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Probing the Earth with WIMPs*

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ABSTRACT

Weakly interacting massive particles (WIMPs), with masses $\gtrsim \mathcal{O}$(GeV), are candidates for the dark matter in galactic halos. We discuss the distribution and detectability of coherently interacting particles (such as massive Dirac or scalar neutrinos, solar cosmions, and some Majorana fermions) that have been captured into orbits within the Earth. Coherent WIMPs in the mass range from 8 to 21 GeV in Earth orbits would give rise to count rates of $\mathcal{O}$(1-1000 /kg/eV/day) in proposed cryogenic detectors operating at thresholds of $\mathcal{O}$(1 eV). Over this mass and energy range, these rates are several orders of magnitude larger than those for direct detection of the corresponding particles coming from the halo. Since they orbit through the Earth's core, these Earth-bound WIMPs can be used to probe the temperature structure of the Earth's interior. The temperature of the Earth's inner core can be determined to within 300°K, compared with the present 1000 – 2000°K uncertainty.

1. Introduction

If the dark matter in galaxy halos is non-baryonic, as suggested by a variety of arguments,[1] it may consist of one of several exotic candidates. Weakly interacting massive particles (WIMPs) are cold dark matter candidates in the GeV mass range which interact with ordinary matter with cross-sections of $\mathcal{O}(10^{-38}\mathrm{cm}^2)$. Experiments are now underway to detect such particles in the Galactic halo[2] and more such experiments are planned.[3] As first discussed by Freese[4] and by Krauss, Srednicki, and Wilczek,[4] halo WIMPs may be captured in appreciable numbers into the Earth. In this paper, we investigate the distribution of WIMPs captured into orbits within Earth in order to determine their number density and detection rate at the Earth's surface.

For our purposes, WIMPs can be divided into two classes: (1) Particles with coherent interactions with nuclei, *i.e.*, cross-sections $\propto N^2$ where N is approximately the atomic number of the scattering nucleus. Particles in this class include massive Dirac and scalar neutrinos, and several proposed versions of the cosmion, a light (3 GeV $\lesssim m_W \lesssim$ 10 GeV) particle with a cross-section on hydrogen, $\sigma_p \sim 10^{-36}\mathrm{cm}^2$, large enough to potentially solve the solar neutrino problem.[5] (2) Particles with only spin-dependent interactions, i.e., cross-sections proportional to nuclear spin. Particles in this class usually include Majorana fermions, such as photinos and Majorana neutrinos. However, it has recently been pointed out[6] that the lightest supersymmetric fermion (LSP) is likely to be a combination of the photino, higgsinos and zino; in this case the LSP can have substantial spin-independent (coherent) interactions as well. Thus, the list of candidates in class (1) may be quite large.

Since the Earth is composed primarily of spinless even-even nuclei, it does not interact appreciably with or capture particles of type (2). We shall therefore focus on the first class of particles, *i.e.*, coherent WIMPs. Further, as we are interested in detecting such particles which are captured within Earth, we will not consider those WIMPs that annihilate with one another on a timescale shorter than the Earth's lifetime. [Indeed, because of their annihilation products, some of these particles can already be ruled out as halo candidates.[4,7]] Thus, we are interested in coherent WIMPs which have either a cosmic asymmetry between particles and antiparticles (*e.g.*, for Dirac neutrinos) or a suppressed annihilation cross-section.[5,8]

Very light particles captured by the Earth tend to "evaporate", *i.e.*, get kicked to escape velocity by collision with nuclei, before they can accumulate in appreciable numbers.[4,9] On the other hand, very massive WIMPs sink to the center of the Earth and could not be detected at the surface. As we will show, the detection rate peaks for WIMPs with the evaporation mass, $m_{\rm ev} = 9\text{-}16$ GeV, where the evaporation and capture rates are equal. (The precise value of $m_{\rm ev}$ depends on the temperature structure of the Earth; see below.) At this mass, the WIMPs are heavy enough that few escape, but still light enough that many bubble up to the surface. Thus, we will focus on particles in the mass range 8-21 GeV, near the evaporation mass, for which the detection rate is appreciable. (The mass range is skewed toward the high end from $m_{\rm ev}$ because the capture rate rises there.) Fortuitously, this covers much of the mass range for which cosmological arguments suggest that coherent WIMPs (with or without an asymmetry) could be the dark matter, $\Omega_W \simeq 0.1 - 1.0$. We note that Dirac and scalar neutrinos of mass $m \gtrsim 20$ GeV have probably been ruled out as the dark matter in the halo by double-beta decay experiments.[2,10]

WIMPs captured in the Earth thermalize with the Earth's core by weak interaction scattering with core nuclei, particularly iron. To first approximation, the bound WIMPs relax to an isothermal distribution, with scale height determined by the Earth's gravitational potential. With this simple model, corrections to which we discuss at length below, one can calculate the density of WIMPs at the Earth's surface. For WIMP masses near the evaporation mass, $m_W \sim m_{\rm ev}$, we find that the number density at the Earth's surface of particles in Earth orbit is significantly enhanced over the local number density of particles streaming through the halo. For example, for 12 GeV Dirac neutrinos and a central temperature of 5300 °K, the number density of trapped WIMPs at the Earth's surface is $n(R_\oplus) \simeq 10^2$ cm^{-3}; the corresponding number density of halo WIMPs is $n_{halo} = \rho_{halo}/m_W \simeq 3 \times 10^{-2}$ cm^{-3}. Despite this enhancement, because the Earth-bound particles must be moving slowly, with velocities less than the escape velocity from the Earth, $v_{esc} = 11.2$ km/sec, the detection rate for these particles is appreciable only at very low nuclear recoil energies, $\Delta E = \mathcal{O}(\text{eV})$. As the halo WIMPs are moving faster, $v_{halo} \sim 270$ km/sec, they can be found with a detector operating at a higher energy threshold, $\Delta E \sim$ hundreds of eV, and the first detectors are likely to find the halo WIMPs rather than those in bound Earth orbits. On the other hand, the count rates for Earth-bound WIMPs are potentially much higher than for halo WIMPs; since the signal is roughly proportional to the flux, the ratio of Earth-bound to halo detection rates for the example above is $S_\oplus/S_{halo} \simeq n_\oplus v_{esc}/n_{halo}v_{halo} \sim 10^2$. It is therefore tempting to imagine designing a detector to look for the Earth-bound WIMPs first. An additional incentive is that $\mathcal{O}$(eV) thresholds are also necessary to detect low energy (pp and ^{7}Be) solar neutrinos.[11] However, the more difficult technical requirement of achieving a very low threshold in a reasonably large detector makes it unlikely that this course will be followed immediately. For purposes of this paper, we therefore assume that, say, Dirac neutrinos of known mass m_W *have already* been discovered, and that their distribution in the neighborhood of the Sun (*i.e.*, the local halo density and approximate speed distribution) is already known by direct measurement. This assumption is testable: if coherent particles in the mass range $8 - 21$ GeV compose the halo, they should be detected within the next few years by either ionization or cryogenic detectors.[2,3,10]

Although WIMPs in Earth orbit are not likely to be the first ones detected, one can use them to probe the temperature structure of the Earth. This potential geophysical application of WIMPs may be quite useful, since in percentage terms, the Earth's central temperature is known much less accurately than the sun's. Present estimates of the Earth's central temperature have claimed uncertainties of $\pm 1000°$K, while the most recent determinations differ from estimates of ten years ago by up to 2500°K. The uncertainties arise from several factors. First, the Earth's mantle and outer core are assumed to be in convective equilibrium, so that the temperature gradient in those regions is approximately adiabatic. (The solid inner core is assumed to be almost isothermal.) Estimates of the adiabatic thermal gradient have fluctuated significantly with time. Twenty years ago, typical estimates of the mantle adiabat were $dT/dr \simeq 1°$K/km, giving central temperature estimates of[12] $T_c = 6400°$K. More recent estimates of the mantle gradient are substantially lower,[13] so that reviews of ten years ago[14,15] gave values as low as $T_c = 4400°K$.These estimates all assume a smooth temperature temperature distribution across the core-mantle boundary. A second uncertainty in T_c arises from the possible existence of a thermal boundary layer at the base of the mantle, a seismically anomalous layer which may support a large (of order 1000°K) jump in temperature (over a range of only 200 km in depth).

An additional constraint on T_c comes from attempts to directly model conditions in the iron-rich core. The inner core-outer core boundary is thought to mark the transition from solid to liquid Fe. Thus measurements of the melting point of Fe at a pressure of 330 GPa (the pressure at the inner core outer core boundary) would give an estimate of the central temperature. Recently, the melting curve of iron was measured[16] to 250 GPa. Extrapolation to higher pressures yields a central temperature estimate of $T_c = 6900 \pm 1000°K$. The large uncertainty arises from statistical errors in the melting point data and from the fact that a lighter alloying component of the outer core may depress the melting temperature of Fe by up to 1000°K. From seismological measurements, the outer core is thought to contain 5-12% by weight of such a light element, which is usually assumed to be sulfur, oxygen, silicon, or hydrogen. (In addition, a pure Fe outer core, with its undepressed melting point, would likely have frozen long ago.[15]) We note that this high value of T_c, coupled with the lower mantle adiabat, strongly suggests the above-mentioned boundary layer at the core-mantle boundary. In any case, it is clear that the central temperature of the Earth remains uncertain.

How might WIMPs improve this situation? Remarkably, if coherent WIMPs make up the dark matter in the Milky Way, then their abundance at the surface of the Earth is a highly sensitive indicator of the Earth's central temperature. Before discussing a more detailed model, we pause here to give a heuristic demonstration. While, for the most part, captured WIMPs would remain in the Earth's core, they would occasionally be kicked up to the Earth's surface, and still less frequently, would be given a strong enough kick (by an iron nucleus in the core) to actually escape the Earth's gravitational field. For a given WIMP mass m_W, there is an "evaporation temperature", T_{ev}, which we estimate below [eqn (2.27)] to be

$$T_{ev} \simeq m_{12} \cdot 5300^{\circ}\mathrm{K}, \tag{1.1}$$

where $m_W = 12 m_{12}$ GeV. T_{ev} is defined to be the temperature of the Earth's core at which the evaporation time (inverse evaporation rate), τ_{ev}, would be equal to an Earth lifetime, $\tau_{\oplus} = 4.6$ Gyr.

If the temperature of the Earth's core is more than a few per cent under T_{ev} (*i.e.*, $m_W > m_{\mathrm{ev}}$), then evaporation will be insignificant. In this case, the number of WIMPs collecting in the Earth's core is independent of the core temperature. Consequently, the number density of WIMPs present at the Earth's surface, n_s, will be roughly proportional to the Boltzmann suppression factor,

$$n_s \propto \exp\left(-\frac{m_W \phi_1}{kT_c}\right), \tag{1.2}$$

where T_c is the temperature of the Earth's core, and ϕ_1 is the gravitational potential difference between the surface of the Earth and its center,

$$\phi_1 \equiv \phi(R_{\oplus}), \tag{1.3}$$

where

$$\phi(r) = \int_0^r \frac{GM(r')}{r'^2} dr'. \tag{1.4}$$

It is convenient to express ϕ_1 as

$$\phi_1 \simeq .8\phi_0, \tag{1.5}$$

where

$$\phi_0 = \frac{1}{2} v_{esc}^2 \tag{1.6}$$

is the potential difference between infinity and the surface of the Earth. (Here $v_{esc} = 11.2$ km/s is the escape velocity from the Earth.) Expression (1.2) may be written

$$n_s = K \cdot \exp(-15T_{ev}/T_c) \ , \qquad (T_c < T_{ev}) \ , \tag{1.7}$$

where the coefficient K is discussed below. Thus, for $T_c < T_{ev}$, the number density at the Earth's surface is an extremely rapidly rising function of the core temperature. If the local halo distribution is measured, and if the theoretical coefficient K can be determined to within a factor of 2, then a measurement of the number density n_s would yield the core temperature to within

$$\frac{\Delta T_c}{T_c} \sim \frac{\ln 2}{15} \left(\frac{T_c}{T_{ev}} \right) \sim 5\% \ , \tag{1.8}$$

or $\Delta T_c \simeq 300°$K (assuming m_W is known).

