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APPLICATION OF THE BOOTSTRAP STATISTICAL METHOD TO THE TAU DECAY MODE PROBLEM*

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ABSTRACT

The bootstrap statistical method is applied to the discrepancy in the 1-charged particle decay modes of the tau lepton. This eliminates questions about the correctness of the errors ascribed to the branching fraction measurements and the use of gaussian error distributions for systematic errors. The discrepancy is still seen when the results of the bootstrap analysis are combined with other measurements and with deductions from theory. But the bootstrap method assigns less statistical significance to the discrepancy compared to a method using gaussian error distributions.

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I. INTRODUCTION

At present there is a problem[1,2] in fully understanding the decay modes of the tau lepton to 1-charged particle. The average directly measured value[1] of the inclusive, 1-charged particle, branching fraction, B_1, is $(86.6 \pm 0.3)\%$. The same number should be obtained by adding up the branching fractions of the individual 1-charged particle modes. Examples of these individual branching fractions are:

$$B_e : \quad \tau^- \to \nu_\tau + e^- + \bar{\nu}_e$$

$$B_\mu : \quad \tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu$$

$$B_\pi : \quad \tau^- \to \nu_\tau + \pi^-$$

$$B_\rho : \quad \tau^- \to \nu_\tau + \rho^- \to \nu_\tau + \pi^- + \pi^0$$

$$B_{\pi 2\pi^0} : \quad \tau^- \to \nu_\tau + \pi^- + 2\pi^0$$

$$B_{\pi 3\pi^0} : \quad \tau^- \to \nu_\tau + \pi^- + 3\pi^0$$

As shown in Table 1 from Ref. 2, this sum is less than $(80.6 \pm 1.5)\%$, 6% less than the directly measured value of B_1. This is the τ decay mode problem.

Attempts to resolve the problem include: questioning[1] the errors ascribed by the experimenters to B_1, B_e B_μ, B_π and B_ρ; searching for bias in the measurements[1]; questioning the theory used to derive upper limits; and searching for unconventional explanations.[2] The error and bias studies in Ref. 1 assumed the errors are gaussian distributed, an assumption which cannot be justified for systematic errors. In this paper we apply the bootstrap[3,4] method of statistical analysis which does not use the ascribed errors.

The bootstrap method, Sec. II, requires multiple measurements of a quantity. We can apply it to B_1, B_e, B_μ, B_π, and B_ρ which have been repeatedly measured, Tables 2–6 and Sec. III. But we cannot apply it to the smaller branching fractions such as $B_{\pi 2\pi^0}$ and $B_{\pi 3\pi^0}$, which have few or no measurements. The branching fraction for the 1-charged particle decay modes containing K mesons is based on a few connected measurements, and is again not suitable for the bootstrap method. Fortunately this branching fraction is small.

In Sec. IV we apply the bootstrap method to the quantity

$$\Delta B = B_1 - (B_\rho + B_\pi + B_e + B_\mu) \quad ,$$

obtaining means and confidence levels for ΔB. We compare these findings with the upper limit on ΔB given by the sums of the branching fractions in rows 3 and 4 of Table 1. This comparison is done in Sec. V. We find that there is still a discrepancy in understanding the 1-charged particle decay modes, but the discrepancy is less striking than when studied using normal error analysis.[1]

II. THE BOOTSTRAP STATISTICAL METHOD

Consider a set of N measurements y_1, $y_2 \ldots y_N$ of a quantity y. Randomly select one of the set, note it, and replace it in the set. Carry out N such random selections with replacement forming a bootstrap replication set, $r(1)$, with N members: y_1^*, $y_2^* \ldots y_N^*$. Some of the y_n will appear as several y_n^*'s, some y_n will not appear at all in $r(1)$. Let $\bar{y}^*(1)$ be the mean value of the y_n^*'s in $r(1)$ calculated by giving each y_n^* the weight $1/N$.

