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Superheavy Magnetic Monopoles and Main Sequence Stars*

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ABSTRACT

We investigate the interactions of superheavy monopoles with stars of mass $0.6 - 25\ M_{\odot}$. Over the main sequence lifetime, stars accumulate significant numbers of monopoles less massive than $\simeq 5 \times 10^{17}$ GeV; e.g., for $m_{mon} = 10^{16}$ GeV, the number captured is of order $10^{41} F_M (M/M_{\odot})^{-0.4}$, where F_M is the monopole flux in $\mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1}$. Captured monopoles cluster near the stellar center; there they generate heat by annihilating and, possibly, by catalyzing the decay of baryons. Their contribution to the total stellar luminosity, and their effects on the structure of stars, are likely to be unobservable as long as the flux is substantially below the Parker bound, $F_M < 10^{-16}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1}$. Although a monopole flux as low as $F_M \sim 10^{-24}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1}$ could give rise to a high energy neutrino flux (from catalysis) above atmospheric background, due to $M\bar{M}$ annihilation this does not translate into a reliable flux bound stronger than the Parker limit. We also argue that all but the strongly magnetic stars will retain a substantial fraction of the monopoles they capture. As a result, including monopoles captured on the main sequence strengthens the upper bound on the flux due to monopole-catalyzed nucleon decay in neutron stars by seven orders of magnitude, $F_M \sigma_{-28} \lesssim 10^{-28}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1}$ (where the catalysis cross-section is $\sigma_c(v/c) = \sigma_{-28} 10^{-28}\ \mathrm{cm}^2$).

Subject headings:

elementary particles - nuclear reactions - stars: interiors

1. Introduction

Since magnetic monopoles were discovered to be a generic feature of grand unified gauge theories (GUTs) ('t Hooft 1974, Polyakov 1974), there has been considerable interest in their astrophysical and cosmological implications. GUTs predict that the very early universe underwent a symmetry-breaking phase transition at a temperature $T_c \sim M_X \sim 10^{14}$ GeV, where M_X is the mass of an associated gauge boson. When the symmetry is broken, monopoles of mass $m_M \sim M_X/\alpha \sim 10^{16}$ GeV form as topologically stable defects (where α is the gauge coupling constant). Their magnetic charge g satisfies Dirac's (1931) quantization condition $g = ng_D$ (for n integer), where $g_D = \hbar c/2e \simeq 69e = 3.3\times10^{-8}$ esu, and their mass $m_M \sim 10^{-8}$ gm is almost macroscopic. In addition, GUT monopoles are distinguished by the remarkable ability to catalyze baryon number violating reactions at a rate characteristic of the strong interactions, $\sigma_c\,(v/c) \sim 10^{-28}\ \mathrm{cm}^2$ (Rubakov 1981,1982, Callan 1982a,b; see also (, Callan and Witten 1984, Bennett 1985, Sen 1985). These features suggest that monopoles may have unusual astrophysical signatures if they are abundant today.

The theoretical predictions for the monopole abundance are problematic: in the standard cosmology, far too many monopoles survive annihilation for the universe to have reached its present state (Zel'dovich and Khlopov 1979; Preskill 1979,1983; Weinberg 1983; Lee and Weinberg 1984), while inflationary models (Guth 1981; Linde 1982; Albrecht and Steinhardt 1982), which were designed in part to alleviate this overabundance, generally leave no trace of monopoles at all (but see Turner 1982; Lazarides and Shafi 1983; Collins and Turner 1984; Lindblom and Steinhardt 1984). Although neither of these extremes is astrophysically promising, we can alternatively consider the cosmological monopole density as a free parameter and ask what observational consequences follow. That is the approach of this paper. So far, this line of reasoning has led to several theoretical arguments placing upper limits on the monopole flux. These generally fall into two categories (for reviews, see, e.g., Turner 1983a,1984). In the first, the survival of magnetic fields (either primordial or dynamo-generated) over long periods limits the rate at which they can be destroyed by monopoles; for example, the galactic field yields the Parker bound $F_M \lesssim 10^{-16}$ cm^{-2} sec^{-1} sr^{-1} (Parker 1970,1971; Turner,Parker, and Bogdan 1982). In the second category, the requirement that the luminosity produced by monopole-catalyzed nucleon decay in, e.g., neutron stars be unobservable gives a limit on the product of the flux and the catalysis cross-section $F_M \sigma_{-28} \lesssim 10^{-21}$ cm^{-2} sec^{-1} sr^{-1} (where $\sigma_c\,(v/c) = \sigma_{-28}\, 10^{-28}$ cm^2; Kolb, Colgate, and Harvey 1982; Dimopoulos, Preskill, and Wilczek 1982; Freese, Turner, and Schramm 1983; for a recent review, see Kolb and Turner 1984). Both of these limits are subject to controversy involving such issues as Landau damping of plasma oscillations (Arons and Blandford 1983; Salpeter, Shapiro and Wasserman 1982; Turner, Parker, and Bogdan 1982; Parker 1984) and the structure of neutron star interiors (Harvey 1984a,b; Harvey, Ruderman, and Shaham 1986.). [In the second category, a slightly weaker but better understood limit comes from catalysis in white dwarfs, $F_M\, \sigma_{-28} \lesssim 10^{-18}$ cm^{-2} sec^{-1} sr^{-1} (Freese 1984).] In the next few years, monopole detectors should be able to probe a flux just below the Parker limit (see Stone 1984); on the other hand, if the catalysis limits are valid, and if $\sigma_{-28} \sim 1$, the prospects for direct detection of monopoles are nonexistent.

Wherefore, then, monopoles and main sequence stars ? This paper serves two purposes. First, since stars on the main sequence (MS) have much larger surface areas, one might expect they capture many more monopoles during this period than they do as neutron stars. Assuming these monopoles survive in the star throughout the MS phase, the flux limits due to neutron star catalysis can be strengthened by up to seven orders of magnitude (Freese, Turner, Schramm 1983). Thus our first goal is to examine the conditions under which monopoles captured on the MS in fact survive. This includes consideration of such processes as $M\overline{M}$ annihilation and ejection by stellar magnetic fields. Second, given that monopoles may survive in reasonable numbers in a stellar core, we investigate their effects on the structure of the star.

The paper is organized as follows. In § II, we discuss the capture of monopoles by main sequence stars. A qualitative estimate of the number of monopoles captured is confirmed by numerical integration of the equations of motion. The numerical results are summarized in Figs.2 and 3 and Tables II and III. In § III, we describe the monopole distribution in stars for different support mechanisms. In Sec.IV we discuss the interactions of monopoles in stars, particularly the generation of energy by $M\overline{M}$ annihilation and monopole-catalyzed nucleon decay. In Sec.V we give a perturbative treatment of the effects of monopoles on stellar structure, with particular attention to changes in central temperature, luminosity, radiative stability, and neutrino emission. In § VI we derive a criterion on the magnetic field strength and geometry needed to eject monopoles from stars and discuss the result in the context of the theory and observations of stellar fields. A summary and our conclusions follow in § VII.

We establish our notation here for reference. We use cgs units throughout, with the exception that we quote the monopole mass in GeV/c^2. Most quantities will be expressed in terms of their fiducial values: the monopole mass $m = m_{16}\, 10^{16}$ GeV/$c^2 = m_{16}\, 1.77\times10^{-8}$ gm, its charge $g = 69e(g/g_D)$ in units of the Dirac charge g_D; its flux in units of the Parker flux $F_M = F_{-16}\, 10^{-16}$ cm^{-2} sec^{-1} sr^{-1}, and its velocity $\beta_M = v_M/c = \beta_{-3} 10^{-3}$ in units of the galactic virial velocity. N_M denotes the total number of monopoles in the star, n_M the monopole number density, and r_m the radius of the monopole core. The subscript n (e.g. ρ_n, v_n) refers to nucleons. The star's central density and temperature are $\rho_c = 100\, \rho_{100}$gm cm^{-3} and $T_c = 10^7 T_7$ K.

II. Capture on the Main Sequence

During the main sequence (MS) phase of stellar evolution, stars derive their energy from thermonuclear burning of hydrogen to helium. Stars on the main sequence range in mass from about 0.08 $M_\odot$ to 100 $M_\odot$, and are divided into two categories: upper ($M \gtrsim 1.2\ M_\odot$) and lower ($M \lesssim 1.2\ M_\odot$) main sequence. Because of the higher central temperatures in upper MS stars, the carbon-nitrogen-oxygen (CNO) cycle provides a substantial fraction of the nuclear energy release. This energy source is extremely centrally concentrated and gives rise in upper MS stars to a convective core surrounded by a radiative envelope. The temperatures in lower MS stars are lower, and the proton-proton (p-p) cycle serves as the primary energy source. These stars have radiative cores with convective envelopes.

The approximate scaling of various stellar parameters with increasing mass M of the star can be expressed fairly simply: luminosity is given by $L/L_\odot \simeq (M/M_\odot)^n$, where n varies ($3 < n < 5$) somewhat across the MS, the stellar radius $R/R_\odot \simeq (M/M_\odot)^{0.6}$, central temperature $T_c \sim M^{1/3}$, escape velocity from the star $v_{esc} = (2GM/R)^{1/2} \sim M^{1/5}$, and lifetime on the main sequence $\tau_{MS} \simeq 13 \times 10^9\ (M/M_\odot)^{1-n}$ yr. More precise values of these quantities calculated from detailed stellar models are given in Table I. (For further discussion of the MS stage of stellar evolution, see, e.g., Chandrasekhar 1939; Clayton 1968; Schwarzschild 1957.)

Magnetic monopoles typically move through the galaxy with the virial velocity ($v_M \approx 10^{-3}c$), or slightly faster if they have been accelerated by the galactic magnetic field (Turner, Parker, and Bogdan 1982), $v_M \approx 3 \times 10^{-3}c\ m_{16}^{-1/2}$ (for field strength $\approx 3 \times 10^{-6}$ Gauss and coherence length $\approx$ 300pc). As a monopole passes through a MS star it loses energy. If it loses all its initial kinetic energy (i.e., its energy infinitely far from the star), it is captured by the star. Since the energy loss increases with decreasing impact parameter, the number of monopoles captured by a MS star exposed to a monopole flux F for a time $\tau = \tau_{MS}$ is just the number incident upon the star with surface impact parameter less than some critical value, b_{crit}:

$$N_M = (4\pi b_{crit}^2)(\pi \text{ sr}) \left[1 + \left(\frac{v_{esc}}{v_\infty} \right)^2 \right] F\tau_{MS} \tag{2.1}$$

where v_∞ is the monopole velocity far from the star. The factor $1 + (v_{esc}/v_\infty)^2 = 1 + 2GM/Rv_\infty^2$ is just the ratio of the gravitational capture area to the geometric cross section of the star. [All quoted impact parameters refer to values at the surface, not at infinity; the critical impact parameter at infinity is $b_{crit}^\infty = [1 + (v_{esc}/v_\infty)^2]^{1/2} b_{crit}$.]

Monopoles moving through matter lose energy via several mechanisms: (i) electronic interactions; (ii) hadronic interactions; (iii) atomic transitions they induce between Zeeman-split levels (Drell, etal. 1983); (iv) direct ionization. Ahlen and Tarle´(1983) (see also Martem'yanov and Khakimov 1972; Hamilton and Sarazin 1983, and Meyer-Vernet 1985) have calculated the energy loss rate for a monopole passing through a non-degenerate electron gas and find it to be

$$\frac{dE}{ds} = 4.66 \left(\frac{g}{g_D} \right)^2 \frac{(1 + X_H)}{T_7^{\frac{1}{2}}} \ln \left(\sin \left(\frac{\psi_{min}}{2} \right) \right) \times \tag{2.2}$$

$$F(\psi_{min})\, C \left(\frac{34.4\beta}{T_7^{\frac{1}{2}}} \right) \beta\, \rho \frac{\text{GeV}}{\text{cm}}$$

where $\psi_{min} \approx 3.35 \left[\dfrac{\rho(1 + X_H)}{T_7^{\frac{3}{2}} \bar{Z}} \right]^{\frac{1}{3}}$ degrees is the minimum scattering angle (Cf. Clayton § 2.3,3.4), X_H is the hydrogen mass fraction, T_7 is the temperature in units of 10^7K, ρ is the density in gm cm^{-3}, $\bar{Z}$ is the number average atomic charge, $\beta = v_M/c$ is the instantaneous monopole velocity, and in our range of interest $F = C = 1$. Nuclear stopping power contributes at about the 5% level and has been included below.

The energy loss in Eqn.(2.2) is due to close elastic encounters and was cut off at a distance from the monopole comparable to the Debye length λ_D; since electric charge screening has no magnetic analogue, one might expect collective plasma excitations to enhance the energy loss considerably (Hamilton and Sarazin 1984). However, Meyer-Vernet (1985) and Tarle (1985) have argued convincingly that such coherent effects are destroyed by thermal scattering of the electrons. An electron plasma is characterized by the dimensionless plasma parameter $\bar{g} \equiv 1/n_e\lambda_D^3$, where n_e is the electron density (i.e., $1/\bar{g}$ is the number of electrons in a sphere with the Debye radius). The ratio of the electron Coulomb scattering frequency to the plasma frequency $\omega_C/\omega_p \sim \bar{g}$. In many plasmas, $\bar{g} << 1$ and one can treat collision effects by a perturbative expansion in $\bar{g}$. In a stellar interior, however, it is easy to show that $\bar{g} \sim 1$, and the collisionless approximation fails.

If the monopole is electrically charged (a dyon) or has previously picked up a nucleon (Bracci and Fiorentini 1984), the energy loss will be enhanced; in the rest of this paper, we shall focus on monopoles which are non-dyonic and which catalyze nucleon decay, so, modulo the subdominant mechanisms (ii),(iii), and (iv), Eqn.(2.2) represents a reliable estimate of the energy loss rate. At the very least, it represents a lower bound to the energy loss rate.

a) Approximate Analysis

With a simple approximation to Eqn.2.2, we can obtain an order of magnitude estimate for the number N_M of monopoles captured by main sequence stars and understand its dependence on stellar mass. We focus on the range $2M_\odot < M < 10M_\odot$ over which the scaling of stellar parameters (noted above) is reliable. In this interval, Table I shows that $\tau_{MS} \sim M^{-2}$, and for monopole velocity $v_\infty \lesssim v_{esc}$ ($\sim 10^{-3}c$) Eqn.2.1 gives

$$N_M \approx 3\times 10^{25}\left(\frac{b_{crit}}{R}\right)^2 \beta_{-3}^{-2}\left(\frac{M}{M_\odot}\right)^{-0.4} F_{-16}\,. \tag{2.3}$$

We now show that the ratio $(b_{crit}/R)^2$, the fraction of incident monopoles captured, has no monotonic dependence on stellar mass. A rough approximation to Eqn.(2.2) gives $dE/dx \simeq 10\rho\beta(g/g_D)^2$ GeV/cm. For a monopole that is marginally captured, $v_{esc} = (2GM/R)^{1/2}$ is a typical speed on its trajectory inside the star. Neglecting the curvature of the interior trajectory, a monopole incident at surface impact parameter b_{crit} loses a total energy

$$\Delta E \approx 20\rho_c\left(\frac{2GM}{Rc^2}\right)^{1/2}\left(\frac{g}{g_D}\right)^2 \int_{b_{crit}}^{R} f(r/R)\left[1 - \left(\frac{b_{crit}}{r}\right)^2\right]^{-1/2} dr \ \ \text{GeV} \tag{2.4}$$

where the stellar density profile is $\rho(r) = \rho_c f(r/R)$, ρ_c is the central density in gm-cm^{-3}, and the integral is in cm. Since density profiles of zero-age main sequence (ZAMS) stars of different masses appear rather similar to each other (e.g., Schwarzschild 1958, p.251), to our order of approximation we assume the profile function $f(r/R)$ is a universal function, independent of M. Then the integral in (2.4) is of the form $Rh(b_{crit}/R)$. The condition for capture, $\Delta E \geq mv_\infty^2/2 = 5\times 10^9 m_{16}\beta_{-3}^2$ GeV, gives an implicit expression for b_{crit}/R,

$$h\left(\frac{b_{crit}}{R}\right) \approx 2.5\times 10^8 m_{16}\beta_{-3}^2 c\rho_c^{-1}(2GMR)^{-1/2}\left(\frac{g}{g_D}\right)^{-2} \tag{2.5}$$

Since $\rho \sim M/R^3$ and $R \sim M^{0.6}$, the right hand side of Eqn.2.5 is independent of

M, i.e., b_{crit}/R is a function only of m, β_∞, and g/g_D. Inspection of Table II shows that this scaling result is borne out by our numerical analysis.

