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FLAVOR CHANGING DECAYS OF THE Z INTO HEAVY NEUTRINOS*

FREDERICK J. GILMAN AND SUN H. RHIE

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

We consider flavor-changing decays of the Z boson to a fourth-generation heavy neutrino and a light neutrino, which are induced at one loop in the standard model. Such decays have a characteristic monojet signature which makes them readily distinguished experimentally, unlike flavor-changing decays involving quarks. Like other such one-loop processes, however, they are very rare when reasonable mixing angles and intermediate fermion masses are considered.

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I. Introduction

In the standard model, with left-handed quarks and leptons in weak SU(2) doublets and right-handed ones in singlets, there are no flavor-changing couplings of either the photon or the Z at tree level. Such effects can be induced at the one-loop level, but are suppressed in branching ratio compared to tree-level processes by factors of order

$$\left[\frac{g^2}{16\pi^2}\left(\frac{m}{M_W}\right)^2\right]^2 \quad ,$$

where m is the mass of an internal fermion in the loop and g the weak SU(2) gauge coupling constant.

Such rare decays are of interest, because they can produce new heavy quarks and leptons, and also because, even when the decay products are only "old" quarks or leptons, the rate for such processes depends on the properties of the virtual particles present in the loop. Therefore it can tell us about the presence of new physics at a mass scale well beyond the mass of the Z. For these reasons, flavor-changing Z decays have previously been investigated both as a test of the standard model and as a probe of physics that may lie beyond it.

In the case of Z decay to two different flavors of quarks, like[1–4]

$$Z \to d\,\bar{s} \text{ or } Z \to u\,\bar{c}$$

the virtual fermions in the loop will be other quarks whose charge differs by one unit. Inasmuch as the amplitude grows rapidly with the mass of this internal quark, the largest decay rates will occur when the heaviest quark participates, *i.e.*, the t quark, if all other considerations of mixing angles and phase space being the same. With these latter factors taken into account, in the three-generation standard model it is

$$Z \to b\,\bar{s} \text{ and } Z \to s\,\bar{b}$$

which has the largest potential branching ratio because of the t quark in the loop.

Still, the branching ratio is only $\sim 10^{-7}$ for a t-quark mass of 200 GeV. This is not a process to be seen soon!

Moreover, the experimental consideration that it is quark jets and not the quarks themselves which are observed makes it extremely unlikely that decays of this nature with tiny branching ratios could be distinguished from potential backgrounds. More explicitly, since decays like $Z \to b\ \bar{b}$ occur at the 10% level in branching ratio, it seems that rare but completely allowed (in terms of quantum numbers) fluctuations in the appearance of the final particles in such processes will lead to events which masquerade as $Z \to b\ \bar{s}$ at a level which is well above that predicted for the latter, flavor-changing process in the standard model. Even with generous allowance for a fourth generation of quarks (in the loop) with "large mixing" the prospects seem dim. However, once we have introduced a fourth generation, the possibilities do improve when one considers decay into third- and fourth-generation quarks,[4] *e.g.*,

$$Z \to b'\ \bar{b} \quad .$$

Even so, it generally will not be easy to separate such a process, which still has a very small branching ratio, from ordinary flavor-conserving processes that involve the emission of additional gluons.

For flavor-changing Z decay to charged leptons, the rates are infinitesimal if we restrict our attention to three generations, as the rates depend on the masses of the electron, muon and tau neutrinos in the loop. Even with a fourth-generation neutrino with a mass below that of the W, the branching ratio seems too small to be found at SLC or LEP.[5] Moreover, decays like $Z \to \mu\ \bar{\tau}$ are experimentally indistinguishable from the usual decays with an "unusual" configuration, *i.e.*, where $Z \to \tau\ \bar{\tau}$ is followed by $\tau \to \mu\ \bar{\nu}_\mu\ \nu_\tau$, and where the final muon goes in the direction of the parent tau (back-to-back with the $\bar{\tau}$) to within the resolution in angle and energy of a potential detector. At something like the 10^{-6} level in branching ratio, the signal gets lost in the background.[6] Similar remarks hold for $Z \to e\ \bar{\tau}$.