In the simplest form of the bootstrap method this replication is carried out R times, constructing sets $r(1)$, $r(2) \ldots r(\mathrm{R})$ with means $\bar{y}^*(1)$, $\bar{y}^*(2) \ldots \bar{y}^*(R)$. From these sets one directly calculates various properties of the distribution of the R means, properties such as the standard deviation of the mean and various confidence intervals for the mean. Thus the errors assigned by the experimenters to their measurements are not used in this analysis. References 3 and 4 give a general description and a technical description of the bootstrap method.

In this paper we use the trimmed mean concept. Consider a bootstrap replication set, $r(i)$, with N members $y_1^*,\ y_2^* \ldots y_N^*$ now ordered in size from smallest to largest. Select a fraction f with $0 \leq f \leq 0.5$, and remove the fN smallest values of y_n^* and the fN largest values of y_n^*. The mean of the $(1-2f)N$ remaining values of y_n^* is the f trimmed mean. The case $f = 1/2$ gives the median. This procedure reduces the sensitivity of the mean to outlying values of y_n^*.

Our analyses are based on $f = 0.25$, a 25% trimmed mean. The selection of $f = 0.25$ comes from a study of the effect of various f values on the analyses.

III. BRANCHING FRACTION MEASUREMENTS

The branching fraction measurements are given in Tables 2–5 taken from Ref. 1. In Ref. 1 all published measurements of B_1, B_ρ, B_π, B_e, and B_μ are listed, including several published measurements partially based on the same data. Here we use the most complete measurement of such a set.

Table 2 gives 11 measurements of B_1. Measurements published before 1982 are not used for reasons given in Ref. 1. Tables 3 and 4 give 6 measurements of B_ρ and 7 measurements of B_π.

Some of the leptonic branching fraction measurements, B_e and B_μ, are complicated by the conventionally accepted existence of the universality relation

$$B_\mu = 0.973\ B_e \quad , \tag{1}$$

or by strongly correlated errors in the two measurements. Therefore the measurements are divided into two groups. Table 5a lists 14 measurements with independent values of B_e and B_μ. Table 5b lists 7 measurements constrained by Eq. 1 or highly correlated.

IV. ANALYSES

We have conducted two conventional bootstrap analyses of the measurements, the two differing in how B_e and B_μ are treated. We have also conducted one unconventional bootstrap analysis using weighted mneasurements.

A. Analysis Ignoring Correlations Between B_e and B_μ

In this analysis we ignored the correlations between the B_e and B_μ measurements in Table 5b, treating those measurements the same as the independent measurements in Table 5a. This gave 14 values of B_e and 20 values of B_μ. Our first step was to produce 500 bootstrap replications of the five branching fractions B_1, B_ρ, B_π B_e, and B_μ. The means and standard deviations of the bootstrap 25% trimmed means are in Table 6.

The second step in the analysis was to produce 2000 bootstrap replications of ΔB obtained by sampling 2000 times from the 500 bootstrap means of B_e, the 500 bootstrap means of B_μ, and so forth. This gave the 25% trimmed mean and the bootstrap confidence levels of ΔB in Table 7. (The bootstrap confidence level is a bias corrected percentile confidence level[3,4].)

B. Analysis Using Correlations Between B_e and B_μ

Next we took account of the correlations between the 7 (B_e, B_μ) measurements in Table 5b. The bootstrap replication sets for B_e and B_μ were constructed by first drawing with replacement 7 (B_e, B_μ) pairs from Table 5b. Then 7 B_e samples were drawn with replacement from Table 5a. Similarly 13 B_μ samples were drawn from Table 5a. Thus a B_e replication set with 14 values was formed and a B_μ set with 20 values was formed. As before we produced 500 bootstrap replication sets of B_e and B_μ.

These 500 new sets of B_e and B_μ were combined with the previous 500 sets of B_1, B_ρ, and B_π to make 2000 bootstrap replications of ΔB. The properties of this sample of 2000 values of ΔB are given in Table 7. Taking account of the (B_e, B_μ) correlations in Table 5b has a negligible effect on the confidence level intervals.