From Eqns.2.3 and 2.5, the expected number of monopoles captured $N_M \sim M^{-0.4}$. In the mass range 2.8-10 $M_\odot$, a fit to the numerical results shown in Figs. 2 and 3 indeed indicates an approximate power law with exponent $\simeq$ -0.4 . The qualitatively different behavior of N_M for $M \lesssim 2M_\odot$ and $M \gtrsim 10M_\odot$ reflects changes in the scaling behavior of τ_{MS} with M and changes in density profile in these regions.

The fastest monopole that a star can capture obviously passes through the stellar center (b=0) on a straight trajectory. Fitting the density profile with $f(y) = e^{-12y^2}$ (see Appendix III), Eqn.2.4 and the capture condition give the velocity of the fastest monopole stopped

$$\beta_\infty^{max} \approx 4.5\times10^{-8} m_{16}^{-1/2} \rho_c^{1/2} \left(\frac{2\pi GMR}{12c^2} \right)^{1/4} \left(\frac{g}{g_D} \right) \sqrt{\mathrm{erf}\sqrt{12}}$$

$$\approx 3\times10^{-3} \left(\frac{g}{g_D} \right) m_{16}^{-1/2} \tag{2.6}$$

again independent of M. This approximation is in good agreement with the result shown in Table III (especially the independence of M). Thus a significant fraction of monopoles with the galactic virial velocity ($\beta_\infty \sim 10^{-3}$) will be captured if m $\lesssim 10^{17}(g/g_D)$ GeV. Eqn.2.6 can be expressed more transparently as a condition on the kinetic energy

$$E_\infty^{max} \approx 5\times10^{10} \left(\frac{g}{g_D} \right)^2 \mathrm{GeV}\,. \tag{2.7}$$

b) Numerical Results

In order to follow the trajectory of the monopole through the star, we numerically integrated the following equations of motion:

$$\ddot{x} = -\frac{GM(r)}{r^3}x - \dot{x}\left(\frac{1}{mv}\frac{dE}{ds}\right) \tag{2.8a}$$

$$\ddot{y} = -\frac{GM(r)}{r^3}y - \dot{y}\left(\frac{1}{mv}\frac{dE}{ds}\right) \tag{2.8b}$$

where x and y are Cartesian coordinates centered on the star, dE/ds is given by Eqn.(2.2) with the nuclear corrections included, the instantaneous velocity of the monopole $v = (\dot{x}^2 + \dot{y}^2)^{1/2}$, and M(r) represents the stellar mass inside a radius r. The initial conditions at the stellar surface were chosen to be: $y(0) = b_s =$ surface impact parameter, $x(0) = R(1 - (b_s/R)^2)^{1/2}$, $\dot{x}(0) = -v_s =$ impact velocity, $\dot{y}(0) = 0$, $v_s = [\, v_\infty^2 + (2GM/R)\,]^{1/2}$, and, as noted earlier, $b_s = (v_\infty/v_s)b_\infty$, where v_∞ and b_∞ are the monopole velocity and impact parameter at infinity (see Fig.1). The values of M(r), ρ(r), T(r), and X_H(r) (where $r^2 = x^2 + y^2$) are interpolated from the ZAMS stellar structure models of Stromgren (1965) (reprinted in Clayton (1968)), Iben,Jr. (1965,1966) (reprinted in Novotny (1973)), and Woosley (1983) (See Table I). Those monopoles that lose all their initial kinetic energy enroute through the star are captured. In Table II we list the maximum surface impact parameter b_{crit} at which monopoles are stopped for various stellar masses (0.6 - 30 $M_\odot$), monopole masses (10^{15} – 10^{19} GeV), and monopole velocities far from the star (10^{-4} – 10^{-1} c). We also include results for monopoles of 2 units of Dirac charge, $g = 2g_D$. Two models with different compositions ($X_H = 0.6,0.7$) were used in the $7M_\odot$ case; Table II shows the insensitivity of the capture results to this variation. In Figures 2 and 3 we have plotted the number of monopoles captured over the MS lifetime as a function of stellar mass, for monopole masses (10^{15} – 10^{18}) GeV, monopole charge $g = (1\ ,\ 2)g_D$, and $v_\infty = 10^{-3}c$. The error bars in Figure 2 indicate the spread in the number captured between partially evolved and ZAMS models; again, the differences are small. In Table III, we list the velocities of the fastest monopoles stopped for different monopole masses and charges.

III. The Monopole Distribution in Stars

To determine the fate of monopoles captured in stars and to examine their effects on stellar structure, we need to estimate where they congregate in stars. As we will see in this section, monopoles are concentrated deep in the stellar core, generally between 10^2 to 10^8 cm of the center.

Once stopped within a star, monopoles fall to the center, their motion at the same time being damped by the electron drag force $F_d = dE/dx \simeq 10\rho\beta$ GeV/cm (approximating Eqn.(2.2)) on a timescale $t_d = 2m_M v_M/F_d \simeq 7\times10^2\ \rho_{100}{}^{-1}\ m_{16}$ sec. Since the capture and damping times are much shorter than other stellar timescales of interest, we can treat the monopole configuration inside the star as approximately static.

We assume throughout that monopoles never dominate the central mass density of the star, i.e., their number density n_M is less than the critical value

$$n_m{}^{crit} = \frac{\rho_c}{m_M} = 5.6\times10^9 \rho_{100}\ m_{16}{}^{-1}\ cm^{-3} = 4.8\times10^9\ m_{16}^{-1}\ (M/M_\odot)^{-0.8}\ cm^{-3} \tag{3.1}$$

In the next section, we will argue for the plausibility of this condition, which justifies the perturbative approach to the effect of monopoles on stellar structure to be used throughout. (See Fry and Fuller 1984 for a discussion of monopole stars.) Here we use this criterion to determine the monopole distribution to good approximation without considering the higher order effect of the small monopole contamination on the star itself.

The monopole distribution will be supported against gravity by pressure gradients and, in some cases, by magnetic fields. We discuss these support mechanisms in turn. In Appendix II, we show that large-scale convective motion is very probably irrelevant to the monopole distribution.

a) Pressure Support

In the absence of convection and magnetic fields, monopoles reach kinetic equilibrium with the stellar plasma at low thermal speeds, $v_{th} \simeq (3kT_c/m_M)^{1/2} \simeq 0.3(T_7/m_{16})^{1/2}$cm sec^{-1}, so the pressure of a heavy monopole gas is weak. To satisfy the condition of hydrostatic equilibrium, a self-supported monopole distribution must be confined to a region where its thermal energy can balance its gravitational potential energy, i.e., within a characteristic 'thermal' radius

$$r_{th} \approx \left(\frac{3kT_c}{2\pi G m_M \rho_c}\right)^{\frac{1}{2}} = 74\left(\frac{T_7}{m_{16}\rho_{100}}\right)^{\frac{1}{2}} \mathrm{cm} = 100\,(M/M_\odot)^{0.6}\, m_{16}^{-1/2}\ \mathrm{cm} \tag{3.2}$$

from the center of the star (Harvey 1984). Although monopoles interact with the gas and radiation in the star (the exchange of energy in this process allows the monopoles to be captured), they rapidly diffuse to the stellar center: no couplings or constraints, such as electric charge neutrality, can buoy them up. Monopoles supported by thermal pressure alone will cluster at the center of stars.

b) Local Magnetic Fields

Although their magnitude and geometry are largely unknown, the interior magnetic fields of stars are certain to affect the monopole distribution, for it is unlikely that the stellar core is completely field-free. Magnetic fields will act to disperse the monopole population, to push it out of thermal equilibrium with the star, and to separate the monopole and antimonopole populations, at least locally. One might conjecture that hidden stellar fields may be strong enough to eject monopoles from stars completely, rendering bootless the rest of our study. Here, we estimate the field strengths required for local $M\overline{M}$ separation. In §VI, we investigate large-scale fields and argue that complete ejection is unlikely in the majority of stars.

To study small-scale effects, we consider the simple case of a uniform axial field **B** or flux tube passing through the stellar center. (This idealized geometry should give a reasonable approximation over small enough distances, even for tangled fields.) We give a detailed analysis in Appendix I, which we now summarize. Monopoles and antimonopoles are pushed toward opposite poles of the tube against the forces of gravity and Coulomb attraction. Assuming a flat density profile ($\rho(r) \simeq \rho_c$) at small radii and an equal number N of monopoles and antimonopoles, the M and $\overline{M}$ distributions will be separated by an average distance $2r_m$ given by

$$F(r) = gB - \frac{4\pi}{3}G\rho_c m_m r_m - \frac{Ng^2}{4r_m^2} = 0 \tag{3.3}$$

where we have ignored the small pressure force ((a) above). Eqn.3.3 is the force at the center of each of the M and $\overline{M}$ distributions, assuming these do not overlap (see below). This cubic equation will in general have two solutions for positive r_m, the smaller one (r^-) where the **B** field roughly balances the Coulomb attraction, the larger one (r^+) where gB approximately balances gravity. The stable solution is

$$r_m^+ \approx 7\times 10^6\, B_{100}\, \rho_{100}^{-1}\, m_{16}^{-1} \left(\frac{g}{g_D}\right) \text{cm} \tag{3.4a}$$

where $B = 100B_{100}$ Gauss, and we have ignored the Coulomb term. This solution becomes unstable when the two positive roots $r^{\pm}$ become degenerate, which occurs at a minimum critical field strength $B_{crit} = 380\ \text{Gauss}\left[\left(\frac{N}{10^{24}}\right)\frac{\rho_{100}^{2}\, m_{16}^{2}}{(g/g_D)}\right]^{\frac{1}{3}}$, where the factor in brackets is of order one for a Parker flux of superheavy monopoles (recall Fig.2, which shows $N \sim 10^{24}\, F_{-16}$). An internal field of several hundred gauss can separate the M and $\overline{M}$ distributions to distances of order 10^7 cm and, as we shall see, prevent $M\overline{M}$ annihilation for $F_{-16} \lesssim 1$. For a small flux $F \lesssim 10^{-23}\ cm^{-2}\ sr^{-1}\ s^{-1} \simeq 10^{-7}F_{-16}$, a field of only ~ 1 Gauss can stabilize the M's and $\overline{M}$'s at $r_m^+ \sim 10^4$ cm, sufficient to freeze out $M\overline{M}$ annihilation. Note that we have ignored the finite radius d_m of the separated M and $\overline{M}$ distributions due to Coulomb repulsion within each distribution; in the limit $d_m << r_m^+$, we find

$$d_m \approx \left[\frac{Ng^2}{4\frac{\pi}{3}G\rho_c m_M}\right]^{1/3} \approx 10^7\ \text{cm}\left(\frac{N}{10^{24}}\right)^{1/3}\rho_{100}^{-1/3}\, m_{16}^{-1/3}\left(\frac{g}{g_D}\right)^{2/3} \quad (3.4b)$$

and one can show that $d_m <\leq r_m^+$ for $B >\geq B_{crit}$. (The effect on d_m due to non-zero monopole pressure is subdominant, since $r_{th} << d_m$.) Thus, for $B >> B_{crit}$, the separated populations will not overlap, and our use of (3.3) above is self-consistent. It is also easy to check that the magnetic field due to the $M\overline{M}$ dipole moment is negligible at the stellar surface.

In the above analysis, we have considered only static solutions; we have not searched for time-dependent solutions where coherent monopole motions may be important. However, under the assumption $B >> B_{crit}$, and thus $r_m^+ >> d_m$, one can show that the separated, non-overlapping, static M and $\overline{M}$ distributions are perturbatively stable. Therefore, more complex, time-dependent behavior (e.g., magnetic plasma oscillations, see Arons and Blandford 1983; Farouki, Shapiro, and Wasserman 1984; Turner, Parker, and Bogdan 1983) will only arise for $B \lesssim B_{crit}$; although this regime may be interesting, we do not address it here. (See Appendix I.)

For these estimates to be consistent, we should check that the coherence length of the field in the stellar interior is at least as large as r_m^+. The decay time for a field of characteristic length l is $\tau_d = 4\pi\sigma l^2/c^2$, where the plasma conductivity $\sigma \simeq 2\times10^7 T^{3/2}$ sec^{-1}. (In this subsection, we are assuming that the core is radiatively stable, so we only consider primordial magnetic fields, which have these long timescales.) For internal temperature $T_c \sim 10^7$ K, τ_d is longer than a typical lower MS stellar lifetime $\sim 10^{10}$ years only for fields with coherence length $l \gtrsim 5\times10^9$ cm. Thus, Eqn.(3.4) will be a reliable estimate as long as $r_m^+ \lesssim 5\times10^9$ cm, i.e., for field strengths $B \lesssim 50$ kG . In this discussion, we have neglected the instability of the field configuration to buoyancy and diffusion; it is conceivable that these effects could leave the central region relatively field-free for at least some time periods.

IV. Interactions of Monopoles in Stars

Given the monopole distribution as a function of support mechanism outlined above, we can ask how this population evolves over the life of the star. In this section, we consider $M\overline{M}$ annihilation and the generation of energy by various processes involving monopoles (catalysis of nucleon decay and annihilation). We will also find the conditions necessary for the validity of the perturbative criterion $n_M < n_M^{crit}$ of Eqn.3.1 .

a) Annihilation

The rates for monopole-antimonopole annihilation have been given by Dicus, Page and Teplitz (1982); since the thermal monopole velocity $\beta_{th} << \alpha^{5/4}$ (where $\alpha = 1/137$), the relevant 2- and 3-body quantum recombination cross-sections are

$$(\sigma v)_2 = 2^5 3^{-3/2}\pi\, m_M^{-2} g^6 v_M^{-1}\, (\hbar c)^{-1} c^{-2} \ln\, [g^{-5/2}(\hbar c)^{5/4}/(v_M/c)]$$

$$= 1.2\times 10^{31}\, m_{16}^{-3/2}\, T_7^{-1/2}\, (g/g_D)^6\ cm^3\ sec^{-1} \qquad (4.1)$$

$$(\sigma v)_3 = n_M\, m_M^{-1/2}\, \alpha^{-5}\, (kT)^{-9/2}\, (\hbar c)^5 \tag{4.2}$$

$$= 8.6\times 10^{-29} \bar{n}_M\, m_{16}^{-1/2}\, T_7^{-9/2}\ \mathrm{cm}^3\ \mathrm{sec}^{-1}$$

where we have used $v_m = v_{th} \simeq (3kT/m_M)^{1/2}$, the logarithm in Eqn.4.1 was evaluated at $T_7 = m_{16} = g/g_D = 1$, and the monopole number density $n_M = \bar{n}_M\ \mathrm{cm}^{-3}$. From Eqns.4.1 and 4.2, the 3-body rate dominates for $\bar{n}_M m_{16} T_7^{-4} > 1.4\times 10^{-3}$; since, in practice, annihilation is relevant only if $\bar{n}_M >> 1$ (see below), one can show that 3-body recombination is the most important annihilation mechanism for $m_M \gtrsim 10^{10}\ \mathrm{GeV}/c^2$ (using $T_c < 10^9$ K for all stars on the main sequence). That is, when annihilation is important, the 3-body rate dominates.(This is not true in the very advanced stages of stellar evolution or in neutron stars.)