The one remaining possibility is Z decay into a pair of neutrinos of different flavors. Of course with only three generations, it is hard enough to detect $Z \to \nu \, \bar{\nu}$ (for example, by "tagging" the Z with a photon emitted from the incident beams, and seeing "nothing" of its decay products). It is simply impossible to know their flavors. However, if there is a fourth-generation heavy neutrino, then by the same mixing that allows it to be produced in conjunction with a light neutrino in the first place, it will decay in flight near the point of production in Z decay. The resulting event will look like a monojet. This is of no interest for fourth-generation neutrinos with masses below $M_Z/2$, for they can be pair produced in Z decay with tree-level couplings to the Z. However, for those with masses between $M_Z/2$ and M_Z, flavor-changing decays of the Z provide a potentially clear, albeit rare, signal of the existence of such particles.

In this paper we present a short summary of the results of the calculation[7] of such neutrino flavor-changing decays of the Z. As noted in Section II, the calculation is quite complicated in the general case. However, in the limit of large internal fermion mass, it takes on a simple form and we give an analytic result, together with physical arguments as to why it has this form. We emphasize the generality of this result — it is one which is essentially identical for both quarks and leptons and gives a decent estimate for the amplitude even for moderate values of the internal fermion mass. Section III explores the consequences of these calculations for future experiments at Z factories.

II. The Calculation

We now proceed to the actual calculation of these one-loop processes. As we work in the standard model (or rather its slight extension to four generations with some massive neutrinos), the gauge group is $SU(2)_L \times U(1)$ and a single Higgs doublet implements the spontaneous symmetry breaking mechanism and also generates Dirac masses for all the fermions, including the neutrinos. The field content of the lepton sector is:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \quad \begin{pmatrix}\nu_L\\ L\end{pmatrix}_L$$

$$\begin{matrix}\nu_{1R} & \nu_{2R} & \nu_{3R} & \nu_{4R}\\ e_R & \mu_R & \tau_R & L_R\end{matrix} \tag{1}$$

where

$$\nu_{\alpha_L} = \sum_i V_{\alpha i}\, \nu_{i_L} \begin{cases}\alpha = e, \mu, \tau, L & \text{gauge eigenstate index}\\ i = 1, 2, 3, 4 & \text{mass eigenstate index}\end{cases} \tag{2}$$

and V is a unitary mixing matrix. From this point onward, we shall use $N \equiv \nu_L$, in order to make it easy typographically to distinguish the heavy neutrino from the other three.

The Feynman diagrams for $Z^0 \to \nu_i \bar{N}$ are shown in Figure 1. Since ν_1, ν_2 and ν_3 are all known to be very light compared to N, whose mass, m_N, we envisage to be comparable to M_Z, the light neutrino masses are negligible and the effective Langrangian induced at one loop involving ν_i and $\bar{N}$ will involve left-handed light neutrinos and will be of the form:

$$\mathcal{L} = \bar{\nu}_{i_L} \left(F_E\, \epsilon \cdot \gamma + F_M\, \frac{q \cdot \gamma}{M_Z}\, \epsilon \cdot \gamma \right) N \quad , \tag{3}$$

where ϵ and q are the polarization and momentum vectors of Z^0. The dimensionless form factor F_E measures the part of the interaction which is helicity-preserving, while F_M measures the part which flips helicity. Since helicity-flip can only occur through the existence of the mass of the heavy neutrino, N, the form factor F_M is expected to be proportional to m_N and to be appreciable in magnitude only when m_N is comparable to the other masses in the problem. When m_N is negligible, the total current is effectively chiral and the decay amplitude will be dominated by F_E.

With these form factors the partial decay rate takes the form:

$$\Gamma(Z^0 \to N\bar{\nu}_i) = \frac{M_Z}{48\pi}(1-z^2)^2\left(|F_E|^2(2+z^2)+6Re(F_E F_M^*)z+|F_M|^2(1+2z^2)\right) \quad (4)$$

where $z \equiv m_N/M_Z$. For the tree-level, flavor-conserving process $Z^\circ \to \nu\ \bar{\nu}$, the analogous form factors have the values:

$$F_E = \left(\frac{g}{2\cos\theta_W}\right) \quad , \quad F_M = 0 \quad ,$$

and using Eq. (4) with $z = 0$,

$$\Gamma(Z^0 \to \nu\bar{\nu}) = \frac{M_Z}{24\pi}\left(\frac{g}{2\cos\theta_W}\right)^2 \simeq 176 \text{ MeV} \quad . \quad (5)$$

