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MULTIPLE BUNCH CROSSING INSTABILITY*

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ABSTRACT

In a linear collider where multiple bunches are accelerated in every RF pulse, the close encounters between the outgoing bunches and the incoming bunches near the central collision point would cause a growth of any initial offset of the beams. In this paper we analyze such an instability both theoretically and through computer simulations. A condition for negligible growth of the instability is derived, and possible cures are discussed.

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I. INTRODUCTION

Design studies of the next generation of electron-positron linear colliders in the range of 1 TeV center-of-mass energy have been intensive during recent years.[1] It has been generally recognized that it would be difficult to achieve a luminosity beyond $1\times10^{33}\text{cm}^2/\text{sec}$ if the acceleration of a single bunch per RF pulse is assumed. However, recent studies[2] show great promise to accelerate multiple (m) bunches within each RF pulse, in this case a luminosity of the order of $10^{34}\text{cm}^2/\text{sec}$ is conceivable. While it is difficult to improve the luminosity by a factor m^2 from the maximal possible number of collisions between the two bunch trains, one may hope to increase it by at least a factor m.

One of the major problems of such a multibunch operation is the interactions between bunches before and after their collisions at the central collision point. The i^{th} bunch in the electron bunch train will collide not only with the i^{th} bunch in the positron train, but also with the $j(< i)^{th}$ positron bunch before its coming to the central collision point. These undesirable collisions will degrade the beam, and will result in a reduction of luminosity. The idea proposed by Palmer[3] to collide two flat beams at a relatively large crossing angle, can help to avoid unwanted direct encounters between the outgoing bunch debris and the incoming fresh bunches. However, due to the long-range nature of the Coulomb interaction, there still exists undesirable interference between two separated bunches at a distance. Since the crossing angle cannot be made arbitrarily large due to the luminosity consideration, this long range interaction cannot be entirely suppressed. In fact, it imposes a severe restriction on the stability of the beams.

In this paper we estimate the effects of such an instability. The beams are assumed to be so flat that the perturbation on the horizontal motion is negligible, only vertical deflections are considered.

II. GROWTH OF BEAM OFFSET

Let us denote the vertical displacement of the n^{th} electron (positron) bunch at the collision point by Y_n (y_n). The kicking angle of the positron bunch can be written as

$$y'_n = -\frac{1}{2}\frac{D_y}{\sigma_z}\,(y_n - Y_n)\ , \tag{1}$$

where D_y is the vertical disruption parameter defined by

$$D_y = \frac{2Nr_e\sigma_z}{\gamma\sigma_y(\sigma_x+\sigma_y)} \sim \frac{2Nr_e\sigma_z}{\gamma\sigma_y\sigma_x}\ , \tag{2}$$

and $\sigma_x, \sigma_y, \sigma_z$ the horizontal, vertical and longitudinal r.m.s. beam sizes at the collision point, respectively. The above equation is true only when both D_y and $|(y_n - Y_n)|/\sigma_y$ are small. The actual angle of the deflection for finite D_y and $|(y_n - Y_n)|/\sigma_y$ can only be obtained from computer simulations, which will be discussed in the following section. The extra factor 1/2 comes from the fact that we are considering not the deflection of a single particle but that of the center-of-mass of the entire bunch. It is the same factor found for the coherent beam-beam tune shift in storage rings.[1]

Next we consider the encounter between the n^{th} positron bunch after collision and the m^{th} $(m > n)$ electron bunch before collision at a distance L from the collision point. A schematic diagram of the system is shown in Fig. 1. We assume that all the bunch encounters occur within the drift space around the central collision point. The vertical position of the positron bunch at L is thus $y_{n,L} = Ly'_n$. If the crossing angle is θ_c, then the distance between the positron bunch and the electron bunch is $d = L\theta_c$. We assume d is much larger than the beam size so that the Coulomb force between point charges can be applied. Then the m^{th} electron bunch will be kicked vertically by the n^{th} positron bunch by an angle

$$\frac{2Nr_e}{\gamma d}\,\frac{y_{n,L}}{d}\;. \tag{3}$$

By multiplying the drift length L to this expression and by using Eq. (1), we find the contribution of the n^{th} positron bunch to the displacement of m^{th} electron bunch at the central collision point to be

$$\Delta Y_m = -\frac{1}{2}\;C(y_n - Y_n)\;, \tag{4}$$

with

$$C = \frac{2Nr_eD_y}{\gamma\theta_c^2\sigma_z}\;. \tag{5}$$

An important fact is that the distance L does not appear in this expression, i.e., distant encounters are as important as those near the collision point.