C. Analyses Using Weighted Measurements

Although one strength of the bootstrap method is its independence from error estimation for individual measurements, it is interesting to investigate the effect of using the individual errors to weight the measurements. Tables 2–5 give the statistical, systematic, and combined errors quoted by experimenters for their own measurement. Calling σ_i the combined error for measurement i, the relative weight is $1/\sigma_i^2$. We used these weights in the bootstrap analysis by finding a constant C such that an interger

$$n_i \sim C/\sigma_i^2 \tag{2}$$

is associated with measurement i. We then formed a larger measurement set with measurement i repeated n_i times.

Using the enlarged measurement sets, 200 bootstrap replication sets were produced for B_1, B_ρ, B_π, B_e, and B_μ. (The correlations in Table 5b were ignored.) Table 6 gives the 25% trimmed means and standard deviations. Finally, 800 bootstrap values of ΔB were generated from these sets. Table 7 gives the properties of ΔB.

V. DISCUSSION OF THE DECAY MODE PROBLEM

The most reliable application of the bootstrap method is the analysis in Sec. IV.B. From Table 7, middle column, this bootstrap analysis gives

$$\Delta B \text{ (bootstrap) } > 13.9\% \tag{3}$$

with 99% confidence. (The bootstrap analysis in Sec. IV.A gives about the same result.) The quantity ΔB is also given by the sums of the branching fractions in rows 3 and 4 of Table 1. Taking the error on the row 3 modes as gaussian the 95% upper limit on ΔB is

$$\Delta B \text{ (rows 3 and 4 of Table 1)} < 13.6\% \tag{4}$$

Thus the 99% lower limit on ΔB from the bootstrap analysis exceeds the 95% upper limit on ΔB from rows 3 and 4 of Table 1. The discrepancy persists in the 1-charged particle decay modes of the tau.

However the discrepancy is less striking than is found with the normal error analysis method. The right column of Table 6 gives the mean values from that method.[1,2] (The formal error means the combined error calculated by adding the statistical and systematic errors in quadrature.) Then

$$\Delta B \text{ (normal) } = (18.0 \pm 1.2)\% \tag{5a}$$

giving

$$\Delta B \text{ (normal)} > 15.3\% \tag{5b}$$

with 99% confidence. (This assumes the error in Eq. 5a is normally distributed.) Combining Eq. 5b with Eq. 4, we see a more striking discrepancy than that provided by the bootstrap method. But in the normal error distribution comparison we are considering discrepancies at the 3 to 4 standard deviation level, and our understanding of the true error distribution is insufficient to justify the use of so many standard deviations. Hence the bootstrap method provides a less severe, but better justified, description of the decay mode problem.

The difference between the bootstrap result and the normal error distribution result is illuminated by comparing in Table 6 the results from the conventional bootstrap analysis, from the unconventional bootstrap analysis, and from the normal error distribution analysis. As expected the latter two methods give similar means and errors. But the use of weights derived from the individual errors brings us back to the question of whether the individual errors are all correct. We went to the bootstrap method to avoid this question. Therefore we do not rely in this situation on the use of weighted measurements in the bootstrap method.

VI. CONCLUSIONS

We have three conclusions.

1. The mean values of B_1, B_e, B_μ, B_π, B_ρ, and ΔB given by the bootstrap analysis are similar to, but not identical to, the means given by an analysis using normal error distributions. The difference in the means of B_1, B_e, and B_μ accounts for most of the difference between ΔB (normal) and ΔB (bootstrap) for methods A and B. The bootstrap analysis with weighted measurements, method C, gives means very similar to the normal error method.
2. The bootstrap analysis given $\Delta B > 13.9\%$ with 99% confidence. This is a smaller value than the corresponding quantity computed using normal error distributions and quoted errors.
3. The problem of understanding the tau 1-charged particle decay modes persists.

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Table 1. Summary of present knowledge of 1-charged particle branching fraction in percent from Refs. 1 and 2. The numbers in Rows 1, 2, and 3 are the average of measured values and the associated standard deviation. The sum in Row 4 is the 95% upper limit obtained from other data and accepted theory. Note that Refs. 1 and 2 used gaussian error distributions.