The one-loop amplitudes are much smaller. They are obtained by summing the amplitudes, T_n, corresponding to the diagrams in Fig. 1:

$$\begin{aligned} F_E &= \left(\frac{g}{2\ \cos\theta_W}\right)\left(\frac{g^2}{16\pi^2}\right)\sum_{\alpha=e}^{L} V_{\alpha i}^* V_{\alpha N}\sum_{n=1}^{10} F_E^{(n)} \quad , \\ F_M &= \left(\frac{g}{2\ \cos\theta_W}\right)\left(\frac{g^2}{16\pi^2}\right)\sum_{\alpha=e}^{L} V_{\alpha i}^* V_{\alpha N}\sum_{n=1}^{10} F_M^{(n)} \quad , \end{aligned} \quad (6)$$

where the factors due to the leptonic mixing matrix and couplings at the weak interaction vertices have been explicitly pulled out of the amplitudes $F_E^{(n)}$ and $F_M^{(n)}$ coming from diagram n.

These factors alone make the decay width smaller than that for the flavor conserving decay by roughly a factor of:

$$\left(\frac{g^2}{16\pi^2}\right)^2 \left(\sum_{\alpha=e}^{L} V_{\alpha i}{}^* V_{\alpha N}\right)^2 \simeq 0.8 \times 10^{-5} \left(\sum_{\alpha=e}^{L} V_{\alpha i}{}^* V_{\alpha N}\right)^2 .$$

In order to get a measurable branching ratio for neutrino flavor-changing decays of the Z°, $\sum F^{(n)}$ has to be substantially bigger than unity. This may be possible, because in a spontaneously broken gauge theory the longitudinal mode of the $W^\pm$ in the physical gauge (or the unphysical charged scalars in Feynman gauge) have a coupling to fermions which is proportional to the fermion's mass. Thus, there is at least a chance that if the mass of the charged lepton (which is found in the loop) is made large enough, one can compensate for the smallness of the factors that are necessitated by dealing with a one-loop electroweak amplitude.

Therefore, let us first consider this special case, *i.e.*, the asymptotic behavior of the decay amplitude as $m_L^2 \to \infty$. Examining the diagrams in Feynman gauge in Fig. 1, it is the amplitudes T_2, T_6, T_8, and T_{10} which will contain factors of $(m_L/M_W)^2$ from the unphysical scalar coupling twice to the charged lepton found in the loop. When $m_L^2 \to \infty$, all other fermion masses can be neglected, *i.e.*, set to zero, in calculating the leading terms. These terms are:

$$\begin{aligned} F_E^{(2)} &\to \left[(\frac{1}{2} - \sin^2\theta_W) - \frac{1}{2}\sin^2\theta_W(\Delta - \log\ m_L^2 - \frac{1}{2})\right]\left(\frac{m_L^2}{M_W^2}\right) \\ F_E^{(6)} &\to \left[\frac{1}{2}(\sin^2\theta_W - \frac{1}{2})(\Delta - \log\ m_L^2 + \frac{3}{2})\right]\left(\frac{m_L^2}{M_W^2}\right) \qquad (7) \\ F_E^{(8)} + F_E^{(10)} &\to \left[(\frac{1}{4})(\Delta - \log\ m_L^2 + \frac{3}{2})\right]\left(\frac{m_L^2}{M_W^2}\right) \end{aligned}$$

where

$$\Delta = \frac{2}{4-D} + \log\ 4\pi - \gamma$$

is the usual divergence at $D = 4$ in the dimensional regularization scheme. The net coefficient of Δ must vanish to get a finite result — it does on summing the amplitudes. Indeed, combining the amplitudes we find the exceedingly simple result:

$$F_E^{(2)} + F_E^{(6)} + F_E^{(8)} + F_E^{(10)} \to \frac{1}{2}\left(\frac{m_L^2}{M_W^2}\right) \quad . \tag{8}$$