1.Main Sequence

If annihilation occurs, the monopole population in the star reaches a plateau at a value N_M^{eq}, where the annihilation and accretion rates balance,

$$\frac{dN_M}{dt} = \frac{N_M^{cap}}{\tau_{MS}} - N_M^{eq}\, n_M^{eq}\, (\sigma v)_3 = 0\ . \tag{4.3}$$

Here, N_M^{cap} is the total number of monopoles captured over the main sequence lifetime τ_{MS} (see Figs.2 and 3). Using the approximate expression (2.3) for N_M^{cap}, assuming for simplicity that the monopoles are uniformly distributed inside a radius $r_m = \bar{r}_m$ cm, and substituting Eqn.4.2 into Eqn.4.3 gives the equilibrium number density of monopoles

$$n_M^{eq} = 1.2\times 10^{12}\ \mathrm{cm}^{-3}\ \bar{r}_m^{-1} \bar{F}_{-16}^{1/3}\, m_{16}^{1/2}\, (M/M_\odot) \tag{4.4a}$$

and a total number

$$N_M^{eq} = 5.2\times10^{12}\, \bar{r}_m^2\, \bar{F}_{-16}^{1/3}\, m_{16}^{1/2}\, (M/M_\odot) \tag{4.4b}$$

where we have defined the effective flux parameter $\bar{F}_{-16} \equiv F_{-16}\, \beta_{-3}^{-2}\, (b_{crit}/R)^2$. In §III, we estimated the radius r_m of the monopole core for different support mechanisms; using Eqns.3.2 in Eqn.4.4b gives

$$N_M^{eq} \approx 5\times10^{16}\, (M/M_\odot)^{2.2}\, m_{16}^{-1/2}\, \bar{F}_{-16}^{1/3} \qquad \text{(thermal)} \tag{4.5a}$$

$$\approx N_M^{cap}, \qquad B > B_{crit} \qquad \text{(magnetic)} \tag{4.5b}$$

These results are summarized in Figure 4.

Note that, through the use of (2.3), we have assumed the scalings appropriate for the mass range $2M_\odot \lesssim M \lesssim 10M_\odot$ and that $v_\infty \lesssim v_{esc}$. Eqn.4.2 gives the 3-body recombination rate into a $M\bar{M}$ bound state; in applying it to Eqns.4.3 and 4.4, we have implicitly assumed that the timescale for annihilation of a bound $M\bar{M}$ pair is less than or of order the recombination timescale. This is readily verified: the capture timescale $\tau_3 \sim (n_M^{eq}\sigma v_3)^{-1} \sim 3\times10^4\, \bar{r}_m^2\, \bar{F}_{-16}^{-2/3}$ sec while the annihilation timescale is roughly the plasma damping time for bound $M\bar{M}$ orbital motion (see § III), $t_d \sim 7\times10^2$ sec.

Annihilations are unimportant if $N_M^{eq} >> N_M^{cap}$. For a thermally supported monopole distribution, comparison of Eqns.2.3 and 4.5a shows that annihilations significantly reduce the number of monopoles in the star unless $F_M \lesssim 10^{-28}$ cm^{-2} sec^{-1} sr^{-1}. Since N_M^{eq} is quadratic in r_m, however, any mechanism for dispersing monopoles strongly reduces the annihilation rate. For example, for a central magnetic field $B > B_{crit}$ (the region of validity of our previous analysis, typically of order several hundred Gauss, see § III.b), annihilations are rendered impotent.

Since annihilations are irrelevant for $B > B_{crit}$, we can use Eqn.2.3 to reexpress the critical field as

$$B_{crit} \approx 1.1\times 10^{3} \bar{F}_{-16}^{1/3} (M/M_{\odot})^{-2/3} (g/g_D)^{-1/3} m_{16}^{2/3} \text{ Gauss.}$$

For future reference, it is convenient to express the above relation $N_M^{eq} >> N_M^{cap}$ as a condition on the monopole radius r_m ; from Eqns. 2.3 and 4.4b this is

$$r_m >> r_m^{ann} \equiv 2\times 10^{6} \text{ cm } \bar{F}_{-16}^{1/3} m_{16}^{-1/2} (M/M_{\odot})^{-0.7} . \tag{4.6}$$

2.Advanced Stellar Evolution

In the advanced stages of stellar evolution, the core undergoes a series of contractions, heating up to ignite the nuclear burning of heavier elements. First consider the case of thermal support. As the density and temperature increase, r_m^{th} and thus N_M^{eq} are reduced from their main sequence values. Although a smaller number of monopoles survives, typically the number is reduced by no more than two orders of magnitude. From Eqn.(3.2), $r_m \sim (T/\rho)^{1/2}$ and from Eqn.(4.2), $N_M^{eq} \sim r_m^2 T^{3/2} \sim T^{5/2}/\rho$. Assuming the core contracts approximately uniformly and adiabatically and using an ideal gas equation of state, $\rho \sim R^{-3}$, $T \sim R^{-1}$, so $N_M^{eq} \sim T^{-1/2}$. The central temperature in these stages of stellar evolution is generally less than $\sim$ 1000 times the average main sequence central temperature, so N_M^{eq} is reduced by less than a factor of $\sim$ 30. Eventually, the simple scaling approximation above breaks down; the post-Helium-burning evolution of massive stars consists of a complex series of convective, radiative, and degenerate burning phases. Consider the case of an $8-10\ M_\odot$ star (Nomoto 1984). During Carbon and Oxygen burning, N_M^{eq} is reduced by a factor $\simeq 50$ from its main-sequence value, and by $\simeq 275$ when the core is O + Ne + Mg. During subsequent phases, N_M^{eq} may be reduced by up to another order of magnitude; for example, during Si burning in a $25 M_\odot$ star (Weaver, Zimmermann, and Woosley 1978), $T_{Si} \simeq 10^2\ T_{MS}$ while $\rho_{Si} \simeq 10^9 \rho_{MS}$, so N_M^{eq} is reduced by $\sim 10^4$ from its main sequence value. These very advanced phases are so rapid, however, that the number of monopoles N_M does not have time to relax to the suddenly smaller values of N_M^{eq}; the actual drop in N_M through the pre-collapse phase is always less than a factor of 1000.

Since the behavior of central magnetic fields during advanced evolution is not well understood, it is not obvious what will happen to magnetic field support during core contraction. In the absence of complicating factors (see below), the high stellar conductivity will freeze the field into the fluid, and magnetic flux should be conserved; then in an adiabatic contraction $B \sim R^{-2}$, $\rho \sim R^{-3}$, so $B \sim \rho^{2/3}$; since $B_{crit} \sim \rho^{2/3}$ as well (see § III), the magnetic force remains sufficient to overcome gravity and the Coulomb attraction if it was initially. Also, from Eqns.3.4, $r_m^+ \sim B/\rho$ and $d_m \sim \rho^{-1/3}$, so $d_m/r_m^+ \sim B^{-1}\rho^{2/3} \sim$ const. Thus, if $d_m < r_m^+$ initially, it remains so, and annihilations will still be prevented by the magnetic field. In the collapse to an object of neutron star dimensions ($R \sim 10^6$ cm), the initial separating field of several hundred gauss (§ III) is amplified to $\sim 10^{12}$G, a typical value for observed pulsar fields. The actual evolution of the field is likely to be much more complicated than this (Ruderman and Sutherland 1973, Levy and Rose 1974a,b); in particular, more or less independently of the initial main sequence field, the convective motions set up in the advanced core may generate a strong central magnetic field $B \sim 10^8 - 10^9$G (which subsequently collapses to $\sim 10^{12}$G) by dynamo action.

Now consider the fate of captured monopoles upon collapse of the core to a neutron star. (The case of white dwarfs is discussed by Freese 1984.) In the absence of a central magnetic field $B_c \gtrsim 10^8$G, annihilation will catastrophically reduce the surviving population (Harvey 1984). On the other hand, if a large central field has been built up by a late-stage dynamo or by collapse amplification of an initial seed field, annihilations will be prevented in the usual way. This simple picture is complicated by the uncertainties regarding the interior structure of neutron stars. For example, in most models, the degenerate protons in the core pair to form a Type II superconductor; magnetic fields must thread the superconductor in thin vortex tubes. If a large field is present in the core before it goes superconducting, monopoles can disperse into the tubes; M's and $\overline{\mathrm{M}}$'s will occasionally annihilate if they find themselves in the same tube, but a large fraction will survive. Their subsequent evolution is discussed by Harvey (1984). If the core is superconducting but without a large field, there are three possibilities: i) superconductivity (SC) occurs well after the collapse,

in which case annihilation destroys the population in the meantime; ii) SC occurs quickly and nucleates from the center outward, expelling monopoles from the core or surrounding them with flux tubes; iii) SC occurs quickly, nucleating inward. In the last case, since the majority of monopoles have field lines which penetrate the surface of the star and since flux lines cannot be broken, monopoles will also presumably form into their own flux tubes, thereby escaping annihilation. In both cases (ii) and (iii), the majority of monopoles survive.

(For further discussion of monopoles in neutron stars, see Harvey, Ruderman, and Shaham 1986.)

b) Self-Consistency of the Perturbative Treatment

We can now verify the claim of § III that the condition $\rho_{\rm mon} < \rho_{\rm nuc}$ is very likely to hold; this requires $\min[n_M^{\rm cap}, n_M^{\rm eq}] < n_M^{\rm crit} = 5.6\times10^9\, m_{16}^{-1}\, \rho_{100}\,{\rm cm}^{-3}$. From Eqn.4.6, for $r_m > r_m^{\rm ann}$ we have $n_M^{\rm cap} < n_M^{\rm eq}$; in this case, $n_M^{\rm cap} \approx N_M^{\rm cap}/r_m^3 < N_M^{\rm cap}/(r_m^{\rm ann})^3 \approx 10^5\,{\rm cm}^{-3}$ (independent of F_M, where we have taken $N_M^{\rm cap} \simeq 10^{25}\,F_{-16}$), which is less than $n_m^{\rm crit}$ as long as $m_{16}/\rho_{100} \lesssim 5.6\times10^4$.

Thus, Eqn.3.1 could only be violated for $r_m < r_m^{ann}$,i.e., annihilations always set in before monopoles reach the critical density n_m^{crit}. To satisfy (3.1), we need only impose $n_M^{eq} < n_M^{crit}$, which becomes, from (4.4a),

$$r_m > r_m^{crit} \approx 2\times 10^2 \text{ cm } \bar{F}_{-16}^{1/3}\, m_{16}^{7/6}\, (M/M_\odot)^{1.8} \tag{4.7}$$

For $F_{-16} \lesssim 1$, Eqns.3.4 show that this condition is easily satisfied if magnetic fields are present. For a thermal monopole distribution, Eqns.3.2 and 4.6 imply that the ratio

$$\frac{r_m^{th}}{r_m^{crit}} \approx 0.4\, \bar{F}_{-16}^{1/3}\, m_{16}^{-5/3}\, (M/M_\odot)^{-1/2} \tag{4.8}$$

is typically larger than unity for $F_M \lesssim 10^{-17}$ cm^{-2} sec^{-1} sr^{-1}. Thus rather large monopole fluxes, which appear to be ruled out on both theoretical and experimental grounds (see § I and references therein), would be needed for a critical monopole density.

In the unlikely case that $n_M > n_M^{crit}$, straightforward arguments (Seckel 1982) indicate that an approximately isothermal monopole-dominated core is unstable to gravitational collapse. (The virial equation for the monopole-nucleon gravitational system is a cubic, so it displays the 'cusp' catastrophe.) As a stellar core approaches monopole domination, however, it will relax to a stable nucleon-dominated configuration. Assuming a thermally supported nucleon-dominated system, Eqns.3.1,3.2, and 4.4a indicate that the ratio $P \equiv n_{M(th)}^{eq}/n_M^{crit} \sim T_c^{3/2}/r_m^{th}\rho_{nuc} \sim T_c/\rho_{nuc}^{1/2}$. Under a uniform contraction of the 'core' radius from R_i to R_f, $T \sim R^{-1}$, $\rho \sim R^{-3}$, and this ratio decreases by $P_f/P_i = (T_f/T_i)\,(\rho_{nuc}^i/\rho_{nuc}^f)^{1/2} = (R_f/R_i)^{1/2}$, where we have assumed the nucleons form an ideal non-degenerate gas. As n_M approaches n_M^{crit}, gravitational contraction of the monopole-nucleon core reduces the monopole density relative to the critical density, rendering the system marginally stable against further collapse; for n_M near n_M^{crit}, we expect the star to relax by slow secular mini-core contraction instead of runaway collapse. There is no difficulty with competing timescales here: for $n_M \sim n_M^{crit}$, annihilations keep n_M from growing very fast, so a slow contraction can indeed keep n_M below n_M^{crit}.

c) Energy Generation

Monopoles at the centers of stars will act as a source of energy through their annihilations and, possibly, through their catalysis of nucleon decay. [We neglect here the possibility of M catalysis of fusion (Bracci and Fiorentini 1984) and other effects connected with the binding of monopoles to nuclei (Lipkin 1983), because we assume monopoles catalyze baryon decay with a strong interaction cross section.] When monopoles and antimonopoles annihilate, at the rate $dn_M/dt = -n_M^2(\sigma v)_3$, their rest energy is thermalized, heating the core at a rate $\epsilon_{ann} \simeq -(d\rho_m/dt)1/\rho_c = m_M c^2 n_M^2(\sigma v)_3 / \rho_c$ per gram of stellar material. [ϵ is the energy released per unit mass per second.] It is clear that ϵ_{ann} will only be appreciable in the regime where annihilations are important, that is, for $r_m < r_m^{ann}$ and $n_M = n_M^{eq}$. In this case, Eqns.4.2 and 4.4a give

$$\epsilon_{ann}^{eq} \approx 3\times 10^{20} \frac{\text{erg}}{\text{gm-sec}} \,\bar{r}_m^{-3}\, m_{16}\, \bar{F}_{-16}\, (M/M_\odot)^{2.4} \tag{4.9}$$

Since it depends strongly on $\bar{r}_m$, the annihilation heat has a wide range: $\epsilon_{ann} \sim 0$ for $r_m \gtrsim r_m^{ann}$,

and for thermally supported monopoles $\epsilon_{ann}^{th} \simeq 3.8\times 10^{14}\, m_{16}^{5/2}\, \bar{F}_{-16}\, (M/M_\odot)^{6/10}$ erg/gm-sec from Eqn.3.2 . By comparison, typical core nuclear energy generation rates are $\epsilon_{pp} \simeq 10\rho_{100}T_7^4$

erg/gm-sec (for $1.1 \lesssim T_7 \lesssim 1.7$) and $\epsilon_{CNO} \simeq 8\rho_{100}T_7^{16}$ erg/gm-sec (for $2.1 < T_7 < 3.1$) for the proton-proton and CNO cycles (e.g., Schwarzschild 1957). (The effects on the star of the extra monopole heat sources will be discussed in the next section.)

As discussed in the Introduction, it is thought that grand unified monopoles catalyze the decay of nucleons with a cross-section $\overline{\sigma} \sim 10^{-28}$ cm^2 characteristic of the strong interactions. Since the energy liberated per decay is roughly the nucleon rest energy m_nc^2, and the central mass density $\rho_c \simeq \rho_n \simeq m_nn_n$ (where n_n is the nucleon number density), the power per unit mass produced by catalyzed decay is $\epsilon_{cat} \simeq n_M\sigma_c vc^2$, where $v \simeq v_n$ is a typical monopole-nucleon relative velocity. At the stellar center, the average nucleon velocity $v_n \simeq (3kT_c/m_n)^{1/2} \simeq 10^{-3}T_7^{1/2}c$. Arafune and Fukugita (1983) have shown that, for a relative velocity $\beta \sim 10^{-3}$ in Hydrogen, the catalysis cross-section is enhanced by a factor $F(\beta) \simeq 1.7\times10^2(\beta/10^{-3})^{-1}$ due to the angular momentum carried in the electromagnetic field of the monopole-nucleus system. (Note that it is possible that for $v \simeq 10^{-3}c$ there are strong interaction barriers which might depress the rate.) Since it is usually assumed that the cross-section otherwise scales with velocity as $\sigma_o = \overline{\sigma}/\beta$, where $\overline{\sigma} = \sigma_{-28}10^{-28}$ cm^2, we have $\sigma_c\beta = \overline{\sigma}\, F(\beta) = 1.7\times10^{-26}\sigma_{-28}T_7^{-1/2}$ cm^2.