Rows	Symbol	Decay Mode of τ^-	Branching Fraction (%)
1	B_1	1-charged particle inclusive	86.6 ± 0.3
2	B_e	$\nu_\tau + e^- + \bar{\nu}_e$	17.6 ± 0.4
	B_μ	$\nu_\tau + \mu^- + \bar{\nu}_\mu$	17.7 ± 0.4
	B_π	$\nu_\tau + \pi^-$	10.8 ± 0.6
	B_ρ	$\nu_\tau + \rho^-$	22.5 ± 0.9
		Sum for modes in Rows 2	68.6 ± 1.2
3	$B_{\pi 2\pi^0}$	$\nu_\tau + \pi^- + 2\pi^0$	7.6 ± 0.8
		$\nu_\tau + mK + n\pi^0$ $\longrightarrow$ 1-charged particle $m \geq 1,\ n \geq 0,\ K = K^0$ or K^-	1.8 ± 0.3
		Sum for modes in Rows 3	9.4 ± 0.9
4		$\nu_\tau + \pi^- + n\pi^0$ $n \geq 3$ $\nu_\tau + \pi^- + m\eta + n\pi^0$ $\longrightarrow$ 1-charged particle $m \geq 1,\ n \geq 0,$	
		Sum for modes in Rows 4	≤ 2.7
5		Sum for modes in Rows 2, 3, and 4	80.7 ± 1.5

Table 2. B_1 topological branching fractions in percent. The statistical error is given first, the systematic error second.

B_1 Measurement	B_1 Combined Error	Energy (GeV)	Experimental Group	Reference
84.0	± 2.0	32.0 to 36.8	CELLO	H. J. Behrend *et al.*, Phys. Lett. **114B**, 282 (1982)
$85.2 \pm 2.6 \pm 1.3$	± 2.9	14.0	CELLO	H. J. Behrend *et al.*, Z. Phys. **C23**, 103 (1984)
$85.1 \pm 2.8 \pm 1.3$	± 3.1	22.0	CELLO	H. J. Behrend *et al.*, Z. Phys. **C23**, 103 (1984)
$87.8 \pm 1.3 \pm 3.9$	± 4.1	34.6 average	PLUTO	Ch. Berger *et al.*, Z. Phys. **C28**, 1 (1985)
$84.7 \pm 1.1^{+1.6}_{-1.3}$	$^{+1.9}_{-1.7}$	13.9 to 43.1	TASSO	M. Althoff *et al.*, Z. Phys. **C26**, 521 (1985)
$86.7 \pm 0.3 \pm 0.6$	± 0.7	29.0	MAC	E. Fernandez *et al.*, Phys. Rev. Lett. **54**, 1624 (1985)
$86.9 \pm 0.2 \pm 0.3$	± 0.4	29.0	HRS	C. Akerlof *et al.*, Phys. Rev. Lett. **55**, 570 (1985)
$86.1 \pm 0.5 \pm 0.9$	± 1.0	30.0 to 46.8	JADE	W. Bartel *et al.*, Phys. Lett. **161B**, 188 (1985)
$87.9 \pm 0.5 \pm 1.2$	± 1.3	29.0	DELCO	W. Ruckstuhl *et al.*, Phys. Rev. Lett. **56**, 2132 (1986)
$87.2 \pm 0.5 \pm 0.8$	± 0.9	29.0	MARK II	W. B. Schmidke *et al.*, Phys. Rev. Lett. **57**, 527 (1986)
$84.7 \pm 0.8 \pm 0.6$	± 1.0	29.0	TPC	H. Aihara *et al.*, Phys. Rev. **D35**, 1553 (1987)

Table 3. The $\tau^- \to \rho^- \nu_\tau$ branching ratio, B_ρ, in percent. The statistical error is given first, the systematic error second.