Therefore, as $m_L^2 \to \infty$,

$$F_E \to \left(\frac{g}{2\cos\theta_W}\right)\left(\frac{g^2}{16\pi^2}\right) V_{Li}^* V_{LN} \, \frac{1}{2} \, \left(\frac{m_L^2}{M_W^2}\right) \tag{9}$$

and $F_M/F_E \to 0$ in the same limit, so that

$$\begin{aligned} &\frac{\Gamma(Z^0 \to N\bar{\nu}_i) + \Gamma(Z^0 \to \bar{N}\nu_i)}{\Gamma(Z^0 \to \bar{\nu}_i\nu_i)} \\ &\quad \to \frac{1}{2}\left(\frac{g^2}{16\pi^2}\, V_{Li}^* V_{LN}\right)^2 \left(\frac{m_L^2}{M_W^2}\right)^2 \left(1 - \frac{m_N^2}{M_Z^2}\right)^2 \left(1 + \frac{m_N^2}{2M_Z^2}\right) \quad . \end{aligned} \tag{10}$$

Thus the asymptotic behavior is determined by the "electric" form factor, F_E, and the leading term is independent of $\sin^2\theta_W$. These two features have a simple physical explanation. First, as already noted, in the asymptotic limit as $m_L^2 \to \infty$, m_N can be set to zero in calculating the leading term. It is m_N which is responsible for allowing helicity flip; with $m_N = 0$, the current is chiral and $F_M = 0$. Second, in this same asymptotic limit the calculation of the leading term is the same as it would be if we neglected the mass of the bosonic as well as fermionic propagators. In other words, for the leading term as $m_L^2 \to \infty$, the calculation of the relevant one-loop amplitudes proceeds in the same manner as if the broken SU(2) $\times$ U(1) symmetry is restored; the calculation is "indifferent" as to how the $\mathrm{U}(1)_{e.m.}$ of electromagnetism is embedded in SU(2) $\times$ U(1). The result for the leading term therefore can not depend upon the value of $\sin^2\theta_W$.

Similarly, the asymptotic result must be independent of the electric charge of the fermions involved. This implies that the one-loop amplitude for the flavor-changing decay of the Z° to any pair of fermions from different generations must have the same asymptotic form when the mass of the internal fermion (in the loop) goes to infinity. It can be checked that with appropriate changes in the names of the fermions and the corresponding mixing matrix elements, Eq. (9) agrees with explicit asymptotic formulas found previously for quarks[1,3,4] and for charged leptons.[5] Given the remarks above, we emphasize that all these previous asymptotic results are essentially the same calculation as the simple one for neutrinos given here.

When the mass of the internal fermion is not large compared to M_W, the various calculations still bear a strong resemblance, but they do in fact differ, with the result having a functional dependence upon $\sin^2\theta_W$ and the charge of the fermions involved. In computing the decay rate, the color degrees of freedom enhance that for quarks by the usual factor of three.

The calculation of $Z \to N\bar{\nu}_i$ for arbitrary m_L and m_N is found in its full glory in Ref. 7. In this general case, the amplitudes $F_E^{(n)}$ and $F_M^{(n)}$ were calculated in 't Hooft-Feynman gauge using dimensional regularization and standard techniques.[8] The answer can be expressed in terms of the B and C functions of 't Hooft and Veltman,[8] which in the general case involve dilogarithms and are functions of both the internal and external masses. One simplification that does occur here is that the masses (and contributions) of the three light charged leptons are completely negligible compared to that of the heavy, fourth-generation charged lepton and the electron, muon and tau masses may be set to zero. The unitary sum over the charged leptons simplifies to

$$\sum_{\alpha=e}^{L} V_{\alpha i}{}^{*} V_{\alpha N}\ F_{E,M}^{(n)}(m_\alpha) = V_{Li}{}^{*} V_{LN} \left[F_{E,M}^{(n)}(m_L) - F_{E,M}^{(n)}(m=0) \right] \qquad (11)$$

on using the unitarity of V. We expect that the off-diagonal elements of the mixing matrix are small and the diagonal ones near unity, so that $|V_{Li}^{*} V_{LN}| \sim |V_{Li}|$.