The cumulative displacement of the m^{th} electron bunch at the central collision point arises from the encounters with all the positron bunches, which gives

$$Y_m = -\frac{1}{2}C\sum_{n<m}(y_n - Y_n) + \delta Y_m\;. \tag{6}$$

Here we have introduced a source term, δY_m, for the m^{th} electron bunch to represent any initial alignment error induced by elements upstream along the linear collider. A similar equation holds for the effect of electron bunches onto positron bunches. Summation of the two equations shows that the center-of-mass of the system is not affected by the encounters. On the other hand, by subtraction we find the equation for the relative motion between the m^{th} electron and positron bunches:

$$\Delta_m = C \sum_{n<m} \Delta_n + \delta_m \ , \tag{7}$$

where $\Delta_m = (Y_m - y_m)/\sigma_y$ is the distance between the m^{th} electron and positron bunches in units of vertical beam size, and $\delta_m = (\delta Y_m - \delta y_m)/\sigma_y$ is the distance in the absence of the interaction. This leads to a difference equation

$$\Delta_{m+1} = (1 + C)\ \Delta_m + (\delta_{m+1} - \delta_m) \ , \tag{8}$$

which has the eigenvalue $(1 + C)$. Since C is positive definite, the system is unstable. Note that the collision of bunches of like charges is also unstable because the signs of both Eqs. (1) and (3) are changed. The solution to Eq. (7) for given δ's is

$$\Delta_m = \delta_m + C \sum_{n=0}^{m-2} (1 + C)^n\ \delta_{m-1-n} \ . \tag{9}$$

In the special case where all the δ's are equal, we have

$$\Delta_m = (1 + C)^{m-1} \delta \ . \tag{10}$$

Therefore, the constant component of the initial offset δ will be enhanced by a factor $(1 + C)^{N_B - 1}$, N_B being the number of bunches. Although C is usually not a big quantity, the cumulative displacement can become large when there involves a large number of bunches.

On the other hand, if δ's are random with standard deviation $\delta_{\rm rms}$, then the r.m.s. of Δ's are given by

$$\Delta_{m,\rm rms} = \sqrt{1 + \frac{C}{2}(1 + C)^{2m-2}}\ \ \delta_{\rm rms} \ , \qquad \text{for} \qquad C \ll 1 \ . \tag{11}$$

which is suppressed by a factor $\sqrt{C/2}$ from the case for constant initial offsets. Thus the oscillating component of δ's, such as the effect from multibunch instability in the linac, will not have an important role in the issue. Only the constant component, e.g., due to field errors in the final focusing system, the ground motion, etc., will be of concern.

In order that the growth of the offset is negligible, the condition $(N_B - 1)C \lesssim 1$ must be satisfied. By using the horizontal disruption parameter

$$D_x = \frac{2Nr_e\sigma_z}{\gamma\sigma_x(\sigma_x + \sigma_y)} \sim \frac{2Nr_e\sigma_z}{\gamma\sigma_x^2} \ , \tag{12}$$

we can simplify the expression for C, i.e.,

$$C = D_x D_y \left(\frac{\theta_d}{\theta_c}\right)^2 \ , \tag{13}$$

where $\theta_d = \sigma_x/\sigma_z$ is the diagonal angle of the beam. Since the factor in the parenthesis must be larger than unity in order that the crossing angle does not reduce the luminosity significantly, the condition for the negligible growth of the instability becomes

$$(N_B - 1)\ D_x D_y \lesssim 1 \ , \tag{14}$$

which imposes a severe limitation in the design of multibunch linear colliders.