On the other hand, if the temperature of the core is a few percent above T_{ev} (*i.e.*, $m_W < m_{\rm ev}$), evaporation will be highly significant. The evaporation rate (and thus the depletion of the population of WIMPs inside the Earth) is inversely proportional to the Boltzmann factor at infinity,

$$\frac{1}{\tau_{\text{ev}}} \propto \exp\left[-\frac{m_W(\phi_0+\phi_1)}{kT_c}\right]. \tag{1.9}$$

The fraction reaching the surface is still given by equation (1.2), so that in this temperature range, the number density at the surface of the Earth is proportional to

$$n_s \propto \exp(m_W\phi_0/kT_c) \sim \exp(18T_{ev}/T_c), \qquad (T_c > T_{ev}), \tag{1.10}$$

a very rapidly falling function of T_c. Thus, again, a determination of this number density to within a factor of 2 gives the central temperature to within a few per cent.

Finally, if the core temperature is in the immediate neighborhood of the evaporation temperature, then the number density at the surface is only weakly dependent on T_c. However, the peak of the function $n_s(T_c)$ about the point $T_c = T_{ev}$ is itself very narrow, so that, in this case also, the central temperature could be determined to within a comparatively small range, $\lesssim 800°$K. The qualitative behavior of n_s is shown in Figure 1 below, a plot of the WIMP detection rate as a function of central temperature.

The above argument shows how sensitive the density of WIMPs at the Earth's surface is to its central temperature. However, in order to make use of these arguments to actually measure the Earth's central temperature, one must know the *theoretical* dependence of this density on various *measurable* quantities as well as on the *unknown* central temperature. In particular, the theoretical uncertainty must be kept to within a factor of 2. The remainder of this paper is devoted to an analytic approximation of our proposed experiment, and to a description of the numerical methods necessary for reducing the theoretical uncertainty to within the prescribed limits.

In Section II, we describe our proposed experiment for detecting WIMPs bound in the Earth using a low threshold detector at the surface. We give a simple analytic discussion of the distribution of WIMPs in the Earth, and an estimate of the detection rate for a given halo density of WIMPs in the neighborhood of the Sun. This simplified analysis confirms the above qualitative treatment and yields the mass range for which WIMPs are detectable and, therefore, useful as a probe of the Earth's core temperature. However, the analysis is founded on the assumption that the Earth is in free space, and this assumption introduces a number of errors.

The most important of these errors is that WIMPs which "evaporate" with low velocities do not, in fact, entirely escape from the Earth's neighborhood. Instead, they go into solar orbit and have a substantial probability of being recaptured. In Section III we give an analytic treatment of this effect and show that recapture can be understood as raising the *effective* gravitational potential of the Earth, ϕ_0, and hence the effective evaporation temperature.

Our conclusions follow in Section IV. In the Appendix, we discuss several additional sources of uncertainty in the Earth-bound WIMP distribution, coming from incomplete knowledge of the solar-bound and Galactic halo populations. We also outline the numerical methods which are required both to take these effects into account and to improve on the accuracy of our analytic calculations. To carry out these numerical calculations would require substantial amounts of computer time, so we have restricted ourselves to demonstrating that these calculations are practical.

2. Detection of Earth-bound WIMPs

To detect captured WIMPs at the Earth's surface, one must employ a detector which is sensitive to $\mathcal{O}$(10 GeV)-mass particles travelling with speeds $w \ll v_{esc}$. Thus the detector must have sensitivity to nuclei recoiling with energies of $\mathcal{O}$(eV) due to elastic scattering with incident WIMPs. We will assume that the detector registers a signal if a WIMP transfers at least kinetic energy ϵ to a nucleus. In our numerical calculations, we will use a fiducial threshold energy of $\epsilon = 1$ eV. The mass and WIMP scattering cross-section of the nuclei in the detector will be designated M and σ. For WIMPs, it is convenient to write the cross-section as

$$\sigma = 5.2 \times 10^{-40} g \frac{m_W M}{(\text{GeV})^2} Q^2 \text{ cm}^2, \tag{2.1}$$

where

$$g(m_W, M) \equiv \frac{4 m_W M}{(m_W + M)^2}, \tag{2.2}$$

and Q^2 is a dimensionless parameter which depends on the nucleus and WIMP identities.[17]

The signal rate from such a detector is

$$S = V_d \int dw f(R_\oplus, w) \Omega_\epsilon(w), \tag{2.3}$$

where $f(R_\oplus, w)$ is the speed distribution of the WIMPs at the Earth's surface [the number density at $R_\oplus$ with speed in the range $(w, w + dw)$], V_d is the volume of the detector, and $\Omega_\epsilon(w)$ is the rate per unit time that a WIMP traveling through the detector at speed w transfers energy of at least ϵ to a nucleus. The scattering rate can be shown to be[17]

$$\Omega_\epsilon(w) = \frac{N_d\sigma}{wV_d}\left(w^2 - \frac{2\epsilon}{m_W g}\right), \tag{2.4}$$

where $g(m_W, M)$ is given by equation (2.2), and N_d is the total number of nuclei in the detector.

In principle $f(w)$ should be determined by solving the Boltzmann collision equation,[9] and we discuss this in the Appendix. In this section, we adopt simple analytic models both of the structure of the Earth and of the resulting distribution of WIMPs in the Earth. We assume the Earth is composed of two regions, a core and a mantle. Like the Earth's actual core, our model core has a mass, M_c, and radius, R_c, given by

$$M_c = 0.32 M_\oplus \ , \qquad R_c = 0.55 R_\oplus, \tag{2.5}$$

where the Earth's mass and radius are $M_\oplus = 5.98 \times 10^{27}$ gm and $R_\oplus = 6.38 \times 10^3$ km; the composition of the core is taken to be 85% iron and nickel. By equation (2.5), the core has an average density of $\rho_c \simeq 11$ gm/cm^3. Corresponding to this, the potential difference between the edge of the core and the center, ϕ_c, is approximately given by [see equation (1.6)]

$$\phi_c = \phi(R_c) \simeq 0.3\phi_0. \tag{2.6}$$

Unlike the actual Earth core, our model core is of uniform density and is at a uniform temperature, T_c. T_c will be taken to be an "average" of the temperatures of the various regions of the actual core. This averaging process will be discussed below. The mantle will be assumed to have the same density distribution as the actual mantle, but will be assumed to be composed of materials entirely transparent to WIMPs. This renders the temperature of the mantle irrelevant, and implies that *all* bound WIMP orbits pass through the core. As we discuss at the end of this section, this model yields results which are accurate to half an order of magnitude (within the framework that the Earth is in free space). It is therefore useful for a semi-quantitative discussion, but not for an actual measurement of the temperature of the core.

WIMPs interacting with such an isothermal core will assume a distribution *in the core region* which is likewise isothermal (Maxwell-Boltzmann),

$$f_{\rm iso}(r,w,\theta)(4\pi r^2 dr)dw\,d\cos\theta$$
$$\equiv N_W \frac{4\pi r^2 dr}{V_c}\exp\left[-\frac{m_W\phi(r)}{kT_c}\right]\cdot\frac{4}{\pi^{\frac{1}{2}}}\left(\frac{m_W}{2kT_c}\right)^{\frac{3}{2}} w^2 dw\exp\left(-\frac{m_W w^2}{2kT_c}\right)\frac{d\cos\theta}{2}, \tag{2.7}$$

where N_W is the total number of WIMPs in the Earth, $\phi(r)$ is the gravitational potential difference between r and the center [eqn.(1.4)], and V_c is a normalization constant which will be evaluated below. In eqn.(2.7), we have ignored the cut-off of the velocity distribution of Earth-bound WIMPs at the local escape velocity from the Earth. This simplification is justified in practice: in all cases where the detection rate is appreciable, the contribution to the rate from the part of the distribution (2.7) with $v > v_{esc}$ is negligible (see Fig.2 below). One caveat should, however, be emphasized: if the detector threshold is set so high (say, above 10 eV) that *only* WIMPs with $v > v_{esc}$ could be detected, then there will be no enhancement in the rate above that of the halo.

In the mantle and above the Earth's surface, the WIMPs will assume a modified isothermal distribution,

$$f(r,w,\theta) = f_{\rm iso}(r,w,\theta), \qquad r < R_c$$
$$f(r,w,\theta) = f_{\rm iso}(r,w,\theta)\cdot\theta\{R_c^2\left[w^2+2\phi(r)-2\phi_c\right]-r^2w^2\sin^2\theta\}, \qquad r > R_c. \tag{2.8}$$

Here, the Heavyside θ-function is unity if the orbit intersects the core and vanishes otherwise; it arises by angular momentum conservation from our assumption above that all bound WIMP orbits pass through the core. The normalization constant V_c [which enters through eqn.(2.7)], the "effective" volume of the Earth, may be evaluated by the condition

$$N_W \equiv \int_0^\infty dr 4\pi r^2 \int_0^\infty dw \int_0^\pi d\cos\theta\ f(r,w,\theta). \tag{2.9}$$

In all the cases we will be considering, the bulk of the distribution will actually be in the core, so that

$$V_c \simeq \int_0^\infty dr 4\pi r^2 \exp\left(-\frac{1}{2}\cdot\frac{4\pi G m_W \rho_c}{3kT_c}r^2\right) = \left(\frac{3kT_c}{2Gm_W\rho_c}\right)^{\frac{3}{2}}, \tag{2.10}$$

where ρ_c is the average density of the core discussed above. Thus, V_c is approximately the volume contained within a radius equal to the WIMP scale height.

The speed distribution at the surface of the Earth is then found by integrating equation (2.8) over all angles,

$$f(R_\oplus,w) = f_{\rm iso}(R_\oplus,w), \qquad w < w_0$$
$$f(R_\oplus,w) = f_{\rm iso}(R_\oplus,w)\cdot\left\{1-\left[1-\frac{R_c^2(w^2+2\phi_1-2\phi_c)}{R_\oplus^2 w^2}\right]^{\frac{1}{2}}\right\}, \quad w > w_0 \tag{2.11}$$

where ϕ_1 is given by eqn.(1.3), and $f_{\rm iso}$ and w_0 are defined by

$$f_{\rm iso}(R_\oplus, w)dw \equiv \frac{N_W}{V_c}\exp\left(-\frac{m_W\phi_1}{kT_c}\right)\cdot\frac{4}{\pi^{\frac{1}{2}}}\left(\frac{m_W}{2kT_c}\right)^{\frac{3}{2}} w^2 dw \exp\left(-\frac{m_W w^2}{2kT_c}\right), \tag{2.12}$$

$$w_0^2 \equiv \frac{2R_c^2(\phi_1 - \phi_c)}{R_\oplus^2 - R_c^2}. \tag{2.13}$$

In our model, $w_0 = 5.2$ km/sec. For reference, we note that

$$\frac{m_W w_0^2}{2kT_{ev}} \simeq 4. \tag{2.14}$$

It is amusing to note that the distribution (2.11) is qualitatively similar to that for anisotropic star clusters;[18] in that case, a core region has a sufficiently high density of stars to thermalize, with an outer region which depends almost entirely on stars orbiting from the core for its population.

Because $f(R_\oplus, w)$ is so complicated, equation (2.3) for the detection rate cannot be evaluated analytically. However, it may be formally rewritten,

$$S = \eta V_d \int dw\, f_{\rm iso}(R_\oplus, w)\Omega_\epsilon(w), \tag{2.15}$$

where $f_{\rm iso}$ is defined by equation (2.12) and η is a correction factor relating the true detection rate to that for an isothermal distribution,

$$\eta \equiv \frac{\int dw\, f(R_\oplus, w)\Omega_\epsilon(w)}{\int dw\, f_{\rm iso}(R_\oplus, w)\Omega_\epsilon(w)}. \tag{2.16}$$

Equation (2.15) is easily evaluated using equations (2.4) and (2.12),

$$S = \eta\frac{N_W}{V_c}N_d\sigma\left(\frac{8kT_c}{\pi m_W}\right)^{\frac{1}{2}}\exp\left(-\frac{T_1 + \frac{\epsilon}{kg}}{T_c}\right), \tag{2.17}$$

where

$$kT_1 \equiv m_W \phi_1. \tag{2.18}$$

In our example ($M = 26.1$ GeV for ^{28}Si),

$$T_1 = m_{12} \cdot 77600°\text{K}; \qquad \frac{\epsilon}{kg} = \frac{1}{g_{12}} \cdot 13500°\text{K} \left(\frac{\epsilon}{1\text{ eV}}\right), \tag{2.19}$$

where g_{12} is the value of g for Si, normalized to its value at $m_W = 12$ GeV (*i.e.*, $g(12, 26.1) = 0.863$ and $g(m_W, 26.1) = 0.863 g_{12}$).