The resulting form factors in the representative case where $m_N = 50$ GeV (and the process $Z \to N\bar{N}$ is forbidden kinematically) are shown in Fig. 2. The dotted line shows the asymptotic (as $m_L^2 \to \infty$) result derived previously, with the quadratic dependence on m_L showing up as a straight line on this log-log plot. We note the following features of the form factors:

- $|F_E|$ everywhere dominates $|F_M|$. We already know this is true in the asymptotic region, where $F_M/F_E \to 0$, but it turns out to be true for all values of m_L.
- There is an abrupt change in the imaginary part of the form factors when the mass of the charged heavy lepton exceeds $M_Z/2$ and the physical process $Z \to L^+L^- \to N\bar{\nu}_i$ is no longer possible. When we recall that the unitarity sum in Eq. (11) always results in a difference of amplitude for the massive and the light charged leptons (see above), it becomes clear that for $M_L > M_Z/2$ the constant, negative imaginary part shown in Fig. 2 is due to the intermediate state consisting of a pair of light charged leptons.
- An abrupt change in the behavior of the imaginary part of an amplitude produces a cusp in the real part. This is true for both F_E and F_M at $M_L = M_Z/2$.
- As the real part of F_E grows beyond the imaginary part, it also quickly converges to its asymptotic behavior. Beyond $m_L \sim 100$ GeV, ReF_E becomes the dominant contribution to the decay rate.

The decay rate itself, modulo the square of the mixing matrix elements $V_{Li}^* V_{LN}$, is shown in Fig. 3 as a function of charged lepton mass when $M_N = 0,\ 50$ and 75 GeV. The straight lines on this log-log plot represent the prediction from the asymptotic form in Eq. (9). Their dependence on m_N is given entirely by the phase-space factor

$$\left(1 - \frac{m_N^2}{M_Z^2}\right)^2 \left(1 + \frac{m_N^2}{2M_Z^2}\right) \quad .$$

If the curves which result from the full calculation are adjusted for this same phase-space factor, they are also not far from congruency with one another. A small but noticeable difference occurs in the shape of the plateau above the cusp at $M_Z/2$.

This is principally due to the presence of the form factor F_M (which is proportional to M_N), even though in general the contribution from $|F_M|^2$ to the decay rate is much smaller than that from $|F_E|^2$.

III. Discussion

We have seen that the amplitudes for the flavor-changing decay of the Z take on a very simple form as m_L, the mass of the internal lepton in the one-loop diagram, becomes large. This is especially relevant physically because it is only then that the predicted rate becomes large enough to result in a detectable number of decays of this kind. Moreover, the asymptotic form turns out to yield a good approximation to the exact decay rate for values of m_L down to ~ 100 GeV, and is even a reasonable semi-quantitative guide well below that.

The same asymptotic form, being independent of $\sin^2\theta_W$ and the electric charge on the internal fermion, is a good approximation to the amplitude for flavor-changing quark and charged lepton decays of the Z as well. Unfortunately, this tells us that in all these cases there are neither miracles due to fortuitous enhancements nor disasters due to "accidental" cancellations: the one-loop amplitude in this case can be estimated in a straightforward way based on the mixing matrix and the mass of the internal fermion. Nor does taking the neutrinos to be Majorana rather than Dirac particles provide any substantial difference in the decay rates.[7]

In order to get a quantitative look at the expected decay rates, let us focus on the number of decays of the form $Z \to N\bar{\nu}_i$ in 10^7 Z decays. Since the light neutrinos all have very small masses, we lump the ν_i together, and set

$$\sum_i |V_{Li}|^2 \equiv \theta^2 \quad .$$

A not unreasonable choice for θ is the value of the quark mixing matrix element $V_{us} \sim 0.22$ or that of $V_{cb} \sim 0.05$. We obtain the following table for $m_N = 50$ GeV and a total Z width of 2.7 GeV.

Table 1. The number of $Z \to N\bar{\nu}_i$ decays in 10^7 Z decays.

θ	m_L	
	200 GeV	500 GeV
0.22	4.0	130
0.05	0.2	7.1

These numbers, which include a factor of two to include the charge conjugate reaction, are very small. The advantage of neutrinos over quarks or charged leptons as the decay products is that even a few events of this type will stand out as monojets, for the same mixing matrix elements that allow for a non zero, one-loop amplitude will cause the heavy neutral lepton, N, to decay near the point of production, thus making the entire event look like a monojet with at least one charged lepton. If we look to the far future and can envisage colliders that produce 10^8 to 10^9 Z's per year, then even with rather small mixing angles such processes become a quite useful tool in looking for fourth-generation heavy neutral leptons, and indirectly, their charged partners.

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FIGURE CAPTIONS

1) The Feynman diagrams for $Z^0 \to \nu_i \bar{N}$ in general linear gauge.

2) The real and imaginary parts of the form factors F_E and F_M as a function of the heavy charged lepton mass, m_L. The dotted line corresponds to using the asymptotic form in Eq. (9).