III. THE EFFECTIVE DEFLECTION

As mentioned in the previous section, the actual deflecting angle deviates from the simple expression that we gave in Eq. (1). Actually, Eq. (1) is an overestimation of the kicking effect at the collision point. The effective deflection can be written as

$$y'_n = -\frac{1}{2}\frac{D_y \sigma_y}{\sigma_z}\ H_c(D_y, \Delta_n) \ , \tag{15}$$

where the function $H_c(D_y, \Delta_n)$ approaches Δ_n in the limit of small D_y and Δ_n. An analytic form of H_c in the limit of small D_y but for arbitrary Δ can be derived for Gaussian bunches as

$$H_c(0, \Delta) = \int_0^{\Delta} e^{-x^2/4}\ dx \ . \tag{16}$$

For finite D_y, computer simulation is needed.

Figure 2 shows the function $H_c(D_y, \Delta)$ computed by the beam-beam interaction code ABEL.[5] In this case $A \equiv \sigma_z/\beta_y = 0.8$ is used, where β_y is the Twiss parameter at the central collision point. It can be shown, however, that the result is not very sensitive to the values of A. Notice that the coefficient which appears in the analytic formula in Eq. (1) corresponds to the tangent slope of the $D_y = 0$ curve near the origin in Fig. 2 (the dashed line). For finite D_y, the deflecting angle can be seen to be suppressed, especially in the large D_y limit. Furthermore, Fig. 2 also clearly shows that when the offset gets large, the deflecting angle tends to gradually saturate.

Using Eq. (15), one can repeat the previous exercise to obtain an equation for the cumulative offset for the m^{th} bunch,

$$\Delta_m = C \sum_{n<m} H_c(D_y, \Delta_n) + \delta_m \ . \tag{17}$$

Based on the numerical values obtained in Fig. 2, the cumulative offset Δ_m (in units of the theoretical offset, $\delta(1+C)^{m-1}$) is plotted as a function of the number of bunches in Fig. 3. We see that the offset for $D_y = 0$ is reasonably close to what is predicted theoretically, especially for smaller number of bunches. Actually, even for bunch number as large as 20, the cumulative offset is still about a factor 0.8 of the theoretical value. The reduction is clearly seen, however, when D_y is finite. This is true especially when D_y is much larger than unity. This means that the condition for negligible growth of the offset given in Eq. 14 is somewhat too pessimistic. Although the correct constraint depends on the specific parameters of the problem, it seems, according to Fig. 3, that one may be safe to relax the constraint to

$$(N_B - 1)\ D_x D_y \lesssim 2\ , \tag{18}$$

as a rule of thumb.

IV. DISCUSSION

One possible cure to the multiple bunch crossing instability is to collide bunches with alternating charges in both bunch trains.[6] The eigenvalue in this case turns out to be $-(1+C)$. This means that although the constant component of the offset may be suppressed, the oscillating component will still grow. It also appears that technically it is rather difficult to implement such a scheme. Another possibility is to partition the outgoing bunch train from the incoming bunch train by a septum.[7] If the bunch spacing is too small, it may be difficult to shield away all the encounters. However, it is clear that even a partial shielding would be beneficial. Consider, for example, that one is able to shield away all the long-range interactions except one, then Eq. (7) becomes

$$\Delta_m = C\Delta_{m-1} + \delta_m\ , \tag{19}$$

which does not induce any enhancement. In general, if one allows for N encounters under a partial shielding, and if $C < 1/N$, then there is a finite limit for the constant component Δ_m of the m^{th} bunch, no matter how large m is:

$$\lim_{m\to\infty} \Delta_m = \frac{\delta}{1-CN}\ . \tag{20}$$

Therefore, if C is much less than unity, this method can be very effective.

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FIGURE CAPTIONS

Fig. 1. Schematic diagram of the system around the Central Collision Point (CCP). Both electron and positron bunch trains travel on the x-z plane, with a crossing angle θ_c. The closest encounter between the mth electron bunch and the nth positron bunch occurs at a distance $z = L$ from CCP with a separation distance d. The vertical displacement at CCP of the positron and the electron bunches are Y_{n+2} and y_{m-2} $(n+2 = m-2$ in this case), respectively.

Fig. 2 The parameter $H_c(D_y, \Delta)$ for the effective deflection as a function of offset Δ for different values of D_y. The curve for $D_y = 0$ is from the theoretical expression of Eq. (16), while the tangent slope (= 1) of this curve near the origin (the dashed line) corresponds to the coefficient that appears in the analytic formula, Eq. (1).