In general, the correction factor η must be evaluated numerically. However, one may easily obtain the following analytic relations for it:

$$1 > \eta > 1 - \left(1 - \frac{R_c^2}{R_\oplus^2}\right)^{\frac{1}{2}} \simeq .16, \tag{2.20}$$

$$\eta \simeq$$

$$1 - \frac{\pi^{\frac{1}{2}}}{2}\left(1 - \frac{R_c^2}{R_\oplus^2}\right)^{\frac{1}{2}} \left(\frac{2kT_c}{m_W w_0^2}\right)^{\frac{1}{2}} \left(\frac{m_W w_0^2}{2kT_c} - \frac{\epsilon}{gkT_c} + \frac{3}{2}\right) \exp\left[-\left(\frac{m_W w_0^2}{2kT_c} - \frac{\epsilon}{gkT_c}\right)\right]$$

$$\text{if } \frac{m_W w_0^2}{2kT_c} \gg 1, \quad \frac{m_W w_0^2}{2kT_c} > \frac{\epsilon}{gkT_c}. \tag{2.21}$$

From expression (2.14), it is clear that, in most cases of interest, the limit in equation (2.21) is reasonably well satisfied, so that η is typically in the range

$$0.5 \lesssim \eta \lesssim 0.9. \tag{2.22}$$

— The number of WIMPs in the Earth, N_W, is determined by competition between capture and evaporation. [Recall that we are assuming that there is either a cosmic asymmetry of WIMPs or a suppressed annihilation cross-section, so that there is no annihilation in the Earth's core.] In this section, we are treating the Earth as though it were in free space. In this approximation, the capture rate, C, is virtually constant over the lifetime of the Earth, and may be calculated exactly if the mass and halo distribution of the WIMPs are known, and if the composition of the Earth is known. For now, we assume the mass and halo distribution of the WIMPs are known; later, we will discuss the effects of uncertainties in these two quantities. Uncertainties in the Earth's composition give rise to errors of at most $\sim 10\%$ in the capture rate.

Although the total number of WIMPs captured by the Earth is simply $C\tau_{\oplus}$, some of these WIMPs evaporate. In the approximation that the Earth is in free space, the evaporation rate can be calculated using the Boltzmann collision equation, as was done for the Sun by Gould.[9] This calculation may be carried out with arbitrary accuracy. Here we use an analytic approximation[9] which, incidentally, is based on essentially the same core model as we used above. In this approximation, the evaporation time τ_{ev} (inverse evaporation rate), is given by

$$\tau_{ev} = \frac{V_c}{N_c\sigma(\mathrm{Fe})}\left(\frac{\pi m_W}{8kT_c}\right)^{\frac{1}{2}}\frac{T_c}{(T_0+T_1)}\exp[(T_0+T_1)/T_c], \qquad (2.23)$$

where

$$kT_0 \equiv m_W\phi_0, \qquad (2.24)$$

ϕ_0 is defined by eqn. (1.6), $\sigma(\mathrm{Fe})$ is the scattering cross-section of the WIMP with iron, and N_c is the number of iron nuclei in the core. T_0 is found to be

$$T_0 = m_{12} \cdot 97000°\mathrm{K}. \tag{2.25}$$

The evaporation time may be evaluated,

$$\frac{\tau_{\mathrm{ev}}}{\tau_\oplus} \simeq \frac{1}{\sigma_{12}(\mathrm{Fe})} e^{-26} \left(\frac{T_c}{T_0 + T_1} \right)^2 \exp[(T_0 + T_1)/T_c], \tag{2.26}$$

where $\sigma_{12}(\mathrm{Fe})$ is $\sigma(\mathrm{Fe})$ normalized to its value at $m_W = 12$ GeV. The evaporation temperature may be evaluated from the above expression by setting the right hand side equal to unity ($\tau_{\mathrm{ev}} = \tau_\oplus$),

$$T_{ev} \simeq \frac{T_0 + T_1}{33} \simeq m_{12} \cdot 5300°\mathrm{K}. \tag{2.27}$$

(Thus the evaporation mass is $m_{ev} = 12(T_c/5300°\mathrm{K})$ GeV.) In the above equation, we have ignored an additional very slight higher order dependence of T_{ev} on the mass. The important energies in the problem may now be expressed in terms of the evaporation temperature

$$T_0 \simeq 18.3\ T_{ev}; \qquad T_1 \simeq 14.7\ T_{ev}; \qquad \frac{\epsilon}{kg} \simeq \frac{2.5}{m_{12} g_{12}} \left(\frac{\epsilon}{1\ \mathrm{eV}} \right) T_{ev}. \tag{2.28}$$

The number of WIMPs in the Earth obeys the differential equation

$$\frac{dN_W}{dt} = C - \frac{N_W}{\tau_{\mathrm{ev}}}, \tag{2.29}$$

so that after an Earth lifetime,

$$N_W = C\tau_{\mathrm{ev}}[1 - \exp(-\tau_\oplus/\tau_{\mathrm{ev}})]. \tag{2.30}$$

This equation has the limiting forms

$$
\begin{aligned}
N_W =& C\tau_{\text{ev}}, \text{ for } \quad \tau_{\text{ev}} \ll \tau_{\oplus} \quad (T_c > T_{ev}) \\
=& C\tau_{\oplus}, \text{ for } \quad \tau_{\text{ev}} \gg \tau_{\oplus} \quad (T_c < T_{ev}).
\end{aligned}
\tag{2.31}
$$

Because τ_{ev} depends exponentially on T_c [see eq. (2.23)], the transition from one limiting region to the other takes place over a comparatively small range of temperatures.

Below the evaporation temperature ($T_c < T_{ev}$), equation (2.17) may be evaluated using equations (2.10) and (2.31)

$$
S(\epsilon) = \frac{8}{\pi^{1/2}} \eta C \tau_{\oplus} N_d \sigma \left(\frac{m_W}{kT_c} \right) \left(\frac{G\rho_c}{3} \right)^{\frac{3}{2}} \exp\left(-\frac{T_1 + \frac{\epsilon}{kg}}{T_c} \right), \qquad (T_c < T_{ev}).
\tag{2.32}
$$

Above the evaporation temperature, equation (2.17) may be evaluated using equations (2.23) and (2.31),

$$
S(\epsilon) = \eta C \frac{N_d}{N_c} \frac{\sigma}{\sigma(\text{Fe})} \frac{T_c}{(T_0 + T_1)} \exp\left(\frac{T_0 - \frac{\epsilon}{kg}}{T_c} \right), \qquad (T_c > T_{ev}).
\tag{2.33}
$$

For illustration, we will consider Dirac neutrinos with mass $m_W = 12m_{12}$ GeV. In this case, the cross section parameter Q [eqn. (2.1)] is given by

$$
Q \equiv N_N - (1 - 4\sin^2\theta_w) Z_N \simeq N_N - .12 Z_N,
\tag{2.34}
$$

where N_N and Z_N are respectively the number of neutrons and protons in the target nucleus. In our calculations we will take the detector to be made of ^{28}Si.

We now pause to estimate how large a detector could be built. Presently envisioned detectors of *halo* particles consist of a few kg of active material, with thresholds of a few hundred eV. In principle, detectors of smaller mass can be operated at lower thresholds. For example, for bolometric (phonon) detectors operating at temperature T, thermal fluctuations give rise to an uncertainty in the energy of $\delta E = \xi (kT^2 C_p)^{1/2}$, where C_p is the heat capacity, and $\xi \simeq 2$ has been achieved in doped semiconductor thermistors with Si detectors.[19] Since the heat capacity scales as $C_p \propto M_{det}(T/\theta_D)^3$, we have $\delta E \sim \xi T^{5/2} M_{det}^{1/2}$, where M_{det} is the detector mass and θ_D is its Debye temperature. Thus, very low noise energy can be achieved at low operating temperatures, assuming thermistors of sufficient sensitivity can be developed. For example, for a Si detector ($\theta_D = 636°\mathrm{K}$), the thermal noise limit is

$$\delta E_{Si} = 1 \text{ eV} \left(\frac{M_{det}}{10 \text{ gm}}\right)^{1/2} \left(\frac{T}{15 \text{ mK}}\right)^{5/2} . \tag{2.35}$$

Since commercial dilution refrigerators routinely operate at temperatures down to $10 - 15$ mK, one could in principle monitor a single 10 gm block of Si with a threshold of $\mathcal{O}(\text{eV})$. An array with 20 electronics channels, each monitoring a single 10 gm segment, is a plausible experimental design, so that a detector with a total mass of 200 grams could be implemented. In our numerical work below, the fiducial detector will be taken to have a total mass of 100 grams.

For Dirac neutrinos and a 100 gm silicon detector with a 1 eV fiducial threshold, the rates in equations (2.32) and (2.33) become

$$S_{Si} = \frac{32}{100\ \mathrm{gm-day}}\,\eta C_{17}\sigma_{12}\frac{T_{ev}}{T_c}\exp\left[17.2 - \left(14.7 + \frac{2.5(\epsilon/1\ \mathrm{eV})}{m_{12}g_{12}}\right)\frac{T_{ev}}{T_c}\right],$$

$$(T_c < T_{ev}), \tag{2.36}$$

and

$$S_{Si} = \frac{33.}{100\ \mathrm{gm-day}}\,\eta C_{17}\frac{\sigma_{12}}{\sigma_{12}(\mathrm{Fe})}\frac{T_c}{T_{ev}}\exp\left[\left(18.3 - \frac{2.5(\epsilon/1\ \mathrm{eV})}{m_{12}g_{12}}\right)\frac{T_{ev}}{T_c} - 15.8\right],$$

$$(T_c > T_{ev}), \tag{2.37}$$

where C_{17} is the capture rate in units of $10^{17}s^{-1}$ and σ_{12} is the silicon-WIMP cross section normalized to its value at 12 GeV. If the Earth is treated as being in free space, then for Dirac neutrinos of mass 8, 12, and 15 GeV, C_{17} is respectively[20,21] 0.1, 0.4, and 5.6. (The capture rate is discussed in more detail in the following sections, and in refs.20,21.)

These detection rates as a function of central temperature are shown in Figure 1 for Dirac neutrinos of 8,12, and 15 GeV, and a threshold of $\epsilon = 1$ eV. (In the figure, we have set $\eta = 1$, so these rates should be corrected downward by up to a factor of 2.) For the case of 12 GeV neutrinos, we have also plotted twice the expected signal; we use this below to estimate the effect of a factor two uncertainty in the capture rate. From equations (2.36) and (2.37) we note the steep dependence of the detection rate on the threshold energy ϵ. This is reflected in Fig.1, which also shows the detection rate for $m_W = 15$ GeV, assuming a threshold of $\epsilon = 4$ eV.

Figure 1 shows how the central temperature of the Earth can be estimated from the WIMP detection rate at the surface. For example, suppose $m_W = 12$ GeV, and a rate of $S = 1$ per day is found in a detector with a 1 eV threshold. Then, from Fig.1, T_c lies in either of the two bands $4400 - 4600°$K or $6400 - 6800°$K. To eliminate this degeneracy in T_c for a given signal rate, we can use the spectrum of the signal; this yields extra information in addition to that contained in the energy-integrated rate. For the Si detector, the nuclear recoil spectrum is

$$-\frac{dS}{dE} = \frac{80\eta C_{17}}{100 \text{ gm} - \text{eV} - \text{day}} \left(\frac{N_W}{C\tau_\oplus}\right) \left(\frac{\sigma_{12}}{m_{12} g_{12}}\right) \left(\frac{T_{ev}}{T_c}\right) \times \exp\left[17.2 - 14.7\frac{T_{ev}}{T_c}\right] \exp\left[-\frac{2.5}{g_{12}}\left(\frac{E}{\text{eV}}\right)\frac{5300°\text{K}}{T_c}\right]. \tag{2.38}$$

This is shown in Figure 2 for the two values of T_c which correspond to $S = 1$, $T_c = 4600°$K and $6400°$K. Suppose the data is taken in bins of width 1 eV. If the experiment runs for a year, the three or four lowest energy bins will each have at least a few counts, while the lowest two bins will typically have tens to hundreds of counts. We can thus fit the slope and intercept of the log of the spectrum with three or four data points of decreasing weight. In particular, there are ample statistics to decide between fits of the spectrum with the two alternative values of T_c. This method can be used in general as long as the alternative T_c values are well separated, *i.e.*, as long as the signal does not lie very close to the peak of the theoretical curve of Fig.1. An additional advantage of this method is that the log of the spectrum depends principally on T_c and only logarithmically on the capture rate and corrections to the isothermal model.