3) The decay rate for $Z \to N\ \bar{\nu}$ plus the charge conjugate reaction as a function of charged lepton mass for $m_N = 0$ (dotted curve), 50 (dashed curve), and 75 GeV (solid curve). The straight lines represent the corresponding curves when the asymptotic form in Eq. (9) is used for the form factors.

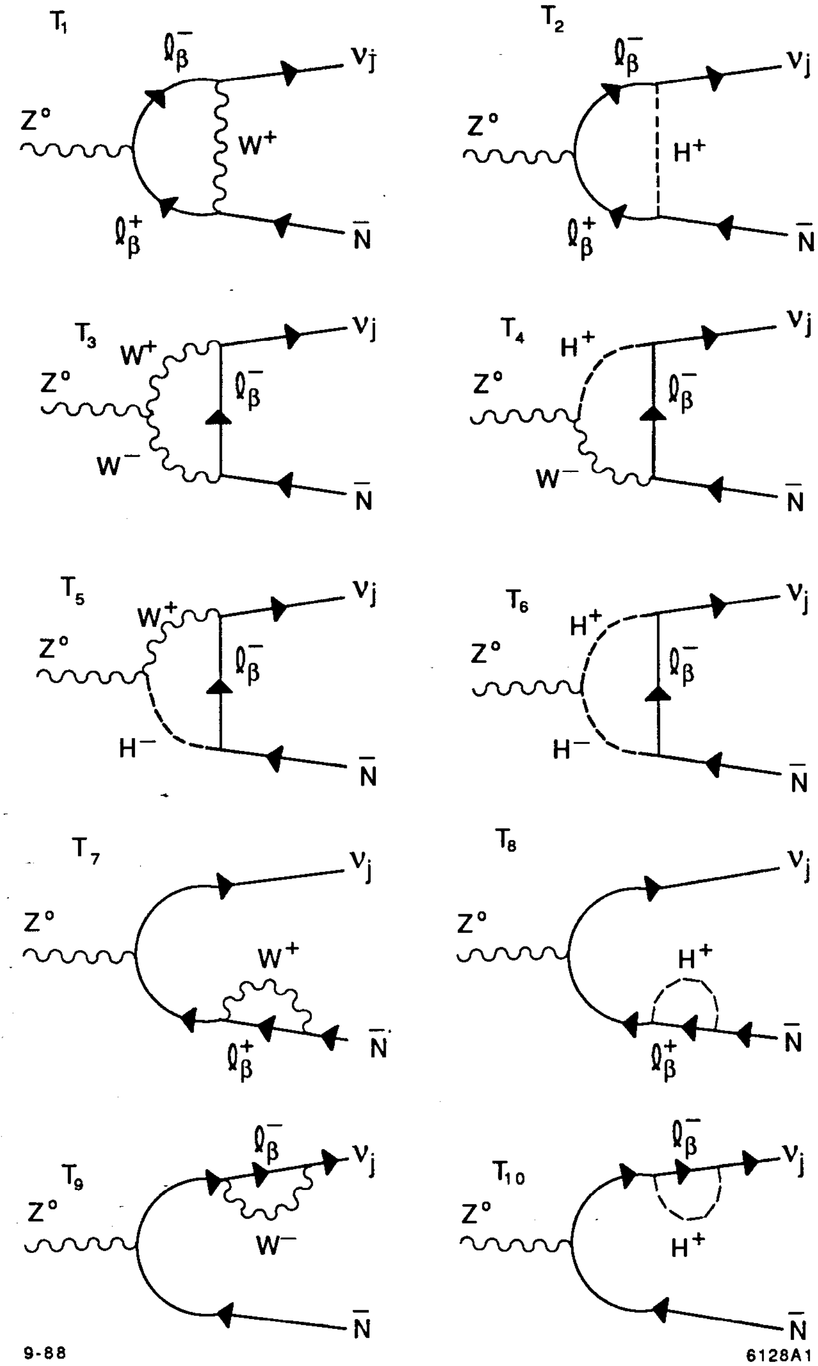
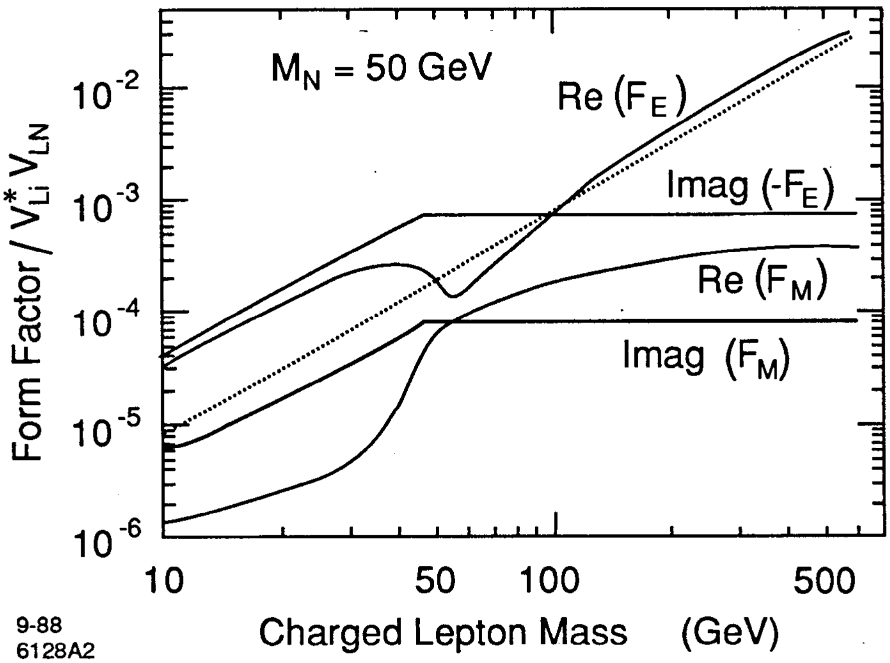