Measurement	Combined Error	Energy (GeV)	Experimental Group	Reference
$24. \pm 6. \pm 7.$	$\pm 9.$	3.6 to 5.2	DASP	R. Brandelik *et al.*, Z. Phys. **C1**, 233 (1979)
$21.5 \pm 1.7 \pm 3.0$	± 3.4	3.7 to 6.0	MARK II	C. A. Blocker, Thesis, LBL-10801 (1980)
$22.1 \pm 1.9 \pm 1.6$	± 2.5	14.0 to 34.0	CELLO	H. J. Behrend *et al.*, Z. Phys. **C23**, 103 (1984)
$22.3 \pm 0.6 \pm 1.4$	± 1.5	29.0	MARK II	J. M. Yelton *et al.*, Phys. Rev. Lett. **56**, 812 (1986)
$23.0 \pm 1.3 \pm 1.7$	± 2.1	3.8	MARK III	J. Adler *et al.*, Phys. Rev. Lett. **59**, 1527 (1987)
$22.6 \pm 0.5 \pm 1.4$	± 1.5	9.4 to 10.6	CRYSTAL BALL	S. T. Lowe *et al.*, SLAC-PUB-4449 (1987)

Table 4. The $\tau^- \rightarrow \pi^- \nu_\tau$ branching ratio, B_π, in percent. The statistical error is given first, the systematic error second.

Measurement	Combined Error	Energy (GeV)	Experimental Group	Reference
$9.0 \pm 2.9 \pm 2.5$	± 3.8	4.1 to 5.0	PLUTO	G. Alexander *et al.*, Phys Lett. **78B**, 162 (1978)
$8.0 \pm 3.2 \pm 1.3$	± 3.5	3.6 to 7.4	DELCO	W. Bacino *et al.*, Phys Lett. **42**, 6 (1978)
$11.7 \pm 0.4 \pm 1.8$	± 1.8	3.5 to 6.7	MARK II	C. A. Blocker *et al.*, Phys. Lett. **109B**, 119 (1982)
$9.9 \pm 1.7 \pm 1.3$	± 2.1	34.0	CELLO	H. J. Behrend *et al.*, Phys. Lett. **127B**, 270 (1983)
$11.8 \pm 0.6 \pm 1.1$	± 1.3	34.6 average	JADE	W. Bartel *et al.*, Phys. Lett. **182B**, 216 (1986)
$10.7 \pm 0.5 \pm 0.8$	± 0.9	29.0	MAC	W. T. Ford *et al.*, Phys. Rev. **D35**, 408 (1987)
$10.0 \pm 1.1 \pm 1.4$	± 1.8	29.0	MARK II	P. R. Burchat *et al.*, Phys. Rev. **D35**, 27 (1987)

Table 5a. Independent measurement of the $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ branching fractions, B_e and B_μ, in percent. The statistical error is given first, the systematic error second.

B_e		B_μ		Energy (GeV)	Experimental Group	Reference
Measurement	Combined Error	Measurement	Combined Error			
		17.5±2.7±3.0	±4.0	3.8 to 7.8	MARK I	M. L. Perl *et al.*, Phys. Lett. **70B**, 487 (1977)
		22.	$^{+10.}_{-7.}$	4.8		M. Cavalli-Storza *et al.*,, Lett. Nuovo Cimento **20**, 337 (1977)
		15.	±3.0	3.6 to 5.0	PLUTO	J. Burmester *et al.*, Phys. Lett. **68B**, 297 (1977)
16.0	±1.3			3.1 to 7.4	DELCO	W. Bacino *et al.*, Phys. Rev. Lett. **41**, 13 (1978)
		22.	$^{+7.}_{-8.}$	6.4 to 7.4	Iron Ball	J. G. Smith *et al.*, Phys. Rev. **D18**, 1 (1978)
		21±5±3	±6.	3.6 to 7.4	DELCO	W. Bacino *et al.*, Phys. Rev. **42**, 6 (1979)
19.	±9.0	35.	±14.	12 to 31.6	TASSO	R. Brandelik *et al.*, Phys. Lett. **92B**, 199 (1980)
		17.8±2.0±1.8	±2.7	9.4 to 31.6	PLUTO	Ch. Berger *et al.*, Phys. Lett. **99B**, 489 (1981)
18.3±2.4±1.9	±3.1	17.6±2.6±2.1	±3.3	34.0	CELLO	H. J. Behrend *et al.*, Phys. Lett. **127B**, 270 (1983)
$20.4\pm3.0^{+1.4}_{-0.9}$	$^{+3.3}_{-3.1}$	$12.9\pm1.7^{+0.7}_{-0.5}$	±1.8	13.9 to 43.1	TASSO	M. Althoff *et al.*, Z. Phys. **C26**, 521 (1985)
13.0±1.9±2.9	±3.5	19.4±1.6±1.7	±2.3	34.6 average	PLUTO	Ch. Berger *et al.*, Z. Phys. **C28**, 1 (1985)
		17.4±0.6±0.8	±1.0	14.0 to 46.8	MARK J	B. Adeva *et al.*, Phys. Lett. **179B**, 177 (1986)
17.0±0.7±0.9	±1.1	18.8±0.8±0.7	±1.1	34.6 average	JADE	W. Bartel *et al.*, Phys. Lett. **182B**, 216 (1986)
19.1±0.8±1.1	±1.4	18.3±0.9±0.8	±1.2	29.0	MARK II	P. R. Burchat *et al.*, Phys. Rev. **D35**, 27 (1987)