Incidentally, we can now justify our earlier claim that neglecting the escape velocity cut-off yields a negligible overestimate of the integrated signal. For a 12 GeV WIMP travelling at the Earth's escape velocity, the maximum energy transfer to a Si nucleus is $E_{max} = gm_W v_e^2/2 = 7.2$ eV. With a cut-off, the signal due to *bound* WIMPs should drop smoothly to zero at E_{max}, but Figure 2 shows that even the untruncated distribution gives an unmeasurably low count rate at such 'high' energy.

So far in our discussion, we have assumed that the Earth capture rate and WIMP mass are precisely known. In the next section, we discuss uncertainties in and corrections to the capture rate C_{17}; here we briefly study the effect of uncertainty in m_W. From equations (2.32) - (2.37), we see that, aside from the mass dependence of the capture rate and cross-section, the WIMP mass enters predominantly through the ratio T_{ev}/T_c. Crudely, then, a change in central temperature δT_c can be compensated by a change in WIMP mass, $(\delta T_c/T_c) \simeq (\delta m_{12}/m_{12})$, or $\delta T_c \simeq 450°\mathrm{K}(\delta m_W/\mathrm{GeV})$. (Inclusion of the capture rate and cross-section dependence will reduce slightly the coefficient in the last equality.) Thus, the WIMP mass must be determined to an accuracy of a GeV or better in order to determine T_c to the accuracy discussed in the Introduction. This can be achieved by detecting WIMPs from the halo:[17] the nuclear recoil spectrum from halo WIMPs depends sensitively on m_W, a 1 GeV change in m_W leading to a $10 - 20\%$ change in detection rate at energies $\mathcal{O}(\mathrm{keV})$. (Note that a change in the halo density shifts the energy-integrated rate, but does not affect the spectrum.) For coherent WIMPs, with detection rates of several per day from the halo, the rate can be determined to within a few percent with a year of data. Further information on the mass can be obtained by using several different detector elements. We conclude that the WIMP mass can be determined with sufficient accuracy for our purposes.

Using equations (2.36) and (2.37), one may estimate the approximate mass range over which Dirac neutrinos could be detected (and serve as a probe of the Earth's temperature). We assume the experiment can be carried out if the expected signal is at least 25 events per year in the 100 gm detector. Much below this rate, there would probably be mounting problems with backgrounds (see below), and the experiment would have to be carried out over too long a period to obtain reasonable statistics. We examine two 'extreme' values for the core temperature, $T_c = 4000°K$ and 6500°K. (Recall that, in our model, T_c is an average over part of the core and so is somewhat lower than the central temperature.)

At $T_c = 4000°K$, the counting rate is above 25/(100 gm-yr) for neutrino masses in the range 8 GeV $\lesssim m_W \lesssim$ 15 GeV. At $T_c = 6500°K$, the same rate is reached for 10 GeV $\lesssim m_W \lesssim$ 21 GeV. Thus, the core temperature can either be measured or restricted by non-trivial limits if the Galactic halo comprises coherent WIMPs in the mass range 8 to 21 GeV. As mentioned in the Introduction, halo Dirac or scalar neutrinos in this mass range should be detectable in the near future with improved Ge or Si detectors, or with other proposed cryogenic detectors.

In the mass range discussed above, the detection rate for Earth-bound WIMPs is of order 10^{-1} to $\mathcal{O}(10^2)$ per 100 gm per day, with all of the events clustered in a bandwidth of a few eV. We should compare these rates with expected backgrounds, which come from several sources. First, although WIMPs from the halo typically deposit $\mathcal{O}$(keV) in the detector, they will occasionally deposit only a small fraction of their kinetic energy. Using the results of ref. 18, since halo

WIMPs have velocity with respect to the Earth of at least 42 km/sec (the escape velocity from the sun at the Earth's position), one can show that the halo recoil energy spectrum is approximately flat below $E \simeq 100$ eV. In this energy range, the detection rate of halo WIMPs is $\lesssim 10^{-3}$ eV^{-1} (100 gm)$^{-1}$ day^{-1}, far below the rate for Earth-bound WIMPs. Since the bandwidth for halo WIMPs is much larger than that for their Earth-bound counterparts, however, it is crucial that the detector have reasonable energy resolution in order to distinguish the two. Fortunately, we can expect this of bolometric detectors.[3]

Another source of background is detector radioactivity. Presently operating ultra-pure Ge detectors have achieved background rates of order 0.5/keV/kg/day at recoil energies $E \simeq 10$ keV. The dominant contribution is believed to come from Compton scattering, which has a flat spectrum. If the background rate at energies $\mathcal{O}$(eV) is of the same order, *i.e.*, 5×10^{-5} per eV per 100 gm per day, it is utterly negligible compared to the rate for Earth-bound WIMPs. In fact, we can tolerate a rise of almost three orders of magnitude in background before running into difficulty; in addition, such a 'dirty' detector might be less expensive to implement. On the other hand, it would certainly be naive to trust our extrapolation of present background levels down to such low energies. Therefore, if a WIMP signal were suspected, other methods of background rejection, such as the use of different detector materials, would be called for.

The final source of background comes from solar neutrinos. In a Si detector with a 1 eV threshold, solar neutrinos are expected to have an energy-integrated rate[11] of order 10^{-3} per 100 gm per day. Again, this is well below the rates for WIMPs. Thus, if Earth-bound WIMPs exist in the mass range above (so that their count rates are statistically significant), there is likely to be little competition from background sources, unless radioactive backgrounds grow very steeply at low energy.

How reasonable is the simplified core model used in this section? The approximation that the core has uniform density is a very good one: The density of the actual core assumes values in the comparatively narrow range of 10 to 13 gm cm^{-3}, and the entire calculation depends only weakly on variations in this density. The assumption that WIMPs interact only with the core is likewise very good, for the mantle is, on average, only about 40% as dense as the core. More importantly, the mantle is only $\sim 9\%$ iron by mass, while almost half the mantle is composed of oxygen. Since the iron-WIMP cross-section (per unit mass) is ~ 10 times greater than the oxygen-WIMP cross-section, the mantle has only a tiny fraction of the cross-section per unit volume of the core. Nevertheless, if the WIMPs spent a considerable fraction of their time in the mantle, one would have to take account of their interactions with the material there, even though the scattering rate is low. Since, however, in all the cases we consider, the WIMPs spend most of their time in the core, one is justified in ignoring the effect of the mantle.

The one assumption of our simple model which is bound to introduce significant errors is that the Earth's core (and therefore the WIMP population) has a uniform temperature. A perusal of core models[13–16] shows that the temperature gradient from the Earth's center to the outer core-mantle boundary is typically $\mathcal{O}(2000)$ °K (excluding boundary layer effects). Thus, even though WIMPs spend most of their time in the core, they still sample regions of varying temperature. To quantify this effect, one must study the WIMP distribution in more detail, by numerically solving the Boltzmann collision equation for Earth models with realistic density and temperature profiles. Using the methods developed by Gould,[9] we have done this for several temperature models.[12–16] We find that the correction factor η is typically a factor of 2-4 lower than the value given by expression (2.22), and the evaporation time τ_{ev} is a factor of ~ 2 greater than the value given by equation (2.23). Thus, the isothermal core model gives a good semi-quantitative picture of the dependence of the WIMP detection rate on the Earth's central temperature, but it could not serve as a basis for *measuring* the central temperature. Numerical methods would still have to be used.

Since the Earth's core is not isothermal, the quantity we called "T_c" above must represent some weighted average of the temperature over the core. The numerical Boltzmann solution yields the average kinetic energy of the WIMPs, and this serves to define the averaging process. We emphasize that it is *this* average temperature of the WIMP distribution which is measured by our experiment. How do we extract the true central temperature of the Earth from this quantity? The average WIMP temperature (kinetic energy) is given roughly by

$$T_c \simeq \int_0^{R_c} dr 4\pi r^2 T(r) n_W(r) \Big/ \int_0^{R_c} dr 4\pi r^2 n_W(r) \ , \tag{2.39}$$

where $T(r)$ is the actual temperature of the core as a function of radius, and $n_W(r)$ is the WIMP number density. Near the center, by continuity, $T(r)$ falls quadratically with increasing r, while $n_W(r)$ drops approximately exponentially with r^2. Then equation (2.39) predicts

$$T_c \simeq T(R_0), \tag{2.40}$$

where the WIMP scale height is

$$R_0 = \left(\frac{6kT_c}{\pi^2 G m_W \rho_c}\right)^{\frac{1}{2}} \simeq .26\left(\frac{T_c}{T_{ev}}\right)^{\frac{1}{2}} R_\oplus. \tag{2.41}$$

For light WIMPs and a hot core, *i.e.*, $T_{ev} \ll T_c$, R_0 would lie substantially out in the outer core. In this case, extraction of the central temperature would depend on the model used for the Earth's core temperature profile. However, since such WIMPs are substantially below the evaporation mass, the detection rate would be negligible. On the other hand, for $T_c \lesssim T_{ev}$, R_0 corresponds to a region within the inner core or very near the inner core-outer core boundary. Since the inner core is nearly isothermal, supporting an adiabatic rise of at most 300 °K from boundary to center, the average WIMP temperature would give a reliable estimate of the Earth's central temperature. Thus, in all cases where the detection rate is measurable, WIMPs will provide useful information on the Earth's core.

3. Refined Analysis of the Experiment

A number of refinements must be made to the above calculation before it could be used to analyze an actual experiment. These may broadly be grouped into two categories. First, numerical methods must be used to improve on the analytic approximations made; we discuss these in the Appendix. Second, physical processes which were not taken into account in the above treatment must be included. In this section we give a general discussion of the physical processes which we have so far ignored and give an analytic treatment of the most important one.

The processes we have neglected all arise from the fact that the Earth is not in free space but is moving deep within the gravitational potential well of the Sun. This leads to several changes in both WIMP capture and evaporation. Capture is affected in three distinct ways.

1.) First, direct capture, *i.e.*, capture of WIMPs from *unbound* solar orbits through the halo, is appreciably reduced for WIMP masses below 12 GeV. This reduction arises because the halo WIMP distribution in the neighborhood of the Earth has a paucity of low velocity orbits: unbound WIMPs have velocities greater than the escape velocity from the Sun at the position of the Earth, about 42 km/sec (in the frame of the Sun). The reduction factor has been evaluated analytically with excellent precision in ref. 21. For Dirac neutrinos of mass 8 and 12 GeV, the reduced direct capture rates are approximately $C_{17} = 0.01$ and 0.22.

2.) The second change is that an entirely new process, indirect capture,[21] appears. A WIMP may weakly interact with the Earth and, while not losing enough energy to be captured into the Earth's core, may be deflected into bound solar orbit. In this case, it is said to be "orbit captured". An orbit-captured WIMP may again weakly interact with the Earth and so be *indirectly* captured by it. For our purposes, indirect capture is significant for WIMPs with mass below 12 GeV. Again, for Dirac neutrinos of mass 8 and 12 GeV, the combined direct and indirect capture rates[21] are $C_{17} \simeq 0.02$ and 0.4. These numbers are approximate, because indirect capture gives rise to two types of calculational difficulties. First, the analytic and numerical methods for calculating indirect capture are only partially developed; this problem will be attacked in the Appendix. Second, the indirect capture rate $C_{ind}(t)$ *grows* over the Earth's lifetime, thus changing the evaluation of the differential equation (2.29) for $\dot{N}_W(t)$. This problem can be solved by numerical integration of (2.29).

3.) The third change in the capture rate arises from the presence of *still other* WIMPs in bound solar orbits. These particles may have been present from the time of formation of the solar system, or they may have been captured by the solar system through three-body gravitational interactions.[21,22] This change also poses two distinct difficulties. First, again, the analytic and numerical methods for calculating the bound distribution are poorly developed. Second, three-body gravitational interactions tend to erase traces of the original bound WIMP distribution. This means that one must take special care to show that the Earth did not capture a huge number of WIMPs early in its history from a distribution which is no longer present. We also address these difficulties in the Appendix, where we argue for an "equilibrium" density of solar bound WIMPs.