Fig. 3. The cumulative offset Δ_m in units of the theoretical offset, $\delta(1+C)^{m-1}$, as a function of the number of bunches.

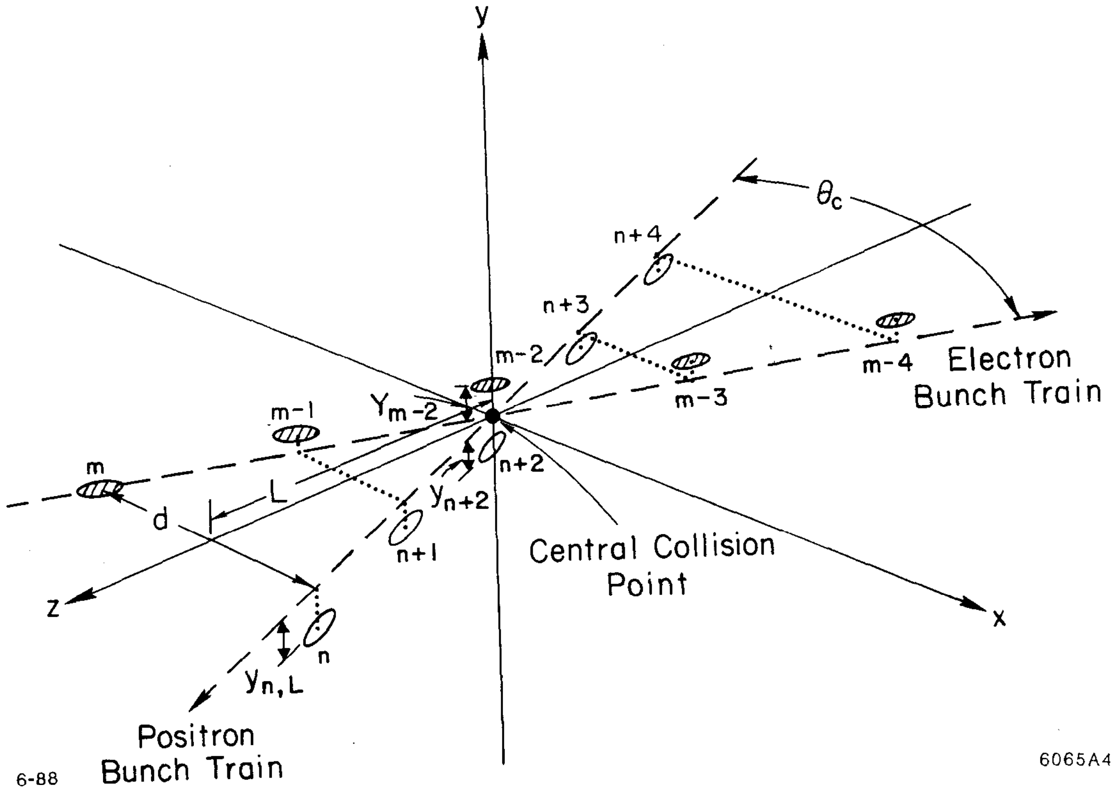
y
θ_c
n+4
n+3
m-2
m-4
Electron
Bunch Train
m-3
m-1
Y_{m-2}
n+2
m
L
y_{n+2}
d
n+1
Central Collision
Point
z
x
n
$y_{n,L}$
Positron
Bunch Train
6-88
6065A4