We evaluate ϵ_{cat} in the two cases of (I) weak ($n_M < n_M^{eq}$, $r_m > r_m^{ann}$) and (II) strong annihilations ($n_M = n_M^{eq}$). In the first case, using $n_M \simeq 3N_M^{cap}/4\pi r_m^3$ with N_M^{cap} given by (2.3),

$$\epsilon_{cat}^{(I)} \approx 4.3\times10^{30}\ \frac{\text{erg}}{\text{gm-sec}}\ \overline{r}_m^{-3}\ \sigma_{-28}\ \overline{F}_{-16}\ (M/M_\odot)^{-0.6} \tag{4.10a}$$

In the strong annihilation regime, Eqn.4.4a gives

$$\epsilon_{cat}^{(II)} \approx 7.4 \times 10^{17} \frac{erg}{gm\text{-}sec} \bar{r}_m^{-1} \sigma_{-28} \bar{F}_{-16}^{1/3} m_{16}^{1/6} (M/M_\odot)^{0.9} \quad (4.10b)$$

and we note that, in general, $\epsilon_{cat}^{(II)} > \epsilon_{cat}^{(I)}$. As before, we can give the catalysis heat for different monopole radii r_m: from Eqn.4.10a, catalysis is negligible ($\epsilon_{cat} \sim 0$) for $r_m \gtrsim 2\times 10^{10}\, \sigma_{-28}^{1/3}\, \bar{F}_{-16}^{1/3}\, (M/M_\odot)^{-2/10}$ cm ; from Eqns.3.2 and 4.10b, for thermal support $\epsilon_{cat}^{th} \simeq 6\times 10^{15}\, m_{16}^{2/3}\, \bar{F}_{-16}^{1/3}\, (M/M_\odot)^{3/10}\, \sigma_{-28}$ erg/gm-sec ;

for the case of magnetic field support, Eqns. 3.4b and 4.10a give $\epsilon_{cat}^{mag} \simeq 7\times 10^{9}\, m_{16}\, (g/g_D)^{-2}\, \sigma_{-28}\, (M/M_\odot)^{-1.4}$ erg/gm-sec.

The preceding analysis shows that, under general conditions, the local energy generation due to annihilation and catalysis can overwhelm the ordinary nuclear rates. In the neighborhood of the small region occupied by monopoles, this will certainly affect the structure of the star (see § V). As far as external observers are concerned, however, the contribution of these processes to the total luminosity of the star is generally negligible because monopoles are confined to such a small region (on a stellar scale, they essentially form a point source). To put an upper limit on the luminosity contributed by monopoles, we assume the structure of the star does not locally adjust to reduce the large monopole heating gradient (e.g., by convection). If the energy generated is efficiently thermalized, then the annihilation luminosity is, using Eqn.4.9,

$$L_{ann} \approx \frac{4}{3}\pi r_m^3 \rho_c \epsilon_{ann}^{eq} \approx 10^{23} \frac{erg}{sec} \bar{F}_{-16} m_{16} (M/M_\odot)^{8/5} \quad (4.11)$$

independent of r_m (and thus of support mechanism). This is negligible compared to the luminosity of even the faintest stars (e.g., $L \simeq 5\times 10^{30}$ erg sec^{-1} for $M = 0.1 M_\odot$, Allen 1973, p. 209). From Eqns.4.10, $L_{cat}^{(I)} \geq L_{cat}^{(II)}$ (even though

$\epsilon_{cat}^{(II)} \geq \epsilon_{cat}^{(I)}$), so the maximum power output due to monopole catalysis of nucleon decay is

$$L_{cat} \approx 1.4 \times 10^{33} \frac{\text{erg}}{\text{sec}} \bar{F}_{-16}\, \sigma_{-28}\, (M/M_\odot)^{-1.4} \tag{4.12}$$

Since stars with mass $M \lesssim 0.08 M_\odot$ are thought to be too cool for nuclear reactions to occur, Eqn.4.12 shows that catalysis also makes no observable contribution to the luminosity of stars which shine if $F_M \lesssim 10^{-20}$ cm^{-2} sec^{-1} sr^{-1}. The effects of monopole luminosity in degenerate black dwarfs, e.g., Jupiter, deserve further study (Turner 1983b); since their structure is rather different from main sequence stars, however, we do not consider them in this paper.

V. Effects of Monopoles on Stellar Structure

In the previous sections, we treated the behavior of monopoles in a fixed background star. Here, we consider the perturbative effects of monopoles on the star itself, and conclude that the star's structure is, on the whole, negligibly changed.

a) Upper Main Sequence

As we show in Appendix II, the convection occurring in the core of a massive star ($M \gtrsim 1.2\ M_\odot$) only drags the monopoles around as a unit, without affecting the monopole distribution itself. Thus, in the following, we will assume that the monopoles act as a heat source in an otherwise convecting star. In Sec. IV, we showed that the local energy generated by thermal monopoles could be large, but that the total monopole luminosity is a negligible fraction of the stellar luminosity (for sufficiently small monopole flux). In addition, despite the fact that $\epsilon_{mon} >> \epsilon_{nuc}$, we now show that even the local effects on the structure of the star are negligible.

We consider the stellar model discussed by Chandrasekhar (1939,ch.9,§ 4), in which the energy generation is completely confined to a convective core occupying a fraction 0.17 of the stellar radius. From the condition of hydrostatic equilibrium, Chandrasekhar shows that the total luminosity $L \sim T_c^{7.5}/\rho_c^2$ (see his Eqn.200). Since radiation pressure is ignored in this model, the polytropic and ideal gas equations of state are $p_c \sim \rho_c^\gamma \sim \rho_c T_c$, so that $\rho_c \sim T_c^{1/(\gamma-1)} \sim T_c^{3/2}$, where we have used $\gamma = 5/3$ for adiabatic convection. Using this above gives $L \sim T_c^{4.5}$. Now consider the energy generation itself: from,e.g., Schwarzschild (1957) Eqn.10.15, the CNO rate can be written $\epsilon_{nuc} \simeq \rho T^\nu$ with typically $\nu \sim 16$; assuming a temperature profile $T(r) = T_c\, f(r/R)$ gives

$$L = \int_0^{0.17R} 4\pi r^2 \rho \epsilon dr = g(R)\, T_c^{\nu+3} \tag{5.1}$$

where all the radial dependence has been absorbed into g. Equating the two expressions for the luminosity, we have $gT_c^{\nu+3} = dT_c^{4.5} \equiv L_o$ (where d is an irrelevant proportionality constant). In the presence of the monopole source $L_{mon} = \alpha L_o$ ($\alpha << 1$), the luminosity of the star is modified to $g\overline{T}_c^{\nu+3} + \alpha L_o = d\overline{T}_c^{4.5} \equiv \overline{L}$, so the temperature and luminosity perturbations $\delta T_c = \overline{T}_c - T_c$, $\delta L = \overline{L} - L_o$ are, to first order,

$$\frac{\delta T_c}{T_c} = \frac{-\alpha}{\nu - 1.5}$$
$$\frac{\delta L}{L} = 4.5\,\frac{\delta T_c}{T_c} \tag{5.2}$$

The surprising feature is that, since equilibrium demands that the total luminosity rise as only a small power of T_c, the star adjusts so that the temperature and luminosity decrease in the presence of the monopole heat source. From Eqn.4.11, since $L_o \gtrsim 10^{30}$ erg sec^{-1} for all stars, $\alpha_{ann} \equiv L_{ann}/L_o \lesssim 10^{-7}\, \bar{F}_{-16}\, m_{16}$, so annihilations have no effect at all. For catalysis,

from § IIIa and IVc we have $L_{cat}^{th} \simeq 2\times10^{24}$ erg sec^{-1} $\bar{F}_{-16}^{1/3}\, \sigma_{-28}\, m_{16}^{-5/6}$, so α_{cat}^{th} is negligible; the change in central temperature

will be insignificant. We conclude that monopoles have essentially no effect on an upper MS star, either globally or locally, for a flux below the Parker bound.

b) Lower Main Sequence

For stars with radiatively stable cores, the picture is potentially more interesting, but as we shall see, the effects remain small. Since monopoles generate heat at a strong, highly localized rate, one might expect them to form a small convective core.

However, if r_m (or, in the case of magnetic support, d_m) is less than $\simeq 10^6$ cm, the analysis of Appendix II shows that dissipative effects are likely to suppress convection. This condition on r_m (d_m) always holds for thermal support and, from (3.4b), it also obtains for magnetic support if the flux is below the Parker bound. (Even if the monopole core did convect, the results of Sec. Va could be applied to show that the effects of monopoles are small, because the Chandrasekhar model also describes the case of a point energy source at the center of a convective core.)

If the monopole core is radiative (either due to dissipative effects or because ϵ_{mon} is below the limit(A.II.9), we can carry out a perturbative analysis similar to that of § Va. We consider Eddington's standard model, an n=3 ($\gamma = 4/3$) polytrope in which the quantities $\beta = p_{gas}/p_{TOT}$ and $\kappa\eta$ (where κ is the opacity and $\eta \equiv$ (L(r)/M(r))/(L/M)) are constant. For any star in radiative equilibrium, we have the luminosity formula (Chandrasekhar 1939)

$$L = \frac{4\pi cGM(1-\beta_c)}{\overline{\kappa\eta}} \tag{5.3}$$

where $\overline{\kappa\eta}$ is the pressure average of $\kappa\eta$ over the star. Using the Kramers opacity $\kappa = \kappa_o \rho T^{-3.5}$, for the standard model we have $\overline{\kappa\eta} = \kappa_c \eta_c = \kappa_o \eta_c T_c^{-3.5}$. In this model, $\rho \sim T^3$, so that $\overline{\kappa\eta} \sim T^{-1/2}$ and Eqn.5.3 gives $L = bT^{1/2}$ (b is an arbitrary constant). From here on, the argument is almost identical to § Va (see Eqns.5.1,5.2), and we only need replace the L exponent there,4.5, with 0.5, and use the fact that here $\rho_c \sim T_c^3$; the analogue of Eqn.5.2 in this case is

$$\frac{\delta T_c}{T_c} = \frac{-\alpha}{\nu + 5.5} \tag{5.4}$$

$$\frac{\delta L}{L} = \frac{1}{2}\frac{\delta T_c}{T_c}$$

where, as before, $\alpha = L_{mon}/L_o$. Again, with the inclusion of monopoles, the central temperature (and temperature gradient) drops very slightly. In this case, α can be somewhat larger than in the convective case, but the effect is still unobservable. From Eqn.4.12, for a $0.1M_\odot$ star, we have $\alpha_{cat} \simeq 4\times10^2\, \overline{F}_{-16}\, \sigma_{-28}$, and for the pp cycle $\nu \simeq 4$, so that $\delta T_c/T_c \simeq -40\, \overline{F}_{-16}\, \sigma_{-28}$, which is still negligible as long as $\bar{F}_M \lesssim 10^{-18}$ cm^{-2} sec^{-1} sr^{-1}. Thus, whether they convect or not, lower MS stars are not affected by monopoles. [This result further reinforces the monopole flux limits based upon monopole-catalyzed nucleon decay, which we discussed in § I.]

In Sec.IV, we saw that, for a Parker flux, a thermal monopole distribution can generate energy $\epsilon_{ann}^{th} \sim 10^{14}$ erg gm^{-1} sec^{-1} while even a magnetically supported distribution releases catalysis heat at a rate $\epsilon_{cat}^{mag} \sim 10^{10}\,\sigma_{-28}$ (again for $F_{-16} \sim 1$ and $B \sim 100$ G). These values are so large that one might consider the possibility of mini-explosion of the monopole core.
In fact, one can show that the expected motions are always subsonic. As a result, the monopole core will perhaps be surrounded by a heated 'deflagration' region, and it is very unlikely that a propagating shock wave develops.

c) Solar Neutrinos

The presence of monopoles will also alter the emission of neutrinos by stars, both directly, through catalysis and annihilation neutrino production, and indirectly through the change in nuclear neutrino luminosity due to the perturbation in central temperature. As in the previous sections, the latter effect is easily seen to be unimportant: assuming the neutrino luminosity $L_\nu \sim T_c^n$, the perturbation due to monopole heating is

$$\frac{\delta L_\nu}{L_\nu} = n\frac{\delta T_c}{T_c} = \frac{-n\alpha}{\nu + C} \tag{5.5}$$

where C = -1.5 (+5.5) in the convective (radiative) case (Eqns. 5.2,5.4), $n \sim 13$, and we saw before that $\alpha << 1$.

Now consider the direct high-energy neutrino flux due to catalysis in the sun. Given a solar nucleon density n_n, the catalysis rate per monopole is just $n_n \langle \sigma_c v \rangle$. Since the typical energy released in a catalysis reaction is of order the nucleon rest mass, the resulting neutrino flux at the earth is approximately

$$F_\nu^{cat} \simeq \frac{N_M^\odot n_n m_n c^2 \langle \sigma_c v \rangle f_c}{4\pi R_{es}^2 \langle E_\nu \rangle} , \tag{5.6}$$

where $f_c \sim 1$ is the average number of neutrinos produced per nucleon decay, $\langle E_\nu \rangle \sim 200$ MeV is the average neutrino energy, and $R_{es} = 1.5 \times 10^{13}$ cm is the mean earth-sun distance. Using the catalysis cross-section discussed previously, we find an expected neutrino flux at the earth $F_\nu^{cat} \simeq (N_M^\odot \sigma_{28}/10^{17})$ cm^{-2} sec^{-1}. From proton decay experiments, the flux of high energy neutrinos is known to have an approximate upper bound (comparable to the expected atmospheric neutrino flux), $F_\nu(E_\nu \gtrsim 200 \text{ MeV}) \lesssim 1$ cm^{-2} sec^{-1}. We can thus obtain an upper limit on the number of monopoles in the sun,

$$N_M^\odot \sigma_{28} \lesssim 10^{17} . \tag{5.7}$$

If monopoles and antimonopoles are magnetically separated, the number of monopoles in the sun is just the number captured, $N_M^\odot = N_\odot^{cap} \simeq 10^{41} F_M$; in this case, from (5.7), one would find an observable neutrino signal for a flux as low as $F_M \sigma_{28} \lesssim 10^{-24}$ cm^{-2} sec^{-1} sr^{-1} (Arafune and Fukugita 1983, Dar and Rosen 1984). However, this does not translate into a comparable monopole flux bound, because the monopoles may only be thermally supported. In that case, $M\bar{M}$ annihilation reduces the solar monopole number to $N_M^\odot = N_M^{eq} \simeq 10^{22} m_{16}^{-1/2} F_M^{1/3} \ll N_{cap}^\odot$ (Eqn.4.5a). Combining this with the limit of Eqn. (5.7) yields a much weaker but more reliable solar catalysis bound on the monopole flux, $F_M \sigma_{28} \lesssim 10^{-15}$ cm^{-2} sec^{-1} sr^{-1} .

If $M\bar{M}$ annihilation is so prolific in the sun, we may wonder whether annihilation rather than catalysis could generate an observable solar neutrino signal.

The neutrino flux from monopole annihilation is obtained from Eqn. (5.6), with the replacements $n_n \rightarrow n_M$, $m_n \rightarrow m_M$, $\sigma_c \rightarrow \sigma_{ann}$, $f_c \rightarrow f_{ann}$, and using $N_M^{\odot} = N_M^{eq}$. Assuming $f_{ann} \sim 1$, we find a neutrino flux $F_\nu^{ann} \simeq m_{16}^{3/2}(F_M/10^{-16})^{2/3}$. Again applying the detector limit on high energy neutrinos, one obtains a 'solar annihilation bound' comparable to the Parker limit.

VI. Ejection by Magnetic Fields

In this section, we offer some estimates of the magnetic field strength and configuration needed to eject monopoles from stars; given the complexity and uncertainty of the theory of stellar magnetic fields, these numbers are necessarily approximate. (Since the order of magnitude estimates we make will turn out to be near interesting thresholds, a more accurate picture would require detailed numerical models of interior stellar fields, models about which there is at present no consensus.)