Including all three corrections, we can give our best estimates for the WIMP capture rate. The effect of the equilibrium distribution (3) is to remove the suppression of the direct capture rate (1) noted above, *i.e.*, the direct capture rates would be just that for the Earth in free space, $C_{17} = 0.1$ and 0.4 for 8 and 12 GeV neutrinos. Including indirect capture (2), although it is somewhat uncertain, yields only a small change in the total rate for 8 GeV neutrinos, while increasing the 12 GeV rate by a factor of two.

WIMP evaporation is affected by the sun's gravitational potential as well. Recall that, in evaporating, a WIMP attains sufficient speed to escape the Earth's gravity and never return. If the Earth were in free space, "escaping the Earth's gravity" would imply "never returning". However, since the Earth is in bound solar orbit, virtually all the WIMPs which escape the Earth's gravity also go into bound solar orbit and so have some finite probability of returning. In this section we give an analytic treatment of this effect by modifying the definition of evaporation. In the Appendix, we indicate the numerical methods necessary for making a more precise analysis of this effect.

For this discussion, we adopt the assumption that once WIMPs "escape" the Earth and go into solar orbit, they retain their initial energy, angular momentum, and angle of inclination until they again suffer a close gravitational encounter with the Earth. This implies that WIMPs which leave the Earth with initial escape velocity u (with respect to the Earth) and then again collide with the Earth, will do so at the same speed u. Further, it implies that the WIMPs of speed u will distribute themselves so that their observed flux at the Earth is isotropic. Under this assumption, WIMPs which leave the Earth at speed u weakly interact with the Earth at an inverse rate,[21]

$$\bar{\tau}_{\text{weak}}(u) = \frac{\tau_\oplus}{2}\left(\frac{\bar{\gamma}(u)}{1+u^2/\overline{v_{esc}^2}}\right)\frac{u^3}{v_\star^3}. \tag{3.1}$$

Here

$$v_\star \equiv \left[\frac{3}{4}\frac{\sigma_E}{\pi R^2}\frac{\tau_\oplus}{\text{yr}}v_\oplus\overline{v_{esc}^2}\right]^{\frac{1}{3}} \simeq 4.7\sigma_{E12}^{\frac{1}{3}}\ \text{km/sec}, \tag{3.2}$$

$R = 1.5 \times 10^{13}$ cm is the mean Earth-sun distance (one astronomical unit), $v_\oplus \simeq 30$ km s^{-1} is the velocity of the Earth about the Sun, and $\overline{v_{esc}^2} \simeq 1.5(2\phi_0) =$ (14 km/sec)^2 is the average square of the escape velocity from the Earth's interior. Also, σ_E is the weak cross section of the Earth, *i.e.*, $\sigma_E = \sum_i N_i\sigma_i$, where N_i is the number of nuclei of type i in the Earth, and σ_i is the WIMP cross-section on a nucleus of type i; σ_{E12} is the same quantity normalized to its value for 12 GeV Dirac neutrinos. $\bar{\gamma}$ is a function of u which is very nearly unity in the range we will consider,[21] so it may be dropped.

The speed $v_\star$, given by equation (3.2), is essentially the speed at which an escaping WIMP will be recaptured in half an Earth lifetime, *i.e.*, as many WIMPs of speed $v_\star$ are recaptured into, as "escape" from the Earth's core. WIMPs escaping with speed $u \gg v_\star$ may be said to truly escape, in the sense that they are unlikely to again enter the Earth's core. On the other hand, WIMPs "escaping" with speed $u \ll v_\star$ may be regarded as part of the Earth's reservoir of WIMPs: they form a constant fraction of all the WIMPs trapped by the Earth and therefore represent an increase in the total number of Earth-bound particles. As an approximation, we will assume that *all* WIMPs with velocity $u > v_\star$ escape, while *all* WIMPs with velocity $u < v_\star$ remain part of the Earth's reservoir. This approximation does introduce some errors, but since $\mathcal{T}_{\text{weak}}(u)$ is such a rapidly rising function of u, and since (as we demonstrate explicitly below) the final results are insensitive to the choice of $v_\star$, these errors are small.

From the analysis above, we may regard the Earth as having an *effective* gravitational potential ϕ_{eff},

$$\phi_0 \to \phi_{\text{eff}} = \phi_0 + \phi_2; \qquad \phi_2 \equiv \frac{v_\star^2}{2}. \tag{3.3}$$

Only WIMPs which escape from this effective potential truly escape. The remaining WIMPs contribute to the Earth's reservoir, and so the effective volume of the Earth must be recomputed, $V_c \to V_{\text{eff}}$. We can then substitute ϕ_{eff} and V_{eff} for ϕ_0 and V_c in the expressions of section 2 and obtain a revised estimate for the detection rate.

By a change of variables in equation (2.7), equation (2.10) for the effective volume may be rewritten

$$V_{\text{eff}} = 4\pi^{\frac{1}{2}} \left(\frac{m_W}{2kT_c}\right)^{\frac{3}{2}} \int dE \exp(-m_W E/kT_c) \int_0^{J_{\max}(E)} dJ^2 \mathcal{T}(E, J), \tag{3.4}$$

where $E(= \phi_0 + \phi_1 + u^2/2)$ and J are the energy and angular momentum per unit mass, and $\mathcal{T}(E, J)$ is the orbit period.[23] If equation (3.4) is integrated over all energies *less than* "escape" energy, we recover the previous evaluation given by equation (2.10). However, we should now also integrate over all energies *greater than* "escape" energy provided that the WIMPs are ultimately recaptured, that is, $u < v_\star$. Thus

$$V_{\rm eff} = V_c + 4\pi^{\frac{1}{2}}\left(\frac{m_W}{2kT_c}\right)^{\frac{3}{2}} \int\limits_{u=0}^{v_\star} dE[J_{\rm max}(E)]^2 \bar{\mathcal{T}}(u) \exp\left(-\frac{m_W u^2/2k + T_0 + T_1}{T_c}\right), \tag{3.5}$$

where $\bar{\mathcal{T}}(u)$ is the average of $\mathcal{T}(E,J)$ over all angular momenta J.

To evaluate equation (3.5), we need to relate the average period $\bar{\mathcal{T}}(u)$ to the interaction period $\bar{\mathcal{T}}_{\rm weak}(u)$ of equation (3.1), and we also require a convenient expression for $J_{max}(E)$. To do this, we again adopt the Earth model of section 2: WIMPs in this energy range ($\phi_0 + \phi_1 < E < \phi_{\rm eff} + \phi_1$) will be assumed to be generated by an isothermal core of radius R_c. The escape velocity from R_c will be designated v_c,

$$\frac{v_c^2}{2} = \phi_0 + \phi_1 - \phi_c \simeq 1.5\phi_0. \tag{3.6}$$

Since, to a good approximation,

$$v_c^2 \simeq \overline{v_{esc}^2}, \tag{3.7}$$

we will take this to be an identity. Using this model, we find

$$[J_{\rm max}(E)]^2 = \left(u^2 + v_c^2\right)R_c^2 \simeq \left(u^2 + \overline{v_{esc}^2}\right)R_c^2, \tag{3.8}$$

and

$$\bar{\mathcal{T}}(u) = \left(\frac{\sigma_E}{\pi R_c^2}\right)\left(\frac{N_c\sigma({\rm Fe})}{\sigma_E}\right)\bar{\mathcal{T}}_{\rm weak}(u). \tag{3.9}$$

In equation (3.9), the first factor occurs because the orbit time is determined by the time it takes to again intersect the core, not the time it takes for the WIMP to *weakly* interact with the Earth. The second factor arises from the fact that, in our model, only the core can *generate* escaping WIMPs, while the whole Earth can recapture them. (Recall that N_c is the number of Fe nuclei in the core.)

Using equations (3.8) and (3.9), equation (3.5) can thus be rewritten

$$V_{\text{eff}} = V_c + \pi^{\frac{1}{2}} \left(\frac{2kT_c}{m_W} \right) \frac{N_c \sigma(\text{Fe}) \overline{v_{esc}^2}}{v_\star^3} \tau_\oplus \exp\left(-\frac{T_0 + T_1}{T_c} \right) \int_0^{\tilde{x}^2} dx^2 \; x^3 \exp(-x^2), \tag{3.10}$$

where

$$\tilde{x}^2 \equiv \frac{T_2}{T_c} \simeq 3.2 \sigma_{E12}^{\frac{2}{3}} \frac{T_{ev}}{T_c}, \qquad kT_2 \equiv \frac{m_W v_\star^2}{2}. \tag{3.11}$$

Clearly, equation (3.10) would be easier to evaluate if the upper limit of the integral could be taken to be infinity. Physically, this would correspond to a situation where essentially all the WIMPs "escaping" from the Earth were eventually recaptured, so that the entire population in solar orbit could be considered part of the Earth's reservoir. In this limit the density of WIMPs at the Earth's surface would still decline at high temperatures, but this would be due to the fact that the bulk of the WIMPs in the Earth's reservoir were "temporarily" away from the Earth in solar orbit rather than because they had escaped permanently. In some sense 3.2 is "almost" infinity. That is, the physical situation described above essentially prevails. In order to make this statement precise, we will carry forward the analysis as though the upper limit were infinity, but we will also carry along a correction factor in parentheses which is based on the upper limit being 3.2. Thus

$$\int_{x^2=0}^{\tilde{x}^2} dx^2\, x^3 \exp(-x^2) = \frac{3\pi^{\frac{1}{2}}}{4} - \tilde{x}^3 \exp(-\tilde{x}^2)\left(1+\frac{3}{2\tilde{x}^2}+\cdots\right) \simeq (.731)\frac{3\pi^{\frac{1}{2}}}{4}. \quad (3.12)$$

With this convention, equation (3.10) may be evaluated,

$$V_{\text{eff}} = V_c + (.731)\frac{3}{4}\left(\frac{2kT_c}{m_W}\right)\frac{N_c\sigma(\text{Fe})\overline{v_{esc}^2}}{v_\star^3}\tau_\oplus \exp\left(-\frac{T_0+T_1}{T_c}\right). \quad (3.13)$$

From equation (2.23) and the definition of the evaporation temperature T_{ev}, one may write $\tau_\oplus$ as

$$\tau_\oplus = \frac{\pi^{\frac{1}{2}}}{2}\left(\frac{T_{ev}}{T_c}\right)^{\frac{3}{2}}\frac{V_c}{N_c\sigma(\text{Fe})}\left(\frac{m_W}{2kT_{ev}}\right)^{\frac{1}{2}}\frac{T_{ev}}{(T_0+T_1)}\exp[(T_0+T_1)/T_{ev}]. \quad (3.14)$$

Thus,

$$\begin{aligned} V_{\text{eff}} =& V_c\left\{1+(.731)\frac{3\pi^{\frac{1}{2}}}{8}\frac{\overline{v_{esc}^2}/2}{(\phi_0+\phi_1)}\left(\frac{2kT_{ev}}{m_W v_\star^2}\right)^{\frac{3}{2}}\left(\frac{T_{ev}}{T_c}\right)^{\frac{1}{2}}\exp\left[(T_0+T_1)\left(\frac{1}{T_{ev}}-\frac{1}{T_c}\right)\right]\right\} \\ =& V_c\left[1+(.731)\frac{1}{10\sigma_{12}(\text{Fe})}\left(\frac{T_{ev}}{T_c}\right)^{\frac{1}{2}}\exp\left(33-\frac{T_0+T_1}{T_c}\right)\right]. \end{aligned} \quad (3.15)$$

Using this version of the effective volume, we now proceed to estimate the detection rate in the limits where the core temperature is below and above the evaporation temperature. To do this we first find the revised evaporation time. This may be calculated by substituting

$$T_0+T_1 \to T_0+T_1+T_2; \qquad V_c \to V_{\text{eff}}, \quad (3.16)$$

in equation (2.23). If the core temperature is below the evaporation temperature, $T_c < T_{ev}$, then V_{eff} will be essentially equal to V_c and so the evaporation time will be roughly $\exp(\tilde{x}^2) \sim 25$ times larger than before. Thus, from equation (2.30), the total number of WIMPs in the Earth will also retain its old value, $N_W = C\tau_\oplus$. Consequently, equation (2.36) is still the appropriate limiting form, *i.e.*, in this case the results of the section 2 are unmodified. This was to be expected: if the WIMPs did not evaporate before, a more accurate treatment of what happens when they do evaporate should not alter the result.