Fig. 1

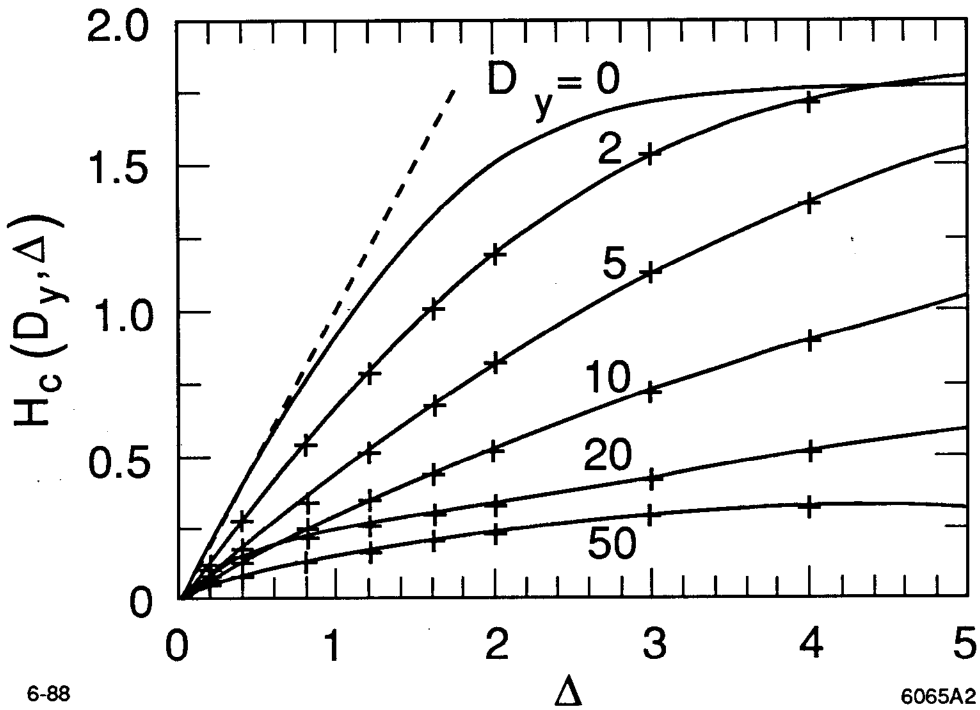

Fig. 2

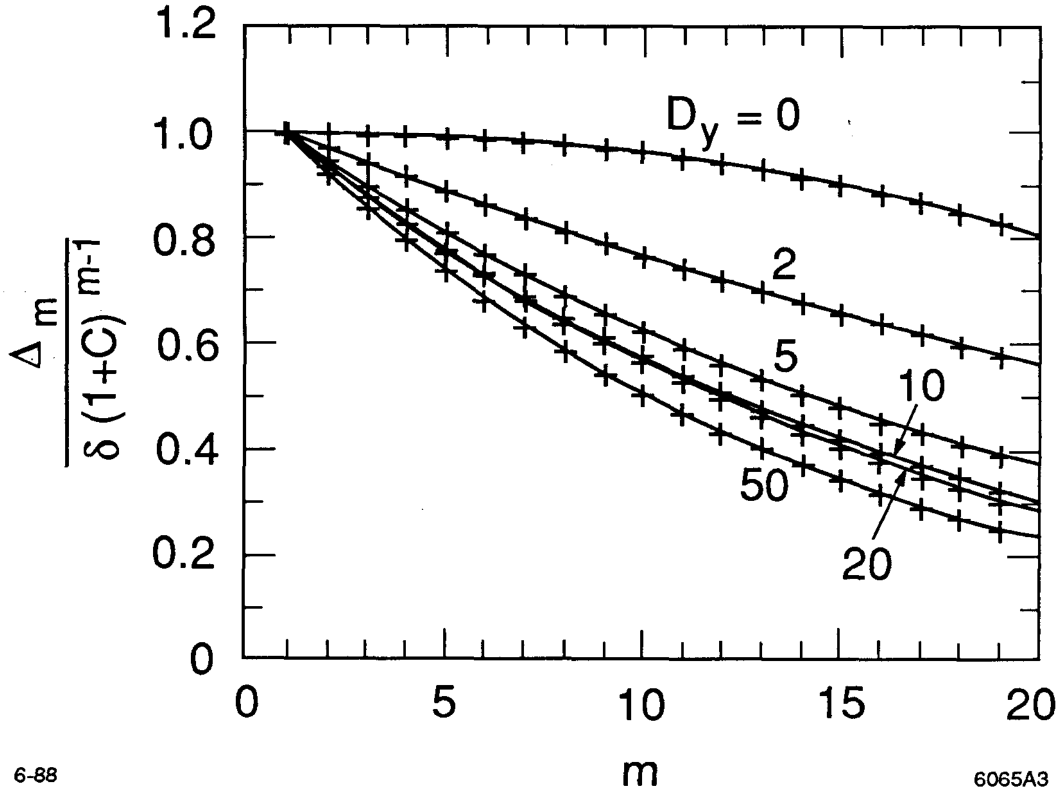

Fig. 3