First, consider the case in which the dominant component of the field **B** (averaged over some coherence length) is radially outward, with magnitude independent of radius. (Although this field is neither divergenceless nor everywhere continuous, it still serves as a simple model.) Monopoles accelerated from rest near the center of the star acquire kinetic energy at the surface $E_s = R\,(gB + \langle F_g\rangle_R\,) - \int_0^R (dE/dx)\,dx$, where the gravitational force averaged over the star $\langle F_g\rangle_R = -(1/R)\int_0^R (GM(r)\,m/r^2)\,dr \simeq -4GMm/R^2$, and (dE/dx) is the drag force given by Eqn.2.2. Approximating the drag integral term by Eqn.2.7, $\Delta E \simeq E_\infty^{max}/2 \simeq 2.5\times10^{10}\,(g/g_D)^2$ GeV, monopoles are ejected with escape velocity if $E_s \geq$ GMm/R , that is, for magnetic fields greater than

$$B_{ej}^{rad} \approx 7.5\times10^4\ G\left(\frac{g}{g_D}\right)^{-1}\left[m_{16} + \frac{6}{5}\left(\frac{g}{g_D}\right)^2\right] \tag{6.1}$$

(We have suppressed a weak dependence on stellar mass M.)

In this calculation, we have idealized capture and expulsion as a two-stage process: initially, with B=0, gravity and drag bring monopoles to rest at the stellar center; subsequently, the magnetic field is switched on, raising the total potential energy at the center to $U(0) > 0$, and monopoles roll (with drag) along the combined magnetic and gravitational potential to infinity (where, by definition, $U(\infty) \equiv 0$). Physically, for a homogeneous field, the total monopole energy does not increase with time, so ejected monopoles are never captured in the first place: they simply bounce off the repulsive magnetic potential at the core. Taking this subtlety into account does not substantially alter the result (6.1), but leads us to reinterpret it as saying that fields $B \sim B_{ej}$ reduce the maximum energy E_∞^{max} of captured monopoles from $\approx 5\times10^{10}$ GeV (see Eqn.2.7) to $\simeq 10^{10}$ GeV (for $g = g_D$). For example, from Eqn.2.6, for $m_{16} = 1$, this reduces the maximum capture velocity from $\approx 3\times10^{-3}$ c to $\lesssim 10^{-3}$c, the galactic virial velocity. The actual fraction of monopoles affected by the magnetic field clearly depends on the monopole velocity distribution at infinity.

Although more realistic field configurations undoubtedly give different values for B_{ej}, most of the added complications (e.g., finite coherence length of the field, incomplete flux coverage of the star, back reaction of accelerated monopoles on the field) lead to larger estimates for B_{ej}; in these cases,Eqn.6.1 represents a lower limit. The majority of stars do not appear to have such strong global magnetic fields at their surfaces: generally $\langle B(R)\rangle \lesssim 100$ Gauss. (They may, however, like the sun, have strong fields confined to a small fraction of the surface area. These are toroidal fields, though, and will be discussed separately below.) A small class of stars, particularly the peculiar A stars, have strong observed fields ranging up to the tens of kiloGauss. These would be the only candidates likely to eject superheavy monopoles by the process described above.

A simple attempt to avoid this conclusion is to invoke a very strong field which extends over a large part of the star's core, dropping near the surface to much lower values. Indeed, it has been argued (e.g., Mestel and Moss 1977) that upper MS stars may generally contain large magnetic fields concentrated deep in their interiors (with surface field anticorrelated with angular velocity). In analyzing the diffusion of dynamo-generated fields to the surface of stars, Schüssler and Pahler (1978) found that the field outside the core falls off exponentially, so it is reasonable to consider a field of the form $B(r) = B_o e^{-A(r/R)}$. (Note that a dipole field, which falls off only as a power of r, will give essentially the same result as the uniform field case considered above.) If we require that monopoles with energy $E_\infty \leq 10^{10}$ GeV (corresponding to $\beta_\infty \leq 1.4\times10^{-3}\, m_{16}^{-1}$) be expelled by the repulsive magnetic core and impose the condition that the surface field $B(R) = B_o e^{-A}$ be small (≤ 100 Gauss), then the approximate analysis of Appendix III gives $A = 14.3$ and a central field $B_o = 1.6\times10^8$ Gauss. Although below the virial limit for stellar stability (Chandrasekhar and Fermi 1953), this field configuration could most likely not be maintained in main sequence stars: the steep gradient would be unstable, and the field would diffuse to the surface in a time $t << \tau_{MS}$. (Schüssler and Pahler find typical values for A of ~ 6.) According to Parker (1979), fields stronger than $\sim 10^7$ Gauss in the solar core would be buoyant and would rapidly escape. More massive stars, with convective cores, would be even less likely to have such strong central fields: fields much greater than the equipartition value $B_{eq} \simeq 10^5$ Gauss (where $B_{eq}^2/8\pi = \rho v_{con}^2/2$ and we have used $\rho = 100$ gm cm^{-3} and the convective velocity $v_{con} = 3\times10^3$ cm sec^{-1}) would have to be primordial (rather than dynamo-generated), and would have had time to be destroyed or to escape. For example, turbulent convection can twist the field lines, reducing the coherence length of the field and causing it to decay (recall $\tau_d \sim L^2$); alternatively, convection might expel the field into the radiative envelope, from which it would diffuse to the surface. (For discussions of these effects, see Parker (1979) and Stothers (1979,1980).) We conclude that ejection by radial fields is unlikely unless the surface field is comparable to $B_{ej} \sim 10^5$ G, which is observationally ruled out in the vast majority of stars.

To model the action of toroidal field components, we consider the effect of a uniform azimuthal field on monopoles confined to the equatorial plane. We treat the magnetic and drag forces as perturbations on circular gravitational orbits. By the virial theorem, monopoles orbiting near the stellar surface at radius $r \lesssim R$ have kinetic energy $\simeq GMm/2r$. (Here and below we use the fact the $M(r) \simeq M$ is generally a very good approximation at radii $r \gtrsim R/2$.) The change in kinetic energy per orbit is just the total work done, or

$$\frac{GMm}{2r^2}\,\Delta r = 2\pi r g B - \int d\vec{r}\cdot\vec{F}_d \tag{6.2}$$

where Δr is the radial increment per orbit, $\vec{F}_d$ is the drag force, the integral is over an (almost) closed orbit, and the change in gravitational potential energy has been included. In the perturbative regime, where $\Delta E/E$, $\Delta r/r << 1$, the drag term is dominated by the azimuthal component. Thus we use (from § II) $F_d \simeq 10\rho\, \beta_\phi$ GeV/cm, where the azimuthal orbital speed is $\beta_\phi \simeq (1/c)\sqrt{GM/r} \simeq \beta_{esc}/\sqrt{2}$. Using the unperturbed orbital period $\tau \simeq 2\pi(r^3/GM)^{1/2}$, Eqn.6.2 gives the radial drift velocity

$$v_r = \frac{\Delta r}{\tau} = \frac{2gB}{m}\left(\frac{R^3}{GM}\right)^{1/2} - \left(10\rho\,\frac{GeV}{cm}\right)\left(\frac{2R}{mc}\right) \qquad (6.3)$$

We are interested in finding a condition on B such that $v_r \geq v_{esc}$. However, at such a high radial speed, we have $v_r \sim v_\phi$ so that $\Delta r/r \sim \Delta E/E \sim 1$, and the perturbative expression (6.3) is no longer reliable; escaping monopoles do not spiral adiabatically outward but instead move on a slightly curved, nearly radial path out of the star. We thus expect the ejection condition on an azimuthal field to be comparable to Eqn.6.1 for a radial field. Ignoring for the moment that Eqn.6.3 breaks down for escaping monopoles, if we set $v_r = v_{esc}$ in Eqn.6.3, we find

$$B_{ej}^{\phi} \approx 10^4\ G\left(\frac{g}{g_D}\right)^{-1}\left[m_{16} + \left(\frac{g}{g_D}\right)\frac{\rho}{10\sqrt{2}}\right] \qquad (6.4)$$

which is almost identical to Eqn.6.1 . Thus, although Eqn.6.4 formally breaks down in the limit of interest, it goes over smoothly to the expression (6.1), which gives the value of B_{ej} in both cases.

As mentioned above, the strong localized fields observed in sunspots and bipolar regions are taken as the signs of a mean dynamo-generated field of 10^3 to 10^4 Gauss in the lower part of the solar convection zone (Parker 1979). These values are close to B_{ej} and indicate that lower main sequence stars (those with dynamos possibly operating in their outer regions) could conceivably eject a significant fraction of incident monopoles. Mitigating this are the facts that the observed solar field is very inhomogeneous, $\Delta B/B \sim 1$, the drag term becomes more important as $r \rightarrow 0$, and field coverage of the convective zone is not complete (although the fields do migrate and reverse on a timescale of 10-11 years). Also, the radiative solar core *may* be relatively field-free; if so, since this is where the majority of monopoles are slowed down, most of them would not be expelled. These uncertainties, coupled to the fact that the inferred solar field values are close to the ejection threshold, make it difficult to give a reliable estimate of the fraction of monopoles ejected. Unless the ejection is so efficient as to make this fraction unity to very high precision (several decimal places), which seems very unlikely, the numbers used elsewhere in this paper will remain accurate to within an order of magnitude.

VII. Conclusions

We have traced the history of monopoles in stars from capture to the end of the main sequence. Numerical results confirm the analytic estimate that, for GUT-scale monopoles with mass $m \lesssim 10^{17}$ (g/g_D) GeV/c^2 traveling with of order the galactic virial velocity $\sim 10^{-3}c$, the number captured over the MS lifetime is $N_M \simeq 10^{25}\, F_{-16}\, (M/M_\odot)^{-0.4}$ (see Figs.2 and 3). This is significantly more monopoles than a typical neutron star captures during its lifetime as a pulsar (few$\times 10^6$ yrs) or even in the age of the galaxy; for example, pulsars capture $N_{PSR}^{cap} \simeq 10^{18}\, F_{-16}$ monopoles (see Fig.4 ; Freese, Turner, and Schramm 1983). Because of their large mass, captured monopoles gravitationally diffuse to the center of stars, forming a core of radius 10^2 - 10^7 cm, depending on support mechanism.

Although this range of five orders of magnitude is 'microscopic' compared to stellar distance scales, it covers a broad range of possibilities for the evolution of the monopole distribution. At one end, for monopoles supported by their own thermal pressure, $r_{th} \simeq 10^2$ cm; unless the flux is very low, $M\overline{M}$ annihilation drastically reduces the number of monopoles, to $N_M^{eq} \simeq 10^{16} F_{-16}^{1/3} m_{16}^{-1/2}$. The prolific annihilations generate heat at a catastrophic rate, $\epsilon_{ann}^{th} \sim 8\times10^{14} F_{-16}$ erg gm^{-1} sec^{-1}, but do not appear to qualitatively affect the luminosity or structure of the star.

At the other end, a central field of several hundred Gauss can support monopoles in a distended configuration with $r_{mag} \sim 10^7$ cm. In this case, annihilations are unimportant, and essentially all the captured monopoles survive. There may still be significant energy generation due to catalysis, $\epsilon_{cat}^{mag} \sim 10^9 m_{16} \sigma_{-28}$ erg gm^{-1} sec^{-1}, but again the star itself is unaffected. The uncertainties surrounding stellar magnetic fields hinder a detailed analysis of monopole ejection. It appears unlikely, however, that non-magnetic stars have strong hidden fields with such complete coverage that they eject all but a minute percentage of monopoles.

We also emphasize that underground observations of the high energy neutrino flux place bounds on the solar monopole abundance which are only competitive with the Parker limit. Due to $M\bar{M}$ annihilation, these solar bounds cannot reliably be made more restrictive. On the other hand, *if* magnetic fields separate monopoles from antimonopoles in the solar core, a monopole flux as low as $F_M \sim 10^{-24}$ cm^{-2} sec^{-1} sr^{-1} would generate a high energy neutrino flux from catalysis which should be separable from atmospheric background. Such a detection could shed light on conditions deep in the solar interior.

Finally, we summarize the relation of this work to the neutron star catalysis limits mentioned in the Introduction. We may interpret the upper bound on the catalysis luminosity of nearby old pulsars as a limit on the number of monopoles present $N_{mon} \lesssim 10^{12}\, \sigma_{-28}^{-1}$. Given the expression above for N_{PSR}^{cap}, this gives the usually quoted bound $F_M \sigma_{-28} \lesssim 10^{-21}$. Inclusion of MS capture strengthens this bound by a factor $\sim 10^6 - 10^7$ (the ratio of N_{MS}^{cap} to N_{PSR}^{cap}), even for thermally supported monopoles (Cf. Figure 4).

This enhancement of the neutron star limit follows even though the monopole population may be depleted during the advanced stages of stellar evolution. Consider an 8 $M_\odot$ neutron star progenitor. The discussion of Sec. 4.1 shows that $N_{eq}^{MS} \simeq 5 \times 10^{18}\, F_{-16}^{1/3} m_{16}^{-1/2}$ and, from Sec. 4.2, a lower bound on the number surviving pre-collapse evolution is $N_{eq}^{adv} \gtrsim 2 \times 10^{16}\, F_{-16}^{1/3} m_{16}^{-1/2}$. For $N_{mon} < 10^{12}$, Fig. 4 shows that $N_{eq}^{adv} > N_{cap}^{MS}$, *i.e.*, annihilations do not affect the limit. We also note that, in arriving at the factor 10^7 improvement in the NS catalysis bound from MS capture, we are assuming that monopoles which survive the MS and post-MS phases also survive the collapse to the neutron star itself. The conditions under which this assumption should hold are discussed in Sec. IVa.2; they appear sufficiently general for one to have confidence in the improved limits.

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Appendix I: Monopole Support by Magnetic Fields

To study the behavior of monopoles supported by magnetic fields at the center of stars, we adopt a two-fluid model of monopoles and antimonopoles. The equations of motion are

$$\frac{\partial \vec{v}_\pm}{\partial t} + (\vec{v}_\pm \cdot \vec{\nabla})\vec{v}_\pm = -\frac{\vec{\nabla} p_\pm}{\rho_\pm} \pm \frac{g\vec{B}}{m} - \frac{(\vec{J}_\pm \times \vec{E})}{\rho_\pm c} + \frac{\vec{F}_v^\pm}{m} + \vec{g} \qquad (A.I.1)$$

and the continuity equations

$$\frac{\partial n_\pm}{\partial t} + \vec{\nabla} \cdot (n_\pm \vec{v}_\pm) = 0 \ . \qquad (A.I.2)$$

Here, $+(-)$ refers to M's($\bar{M}$'s), the monopole mass density is $\rho_\pm = n_\pm m$, and pressure $p_\pm = n_\pm k T_\pm$ (we will assume $T_+ = T_- = T$ from now on). The magnetic field is $\vec{B} = \vec{B}_0 + \vec{B}_m$, where $\vec{B}_0$ is an 'external' field (due to ordinary electric currents) and $\vec{B}_m$ is the field due to the monopoles, and the magnetic current is $\vec{J}_\pm = \pm g n_\pm \vec{v}_\pm$. The viscous drag force, from Sec. II, is $\vec{F}_v^\pm \simeq -10\rho_c \vec{\beta}_\pm (g/g_D)^2$ GeV cm^{-1} $\equiv -K_1 \vec{v}_\pm$, and the gravitational field is $\vec{g} \simeq -(4\pi/3)G\rho_c \vec{r}_\pm \equiv -K_2 \vec{r}_\pm$.