Next consider the limit where the core temperature is above the evaporation temperature. Recall that when the Earth was in free space, "above" meant a few percent above T_{ev}. Now, from equation (3.15), it is clear that "above" means a few per cent above $\tilde{T}_{ev}$, which is defined by

$$\tilde{T}_{ev} \equiv T_{ev}\left\{1 + \frac{\ln\left[\frac{1}{(.731)}10\sigma_{12}(\mathrm{Fe})\right]}{33}\right\} \simeq 1.08\ T_{ev}. \tag{3.17}$$

From equation (2.23) [modified by equation (3.16)] and equation (3.13), one finds

$$\frac{\tau_{\mathrm{ev}}}{\tau_\oplus} \to (.731)\frac{3\pi^{\frac{1}{2}}}{8}\frac{\overline{v_{esc}^2}/2}{(\phi_0 + \phi_1 + \phi_2)}\left(\frac{T_c}{T_2}\right)^{\frac{3}{2}} \exp(T_2/T_c) \simeq 1.58. \tag{3.18}$$

Thus, from equation (2.30), the number of WIMPs in the *whole* reservoir, including those in solar orbit, is

$$N_W \simeq (.741)C\tau_\oplus. \tag{3.19}$$

[The parenthesis surrounding the factor .741 in the above equation has the same sense as that surrounding the factor .731 in previous equations: if $v_\star$ was large enough to set the upper limit in equation (3.12) to infinity, then the quantity in parenthesis would be unity.] Finally, using equation (2.17) [with the modification

(3.16)] and equations (3.19) and (3.13), we find the corrected detection rate [to be compared with eqn. (2.33)]

$$S(\epsilon) = \eta C \frac{N_d}{N_c} \frac{\sigma}{\sigma(\mathrm{Fe})} \frac{T_c}{(T_0 + T_1)} \exp\left(\frac{T_0 - \frac{\epsilon}{kg}}{T_c}\right) \times \frac{(.741)}{(.731)} \frac{8}{3\pi^{\frac{1}{2}}} \frac{(\phi_0 + \phi_1)}{v_{esc}^2/2} \left(\frac{T_2}{T_c}\right)^{\frac{3}{2}},$$

$$(T_c > \tilde{T}_{ev}). \tag{3.20}$$

Equation (3.20) clarifies our contention that the cutoff in equation (3.12) could "almost" be ignored: inclusion of this cutoff results in a correction $\simeq 0.741/0.731 = 1.01$. One may show that this correction factor is a slowly changing function of $\tilde{x}^2$ in the neighborhood of $\tilde{x}^2 = 3.2$.

For the 100 gm Si detector for Dirac neutrinos, equation (3.20) gives

$$S_{Si} = \frac{350}{100\ \mathrm{gm} - \mathrm{day}} \eta C_{17} \frac{\sigma_{12}}{\sigma_{12}(\mathrm{Fe})} \left(\frac{T_{ev}}{T_c}\right)^{\frac{1}{2}} \exp\left[\left(18.3 - \frac{2.5(\epsilon/1\ \mathrm{eV})}{m_{12} g_{12}}\right) \frac{T_{ev}}{T_c} - 15.8\right]. \tag{3.21}$$

We note that, in equation (3.21), one should use the corrected values for the capture rate C_{17} discussed at the beginning of this section. Figure 3 shows the detection rate of 12 GeV Dirac neutrinos for a 100 gram ^{28}Si detector, comparing the corrected rate (3.21) [dotted curve] to that for the Earth in free space, eqns. (2.36), (2.37) [solid curve]. For core temperatures above the evaporation temperature, the corrected rate is roughly an order of magnitude larger, *i.e.*, the dotted curve is displaced $\sim T_{ev}(\ln 10)/15.8 \sim 0.15 T_{ev}$ to the right. Thus, including the recapture of escaped WIMPs raises the evaporation temperature by approximately 15%. As a result, the range of WIMP masses over which the experiment would be viable is slightly extended. In particular, at a core temperature of 4200 °K, WIMPs as light as 7 (as opposed to 8) GeV could be detected.

4. Conclusion

In this paper, we have studied in detail the distribution of weakly interacting dark matter candidates captured into the Earth. Although we have used the example of Dirac neutrinos throughout, we emphasize that our analysis applies to *any* WIMP with appreciable spin-independent nuclear interactions and negligible annihilation rate. The detection rate for such particles, shown in Figs. 1-3, is potentially large over a significant range of WIMP masses [e.g., $7 - 20$ GeV for Dirac neutrinos], provided detectors with sufficiently small thresholds, $\mathcal{O}$(eV), can be implemented. In addition, since the Earth-bound WIMP signal is essentially confined to a narrow $\mathcal{O}$(eV's) peak, the background rejection necessary for its detection is less severe than that for halo WIMPs [where the signal is spread over $\mathcal{O}$(keV's)].

In addition, we have shown that trapped particles can be used to extract useful information about the central temperature of the Earth. To realize such a project in practice, the analytic estimates given here would have to be augmented with the numerical analysis outlined in the Appendix; such an analysis would be warranted if halo WIMPs are detected. In the absence of ultra-high pressure data on the melting point of iron, coupled with the unknown identity of the lighter alloying component in the core, WIMPs may provide the only direct geophysical probe of conditions near the center of the Earth.

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APPENDIX

Numerical Methods

In the body of the paper, we gave an approximate analysis of our proposed experiment. This approximation served to give a good qualitative understanding of the dependence of the detection rate on the Earth's central temperature. However, to actually use such an experiment to determine the central temperature, one must have a better quantitative understanding of the distribution of Earth-bound WIMPs. In this Appendix we present the numerical methods necessary for such a determination.

We may divide the WIMPs whose orbits intersect the Earth into three categories:

i. WIMPs in bound Earth orbit.

ii. WIMPs coming directly from the Galactic halo (in unbound solar orbit).

iii. WIMPs in bound solar orbit.

In principle, all three classes contribute to the detection rate. As discussed in Section 2, in practice it should be straightforward to distinguish the first from the latter two categories because WIMPs in classes *ii*. and *iii*. almost always deposit far more than $\mathcal{O}(\mathrm{eV})$ in the detector. However, as we discuss below, knowledge of the distributions *ii.* and *iii.* is required to accurately determine the population of WIMPs bound to the Earth.

The population of WIMPs trapped in the Earth evolves asymptotically toward a steady state distribution $f(r,w)$ which depends (up to an overall scale factor) only on the Earth's temperature structure and composition. The characteristic time scale for this relaxation is extremely short compared to the other time scales in the problem ($\tau_{\rm ev}$ and $\tau_\oplus$). Thus the distribution may be assumed to instantaneously adjust to its asymptotic form whenever a WIMP is captured by or evaporates from the Earth. The numerical methods for determining this distribution with arbitrary accuracy are given in ref. 9.

The earth-bound WIMP population reaches an equilibrium between capture and evaporation. From the asymptotic earth-bound distribution above, we can calculate the evaporation rate per unit escape velocity u,

$$\frac{d^2N}{dt\,du} = \int_0^{R_\oplus} dr\ 4\pi r^2 f(r,w)\frac{u}{v}R[w \to v; T(r)]; \qquad u^2 = v^2 - [v_{esc}(r)]^2. \qquad \text{(A.1)}$$

Here, $R(w \to v)$ is the the rate at which a WIMP of speed w scatters to a speed v when it is moving through a Maxwell-Boltzmann distribution of temperature $T(r)$. [Of course, $R(w \to v)$ depends on the number density and composition of the medium as well as its temperature.] The formula for $R(w \to v)$ has been given explicitly by Gould,[9] along with its integral over all speeds greater than escape velocity, $v_{esc}(r)$. Thus, once the distribution $f(r,w)$ is known, the evaporation rate can be inferred.

On the other hand, the capture rate into the Earth is a known function of the WIMP distributions in the halo and in solar orbit.[20,21] In calculating the capture rate in Sections 2 and 3, the halo WIMPs were assumed to have a Maxwell-Boltzmann (isothermal) distribution with a velocity dispersion $\bar{v} = 300$ km/sec and a local density $\rho_{halo} = 0.4$ GeV cm^{-3}. All of these assumptions are, in fact, suspect: the actual velocity distribution is expected to be both anisotropic and non-Gaussian, $\bar{v}$ is uncertain by 10%, and ρ_{halo} is uncertain to a factor of 2 or so. At present, more realistic halo models are being constructed,[24] and these should give a more accurate picture of the velocity distribution. The dominant uncertainty, however, arises from ρ_{halo}, since the capture rate is directly proportional to it: as Figure 1 shows, a factor 2 uncertainty in ρ_{halo} (and thus in capture rate) leads to an uncertainty in T_c of a few hundred °K. This source of error should be largely eliminated when halo WIMPs are detected (recall we are assuming this occurs before the detection of Earth bound WIMPs). As a consequence, we shall assume hereafter that the halo distribution is known to good accuracy. Moreover, it may be assumed to be time independent over the lifetime of the Earth: the Galactic distribution evolves slowly, and the Earth only samples this distribution along the Sun's nearly circular orbit about the Galactic center (the halo is presumably symmetric about the Galactic polar axis).

Thus, the principal unknown is the distribution, both present and historic, of WIMPs in bound solar orbit. In the rest of this Appendix, we discuss the calculation of this quantity. In general this will require numerical analysis. However, we will make substantial use of analytic approximations (both our own and those of previous authors) to estimate how much accuracy is required for a particular subcalculation. When not much accuracy is needed, we will use analytic approximations and arguments in place of numerical calculations.

The bound solar orbit population has four sources, three forms of internal evolution, and two sinks. We consider these in turn. The sources are

1. Evaporation from the Earth.

2. Orbit capture from the Galactic halo.

3. Three-body capture from the Galactic halo.

4. WIMPs which were originally present when the Earth was formed.

The first of these is known and is given by equation (A.1). The second is also known analytically and is given by Gould.[21] In general, three-body capture will be dominated by the larger planets, particularly Jupiter. But, *considering only those solar bound WIMPs which eventually pass through the Earth*, one may show that only three-body capture by the Earth is significant. Since the dynamics of this source are intimately related to the internal evolution of the WIMP distribution, we defer discussion of it. The fourth source, the initial solar bound distribution, is a significant uncertainty, since estimates for it have ranged widely. We also postpone discussion of this source until we have discussed internal evolution of the distribution.

The solar bound distribution evolves internally, *i.e.*, without change of number, under the influence of three forces:

1. Scattering via weak interaction with nuclei in the Earth.

2. Close gravitational interaction with the Earth.

3. Long range gravitational interaction with the Earth, Jupiter, and other

planets.

The effect of weak interaction with the Earth has been worked out analytically by Gould.[9,21] There is no hard and fast line separating the latter two forces. In principle, it is possible to take account of them simultaneously by simulating the evolution of a statistical sample of WIMPs (whose initial distribution is given by the sources listed above) on a computer. However, to actually carry out this simulation would require following the orbits of thousands of WIMPs, each over billions of revolutions. (To date, similar undertakings in asteroid physics have succeeded only in following a single particle over tens of millions of revolutions.) We therefore follow the treatment of ref.21, which gives an analytic approximation for the evolution. This method is not sufficiently accurate for present purposes, but it serves as a starting point for a more practical numerical analysis.

We first assume that the long range interactions leave several parameters of a WIMP's orbit fixed, including its angle of inclination, energy, and angular momentum. These interactions only cause the orbit to precess about its axis or the axis of the plane of the orbit to precess about the axis of the ecliptic. With this assumption (and barring for the moment short range interactions), whenever the WIMP approaches the Earth, it always has the same velocity $(u, \theta, \pm\phi)$. Here u is the speed relative to the Earth, θ is the polar angle relative to the Earth's direction of motion, and ϕ is the azimuthal angle. The upper (lower) sign refers to the WIMP's angle of approach in an outbound (inbound) encounter. In a close gravitational encounter the WIMP's relative speed u is unchanged, by conservation of energy, while the angle $\Omega = (\theta, \phi)$ is reoriented.