Fig. 1



M_N = 50 GeV
Re (F_E)
Imag (-F_E)
Re (F_M)
Imag (F_M)
Form Factor / $V^*_{Li} V_{LN}$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10
50
100
500
Charged Lepton Mass (GeV)
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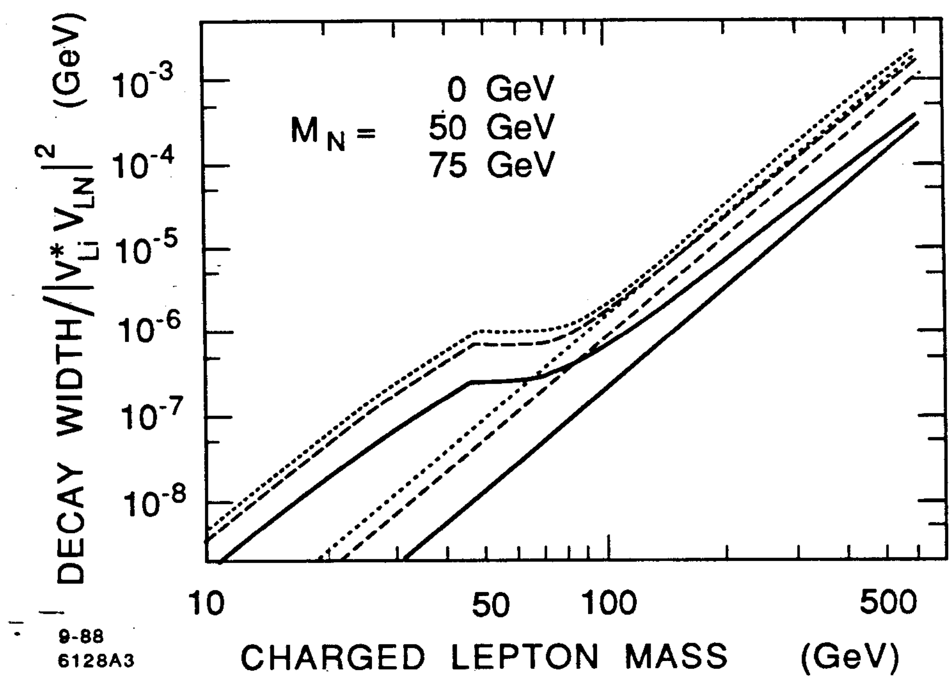
6128A2

Fig. 2



M_N =
0 GeV
50 GeV
75 GeV
DECAY WIDTH/$|V^*_{Li} V_{LN}|^2$ (GeV)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10
50
100
500
CHARGED LEPTON MASS (GeV)
9-88
6128A3

Fig. 3