The electromagnetic fields are governed by the Maxwell equations (assuming vanishing electric charge density),

$$\vec{\nabla} \cdot \vec{E} = 0 \ \ ; \quad \vec{\nabla} \cdot \vec{B}_m = 4\pi g(n_+ - n_-) \qquad (A.I.3)$$

$$\vec{\nabla} \times \vec{B}_m - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = 0 \qquad (A.I.4)$$

$$-\vec{\nabla} \times \vec{E} - \frac{1}{c}\frac{\partial \vec{B}_m}{\partial t} = \frac{4\pi g}{c}(n_+ \vec{v}_+ - n_- \vec{v}_-) \ . \qquad (A.I.5)$$

Now consider the form of the external field $\vec{B}_0$ expected near the center of stars. In this discussion, we focus on lower main sequence stars, with cores which are stable against convection. In this case, no turbulent dynamo operates, and any field present must be primordial. In Sec. III.b, we show that such a field must have a coherence length $l \gtrsim 5 \times 10^9$ cm, in order to be stable against decay over the MS lifetime. A continuous field distribution can be viewed as a system of axial flux tubes, which each exert a pressure $B^2/8\pi$ and tension $B^2/4\pi$ (Parker 1979). If the tube radius $R \lesssim l$, the field strength B_0 can be approximated as uniform over the tube cross-section. We are interested in the dynamics of monopoles over length scales $r_m \lesssim 10^7$ cm, much smaller than the minimum coherence length above and also much less than the pressure scale height in the stellar interior. The monopole-antimonopole population thus lives deep inside a flux tube, $r_m \ll (R, l)$, so the field strength is constant to excellent approximation over the scale r_m. We shall therefore assume a static axial field $\vec{B}_0 = B_0\hat{z}$, with B_0 a constant over the scales of interest. We note that such a field configuration is not absolutely stable: in addition to resistive decay, the flux tube will eventually float away from the core due to magnetic buoyancy. However, for a stable radiative interior and a field of order a few hundred Gauss, the characteristic rise velocity is tiny, and the tube is essentially anchored over the MS lifetime (Parker 1979).

We now look for solutions to Eqns. A.I.1-5 which are static, $v_\pm = 0$, with monopoles and antimonopoles separated (non-overlapping). From Eqn.(3.2) the thermal kinetic energy of a monopole is negligible compared to its gravitational potential energy at radii $r \gg r_{th} \sim 100$ cm. Since the radii and separation of the M and $\bar{M}$ distributions will turn out to be much larger than r_{th}, we can selfconsistently neglect the pressure gradient term in (A.I.1) in determining them. The static equations of motion are then

$$\vec{r}_+ = \frac{g}{K_2}(\vec{B}_0(\vec{r}_+) + \vec{B}_m(\vec{r}_+)) \ \ ; \ \ \vec{r}_- = \frac{-g}{K_2}(\vec{B}_0(\vec{r}_-) + \vec{B}_m(\vec{r}_-)) \qquad (A.I.6)$$

$$\vec{E} = 0 \ \ ; \ \ \vec{\nabla} \times \vec{B}_m = 0 \ \ ; \ \ \vec{\nabla} \cdot \vec{B}_m = 4\pi g(n_+ - n_-) \ . \qquad (A.I.7)$$

These equations possess solutions which are axisymmetric; each distribution is approximately an ellipsoid of revolution about the $\hat{z}$ axis. The mean separation $2r_m$ of the M and $\bar{M}$ distributions is determined by the balance between gravity and the external magnetic field. For well-separated distributions, the size d_m of each distribution is determined primarily by the balance between magnetic Coulomb repulsion and gravity, while the Coulomb attraction between the two distributions tidally distends them along the $\hat{z}$ axis. For large separation, $d_m \ll r_m$, the tidal distortion is small, and we can treat each distribution as spherically symmetric to first approximation. Using the coordinates of Fig. 5, for monopoles Eqns.(A.I.6-7) become

$$F_x = -K_2 x + \frac{g^2(N-1)x}{d_m^3} - \frac{g^2 N x}{[(2r_m + z)^2 + x^2]^{3/2}} = 0 \qquad (A.I.8)$$

$$F_z = -K_2(z + r_m) + \frac{g^2(N-1)z}{d_m^3} - \frac{g^2 N(2r_m + z)}{[(2r_m + z)^2 + x^2]^{3/2}} + gB_0 = 0 \ , \quad (A.I.9)$$

with similar expressions for anti-monopoles.

If the distributions are assumed, in addition, to have uniform density, then Eqns.(A.I.8-9) hold at arbitrary points (x, z) inside each distribution [Fig. 5];

with the weaker assumption of spherical symmetry only, Eqns.(A.I.8-9) hold at points on the surface of each distribution. Assuming well-separated (non-overlapping) distributions, $d_m \ll r_m$, the condition for equilibrium in the x direction becomes $K_2 \simeq g^2(N-1)/d_m^3$, which sets the size d_m of each distribution; this is the origin of Eqn.3.4b. Substituting this into (A.I.9) and again using the fact that $(x,z) \leq d_m \ll r_m$, we obtain

$$\frac{g^2 N}{4r_m^2} + K_2 r_m - gB_0 = 0 \; , \qquad (A.I.10)$$

which is equivalent to Eqn. 3.3. (We note that relations A.I.8,10 hold approximately independently of x and z, so that such extended solutions exist.) Eqn.(A.I.10) may be written as $dU/dr_m = 0$, where the effective potential $U(r_m) = K_2 r_m^2/2 - g^2 N/4r_m - gB_0 r_m$. As $r_m \to 0$, the potential is unbounded from below, and it has in general two extrema for positive radii, $r_m^{\pm}$. An inspection of $U(r_m)$ shows that the stable extremum (local minimum) corresponds to $r_m^{+} \simeq gB_0/K_2$. A sufficient condition for this minimum to exist is that the external field $B_0 \gg B_{crit}$ (see Sec.III). Note that the only other condition on $\vec{B}_0$ is that it vary over a scale much larger than r_m. (As we argued above, this must hold for primordial fields anchored in stellar interiors.) Thus we have demonstrated that the separated solution is stable against 'radial' perturbations of the center-of-mass position of each distribution. A similar analysis shows stability against 'transverse' (*i.e.*, x direction) perturbations. Clearly stability holds only if the amplitude of the radial perturbation is less than $r_m^{+} - r_m^{-}$, where r_m^{-} is the local maximum of U.

To show that the solution above represents a true equilibrium point, we now consider more general perturbations about the separated solution. For small amplitudes, the perturbative solutions are oscillations which are damped by the viscous drag force. Since we find no growing modes, the asymptotic solution at times $t \gg \tau_{damp}$ should be just the static equilibrium found above. Our intuitive picture is that monopoles thermally fluctuate in the effective potential $U(r_m)$; these thermal motions have only small amplitude [Eqn. 3.2], insufficient to surmount the barrier at r_m^-.

Consider the equations of motion (A.I.1-5), linearized about the static solution (A.I.6-7). We neglect perturbations in temperature and, since we assume the mass density of monopoles is everywhere less than that of the star (Eqn. 3.1), we also ignore changes in the gravitational field. The resulting first order equations are

$$\frac{\partial n_+^{(1)}}{\partial t} + n_+^{(0)} \vec{\nabla} \cdot \vec{v}_+ = 0 \tag{A.I.11}$$

$$\frac{\partial \vec{v}_+}{\partial t} + \frac{kT}{m n_+^{(0)}} \vec{\nabla} n_+^{(1)} - \frac{g \vec{B}_m^{(1)}}{m} + \frac{K_1 \vec{v}_+}{m} = 0 \tag{A.I.12}$$

where superscripts (0) and (1) indicate unperturbed and first order perturbative quantities; similar equations apply for antimonopoles. The perturbed field equations are

$$\vec{\nabla} \cdot \vec{E} = 0 \ ; \quad \vec{\nabla} \cdot \vec{B}_m^{(1)} = 4\pi g (n_+^{(1)} - n_-^{(1)}) \tag{A.I.13}$$

$$\vec{\nabla} \times \vec{B}_m^{(1)} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = 0 \tag{A.I.14}$$

$$-\vec{\nabla} \times \vec{E} - \frac{1}{c} \frac{\partial \vec{B}_m^{(1)}}{\partial t} = \frac{4\pi g}{c} (n_+^{(0)} \vec{v}_+ - n_-^{(0)} \vec{v}_-) \ . \tag{A.I.15}$$

Since (A.I.12) does not contain the electric field, we look for solutions which describe longitudinal magnetostatic oscillations, *i.e.*, we set $\vec{E} = 0$. Combining Eqns.(A.I.11-15) yields a second order equation for the density fluctuation

$$\frac{\partial^2 n_+^{(1)}}{\partial t^2} - \frac{kT}{m}\nabla^2 n_+^{(1)} + \frac{4\pi g^2 n_+^{(0)} n_+^{(1)}}{m} + \frac{K_1}{m}\frac{\partial n_+^{(1)}}{\partial t} = 0 \ , \qquad (A.I.16)$$

with a similar equation for $\vec{B}_m^{(1)}$. (Since we are considering only small amplitude (linear) perturbations, we continue to treat the M and $\bar{M}$ distributions as non-overlapping.) The solutions to (A.I.16) are damped waves with dispersion relation

$$\omega(k) = \frac{1}{2}\left[\frac{iK_1}{m} \pm \left(\frac{4kT}{m}k^2 + \frac{16\pi g^2 n_+^{(0)}}{m} - \frac{K_1^2}{m^2}\right)^{1/2}\right] \qquad (A.I.17)$$

For $m \gtrsim 10^{14}$ GeV, the square root is real, so that Im $\omega(k) = K_1/2m$; this corresponds to a damping time $\tau_{damp} \simeq 700\rho_{100}^{-1}m_{16}$ sec (see Sec. III) due to viscous drag. Note that, unlike the case of Landau damping (which will also be present here), Im $\omega(k)$ is independent of wavenumber, so that all modes are damped at the same rate. This conclusion can be extended to transverse waves ($\vec{E} \neq 0$) as well.

Appendix II: Monopoles and Convection

In this Appendix, we discuss the role of convection in determining the stellar distribution of monopoles and conclude that the effects are negligible. Recall that upper main sequence stars ($M \gtrsim 1.2 M_\odot$) are generally believed to have convective cores. In this case, monopoles could potentially be 'mixed' over a volume of radius $r_{con} \gg r_{th}$ due to the drag force exerted by rising bulk fluid elements. Following Schwarzschild (1957), a small superadiabatic temperature gradient leads to an average convective velocity $\bar{v}_{con} \simeq 3 \times 10^3 \ (M/M_\odot)^{5/6}$ cm/sec. The drag force of a rising fluid with this velocity on a stationary monopole, $dE/dx \simeq 10\beta_{con}\rho(g/g_D)^2$ GeV/cm, overpowers the gravitational force $F_g \simeq -(4\pi/3)G\rho m_M r$ out to a radius

$$r_{con} \simeq 3.2 \times 10^5 \left(\frac{M}{M_\odot}\right)^{5/6} m_{16}^{-1} \left(\frac{g}{g_D}\right)^2 \text{ cm} . \qquad (A.II.1)$$

However, we cannot conclude from this that monopoles are in fact mixed out to r_{con}. The relevant quantity for mixing is the velocity gradient. Above, $\bar{v}_{con}$ is the mean velocity gradient on the mixing length scale, $l \sim R_\odot$, but it is well known that the gradient decreases for smaller eddy sizes, eventually dropping to zero at the minimum eddy size r_{crit} due to dissipation. We now show that, in general, $r_{crit} > r_{con}$, *i.e.*, there is no power in the convection on scales small enough to mix the monopoles.

The usual estimate of the minimum eddy scale proceeds from the Kolmogorov-Obukhov law; in that analysis, r_{crit} is determined by the viscosity ν. For convection to occur, however, the superadiabatic gradient

$$D \equiv -\frac{1}{T}\frac{dT}{dr} + \left(1 - \frac{1}{\gamma}\right)\frac{1}{p}\frac{dp}{dr} \qquad (A.II.2)$$

must be large enough to overcome the effects of both viscosity and thermal conduction. (Here, γ is the usual ratio of specific heats.) That is, the usual Schwarzschild criterion for convective instability, $D > 0$, is not *sufficient* for convection to actually take place. Instead, if we define the Rayleigh number for a fluid layer of thickness r,

$$R \equiv \frac{|\vec{g}| D r^4}{\chi \nu} , \qquad (A.II.3)$$

then convection generally sets in for $R \geq \mathcal{O}(10^3) \equiv R_{crit}$ (Chandrasekhar 1961). Consider a $5M_\odot$ star (see Table 1): near the center, the gravitational acceleration $|\vec{g}| \simeq (4\pi/3)G\rho_c r$, and the central density $\rho_c \simeq 21$ gm cm^{-3}; the thermometric conductivity $\chi = K/\rho c_p = (4acT_c^3/3\kappa\rho_c^2 c_p) \simeq 4 \times 10^7$ cm^2 sec^{-1}, where K is the coefficient of heat conduction, κ is the radiative opacity, c_p is the specific heat at constant pressure, and a is the Stefan-Boltzmann constant; the superadiabatic gradient is typically $D \simeq 10^{-16}$ cm^{-1}; the radiative viscosity $\nu_r \simeq (aT_c^4/4c\kappa\rho_c^2) \simeq$ 85 cm^2 sec^{-1}; and the electron viscosity $\nu_e \simeq n_e k T_c \tau \simeq 3.2$ cm^2 sec^{-1}, where τ is the mean free time (relaxation time) for electron-ion collisions in a screened plasma. Putting this all into Eqn. A.II.3 gives a condition on the radius,

$$r_{crit} \simeq \left(\frac{3\chi\nu_r R_{crit}}{4\pi G\rho_c D}\right)^{1/5} \simeq 6 \times 10^6 \text{ cm} , \qquad (A.II.4)$$

which we take as a rough estimate of the minimum eddy scale. Comparison with (A.II.1) shows that this scale is too large to affect the monopoles. The effect of large scale convection will be to move the monopole core as a whole, without disrupting it.

The discussion above applies to the case of monopoles in a star with large-scale convective motions already present. For stars with radiatively stable cores, there is another potential effect to consider. Since monopoles generate heat by annihilation or catalysis at a strong, highly localized rate, one might expect the resulting steep temperature gradient to be convectively unstable. As a consequence, the monopoles would surround themselves with a small convective core. To consider this, we first recall that the equations of stellar structure (assuming radiative stability) can be expanded near the center ($r \sim 0$, $\rho \simeq \rho_c$) as

$$M(r) \approx \frac{4\pi}{3} r^3 \rho_c \tag{A.II.5}$$

$$p(r) \approx p_c - \frac{2\pi}{3} G\rho_c^2 r^2 \tag{A.II.6}$$

$$L(r) \approx \frac{4\pi}{3} r^3 \rho_c \epsilon_c \tag{A.II.7}$$

$$T(r) \approx T_c - \frac{\kappa_c \epsilon_c \rho_c^2 r^2}{8acT_c^3} \tag{A.II.8}$$

As $r \to 0$, both dT/dr and $dp/dr \sim r$ go to zero; at finite r, $dT/dr \simeq (T-T_c)/r$, $dp/dr \simeq (p-p_c)/r$, and using (A.II.6) and (A.II.8), we can reformulate Schwarzschild's criterion, $D > 0$, as

$$\epsilon_c \leq \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{16\pi G}{3}\right)\frac{acT_c^4}{\kappa_c \rho_c} \approx 10 T_7^4 \kappa_1^{-1} p_{17}^{-1} \, \frac{\text{erg}}{\text{gm-sec}} \qquad \text{(A.II.9)}$$

where the central opacity $\kappa_c = \kappa_1 \, \text{cm}^2/\text{gm}$, and the central pressure $p_c = p_{17} 10^{17}$ dyne cm^{-2}. From § IV, the expressions for $\epsilon_{\text{ann}}^{\text{th}}$ and $\epsilon_{\text{cat}}^{\text{th}}$ show that for a flux $F_M \gtrsim 10^{-30}\,\text{cm}^{-2}\,\text{sec}^{-1}\,\text{sr}^{-1}$ (annihilation) or $F_M \gtrsim 10^{-59}\,\sigma_{-28}^{-3}\,\text{cm}^{-2}\,\text{sec}^{-1}\,\text{sr}^{-1}$ (catalysis), a thermal monopole distribution will violate (A.II.9) due to annihilations or catalysis, and the system is convectively unstable.