The solar bound WIMPs may be divided into two velocity regions: an "upper range", with $u > (2^{\frac{1}{2}} - 1)v_\oplus$, and a "lower range", where $u < (2^{\frac{1}{2}} - 1)v_\oplus$; here $v_\oplus = 30$ km/sec is the orbital velocity of the Earth around the Sun. In the lower velocity range, all WIMPs are trapped, *i.e.*, all directions Ω represent bound solar orbits. In the upper range, the bound orbits occupy a smaller and smaller fraction of the angular (Ω) phase space. In the lower range, every close encounter (or every several close encounters) can be regarded as randomly reorienting Ω, due to large angle scattering. As u increases into the upper range, the change in Ω due to close interactions must be treated as slow drift (diffusion) rather than random reorientation. Note that while there may be more or less rapid drift in Ω, u remains absolutely fixed. Then, for fixed u, diffusion in Ω is, to a very good approximation, governed by

$$2\mathcal{T}(u,\theta,\phi)\frac{\partial f(u,\theta,\phi)}{\partial t} = \nabla^2 f(u,\theta,\phi) \qquad \text{(A.2)}$$

where ∇^2 is the two dimensional divergence on the unit sphere, and

$$\mathcal{T}(u,\theta,\phi) \simeq \frac{1}{60}\gamma(u,\theta,\phi)\left(\frac{M_\odot}{M_\oplus}\right)^2\left(\frac{u}{v_\oplus}\right)^5 \text{yr} \simeq 0.4\gamma(u,\theta,\phi)\left(\frac{u}{v_\oplus}\right)^5 \tau_\oplus. \qquad \text{(A.3)}$$

Here $\gamma(u,\theta,\phi)$ is a function which is generally of order unity, but which diverges as Ω approaches the boundary of phase space when u is in the upper range. Gould[21] describes the derivation of γ and gives an analytic estimate of it which is almost certainly accurate enough for present purposes. In any event, it is not difficult to get a more accurate estimate of γ numerically. (The expression for γ is complicated and unenlightening, so we do not display it here.)

– The assumption that long range interactions do not influence certain orbit parameters is only useful as a zeroth order approximation. In general, the angle of inclination, energy, and angular momentum of the orbit will change. This means u will not remain fixed. Our numerical model seeks to retain the conceptual framework of the above analysis, while taking into account the drift in u.

We first adopt the coordinate system above. Every WIMP orbit is described by coordinates (u, θ, ϕ), defined by rotating the orbit ellipse (bound WIMP trajectory) about its major axis until it intersects the Earth's orbit (idealized as a circle one astronomical unit in radius). The coordinates (u, θ, ϕ) are just the components of velocity in an outbound encounter in this rotated orbit. This coordinate system is open to the criticism that it is "pre-Copernican": not all solar orbits can be described with it. In particular, if the perihelion of the orbit is above or the aphelion below an astronomical unit, then no matter how the ellipse is rotated it will never intersect the Earth's orbit. For the moment we ignore this problem.

Next, we seek to describe the diffusion of WIMPs in (u, θ, ϕ) space due to gravitational interactions. First, we "factor" this diffusion process into its u and Ω components. While we will no longer insist that diffusion in the u direction is impossible, we will make the approximation that diffusion in the Ω directions is still given by equation (A.2). To the level of accuracy required here, this assumption is extremely reasonable and, in any event, can be checked for consistency in the course of the numerical calculation below.

We now describe the calculation of the diffusion in the lower range of velocities, $u < (2^{\frac{1}{2}} - 1)v_{\oplus}$. From equation (A.3), it is clear that the diffusion time scale is short compared to $\tau_{\oplus}$, and in the bottom part of this range it is extremely short. Thus, we may assume that WIMPs which have newly diffused to speed u instantaneously redistribute themselves so that their flux at the Earth is isotropic, *i.e.*, they populate the orbit (u, θ, ϕ) with density proportional to $\mathcal{T}(u, \theta, \phi)$. Consequently, we need only calculate the diffusion rate in the (one-dimensional) u-space. This problem is tractable, though by no means trivial, using numerical methods. One would simply track, over say 100,000 revolutions, a statistical sample of $\sim$ 100 WIMPs of speed u drawn randomly from the isotropic distribution described above, and measure their speed distribution as a function of time. Since the diffusion rate is presumably a smooth function of u, this would only have to done for $\sim$ 20 different values of u over the lower range.

How accurately must the diffusion rate in the lower range be calculated? If the core temperature is above the evaporation temperature, then according to equation (3.20), recapture from the lower part of the lower velocity range enhances the detection rate by a factor of $\sim$ 10. It would thus be extremely important to get an accurate estimate of the diffusion rate in this regime. On the other hand, from the analytic estimates of ref.21, the indirect capture rate from orbit-captured WIMPs in the lower range is at most comparable to the direct capture rate from the galactic halo. Thus, if the core temperature is *below* the evaporation temperature, one may be a little more lax about calculating the diffusion rate in this regime.

In the upper range, $u > (2^{\frac{1}{2}} - 1)v_{\oplus}$, unlike the lower range, the problem of diffusion is integrally bound up with the problem of 3-body capture into, and 3-body expulsion from, the solar system. This is because all WIMPs which are 3-body captured have a speed relative to the Earth of at least $(2^{\frac{1}{2}} - 1)v_{\oplus}$, and only WIMPs with at least this relative speed are eligible for expulsion. (Of course, it is also possible for a WIMP to be 3-body captured into the upper range and then diffuse into the lower range.) Thus, before proceeding to the analysis of diffusion in this range, we show that there is a unique solar-bound WIMP distribution for which 3-body capture and 3-body expulsion are in equilibrium.

To see this, first note that by equation (3.18), the effective evaporation time is *at least* of order $\tau_\oplus$, which in turn is very large compared to the $\sim$ 200 million year orbit time of the Sun about the Galactic center. It is therefore appropriate to time-average the Galactic WIMP distribution in the frame of the Sun over the Galactic orbit time. An accidental symmetry renders this time-averaged distribution almost completely isotropic.[21] Moreover, since the WIMPs eligible for orbit capture have speeds $\sim v_\oplus$, while the characteristic speed of the galactic WIMP distribution is $\sim$ 250 km s^{-1}, this time-averaged distribution in the relevant speed range has the form (in the frame of the Sun)

$$f_g(t)dt = \kappa t^2 dt \ , \qquad t \ll v_\odot, \tag{A.4}$$

where κ is a constant and t is the speed of the WIMP far from the Sun. Since the escape velocity from the Sun at the position of the Earth is $2^{1/2}v_\oplus$, in the neighborhood of the Earth (but the frame of the Sun) the unbound WIMPs have a distribution[20,21]

$$f_s^{(\mathrm{unbound})}(s)ds = \kappa s^2 ds \ \theta(s^2 - 2v_\oplus^2), \qquad s \ll v_\odot, \tag{A.5}$$

where

$$s^2 = t^2 + 2v_\oplus^2. \tag{A.6}$$

We now claim that the equilibrium distribution of *solar-bound* WIMPs is isotropic and is given by

$$f_s^{(\mathrm{bound})}(s)ds = \kappa s^2 ds, \qquad s^2 < 2v_\oplus^2. \tag{A.7}$$

The proof of this claim is straightforward. If the bound distribution is given by equation (A.7), then in the frame of the Earth, the total distribution (bound plus unbound) will (by a Galilean transformation) be isotropic and given by

$$f_e(u)du = \kappa u^2 du. \tag{A.8}$$

Because the Earth is spherically symmetric, if the incoming distribution is isotropic, the gravitationally scattered outgoing distribution will likewise be isotropic. Thus the Earth will 3-body capture exactly as many WIMPs as it 3-body expels. The time-scale of the approach to this equilibrium distribution is given by equation (A.3), so equilibrium is reached rather rapidly compared to the lifetime of the Earth for speeds u in the lower part of the upper range. On the other hand, in the upper part of this range (say, $u > v_\oplus$) equilibrium will not be reached even over the lifetime of the Earth.

The equilibrium distribution described above is a useful tool for understanding how accurately the diffusion in the upper range must be calculated. If the equilibrium distribution prevailed over the entire lifetime of the Earth (and if a similar distribution were present in the much smaller lower range), then the *total* capture rate would be virtually the same as if the Earth were in free space. If we assume for the moment that the initial distribution was somewhere between zero and twice the equilibrium distribution, then on very general grounds we may conclude that over most of the lifetime of the Earth and over most of the relevant phase space, the actual distribution was very nearly the equilibrium distribution. Furthermore, by comparing the calculation for direct capture when the Earth is and is not treated as being in free space,[20,21] we can see that it is only for WIMPs below 9 GeV that there is even a factor of 2 difference. From this we may conclude that even a crude calculation of diffusion will be sufficient provided that the WIMPs are heavier than 9 GeV and the initial distribution is less than twice the equilibrium distribution. We will describe such a crude calculation. More sophisticated (but certainly attainable) methods will be required if either of these two criteria is not met.

First consider the lowest part of the upper range (say $u < .6v_{\oplus}$). This regime is quite important because diffusion from the lowest part of the upper range is the principle source of WIMPs in the lower range, which in turn is an important source for capture. In this regime, the time scale [eq (A.3)] is still short enough that the WIMPs of speed u may be assumed to instantaneously redistribute themselves so that their flux at the Earth is "isotropic" (just as was the case for the lower range). We have put "isotropic" in quotes because in the upper range only a part of the 4π incident directions are bound orbits so that the flux is isotropic over only these allowed directions. In addition, since the time scale diverges near the boundary of the bound and unbound orbits, the flux will not have time to achieve the isotropic value in this boundary region. However, the boundary region is precisely the region which reaches the equilibrium distribution first, so that the flux in this region may be considered known. From this description it is clear that the diffusion rates for speeds u in the lowest part of the upper range may be obtained by methods very similar to those for speeds in the lower range.

In the upper parts of the upper range it is not so important to get an accurate fix on the diffusion rate because the capture rate for various speeds, u, does not vary very much. Thus, even though it is not necessarily valid to assume that the WIMPs with these speeds will distribute themselves so that their flux at the Earth is isotropic, this distribution may be used to estimate the diffusion rate. On the other hand, the diffusion in the Ω directions may be obtained by direct numerical integration of equation (A.2), using the initial distribution of WIMPs and the low-speed galactic distribution of WIMPs as the initial and boundary conditions. This completes our discussion of the forms of internal evolution of the bound solar orbit WIMP distribution.

The two sinks of this distribution are

1. WIMPs weakly captured by Earth.
2. WIMPs which are 3-body expelled.

The first sink is understood analytically[20,21] and the second has been covered in the context of forms of internal evolution above.

There are several loose ends yet to be tied up. First, when we discussed evaporation from the Earth as a source of the bound solar distribution, we implicitly assumed that this would be determined by the *current* temperature distribution inside the Earth. One might object that evaporation was far more rapid early in the Earth's lifetime when its core was hotter. If the temperature distribution were known as a function of time over the entire history of the Earth this would pose no problem. However, since it is the aim of the experiment to measure the *current* temperature, this history can hardly be assumed known. Thus the measurement would appear polluted in an unknown way by this history. Actually, this objection is not well founded. There is good reason to believe that the central temperature has not changed significantly over most of the lifetime of the Earth.[15] Also, it is not the absolute magnitude of the Earth's temperature but only its rate of decline which must be known in order to predict the effect on evaporation. Finally, even substantial changes in the Earth's temperature over its lifetime would, in fact, have relatively little effect on our proposed experiment. This can be seen from equation (3.18): the effective evaporation time is actually longer than an Earth lifetime even if the core temperature is well above the evaporation temperature. The great majority of WIMPs which "escape" from the Earth early in its history are recaptured and re-escape several times. Thus, the present day ratio of WIMPs in the two parts of the "effective volume" of the Earth (that inside the Earth and that in solar orbit) reflect the Earth's temperature structure today and in the recent past, not conditions of the early solar system.

Another loose end concerns the initial distribution of WIMPs. As mentioned above, if this initial distribution is very roughly the equilibrium distribution or less, then there is no problem. However, if this distribution is many times the equilibrium distribution, as some authors have claimed,[25] then there would have been an exceptionally large capture rate early in the Earth's history before the distribution was driven toward equilibrium. If we then assumed that the present distribution of bound WIMPs were always present, we would vastly underestimate the capture rate. Recently, Griest[22] has shown that the initial distribution is almost certainly very close to the equilibrium value and, even under the most extreme conditions, cannot exceed it by more than a factor of three. In addition, one can in principle measure the distribution of bound WIMPs in the upper part of the "upper range", $u \gg (2^{1/2}-1)v_{\oplus}$: according to equation (A.3), if $u \gg v_{\oplus}$, then $\mathcal{T} \gg \tau_{\oplus}$, so this part of the distribution should reasonably reflect the initial conditions. Such a measurement would involve determining both the energy and direction of these WIMPs. It remains to be seen whether a device with such capability can be built.

