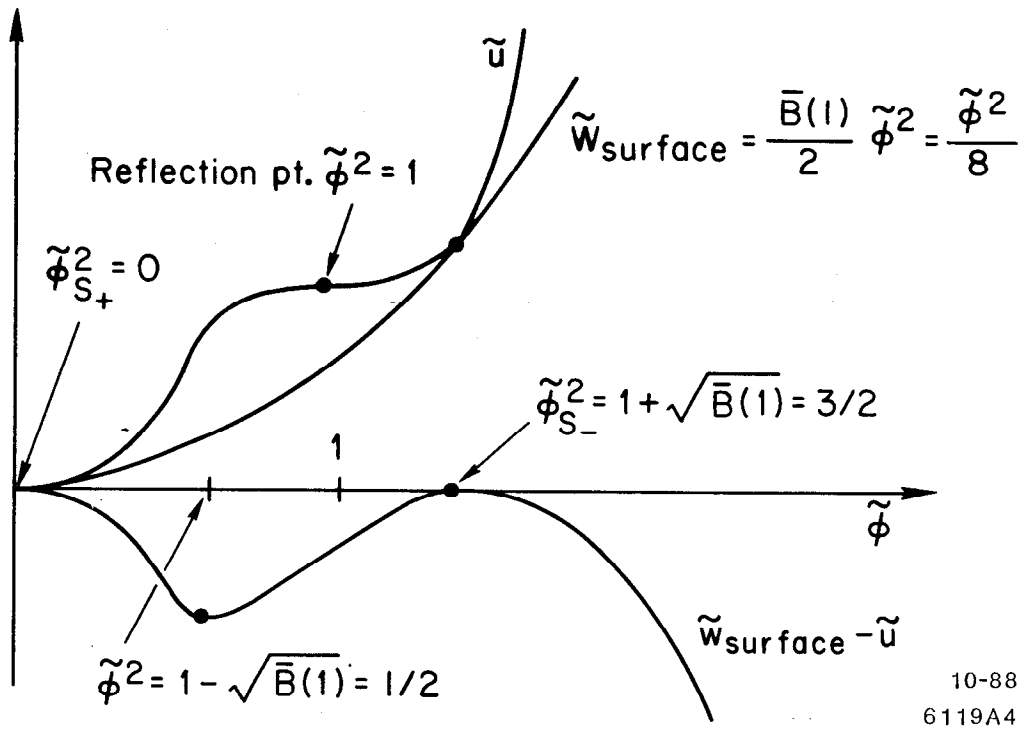


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Figure 3





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Figure 4

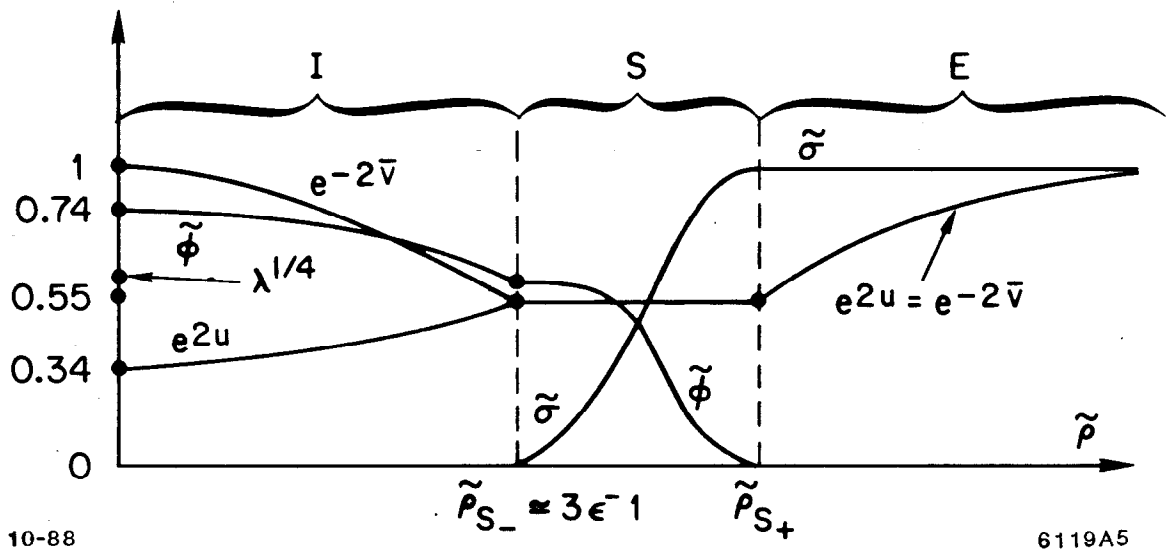
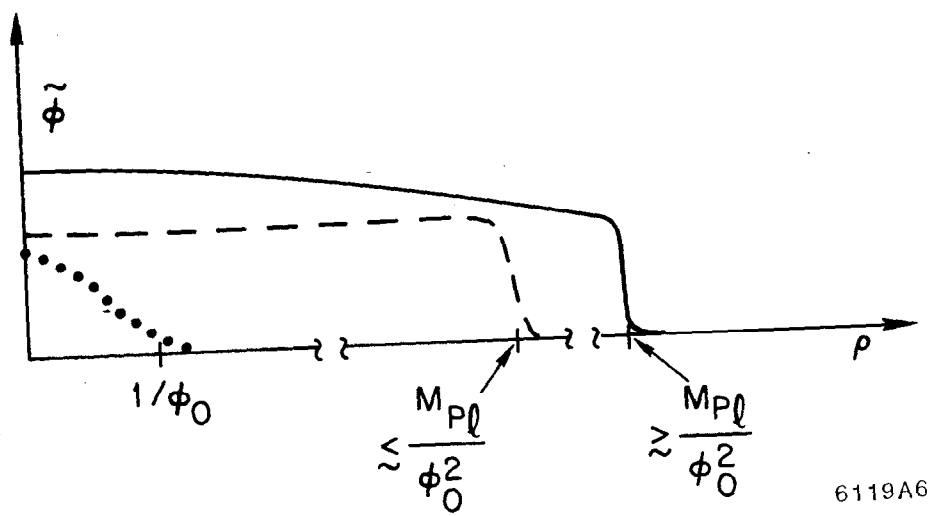


Figure 5



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Figure 6