However, this argument neglects the effects of dissipation. The size of the monopole core is set by r_{th} (thermal support) or d_m (magnetic support), and this radius is also expected to set the scale over which the temperature gradient is large. However, from (3.2) and (3.4b), this scale is always smaller than the minimum eddy size r_{crit} estimated above (assuming a flux $F_M < 10^{-16}\,\text{cm}^{-2}\,\text{sec}^{-1}\,\text{sr}^{-1}$). Thus, we expect dissipation to suppress monopole-induced convection.

Appendix III: Radial Magnetic Field

In this Appendix, we calculate the parameters of an exponentially decreasing radial magnetic field $\mathbf{B}(\mathbf{r}) = B_o\, e^{-A(r/R)}\, \hat{r}$ which would eject a significant fraction of monopoles from the sun. We assume monopole mass $m = 10^{16}\,\text{GeV}/c^2$ and charge $g = g_D$. The repulsive magnetic potential ejects monopoles by keeping them out of the stellar core, where they would otherwise efficiently lose energy to electrons. Recall that a monopole is not captured if its total energy loss through the star is less than its initial kinetic energy, $\Delta E \lesssim E_\infty$. Assuming the monopole velocity distribution is peaked near the galactic virial velocity $v_\infty \approx 10^{-3}$ c, which corresponds to $E_\infty \approx 10^{10}$ GeV for $m_{16} = g/g_D = 1$, a significant number will be expelled if $\Delta E \lesssim 10^{10}$ GeV. This will give a condition on the radius r_b of the core from which monopoles must be excluded for expulsion. From § IIa, the approximate energy loss is $dE/dx \simeq 10\rho\,\beta$ GeV/cm (to within an order of magnitude). For monopoles with the virial velocity, the speed inside the star is always close to β_{esc} ; we approximate the density profile by $\rho(r) = \rho_c\, e^{-12(r/R)^2}$ (typically accurate to 50%, although some upper MS models have broader distributions). The approximate energy lost by a monopole which bounces off the magnetic core at $r = r_b$ is then (in GeV)

$$\Delta E \approx 20\rho_c\beta_{esc}\int_R^{r_b} e^{-12(r/R)^2}\,dr = 5R\beta_{esc}\rho_c\sqrt{\frac{\pi}{3}}\left[1 - \mathrm{erf}(2\sqrt{3}r_b/R)\right]$$

(where R is in cm, ρ in gm/cm^3 , and erf(x) is the error function). Using typical stellar values $R \simeq 7\times10^{10}$, $\beta_{esc} \simeq 2\times10^{-3}$, $\rho_c \simeq 100$, and the previous condition $\Delta E < 10^{10}$ GeV, Eqn.A.1 gives $r_b/R \simeq 0.4$. To be expelled, monopoles must bounce off the potential U(r) at r_b , so we require $U(r_b) \geq E_{mon}(r_b) = E_\infty - \Delta E/2 = 5\times10^9$ GeV. The total magnetic and gravitational potential is $U(r) \simeq (gB_o R/A)\,(e^{-A(r/R)} - e^{-A}) - GMm/r$ (where the approximation to the gravitational term is accurate to $\sim 10\%$ down to $r \sim r_b$). Using the bounce condition on $U(r_b)$, solar values for M and R, and imposing the condition that the surface field be small, $B(R) = B_o\, e^{-A} \leq 100$ G, gives (100 Gauss/A)gR $(e^{0.6A} - 1) \geq 5.5\times10^{10}$, which has the solution $A = 14.3$, $B_o = 100e^A = 1.6\times10^8$ Gauss. This estimate is subject to considerable uncertainty, since B_o is exponentially dependent on A and $\rho(r)$ is also fitted with an exponential. We believe, however, that the value for r_b, and thus for A and B_o, are lower limits.

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Figure Captions

[1] Monopole trajectory through the star and coordinate system described in § II. Here v_∞ and b_∞ (v_s and b_s) are the velocity and impact parameter of the monopole at infinity (surface). Note that the coordinate system is oriented so that $\vec{v}_s = -v_s\hat{e}_x$.

[2] The number of monopoles captured, in units of the Parker flux, as a function of stellar and monopole mass, for monopoles with Dirac charge and velocity far from the star $\beta_\infty = 10^{-3}$. The error bars for the $5M_\odot$ and $15M_\odot$ data indicate the spread in the number captured between evolved and ZAMS models (all other models are ZAMS). In the $25M_\odot$ results, error bars show the spread between the Woosley (1983) model and a polytrope with index $n = 3$.

[3] Same as figure 2, for monopoles with twice the Dirac charge.

[4] The number of monopoles surviving in a $8M_\odot$ star as a function of the monopole flux, in units of the Parker flux. For this plot we have taken $g = g_D$, $m_{16} = 1$, and $\beta_\infty = 10^{-3}$. N_{TH} is the equilibrium monopole abundance in the case that monopoles are supported against gravity by thermal pressure, see § III. N_{MS}^{cap} is the number of monopoles captured by the star during its MS lifetime (as in Fig.2). For reference, the number of monopoles captured by a few$\times 10^6$ yr old pulsar is shown as the broken line labeled N_{PSR}^{cap} . Einstein observations of several old, nearby radio pulsars restrict the number of monopoles in these objects to be $\lesssim 10^{12}\,\sigma_{-28}^{-1}$ (Freese, Turner, and Schramm 1983), resulting in a monopole

flux limit $F_M\,\sigma_{-28} \lesssim 10^{-21}$ cm^{-2} sr^{-1} sec^{-1} (shown by the dotted lines). Taking into account the monopoles captured by the MS progenitors of these pulsars improves this limit by a factor of $O(10^7)$ (shown by the dotted line). The approximate scalings of N_{TH}, and N_{MS}^{cap} with stellar mass and monopole properties are given in Eqns.4.5a and 2.3.

[5] Cross-section of coordinate system for monopole-antimonopole separation by an external magnetic field $\vec{B}_o$. The average $M\bar{M}$ separation is r_m; d_m is the radius of each distribution. The (z, x) coordinates of an arbitrary point P are measured from the center C of the distribution.

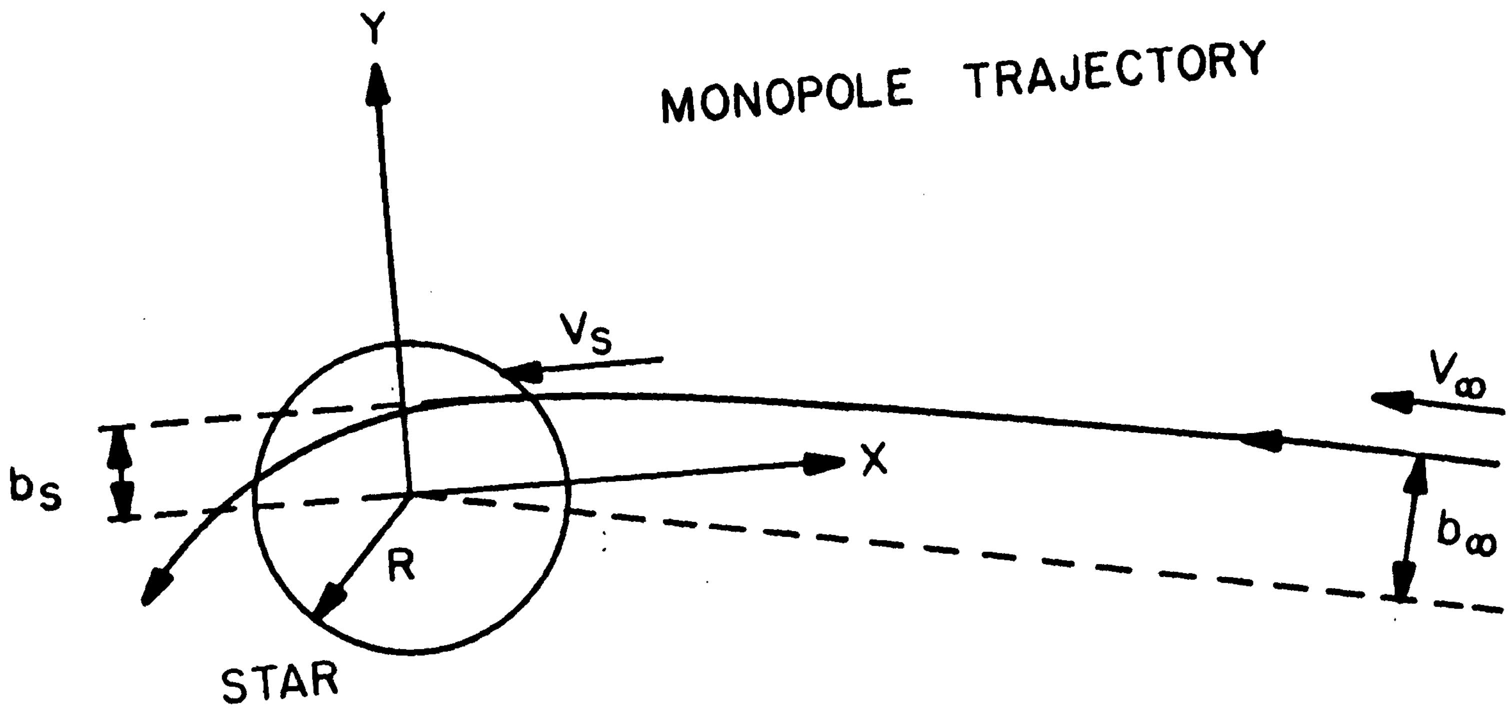


FIGURE 1

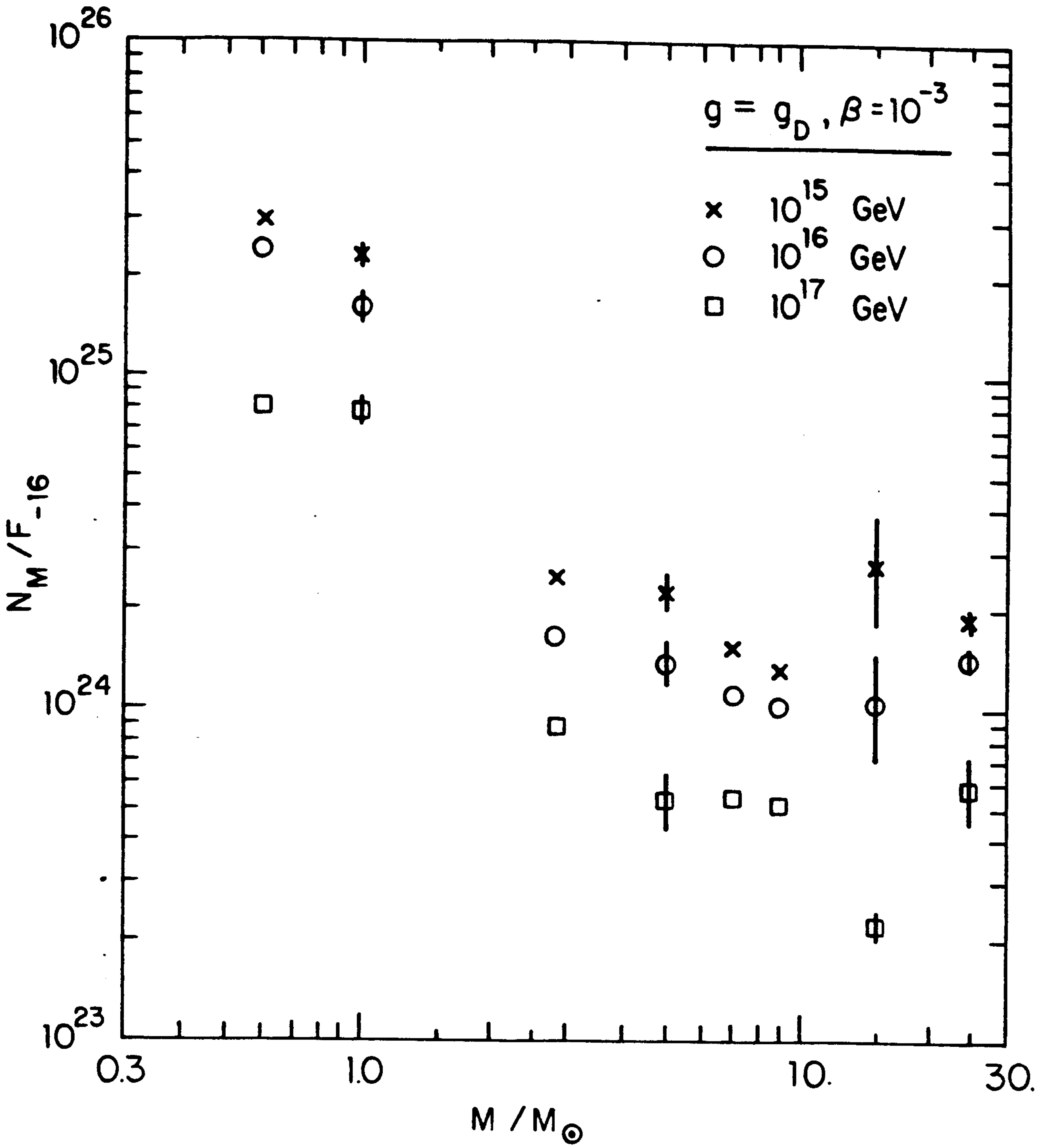
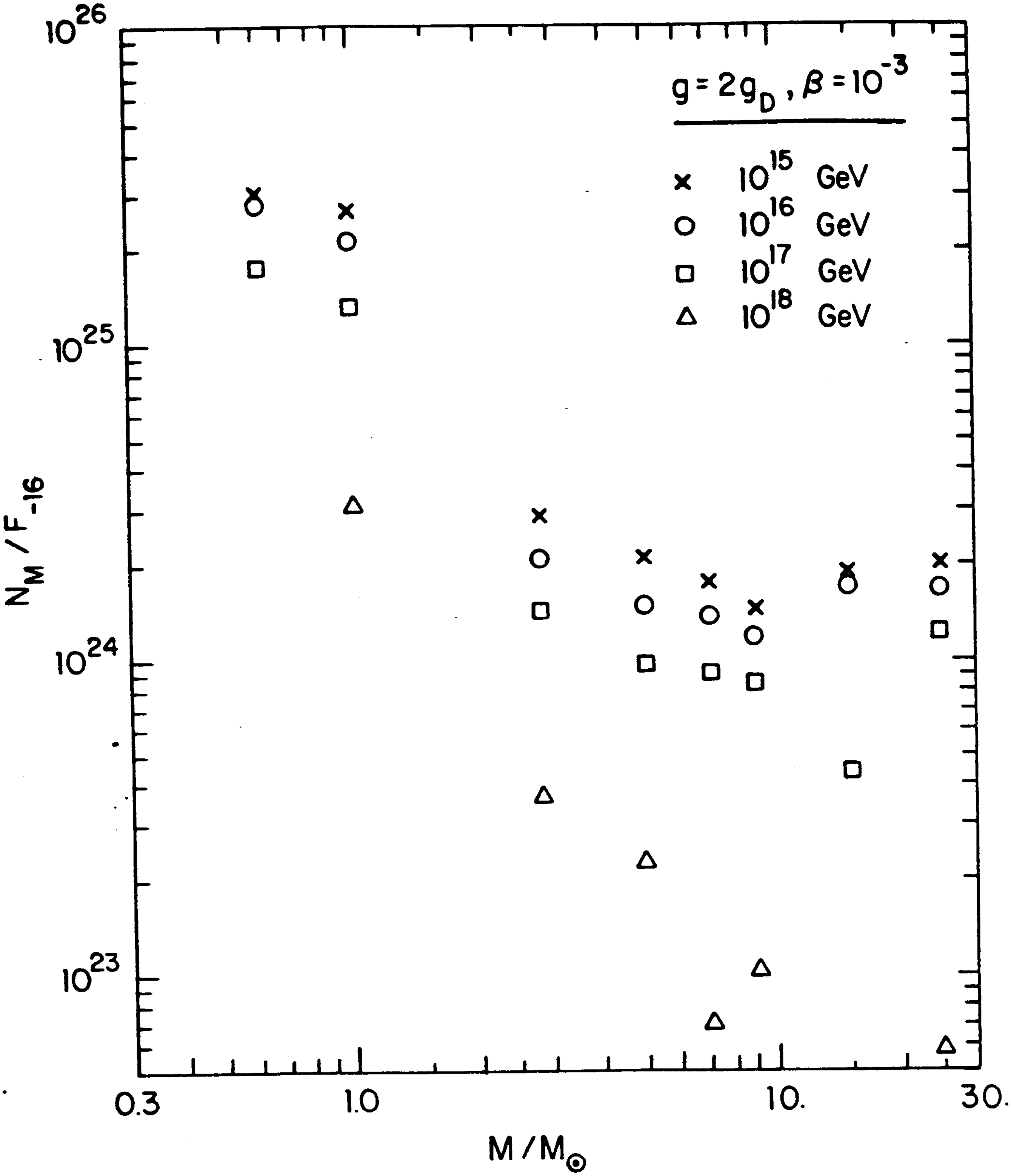
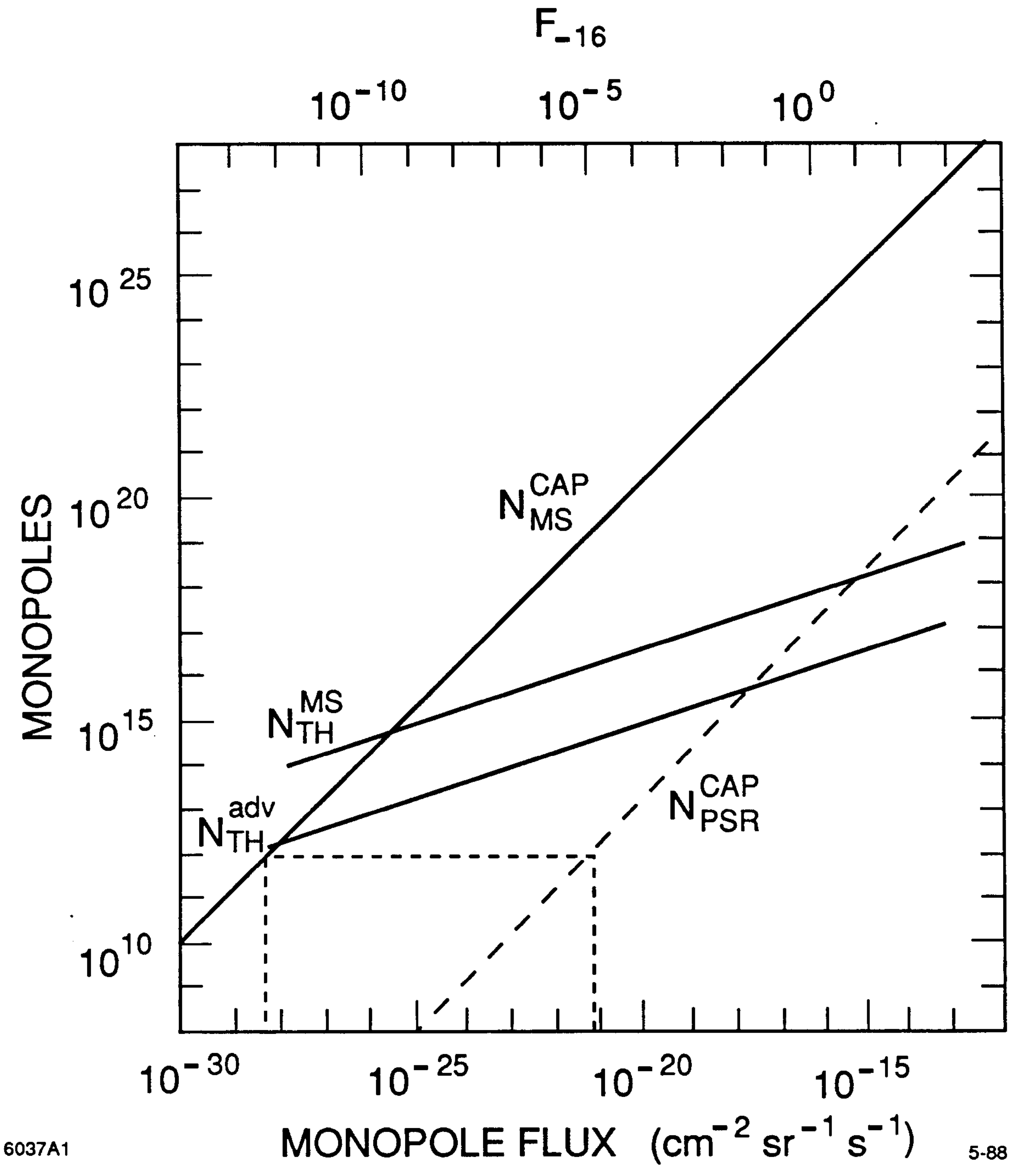