Table 5b. Constrained or correlated measurements of the $\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$ branching fractions, B_e and B_μ, in percent. The statistical error is given first, the systematic error second.

B_e		B_μ		Energy (GeV)	Experimental Group	Reference
Measurement	Combined Error	Measurement	Combined Error			
18.9±1.0±2.8	±3.0	18.3±1.0±2.8	±3.0	3.8 to 7.8	MARK I	M. L. Perl *et al.*, Phys. Lett. **70B**, 487 (1977)
22.7	±5.5	22.1	±5.5	4.1 to 7.4	Lead Glass Wall	A. Barbaro-Galtieri *et al.*, Phys. Rev. Lett. **39** 1058 (1977)
18.5±2.8±1.4	±3.1	18.0±2.8±1.4	±3.1	3.9 to 5.2	DASP	R. Brandelik *et al.*, Phys. Lett. **73B**, 109 (1978)
17.6±0.6±1.0	±1.3	17.1±0.6±1.0	±1.3	3.5 to 6.7	MARK II	C. A. Blocker *et al.*, Phys. Lett. **109B**, 119 (1982)
18.2±0.7±0.5	±0.9	18.0±1.0±0.6	±1.2	3.8	MARK III	R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**,1842 (1985)
17.4±0.8±0.5	±0.9	17.7±0.8±0.5	±0.9	29.0	MAC	W. W. Ash *et al.*, Phys. Rev. Lett. **55**, 2118 (1985)
18.4±1.2±1.0	±1.6	17.7±1.2±0.7	±1.4	29.0	TPC	H. Aihara *et al.*, Phys. Rev. **D35**, 1553 (1987)

Table 6. Means and standard deviations (SD) for B_1, B_ρ, B_π, B_e, and B_μ. Both quantities are in percent.

Branching Fraction	Analysis Method					
	Bootstrap (Method A)		Bootstrap with weighted measurements (Method C)		Normal error method from Ref. 1	
	Mean, 25% trimmed	SD	Mean, 25% trimmed	SD	Mean, not trimmed	Formal error
B_1	85.8	0.63	86.9	0.36	86.6	0.28
B_ρ	22.5	0.35	22.5	0.19	22.5	0.85
B_π	10.2	0.55	10.8	0.45	10.8	0.60
B_e	18.3	0.38	17.8	0.30	17.6	0.44
B_μ	18.2	0.56	17.8	0.21	17.7	0.41

Table 7. The 25% trimmed mean and bootstrap confidence levels for ΔB in percent.

Properties of ΔB	Analysis Method		
	Bootstrap, ignore $B_e - B_\mu$ correlations in Table 5b	Bootstrap, use $B_e - B_\mu$ correlations in Table 5b	Bootstrap with weighted measurements
mean	16.62	16.58	18.01
.01 confidence level	14.12	13.94	15.83
.05 confidence level	14.96	14.86	16.52
.50 confidence level	16.79	16.87	17.86
.95 confidence level	18.59	18.70	18.95
.99 confidence level	19.41	19.43	19.28