Finally, we address the problems posed by our "pre-Copernican" coordinate system introduced above. WIMPs which are near the "boundary" of this coordinate system, *i.e.*, which have aphelions and perihelions near one astronomical unit, may leak out of the system. Conversely, other WIMPs whose orbits have aphelions just below or perihelions just above 1 A.U. may leak into it. These effects are almost certainly small because the boundary region is small compared to the total phase space of orbit-captured WIMPs. The effect may be calculated by integrating WIMP orbits in a standard coordinate system to see how may drift back and forth across the boundary.

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FIGURE CAPTIONS

1) Dirac neutrino signal rate per day for 100 gm silicon detector with 1 eV threshold, plotted as a function of Earth's central temperature. Shown are WIMP masses of 8 GeV (solid line), 12 GeV (dotted curve), and 15 GeV (dashed curve). To illustrate the effect of theoretical uncertainties, the upper dotted curve displays twice the value of the lower 12 GeV curve. Also shown is the rate for a 15 GeV neutrino with a 4 eV detector threshold (dot-dash curve). In all figures, we set $\eta = 1$.

2) Recoil energy spectrum (rate per eV per day) for 100 gm Si detector of 12 GeV Dirac neutrinos for two values of the central temperature, $T_c = 4600°$K and 6400°K. The vertical dashed line indicates a 1 eV detector threshold.

3) Signal rate for 12 GeV Dirac neutrinos for Earth in free space (solid curve) and corrected for solar gravitational potential (dotted curve).

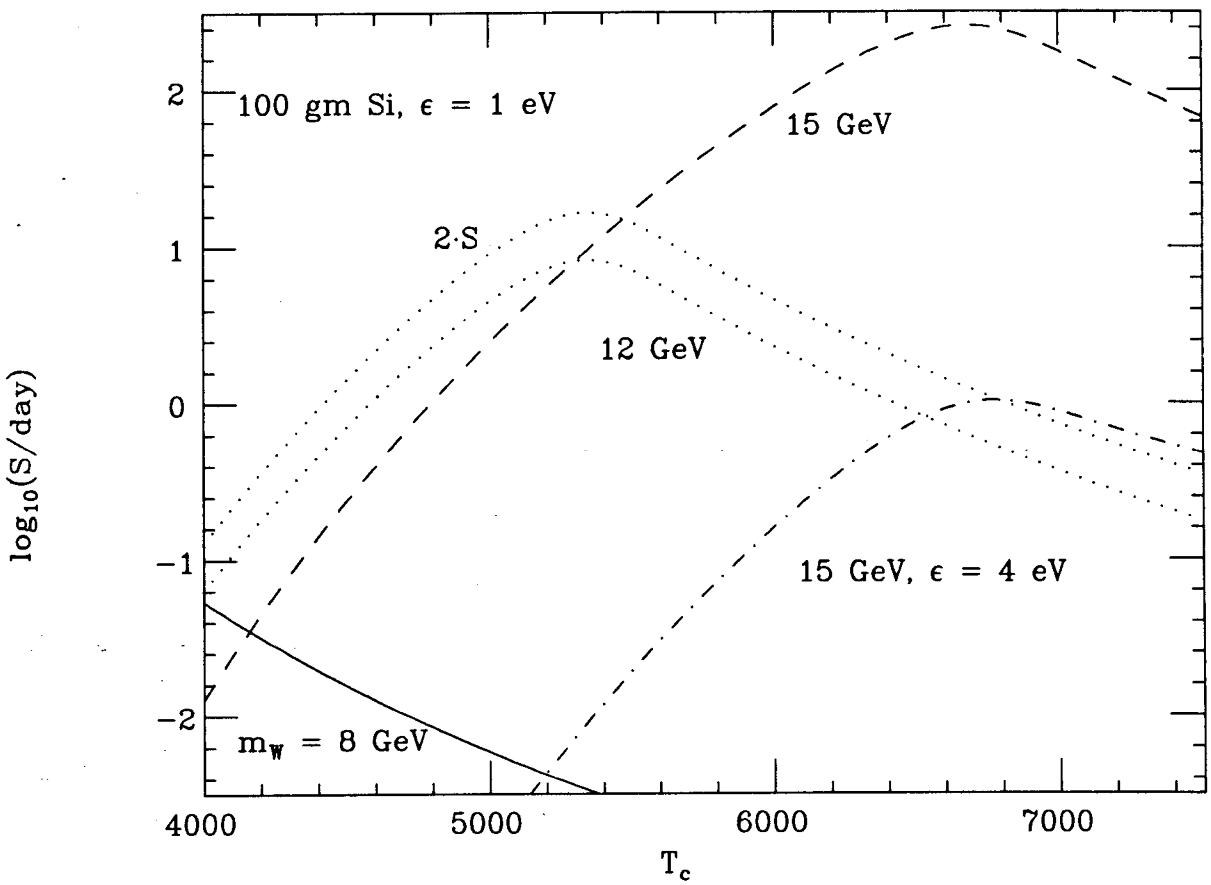
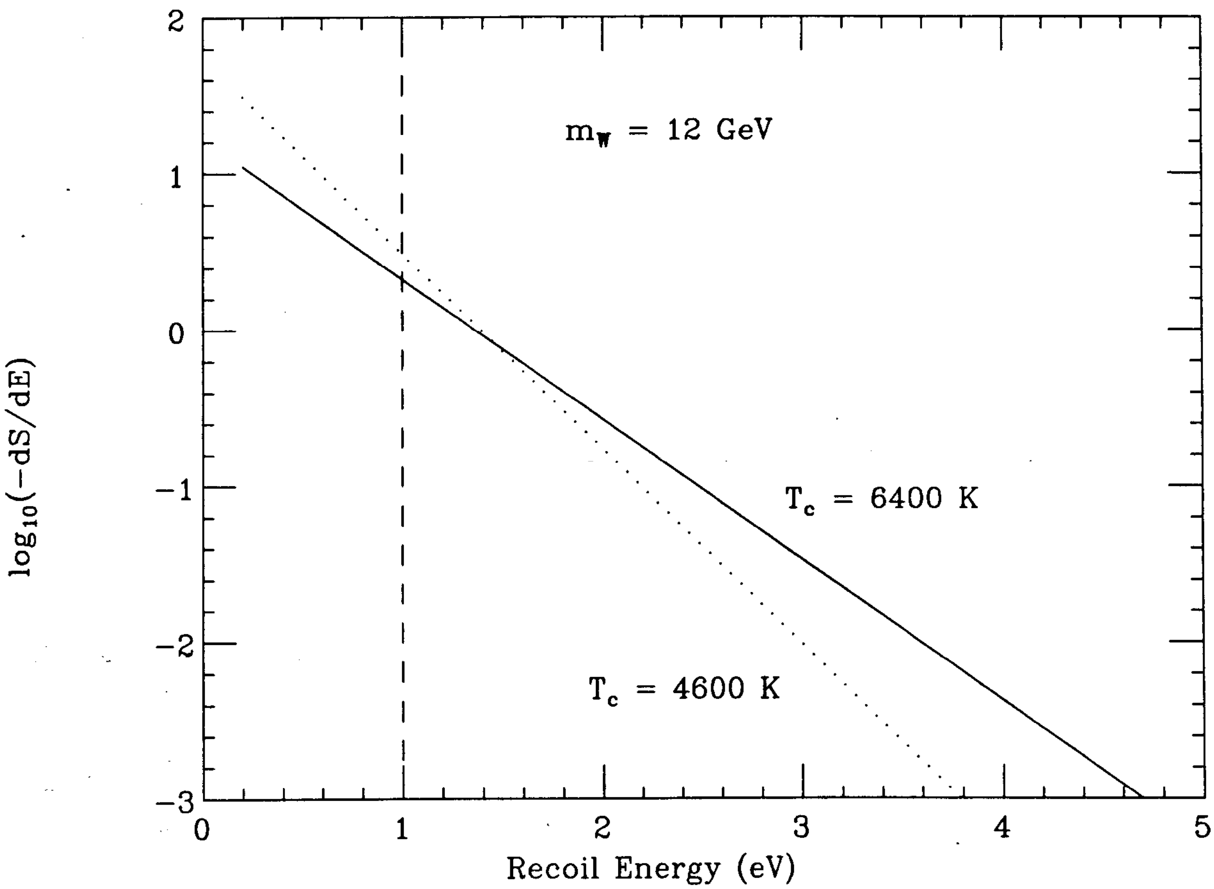

100 gm Si, ϵ = 1 eV
15 GeV
2·S
12 GeV
15 GeV, ϵ = 4 eV
m_W = 8 GeV
log10(S/day)
T_c
2
1
0
−1
−2
4000
5000
6000
7000

Fig. 1



2
1
0
−1
−2
−3
0
1
2
3
4
5
m_W = 12 GeV
T_c = 6400 K
T_c = 4600 K
log$_{10}$(−dS/dE)
Recoil Energy (eV)

Fig. 2

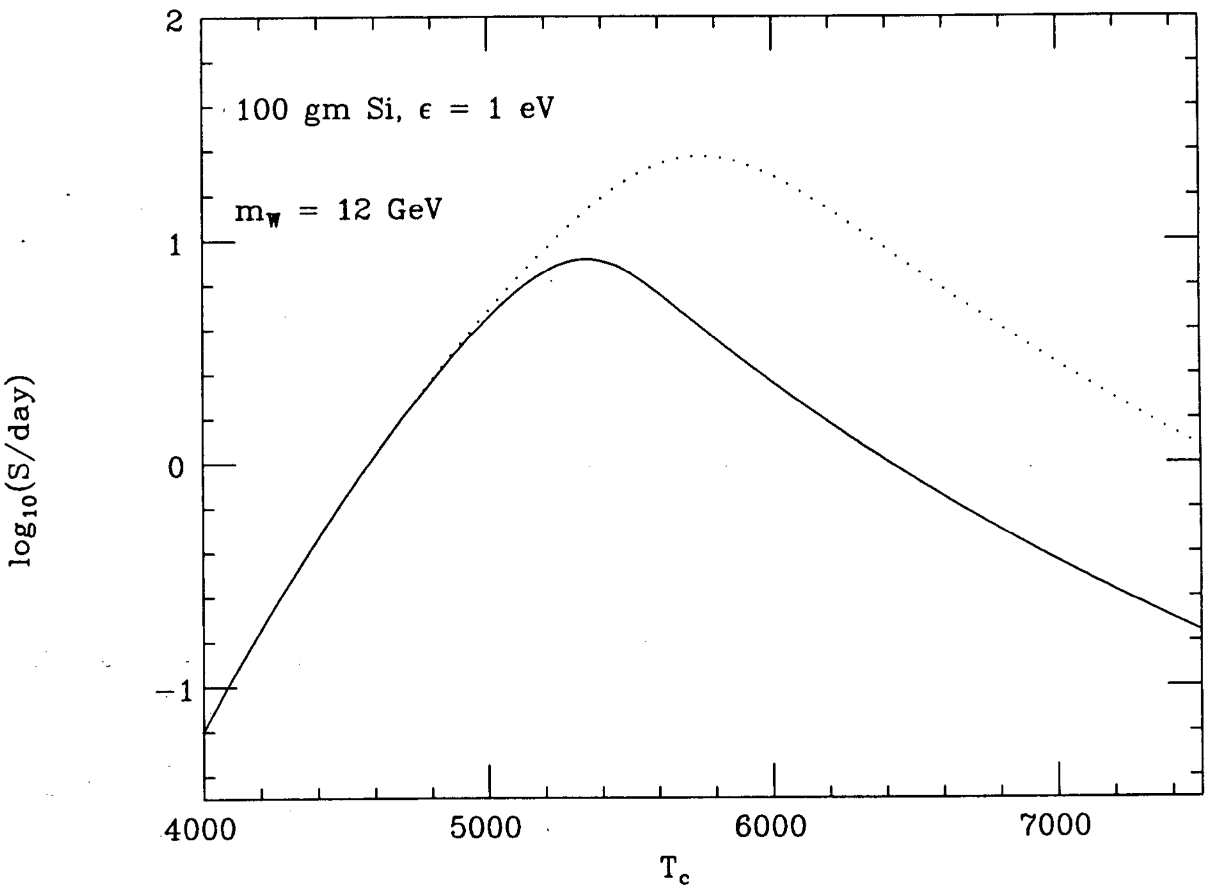


Fig. 3