10^{26}
10^{25}
10^{24}
10^{23}
N_M / F_{-16}
$g = g_D$, $\beta = 10^{-3}$
10^{15} GeV
10^{16} GeV
10^{17} GeV
0.3
1.0
10.
30.
$M / M_\odot$

FIGURE 2



$g = 2g_D$, $\beta = 10^{-3}$
× 10^{15} GeV
○ 10^{16} GeV
□ 10^{17} GeV
△ 10^{18} GeV
10^{26}
10^{25}
10^{24}
10^{23}
N_M / F_{-16}
0.3
1.0
10.
30.
$M / M_{\odot}$

FIGURE 3



F-16
10-10
10-5
100
1025
1020
1015
1010
MONOPOLES
NCAP MS
NMS TH
Nadv TH
NCAP PSR
10-30
10-25
10-20
10-15
6037A1
MONOPOLE FLUX (cm-2 sr-1 s-1)
5-88

Fig. 4

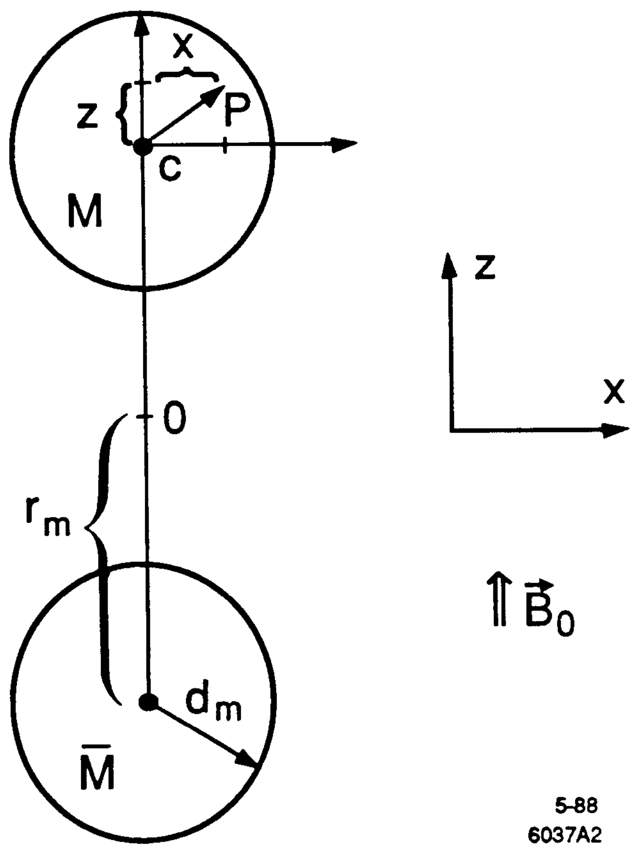


Fig. 5

Table 1 - Zero-age Properties of the MS Models Used[a]

$M/M_\odot$	$R/R_\odot$	β_{esc}	$T_c/10^7K$	$\rho_c(gcm^{-3})$	$L/L_\odot^a$	$\tau_{MS}(yrs)$
0.6[b]	0.64	2.0×10^{-3}	0.81	65	0.56	2.6×10^{10}
1[d]	0.87	2.2×10^{-3}	1.4	85	0.73	1.0×10^{10}
1[c]	0.95	2.1×10^{-3}	1.4	90	0.71	1.0×10^{10}
2.82[h]	2.0	2.4×10^{-3}	2.2	38	63	2.2×10^{8}
5[e]	2.4	3.0×10^{-3}	2.7	21	630	6.4×10^{7}
7.08[j]	3.3	3.0×10^{-3}	2.8	12	2×10^{3}	3.3×10^{7}
9[f]	3.4	3.3×10^{-3}	3.1	11	4.5×10^{3}	2.1×10^{7}
15[g]	4.6	3.7×10^{-3}	3.4	6.1	2.1×10^{4}	1.0×10^{7}
25[k]	6.0	4.2×10^{-3}	3.6	3.9	1.0×10^{5}	5.6×10^{6}

[a]Where $M_\odot = 1.99 \times 10^{33}$gm, $R_\odot = 6.96 \times 10^{10}$cm, and $L_\odot = 3.90 \times 10^{33}$erg sec^{-1}.

[b]Schwarzschild (1957), Table 28.4.

[c]Clayton (1968), Table 6-5 (from Strömgren 1965).

[d]Novotny (1973), Table 7-14 (from Iben 1965).

[e]Novotny (1973), Table 7-25 (from Iben 1966).

[f]Novotny (1973), Table 7-32 (from Iben unpublished).

[g]Novotny (1973), Table 7-35 (from Iben 1965).

[h]Clayton (1968), Table 6-2 (from Strömgren 1965).

[j]Clayton (1968), Table 6-3 (from Strömgren 1965).

[k]Woosley (1983).

Table 2 - Critical impact parameter for monopole capture[a]

m_M (GeV)	$\frac{g}{g_D}$	Stellar Mass (in $M_\odot$) 0.6	1	1	3	5	7^b	7	9	15	25
					$\beta = 10^{-4}$						
10^{15}	1	0.98	0.98	0.98	0.96	0.98	0.98	0.98	0.98	0.98	0.98
	2	0.98	0.98	0.98	0.98	0.98		0.98	0.98	0.98	0.98
10^{16}	1	0.98	0.96	0.98	0.96	0.98	0.96	0.96	0.98	0.98	0.98
	2	0.98	0.98	0.98	0.94	0.98		0.96	0.98	0.98	0.98
10^{17}	1	0.96	0.90	0.92	0.84	0.94		0.86	0.86	0.98	0.90
	2	0.96	0.96	0.94	0.88	0.98		0.90	0.90	0.98	0.94
10^{18}	1	0.88	0.78	0.76	0.70	0.74		0.72	0.76	0.60	0.80
	2	0.94	0.88	0.86	0.78	0.84		0.82	0.84	0.94	0.88
10^{19}	1	0.54	0.56	0.54	0.52	0.54		0.52	0.54	0.38	0.58
	2	0.78	0.72	0.68	0.64	0.68		0.66	0.70	0.48	0.74
					$\beta = 3 \times 10^{-4}$						
10^{15}	1	0.98	0.96	0.98	0.94	0.98	0.94	0.94	0.94	0.98	0.96
	2	0.98	0.98	0.98	0.94	0.98		0.96	0.98	0.98	0.98
10^{16}	1	0.96	0.92	0.92	0.86	0.98	0.88	0.88	0.88	0.98	0.92
	2	0.96	0.96	0.96	0.90	0.98		0.90	0.92	0.98	0.94
10^{17}	1	0.88	0.80	0.78	0.70	0.76		0.74	0.76	0.62	0.82
	2	0.94	0.88	0.86	0.80	0.84		0.82	0.84	0.96	0.88
10^{18}	1	0.56	0.58	0.54	0.52	0.56		0.52	0.56	0.40	0.60
	2	0.78	0.72	0.70	0.64	0.68		0.68	0.70	0.50	0.74
10^{19}	1	---	---	---	---	---		---	---	---	---
	2	0.14	0.38	0.36	0.36	0.36		0.26	0.32	---	0.28

					$\beta = 10^{-3}$						
10^{15}	1	0.96	0.90	0.90	0.82	0.94	0.84	0.84	0.86	0.98	0.90
	2	0.96	0.94	0.94	0.88	0.98		0.90	0.90	0.98	0.94
10^{16}	1	0.86	0.76	0.74	0.68	0.74	0.70	0.72	0.76	0.60	0.82
	2	0.94	0.86	0.84	0.76	0.82		0.80	0.82	0.94	0.86
10^{17}	1	0.50	0.54	0.50	0.48	0.52		0.50	0.54	0.38	0.56
	2	0.74	0.68	0.66	0.62	0.66		0.64	0.68	0.48	0.74
10^{18}	1	---	---	---	---	---		---	---	---	---
	2	---	0.34	0.32	0.32	0.32		0.18	0.24	---	0.16
					$\beta = 3 \times 10^{-3}$						
10^{15}	1	0.82	0.72	0.68	0.60	0.68	0.64	0.66	0.70	0.62	0.76
	2	0.92	0.82	0.82	0.70	0.78		0.76	0.78	0.96	0.84
10^{16}	1	0.46	0.46	0.42	0.42	0.46	0.42	0.42	0.48	0.38	0.54
	2	0.72	0.62	0.58	0.54	0.60		0.58	0.62	0.50	0.76
10^{17}	1	---	---	---	---	---		---	---	---	---
	2	0.22	0.32	0.28	0.28	0.32		0.24	0.28	---	0.26
					$\beta = 10^{-2}$						
10^{15}	1	0.50	0.44	0.40	0.34	0.40	0.34	0.36	0.40	0.36	0.44
	2	0.76	0.60	0.56	0.48	0.54		0.52	0.56	0.54	0.62
10^{16}	1	---	---	---	---	---	---	---	---	---	---
	2	0.32	0.32	0.28	0.24	0.28		0.24	0.26	0.20	0.26
					$\beta = 10^{-1}$						
10^{15}	1	---	---	---	---	---	---	---	---	---	---
	2	0.30	0.28	0.26	0.20	0.20		0.14	0.14	0.20	---

[a]The critical impact parameter (i.e., monopoles with $b > b_{crit}$ are not captured) is given in units of the capture radius ($= R(1+\beta^2_{esc}/\beta^2)^{1/2}$). Dashed line entry means that even a monopole with zero impact parameter will not be stopped.

[b]This $7M_\odot$ model (also from Clayton (1968), Table 6-3) differs from the other $7M_\odot$ model only in composition, and was used to explore the sensitivity of our results to the stellar model used. Because of the close agreement between the two models, we only ran a few values of m_M and β.

Table 3 - Fastest Monopole that can be stopped[a]

m_M (GeV)	$\frac{g}{g_D}$	Stellar Mass (in $M_\odot$): 0.6	1	1	3	5	7^b	7	9	15	25
10^{15}	1	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}
	2	10^{-1}	10^{-1}	10^{-1}	10^{-1}	10^{-1}		10^{-1}	10^{-1}	10^{-1}	3×10^{-2}
10^{16}	1	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}
	2	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}		10^{-2}	10^{-2}	10^{-2}	10^{-2}
10^{17}	1	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
	2	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}	3×10^{-3}		3×10^{-3}	3×10^{-3}	10^{-3}	3×10^{-3}
10^{18}	1	3×10^{-4}	3×10^{-4}	3×10^{-4}	3×10^{-4}	3×10^{-4}		3×10^{-4}	3×10^{-4}	3×10^{-4}	3×10^{-4}
	2	3×10^{-4}	10^{-3}	10^{-3}	10^{-3}	10^{-3}		10^{-3}	10^{-3}	3×10^{-4}	10^{-3}
10^{19}	1	10^{-4}	10^{-4}	10^{-4}	10^{-4}	10^{-4}		10^{-4}	10^{-4}	10^{-4}	10^{-4}
	2	3×10^{-4}	3×10^{-4}	3×10^{-4}	3×10^{-4}	3×10^{-4}		3×10^{-4}	3×10^{-4}	10^{-4}	3×10^{-4}

[a]Velocities are given in units of c and to the nearest 1/2-order-of-magnitude, i.e., 3×10^{-3} means that the fastest monopole that can be stopped has velocity v_{max}: $3\times10^{-3}c \leq v_{max} \leq 10^{-2}c$.

[b]This $7M_\odot$ model (also from Clayton (1968), Table 6-3) differs from the other $7M_\odot$ model only in composition, and was used to explore the sensitivity of our results to the stellar model used. Because of the close agreement between the two models, we only ran a few values of m_M and β.